α-Symmetries, Coloured Dimensions and Gauge-String Correspondence

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•The gauge-string correspondence is a profound hypothesis and a promising approach to important long-standing problems in QCD (such as quark confinement), relating the observables (physical vertex operators) in string theory to local gauge invariant operators in QCD. In particular, such a correspondence identifies open strings with thin tubes of gluon field lines, connecting hadrons, so the Wilson loop's expectation value $< W(C) >$ on the QCD side is identified with the partition function $Z(C)$:

$$Z(C) \leftrightarrow < W(C) > \tag{1}$$

of the open string with the ends attached to the same contour C.

• Once such an isomorphism holds, one could expect that correlation functions

of massless vertex operators in open string theory are to reproduce QCD dynamics. Such a string-theoretic framework would be particularly efficient and natural to address the problem of confinement, as well as other non-perturbative QCD dynamics.

IN PRACTICE THINGS ARE FAR MORE COMPLICATED

PROBLEMS WITH "NAIVE" G/S CORRESPONDENCE:

• There are 8 coloured gluons in QCD vs 1 colourless massless gauge boson (a photon) in perturbative spectrum of open string

• There is an infinite tower of massive intermediate states in Veneziano amplitude, absent in QCD:

$$< V_{ph}(p_1)V_{ph}(p_2)V_{ph}(p_3)V_{ph}(p_4) >\sim \frac{\Gamma(-\frac{s}{2}-1)\Gamma(-\frac{t}{2}-1)}{\Gamma(-\frac{s}{2}-\frac{t}{2}-2)} \tag{2}$$

$$s=-(p_1+p_2)^2; t=-(p_1+p_3)^2$$

- infinite sequence of poles for each integer non-negative s and t vs. single massless pole for QCD amplitude

• The presence of massive intermediate states in stringy amplitudes (destroying the G/S correspondence) is related to higher order terms in the OPE of two photon vertex operators:

$$W_l(k;z_1)W_m(p;z_2) \sim \frac{C^n_{lm}(k,p)W_n(q;\frac{z_1+z_2}{2})}{z_1-z_2} + \sum_{N=0}^{\infty}(z_1-z_2)^{N+\frac{(\vec{q})^2}{2}}C^{(N)}(k,p)W^{(N)}(q) \tag{3}$$

where

$$\begin{aligned} C^n_{lm}(k,p) &= i(k^n\eta_{lm} - q_m\eta^n_l + p_l\eta^n_m) \\ \vec{k}+\vec{p}+\vec{q} &= 0 \end{aligned} \tag{4}$$

and we have skipped BRST trivial tachyonic term of the order of $(z-w)^{-2}$. Here $W^{(N)}$ are the massive operators (intermediate poles) with the on-shell condition

$$\vec{q}^2 = -2N-2 = -m^2 \tag{5}$$

(m is the mass)

• There is an underlying geometrical reason for the appearance of massive poles: the absence of the ZIGZAG SYMMETRY in conventional string-theoretic models (which, however, is present on the QCD side) - invariance of $< W(C) >$ under diffeomorphisms changing the orientation of the Wilson loop. Z(C) on the string theory side is only invariant under worldsheet reparametrizations preserving the orientation of the boundary.

Therefore:

• Confining (QCD) string = string with zigzag symmetry

• Zigzag symmetry = existence of closed subalgebra of
massless open string operators (gluons)
(no massive terms in their OPE).

• Our goal = to construct vertex operators of 8 massless aguge bosons in the adjoint of SU(3), possessing the zigzag symmetry and reproducing field-theoretic QCD amplitudes (no massive poles).

Outline of the construction :

•Special non-linear global space-time symmetries (α-symmetries) in RNS model of superstrings

$$\Updownarrow$$

• 3 underlying extra dimensions (each associated with colour-anticolour pair)

$$\Updownarrow$$

• SU(3) subgroup of α-generators (isometries of hidden coloured dimensions)

$$\Updownarrow$$

• SU(3) multiplet of gluon vertex ops with zigzag symmetry and QCD string obtained as a result of "photon painting" by α-generators

