Nonequilibrium Thermodynamics of the Quantum Universe

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WMAP-COBE experimental data with satisfactory precision predicts that the sufficient model of our Universe is an exact solution of the Einstein field equations of General Relativity - the Friedmann-Lemaître-Robertson-Walker flat Spacetime. In frames of this model we can explain physics of the Cosmic Microwave Background Radiation, but still is not clear the Nature of the CMBR anisotropies.

We try give scenario for the CMBR anisotropies based on application of Quantum Field Theory and Quantum Information Theory language to the classical Friedmann-Lemaître-Robertson-Walker Universe.

In this lecture I present introduction to the strict method for obtain of thermodynamics of the Universe. We will study the cosmological constant approximation - the model of Quintessence.

Classical Friedmann Universe

The Universe is homogenous, flat and isotropic expanding Spacetime founded by A.A. Friedmann, G.-H. Lemaître, H.P. Robertson and A.G. Walker

$$ds^2 = dtdt - a^2(t)dx^i dx^i, \tag{1}$$

where $a(t)$ is cosmological scale factor. Friedmann introduced change of diffeoinvariants $t \longrightarrow \eta$

$$d\eta = dt/a(t)\ . \tag{2}$$

With this the interval (1) takes the pseudo-euclidean form

$$ds^2 = a^2(\eta)\left(d\eta d\eta - dx^i dx^i\right). \tag{3}$$

In *1958* P.A.M. Dirac proposed change *the Friedmann integral measure* (2) by

$$d\eta = N_d(x^0)dx^0, \tag{4}$$

where $N_d(x^0)$ is the lapse function, x^0 is object of diffeomorphisms

$$x^0 \longrightarrow \widetilde{x}^0 = \widetilde{x}(x^0), \tag{5}$$

introduced by Albert Einstein.

The Dirac approach to General Relativity

The General Relativity with Matter fields is described by the Hilbert action

$$\mathcal{A} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{6}\mathcal{R} + \mathcal{L}_M \right\}, \tag{6}$$

where $g = \det g_{\mu\nu}$, $g_{\mu\nu}$ is the metric tensor of the Space-time, $\mathcal{L}_M$ is the Matter fields Lagrangian and $\mathcal{R}$ is the Ricci scalar.

The Hilbert action (6) calculated for the Universe (1) is

$$\mathcal{A}[a] = -V \int dx^0 \left\{ \frac{1}{N_d} \left(\frac{da}{dx^0} \right)^2 + N_d a^4 \langle \mathcal{H}(x^0) \rangle \right\}, \tag{7}$$

where

$$\langle \mathcal{H}(x^0) \rangle \ = \frac{1}{V} \int d^3x \ \mathcal{H}_M(x^i, x^0), \tag{8}$$

$$V = \int d^3x < \infty, \tag{9}$$

are the zeroth Fourier harmonic of the Matter Hamiltonian and spatial volume, respectively. We calculate the canonical conjugate momentum of the theory

$$p_a = -\frac{2V}{N_d}\frac{da}{dx^0}, \tag{10}$$

and using this momentum the Friedmann Universe action (7) becomes

$$\mathcal{A}[a] = -V\int dx^0 \left\{\frac{p_a^2}{4V^2} + a^4\langle\mathcal{H}(x^0)\rangle\right\}. \tag{11}$$

From the Hamiltonian reduction viewpoint reduced action has a form

$$\mathcal{A}[a] = \int dx^0 \left\{p_a\frac{da}{dx^0} - \mathrm{H}(p_a, a)\right\}, \tag{12}$$

where $\mathrm{H}(p_a, a)$ is a fuction on phase space of the system. We obtain

$$\mathrm{H}(p_a, a) = N_d\left[-\frac{p_a^2}{4V} + V\langle\mathcal{H}(x^0)\rangle a^4\right] \tag{13}$$

Dirac said that action principle with respect to N_d applied to the action (12) produce Hamiltonian constraint equation, which in our case is

$$\frac{\delta\mathcal{A}[a]}{\delta N_d} = 0 = -\frac{p_a^2}{4V} + V\langle\mathcal{H}(x^0)\rangle a^4. \tag{14}$$

If we resolve this constraints equation we obtain

$$\frac{a(t)}{a(t_0)} = \exp\left\{\mathrm{sgn}(t-t_0)\int_{t_0}^{t} dx^0 \sqrt{\langle \mathcal{H}(x^0)\rangle}\right\}, \tag{15}$$

and it is the Hubble law.