2. α-Symmetries, Hidden Dimensions and Ghost Cohomologies

Consider full matter + ghost + Liouville RNS superstring action given by:

$$
S = \frac{1}{2\pi}\int d^2z\{-\frac{1}{2}\partial X_m\bar{\partial}X^m
-\frac{1}{2}\psi_m\bar{\partial}\psi^m - \frac{1}{2}\bar{\psi}_m\partial\bar{\psi}^m\} + S_{ghost} + S_{Liouville}
$$

$$
S_{ghost} = \frac{1}{2\pi}\int d^2z\{b\bar{\partial}c + \bar{b}\partial\bar{c} + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma}\}
$$

$$
S_{Liouville} = \frac{1}{4\pi}\int d^2z\{\partial\varphi\bar{\partial}\varphi + \lambda\bar{\partial}\lambda + \bar{\lambda}\partial\bar{\lambda}
-F^2 + 2\mu_0 b e^{b\varphi}(ib\lambda\bar{\lambda} - F)\} \tag{6}
$$

where φ, λ, F are the components of the Liouville superfield, , $X^m, m = 0, ..., d-1$ are the space-time coordinates,$\psi^m, \bar{\psi}^m$ are their worldsheet superpartners; b, c, β, γ are

the fermionic and bosonic

(super)reparametrization ghosts bosonized as

$$
\begin{aligned}
b = e^{-\sigma}; c = e^{\sigma}, \\
\beta = e^{\chi-\phi}\partial\chi \equiv \partial\xi e^{-\phi}; \gamma = e^{\phi-\chi}
\end{aligned}
\tag{7}
$$

• The RNS action (5) is surprisingly invariant (in addition to translations and rotations) under the set of global non-linear space-time transformations (α-symmetries) mixing the matter and the ghost degrees of freedom:

$$
\begin{aligned}
\delta X^m = \epsilon\{\partial(e^{\phi}\psi^m) + 2e^{\phi}\partial\psi^m\} \\
\delta\psi^m = \epsilon\{-e^{\phi}\partial^2 X^m - 2\partial(e^{\phi}\partial X^m)\} \\
\delta\gamma = \epsilon e^{2\phi-\chi}\{\psi_m\partial^2 X^m - 2\partial\psi_m\partial X^m\} \\
\delta\beta = \delta b = \delta c = 0
\end{aligned}
\tag{8}
$$

with the generator of (7) given by

$$
T = \int \frac{dz}{2i\pi} e^{\phi}(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m) \tag{9}
$$

It is straightforward to check that

$$
\begin{aligned}
\delta S_{matter} = -\delta S_{ghost} \\
= -\frac{\epsilon}{2\pi}\int d^2 z(\bar{\partial}e^{\phi})(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m)
\end{aligned}
\tag{10}
$$

under (7), so the overall action S_{RNS} is invariant.

• Equivalent set of transformations leaving S_{RNS} invariant can also be obtained from (7), (8) by replacing

$$\phi \leftrightarrow -3\phi$$

• Given the Liouville mode, there are $(d+1)$ additional α-symmetry generators (one scalar and one d-vector) with the structure similar to (7), (8):

$$L^{m\alpha} = \oint \frac{dz}{2i\pi} e^{\phi}\{\partial^2\varphi\psi^m - 2\partial\varphi\partial\psi^m + \partial^2 X^m\lambda - 2\partial X^m\partial\lambda\} \tag{11}$$

and

$$L^{\alpha-} = \oint \frac{dz}{2i\pi} e^{\phi}\{\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda\} \tag{12}$$

•

The appropriate space-time transformations are given by

$$\begin{aligned} \delta X_m &= \epsilon_{m\alpha}\{\partial(e^{\phi}\lambda) + 2e^{\phi}\partial\lambda\} \\ \delta\lambda &= -\epsilon_{m\alpha}\{2\partial(e^{\phi}\partial X^m) + e^{\phi}\partial^2 X^m\} \\ \delta\gamma &= \epsilon_{m\alpha}e^{2\phi-\chi}\{\partial^2 X^m\lambda - 2\partial X^m\partial\lambda\} \\ \delta\beta &= \delta b = \delta c = \delta\varphi = \delta\psi^m = 0 \end{aligned} \tag{13}$$

and

$$\begin{aligned} \delta\varphi &= \epsilon_{-\alpha}\{\partial(e^{\phi}\lambda) + 2e^{\phi}\partial\lambda\} \\ \delta\lambda &= -\epsilon_{-\alpha}\{2\partial(e^{\phi}\partial\varphi) + e^{\phi}\partial^2\varphi\} \\ \delta\gamma &= \epsilon_{-\alpha}e^{2\phi-\chi}\{\lambda\partial^2\varphi - 2\partial\varphi\partial\lambda\} \\ \delta\beta &= \delta b = \delta c = \delta X^m = \delta\psi^m = 0 \end{aligned} \tag{14}$$

.

(equivalent versions of these generators obtained by $\phi \to -3\phi$ are also available)

• Combined with $(d+1)$ translations and $\frac{(d+1)(d+2)}{2}$ rotations of Poincare (includng the Liouville direction), the $(d+2)$ α-generators (8), (11), (12) extend the full space-time isometry group

$$SO(d,2) \Rightarrow SO(d+1,2)$$

, unsealing the underlying hidden extra dimension (with the index α in (8), (11), (12) referring to this extra dimension)

• The generators (8), (11), (12) of the α-symmetries are the worldsheet integrals of dimension 1 primary fields, i.e. are *physical vertex operators* (one can prove their BRST invariance and non-triviality). Their peculiar property is that they are annihilated by the inverse picture changing operator $\Gamma^{-1} \sim ce^{\chi-2\phi}\partial\chi$, if taken at picture $+1$-representation and by the direct picture-changing $\Gamma =: e^{\phi}(G_{matter}+G_{[1]})$:, if taken in an equivalent picture -3-version. In other words, they violate the equivalence of superconformal ghost pictures, existing at pictures 1 and above and -3 and below, but not in between (including picture 0), so their coupling to superconformal ghost d.o.f. is essential, distinguishing them radically from standard symmetry generators (such as those of Poincare group).

• The α-generators can be classified in terms of ghost cohomologies $H_n \sim H_{-n-2}(n =$

$1, 2, \ldots$); in particular for the generators (8), (11) and (12) $n = 1$.

Definition and brief description of properties of ghost cohomologies $H_n \sim H_{-n-2}$

• The positive number n ghost cohomology H_n $(n = 1, 2, ...)$ consists of physical (BRST invariant and non-trivial) vertex operators violating the picture equivalence, existing at picture n and above, that are annihilated at their minimal positive picture n by the inverse picture changing operator $\Gamma^{-1} = c\partial\xi e^{-2\phi}$ (higher than n pictures of such operators are related by the usual picture changing).

• The negative number $-n$ ghost cohomology H_{-n} $(n \geq 3)$ consists of physical (BRST invariant and nontrivial) operators that exist at picture $-n$ or below, that are annihilated by the direct picture changing operator $\Gamma =: e^{\phi}G :$ at the *minimal* negative picture $-n$ (here G is the full matter + ghost worldsheet supercurrent). The operators of H_{-n} at lower than $-n$ pictures are related by the usual picture-changing.

• There is an isomorphism between positive and negative ghost cohomologies $H_n \sim H_{-n-2}$

as any element of H_{-n-2} (typically having the form $\sim e^{-(n+2)\phi}F_{matter}$ at the minimal negative picture) has a representation in H_n obtained by replacing $e^{-(n+2)\phi} \rightarrow e^{n\phi}$ (with the matter part unchanged) and adding the $b-c$ ghost dependent counterterms in order to protect their BRST invariance.

- The usual picture-independent observables, existing at all pictures, including picture 0 (at which the superconformal ghosts decouple) are by definition the elements of H_0. The cohomologies H_{-1} and H_{-2} are empty.

• The α-generators (8), (11), (12), unsealing the first hidden dimension, are thus the elements of $H_1 \sim H_{-3}$. It is possible to construct two higher order classes of the α-generators inducing space-time symmetries, that are the elements of $H_n \sim H_{-n-2}$ for $n = 2$ and 3, with each n opening up an associate hidden dimension. The overall number of hidden dimensions thus turns out to be 3, with each extra dimension alluding to colour in terms of G/S correspondence (see below)