The Hamilton constraints expressed by conformal time are

$$p_a = -2V\frac{da}{d\eta} = \pm\omega_a, \tag{16}$$

where $\omega_a = 2V\sqrt{\langle \mathcal{H}(\eta)\rangle}a^2(\eta)$. By this we have ODE on $a(\eta)$

$$-\frac{da}{d\eta} = \pm\sqrt{\langle \mathcal{H}(\eta)\rangle}a^2(\eta). \tag{17}$$

In this equation variables can be separated and elementary integration gives

$$a(\eta) = \frac{a(\eta_0)}{1+z(\eta_0;\eta)}, \tag{18}$$

and it is the Hubble law too. We have defined the quantity

$$\begin{aligned} z(\eta_0;\eta) &= a(\eta_0)\mathrm{sgn}(\eta-\eta_0)\int_{\eta_0}^{\eta} d\eta'\sqrt{\langle\mathcal{H}(\eta')\rangle} = \\ &= H_0|\eta-\eta_0| + \left(1+\frac{q_0}{2}\right)H_0^2(\eta-\eta_0)^2 + \ldots \quad , \end{aligned} \tag{19}$$

that is called *redshift*. Constants H_0 and q_0 are called the Hubble parameter and the deceleration parameter

$$H_0 = \sqrt{\langle\mathcal{H}(\eta_0)\rangle}a(\eta_0), \tag{20}$$

$$q_0 = \frac{2}{H_0}\frac{\langle\dot{\mathcal{H}}(\eta_0)\rangle}{\langle\mathcal{H}(\eta_0)\rangle} - 2. \tag{21}$$

Quintessence model

We understand the Quintessence as a kind of Matter characterized by constant energy Λ

$$\langle \mathcal{H}(\eta) \rangle = \langle \mathcal{H}(\eta_0) \rangle = \Lambda, \tag{22}$$
$$\langle \dot{\mathcal{H}}(\eta) \rangle = \langle \dot{\mathcal{H}}(\eta_0) \rangle = 0. \tag{23}$$

For this model we have

$$H_0 = \Lambda^{1/2} a(\eta_0), \tag{24}$$
$$q_0 = -2, \tag{25}$$
$$z(\eta_0; \eta) = H_0 |\eta - \eta_0|. \tag{26}$$

Solution of Hamilton constraints for the Quintessence is

$$p_a = \pm \omega_a(\eta) = \pm 2V \Lambda^{1/2} a^2(\eta) = \pm \omega_a(\eta_0) \left(\frac{a(\eta)}{a(\eta_0)} \right)^2, \tag{27}$$

where

$$\omega_a(\eta_0) = 2V \Lambda^{1/2} a^2(\eta_0) = 2V \frac{H_0^2}{\sqrt{\Lambda}}. \tag{28}$$

Quantization Procedure of the Universe

For quantization of the Universe with Quintessence we use a set of steps

1. First quantization of the classical constraints by Dirac recept. Equation on the wave function Ψ of the Universe,
2. Classical Hamilton equation of motion for field Ψ and their conjugate momentum field Π_Ψ,
3. Quantization of the Hamilton equations by nonfockian distributions in the Fock space of annihilation and creation operators,
4. The Bogoliubov transformation. Diagonalization of quantum Hamilton equations,

Quantum Mechanics of the Universe

Dirac proposed applying of the first quantization to the constraints equation. The Hamiltonian constraint equation for the Universe is

$$p_a^2 - \omega_a^2 = 0, \tag{29}$$

with $\omega_a = \pm 2V\Lambda^{1/2}a^2$. Classical solution of this constraints is given by the Hubble law

$$a(\eta) = \frac{a(\eta_0)}{1 + \Lambda^{1/2}(\eta - \eta_0)a(\eta_0)}. \tag{30}$$

The first quantization is given by CCR

$$i\,[\hat{\mathrm{p}}_a, a] = 1, \tag{31}$$

where $\hat{\mathrm{p}}_a = -i\dfrac{\partial}{\partial a}$. We assume that the wave function $\Psi(a)$ exist. The final result is the Wheeler-DeWitt equation

$$\left(\partial_{aa}^2 + \omega_a^2\right)\Psi(a) = 0, \tag{32}$$

that describes Quantum Mechanics of the Friedmann-Lemaître Spacetime.

Classical field theory of the Universe

The Wheeler-DeWitt equation looks like the Klein-Gordon equation for the boson with mass ω_a. Let's consider the Wheeler-DeWitt equation as an equation of motion for the classical field Ψ.