•The (d+3) α-generators at the $n = 2$ level are given by

$$
\begin{aligned}
L^{\beta+} &= \oint \frac{dz}{2i\pi} e^{-4\phi} F_1(X,\psi) F_1(\varphi,\lambda)(z) \\
L^{\beta-} &= -\oint \frac{dz}{2i\pi} e^{-4\phi} F_{1m}(X,\lambda) F_1^m(\varphi,\psi)(z) \\
L^{\beta m} &= \oint \frac{dz}{2i\pi} e^{-4\phi} (F_1^m(X,\lambda) F_1(\varphi,\lambda) \\
&\quad - F_1(X,\psi) F_1^m(\varphi,\psi))(z) \\
L^{\alpha\beta} &= \oint \frac{dz}{2i\pi} e^{-4\phi} (\frac{1}{2} F_2(\lambda,\varphi) \\
&+ L_1(X,\psi)\partial L_1(\varphi,\lambda) - \partial L_1(X,\psi) L_1(\varphi,\lambda))(z)
\end{aligned}
\tag{15}
$$

with the matter+Liouville structures L and F (L_1, F_1 and F_1^m) being the primary fields of dimensions 2 and $\frac{5}{2}$:

$$
\begin{aligned}
F_1(X,\psi) &= \psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m \\
F_1(\varphi,\lambda) &= \lambda\partial^2\varphi - 2\partial\lambda\partial\varphi \\
F_1^m(X,\lambda) &= \lambda\partial^2 X^m - 2\partial\lambda\partial X^m \\
F_1^m(\varphi,\psi) &= \psi^m\partial^2\varphi - 2\partial\psi^m\partial\varphi \\
L_1(X,\psi) &= \partial X_m \partial X^m - 2\partial\psi_m\psi^m \\
L_1(\varphi,\lambda) &= (\partial\varphi)^2 - 2\partial\lambda\lambda
\end{aligned}
\tag{16}
$$

and $F_2(\lambda, \varphi)$ being the primary field of dimension 5:

$$F_2(\varphi, \lambda) = \frac{1}{4}(\partial\varphi)^5 - \frac{3}{4}\partial\varphi(\partial^2\varphi)^2 + \frac{1}{4}(\partial\varphi)^2\partial^3\varphi$$
$$+\lambda\partial\lambda(\partial^3\varphi - (\partial\varphi)^3) - \frac{3}{2}\lambda\partial^2\lambda\partial^2\varphi$$
$$+3\partial\lambda\partial^2\lambda\partial\varphi\} \equiv i :(\oint e^{-i\varphi}\lambda)^3 e^{3i\varphi}\lambda : \quad (17)$$

• Combined with the matter + Liouville

Poincare generators of $SO(2, d)$ and the α-generators (8),(11),(12) of $H_1 \sim H_{-3}$, the α-generators () of $H_2 \sim H_{-4}$ enhance the space-time symmetry group to $SO(2, d+2)$ launching the second hidden space-time dimension (labelled by the index β in ())

• The $n = 3$ level α-generators of $H_3 \sim H_{-5}$, opening up the third extra dimension (labelled by the space-time index γ) , are constructed as

$$
\begin{aligned}
L^{\gamma +} &= \oint \frac{dz}{2i\pi} e^{-5\phi} \{2F_2(\varphi,\lambda)\partial F_1(X,\psi) \\
&\quad - F_1(X,\psi)\partial F_2(\varphi,\lambda)\} \\
L^{\gamma m} &= \oint \frac{dz}{2i\pi} e^{-5\phi} \{2F_2^m(\psi,\lambda,\varphi)\partial F_1(X,\psi) \\
&\quad - \partial F_2(\psi,\lambda,\varphi) F_1(X,\psi) \\
&\quad + 2F_2(\varphi,\lambda)\partial F_1^m(X,\lambda) - \partial F_2(\varphi,\lambda) F_1^m(X,\lambda)\} \\
L^{\gamma -} &= \oint \frac{dz}{2i\pi} e^{-5\phi} \{2G_2(\psi,\lambda,\varphi)\partial F_1(X,\psi) \\
&\quad - \partial G_2(\psi,\lambda,\varphi) F_1(X,\psi) \\
&\quad + 3F_{2m}(\psi,\lambda,\varphi)\partial F_1^m(X,\lambda) \\
&\quad - 2\partial F_{2m}(\psi,\lambda,\varphi) F_1^m(X,\lambda) \\
&\quad - \partial F_2(\lambda,\varphi) F_1(X,\psi)\} \\
L^{\gamma\beta} &= \oint \frac{dz}{2i\pi} e^{-5\phi} \{F_3(\varphi,\lambda) \\
&\quad + \partial L_1(X,\psi) L_2(\varphi,\lambda) \\
&\quad - \frac{4}{11} L_1(X,\psi)\partial L_2(\varphi,\lambda)\} \\
L^{\gamma\alpha} &= \oint \frac{dz}{2i\pi} e^{-5\phi} L_{2m}(\varphi,\psi) L_1^m(X,\lambda)
\end{aligned} \tag{18}
$$

with the additional matter+Liouville blocks given by:

$$
\begin{aligned}
F_2^m(\psi,\lambda,\varphi) &= \partial^2\psi^m\lambda\partial^2\varphi - \psi^m\partial^2\lambda\partial^2\varphi \\
&\quad +3\partial^2\psi^m\partial\lambda\partial\varphi - 3\partial\psi^m\partial^2\lambda\partial\varphi \\
G_2(\psi,\lambda,\varphi) &= 4\partial\psi_m\partial^2\psi^m\partial\varphi - 2\psi_m\partial^3\psi^m\partial\varphi \\
&\quad +(2d-4)(\lambda\partial^3\lambda\partial\varphi - 2\partial\lambda\partial^2\lambda\partial\varphi) \\
L_2(\varphi,\lambda) &= -\frac{5}{4}(\partial\varphi)^4\partial\lambda + \frac{3}{4}(\partial^2\varphi)^2\partial\lambda \\
&\quad +\frac{3}{2}\partial\varphi\partial^2\varphi\partial^2\lambda - \frac{5}{2}\partial\varphi\partial^3\varphi\partial\lambda \\
&\quad -\frac{1}{4}(\partial\varphi)^2\partial^3\lambda - 4\partial\varphi\partial^2\varphi\partial^2\lambda + \partial^2\varphi\partial^3\varphi\lambda \\
L_2^m(\varphi,\psi) &= -\frac{5}{4}(\partial\varphi)^4\partial\psi^m + \frac{3}{4}(\partial^2\varphi)^2\partial\psi^m \\
&\quad +\frac{3}{2}\partial\varphi\partial^2\varphi\partial^2\psi^m - \frac{5}{2}\partial\varphi\partial^3\varphi\partial\psi^m \\
&\quad -\frac{1}{4}(\partial\varphi)^2\partial^3\psi^m - 4\partial\varphi\partial^2\varphi\partial^2\psi^m + \partial^2\varphi\partial^3\varphi\psi^m \\
L_1^m(X,\lambda) &= \partial^2\lambda\psi^m + \lambda\partial^2\psi^m \\
F_3(\varphi,\lambda) &=: (\oint e^{-i\varphi}\lambda)^4 e^{-5\phi+4i\varphi}\lambda :
\end{aligned}
\tag{19}
$$

• Combined with the space-time Poincare generators (9) and with the α-generators of two lower ghost cohomologies,

the α-generators (14),(15) of $H_{-3} \sim H_{-5}$ extend the space-time isometry group to

$SO(2, d+3)$, giving rise

to the third hidden dimension.

Construction of the octet of gluon vertices

• Consider the subset of the α-generators corresponding to isometries of 3 hidden dimensions (those not mixing with the visible d-dimensional space-time, i.e. without the space-time index m) There are altogether 9 generators:

$$L^{\alpha\pm}, L^{\beta\pm}, L^{\gamma\pm}, L^{\alpha\beta}, L^{\alpha\gamma}, L^{\beta\gamma}$$

• The gluons are constructed by acting with this extra-dimensional subset of 9 α-isometries () on a photon vertex operator:

$$V_{ph}(\vec{p}) = A_m(\vec{p}) \oint dz(\partial X^m + i(\vec{k}\vec{\psi})\psi^m)e^{i\vec{p}\vec{X}} \tag{20}$$

• Remarkably, one of 9 α-generators, $L^{\alpha-}$, drops out as it turns out to commute with the photon:

$$[L^{\alpha-}, V_{ph}(\vec{p})] = 0 \tag{21}$$

producing no new state.