By heuristic analogy with Klein-Gordon mass field we construct the action

$$\mathcal{S}[\Psi] = \frac{1}{2}\int da \left\{ (\partial_a \Psi)^2 - \omega_a^2 \Psi^2 \right\}. \tag{33}$$

It is not difficult to check that this action is correct. Recall that by the Hamilton action principle we have generally

$$\delta\mathcal{S}[\Psi] \equiv 0 \ = \ \left\{ \frac{\delta\mathcal{S}[\Psi]}{\delta\Psi} - \partial_a \frac{\delta\mathcal{S}[\Psi]}{\delta\partial_a\Psi} \right\} \delta\Psi + \partial_a \left\{ \frac{\delta\mathcal{S}[\Psi]}{\delta\partial_a\Psi} \delta\Psi \right\}. \tag{34}$$

The second term vanishes on boundaries and by this we obtain

$$\frac{\delta\mathcal{S}[\Psi]}{\delta\Psi} - \partial_a \frac{\delta\mathcal{S}[\Psi]}{\delta\partial_a\Psi} = 0 \rightarrow \int da \left\{ \omega_a^2 \Psi + \partial_a\partial_a \Psi \right\} = 0 \Rightarrow \left(\partial_a\partial_a + \omega_a^2\right) \Psi = 0,$$

what is exactly the Wheeler-DeWitt equation.

The conjugate momentum field corresponds with the action is

$$\Pi_\Psi = \frac{\delta \mathcal{S}[\Psi]}{\delta\left(\partial_a \Psi\right)} = \partial_a \Psi, \tag{35}$$

With this momentum the action (33) reduces into the form

$$\mathcal{S}[\Psi] = \int da \left\{ \Pi_\Psi \partial_a \Psi - \underbrace{\frac{1}{2}\left(\Pi_\Psi^2 + \omega_a^2 \Psi^2\right)}_{\mathrm{H}(\Pi_\Psi, \Psi)} \right\}, \tag{36}$$

where $\mathrm{H}(\Pi_\Psi, \Psi)$ is the Hamiltonian. The Hamilton equations of motion are

$$\left.\begin{array}{r} \dfrac{\partial \mathrm{H}(\Pi_\Psi, \Psi)}{\partial \Pi_\Psi} = \partial_a \Psi \\ \dfrac{\partial \mathrm{H}(\Pi_\Psi, \Psi)}{\partial \Psi} = -\partial_a \Pi_\Psi \end{array}\right\} \Longrightarrow \boxed{\partial_a \begin{bmatrix} \Psi \\ \Pi_\Psi \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_a^2 & 0 \end{bmatrix} \begin{bmatrix} \Psi \\ \Pi_\Psi \end{bmatrix}}.$$

$\boxed{\text{We will base Quantum Gravity on the Hamilton equations of motion}}$

Quantization of the Hamilton equations of motion

Analogy presented in previous section products conclusion

$$\boxed{\text{QFT of the Universe should be build in language of boson Fock space}}$$

The boson Fock space of creation $\mathcal{G}^\dagger$ and annihilation $\mathcal{G}$ operators is construct by CCRs

$$\begin{aligned} \left[\mathcal{G}(a(\eta)), \mathcal{G}^\dagger(a(\eta'))\right] &= \delta(a(\eta) - a(\eta')), \\ \left[\mathcal{G}(a(\eta)), \mathcal{G}(a(\eta'))\right] &= 0. \end{aligned} \tag{37}$$

Analogy with the Klein-Gordon case gives the 2nd quantization in the form

$$\left[\begin{array}{c} \Psi(a) \\ \Pi_\Psi(a) \end{array}\right] \longrightarrow \left[\begin{array}{c} \mathbf{\Psi}(a) \\ \mathbf{\Pi}_\mathbf{\Psi}(a) \end{array}\right] = \left[\begin{array}{cc} \dfrac{1}{\sqrt{2\omega_a}} & \dfrac{1}{\sqrt{2\omega_a}} \\ -i\sqrt{\dfrac{\omega_a}{2}} & i\sqrt{\dfrac{\omega_a}{2}} \end{array}\right] \left[\begin{array}{c} \mathcal{G}(a) \\ \mathcal{G}^\dagger(a) \end{array}\right]. \tag{38}$$

The matrix depend on a and by this we have nonfockian representation in the Fock space, but correct CCR for $\mathbf{\Psi}$ and $\mathbf{\Pi}_\mathbf{\Psi}$ is preserved

$$\left[\mathbf{\Pi}_\mathbf{\Psi}(a(\eta)), \mathbf{\Psi}(a(\eta'))\right] = -i\delta(a(\eta) - a(\eta')). \tag{39}$$