• The remaining 8 operators, however, do not commute with V_{ph}, their commutators producing 8 new physical states (massless gauge bosons), each inheriting the ghost cohomology of the appropriate α-generator.

• Because of () each of these 8 operators can be shifted by any operator proportional to $L^{\alpha-} \approx 0$

• The SU(3) group is then induced by the following 8 linear combinations of the extra-dimensional α-generators: vfill

$$
\begin{aligned}
F_+ &= -\frac{1}{\sqrt{2}}(L^{\gamma+} + L^{\gamma-}) - \frac{i}{\sqrt{2}}(L^{\beta+} + L^{\beta-}) \\
&\quad + L^{\alpha\beta} - iL^{\alpha\gamma} - \frac{i}{\sqrt{2}}(L^{\alpha+} - L^{\alpha-}) \\
F_- &= -\frac{1}{\sqrt{2}}(L^{\gamma+} + L^{\gamma-}) - \frac{i}{\sqrt{2}}(L^{\beta+} + L^{\beta-}) \\
&\quad - L^{\alpha\beta} + iL^{\alpha\gamma} - \frac{i}{\sqrt{2}}(L^{\alpha+} - L^{\alpha-}) \\
F_3 &= -\frac{1}{\sqrt{2}}(L^{\gamma+} - L^{\gamma-}) - \frac{i}{\sqrt{2}}(L^{\beta+} - L^{\beta-}) \\
L_1 &= \frac{i}{2}L^{\beta\gamma} \\
L_2 &= \frac{i}{\sqrt{2}}(L^{\alpha+} + L^{\alpha-}) \\
G_+ &= -\frac{1}{\sqrt{2}}(L^{\gamma+} + L^{\gamma-}) + \frac{i}{\sqrt{2}}(L^{\beta+} + L^{\beta-}) \\
&\quad + L^{\alpha\beta} + iL^{\alpha\gamma} + \frac{i}{\sqrt{2}}(L^{\alpha+} - L^{\alpha-}) \\
G_- &= -\frac{1}{\sqrt{2}}(L^{\gamma+} + L^{\gamma-}) + \frac{i}{\sqrt{2}}(L^{\beta+} + L^{\beta-}) \\
&\quad - L^{\alpha\beta} - iL^{\alpha\gamma} + \frac{i}{\sqrt{2}}(L^{\alpha+} - L^{\alpha-}) \\
G_3 &= -\frac{1}{\sqrt{2}}(L^{\gamma+} - L^{\gamma-}) + \frac{i}{\sqrt{2}}(L^{\beta+} - L^{\beta-})
\end{aligned}
\tag{22}
$$

$L_{1,2}$ are in Cartan subalgebra;

$G_\pm, G_3$ - raising subalgebra

$F_\pm, F_3$ - lowering subalgebra.

Gluons are constructed as

$$\begin{aligned} V^i &= [T^i, V_{ph}] \\ T^i &\equiv \{F\pm, F_3, L_1, L_2, G\pm, G_3\} \\ i &= 1, ..., 8 \\ [T^i, T^j] &= D_k^{ij} T^k \end{aligned} \tag{23}$$

(D_k^{ij} are the SU(3) structure constants)

Zigzag Invariance of Gluon Vertex Operators

Using $W_m^i(z, \vec{k}) = [T^i, W^m(z, \vec{k})]$, the OPE of two gluon integrands is given by

$$
\begin{aligned}
W_m^i(z_1,\vec{k})W_n^j(z_2,\vec{p})r = \oint_{z_1}\frac{dw_1}{2i\pi}\oint_{z_2}\frac{dw_2}{2i\pi} \\
\{T^i(w_1)T^j(w_2)W_m(z_1,\vec{k})W_n(z_2,\vec{p})\} \\
\sim \frac{D_k^{ij}C_{mn}^p(\vec{k},\vec{p})}{z_1-z_2}W_p(\frac{z_1+z_2}{2};\vec{q}) \\
+\sum_{N=0}^{\infty}(z_1-z_2)^{N+\frac{(\vec{q})^2}{2}}C^{(N)}(\vec{k},\vec{p})D_k^{ij}W_N^i(\vec{q})
\end{aligned}
\tag{24}
$$

where

$$
W_N^k(\vec{q}) = [T^k, W_N(\vec{q})] \tag{25}
$$

is the α-transform of massive operators of $q^2 = -2N-2$ of the photon OPE. vskip 0.2in

• Remarkably, it turns out that the α-transform of any massive physical operator is BRST-trivial

Namely, defining

$$
\begin{aligned}
W_N(q) =^{def} U_N(q)e^{i\vec{q}\vec{X}} \\
[T^k, U_N(q)] =^{def} U_N^k \\
[T^k, e^{i\vec{q}\vec{X}} =^{def} Z^k(q)e^{i\vec{q}\vec{X}}
\end{aligned}
\tag{26}
$$

one can show

$$W_N^k(q) = \frac{2}{(\vec{q})^2}[Q_{brst}, b_0 c U_N Z^k e^{i\vec{q}\vec{X}}] \tag{27}$$

• This ensures the zigzag symmetry of the gluon OPE () and allows to use itto calculate scattering amplitudes reproducing SU(3) QCD dynamics (including the absence of massive poles)

6. Computation of the 4-point Amplitude

• The simple pole in the gluon OPE produces the 3-gluon vertex given by

$$\begin{aligned} &< V_m^i(\vec{k}) V_n^j(\vec{p}) V_p^k(\vec{q}) \\ &= D^{ijk} C_{mnp}(\vec{k}, \vec{p}) \delta(k+p+q) \end{aligned} \tag{28}$$

Using the zigzag invariance of (24) one can apply the bootstrap expansion to calculate QCD scattering amplitudes of higher points. E.g. the 4-point scattering amplitude

$$\begin{aligned} &A^{i_1 \dots i_4}(\vec{p_1}, \vec{p_2}, \vec{p_3}, \vec{p_4}) \\ &=< V_1^i(\vec{p_1}) V^{i_2}(\vec{p_2}) V^{i_3}(\vec{p_3}) V^{i_4}(\vec{p_4}) > \end{aligned} \tag{29}$$

is computed to give

$$A^{i_1 \ldots i_4}(\vec{p_1},\vec{p_2},\vec{p_3},\vec{p_4}) = \prod_{j=1}^{4} A^{m_j}(\vec{p_j})$$
$$\times\{D_j^{i_1 i_2} D^{j i_3 i_4} + D_j^{i_1 i_3} D^{i_2 j i_4} + D_j^{i_2 i_4} D^{i_2 i_3 j}\}$$
$$\times\{\frac{C^n_{m_1 m_2}(\vec{p_1},\vec{p_2}) C_{m_3 m_4 n}(\vec{p_3},\vec{p_4})}{(\vec{p_1}\vec{p_2})(\vec{p_3}\vec{p_4})} \tag{30}$$
$$+\frac{C^n_{m_1 m_3}(\vec{p_1},\vec{p_3}) C_{m_2 m_4 n}(\vec{p_2},\vec{p_4})}{(\vec{p_1}\vec{p_3})(\vec{p_2}\vec{p_4})}$$
$$+\frac{C^n_{m_1 m_4}(\vec{p_1},\vec{p_4}) C_{m_2 m_3 n}(\vec{p_2},\vec{p_3})}{(\vec{p_1}\vec{p_4})(\vec{p_2}\vec{p_3})}\}\delta(\sum_{a=1}^{4}\vec{p_a})$$

• The group-theoretic factor is easily recognized as $\sim Tr(t^{i_1}...t^{i_4})$ (with t^{i_k} being the SU(3) generators), as one would expect for QCD amplitudes; the amplitude is manifestly cross-symmetric and the factor in numerator (quadratic in structure constants and momenta) protects it from double poles. The calculation can be extended to higher number of points showing the constructed gluon operators to reproduce perturbative QCD dynamics

Conclusions

•QCD string turns out to be a special sector of RNS string theory, related to conventional RNS superstring by the α-transform.

•α-isometries of 3 hidden dimensions are translated into SU(3) colour group

• Schematically:

$$\begin{gathered}\text{ghost cohomology}\\ \Updownarrow \\ \text{hidden space-time dimension}\\ \Updownarrow \\ \text{associated colour-anticolour pair}\end{gathered}$$

Future directions:

• Higher cohomologies/dimensions?

• Closed string extension/strongly coupled QCD limit

• Ghost number selection rules vs. conservation laws in QCD?

References

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