We can translate the Wheeler-DeWitt action into the Fock space

$$\mathcal{S}(\mathcal{G},\mathcal{G}^\dagger) = \int \mathcal{D}a \left\{ i\frac{\mathcal{G}^\dagger\partial_a\mathcal{G} - \mathcal{G}\partial_a\mathcal{G}^\dagger}{2} - \mathcal{H} \right\}, \tag{40}$$

where the effective Hamiltonian $\mathcal{H}$ is

$$\mathcal{H} = \left(\mathcal{G}^\dagger\mathcal{G} + \frac{1}{2}\right)\omega_a + \frac{i}{2}\left(\mathcal{G}^\dagger\mathcal{G}^\dagger - \mathcal{G}\mathcal{G}\right)\Delta, \tag{41}$$

where $\Delta = \dfrac{\partial_a\omega_a}{2\omega_a}$ has sense of coupling. The Hamiltonian (41) is well known from the Many Particle Physics as the Hamiltonian of *the boson superfluidity*. Quantization of the Hamilton equations of motion gives

$$\partial_a \left[\begin{array}{c} \mathbf{\Psi} \\ \mathbf{\Pi_\Psi} \end{array} \right] = \left[\begin{array}{cc} 0 & 1 \\ -\omega_a^2 & 0 \end{array} \right] \left[\begin{array}{c} \mathbf{\Psi} \\ \mathbf{\Pi_\Psi} \end{array} \right] \Longrightarrow \boxed{ i\partial_a \left[\begin{array}{c} \mathcal{G} \\ \mathcal{G}^\dagger \end{array} \right] = \left[\begin{array}{cc} -\omega_a & \boxed{2i\Delta} \\ \boxed{2i\Delta} & \omega_a \end{array} \right] \left[\begin{array}{c} \mathcal{G} \\ \mathcal{G}^\dagger \end{array} \right] } \tag{42}$$

These equations are understood as the Heisenberg equations for $\mathcal{G}$ and $\mathcal{G}^\dagger$ with nonlinearity in form of nondiagonal elements in the evolution matrix (42). We see that the quantum evolution (42) is not diagonal.

Diagonalization of equations of motion

Now we diagonalize this evolution. Firstly we use

The boson Bogoliubov transformation We change the basis $(\mathcal{G}^\dagger, \mathcal{G})$ on the other basis $(\mathcal{W}^\dagger, \mathcal{W})$ in the Fock space by general transformation

$$\begin{bmatrix} \mathcal{W}(a) \\ \mathcal{W}^\dagger(a) \end{bmatrix} = \begin{bmatrix} u(a) & v(a) \\ v^*(a) & u^*(a) \end{bmatrix} \begin{bmatrix} \mathcal{G}(a) \\ \mathcal{G}^\dagger(a) \end{bmatrix}. \tag{43}$$

If we want to preserve CCRs

$$\left[\mathcal{W}(a(\eta)), \mathcal{W}^\dagger(a(\eta'))\right] = \delta(a(\eta) - a(\eta')) \tag{44}$$

$$[\mathcal{W}(a(\eta)), \mathcal{W}(a(\eta'))] = 0, \tag{45}$$

we obtain the rotation condition

$$|u(a)|^2 - |v(a)|^2 = 1. \tag{46}$$

After this we apply

Diagonalization of the quantum Hamilton equations of motion Evolution in the basis $(\mathcal{W}^\dagger, \mathcal{W})$ has a form

$$i\partial_a \begin{bmatrix} \mathcal{W} \\ \mathcal{W}^\dagger \end{bmatrix} = \begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix} \begin{bmatrix} \mathcal{W} \\ \mathcal{W}^\dagger \end{bmatrix}, \tag{47}$$

with some diagonalization energies ω_1 and ω_2.

This procedure gives equations for u and v

$$i\partial_a \begin{bmatrix} v \\ u \end{bmatrix} = \begin{bmatrix} -\omega_a & -2i\Delta \\ -2i\Delta & \omega_a \end{bmatrix} \begin{bmatrix} v \\ u \end{bmatrix}. \tag{48}$$

and values of the diagonalization energies ω_1 and ω_2 are trivial

$$\omega_1 = \omega_2 = 0. \tag{49}$$

By this we have solution of the equations (47)

$$\mathcal{W}(a) = \mathcal{W}(a_0), \tag{50}$$

$$\mathcal{W}^\dagger(a) = \mathcal{W}^\dagger(a_0), \tag{51}$$

and the operator $\mathcal{N}_{\mathcal{W}} = \mathcal{W}^\dagger \mathcal{W} = \mathcal{W}^\dagger(a_0)\mathcal{W}(a_0)$ is an integral of motion

$$\partial_a \mathcal{N}_{\mathcal{W}} = 0. \tag{52}$$

By this exist the stable Bogoliubov vacuum state $|0\rangle$

$$\mathcal{W}|0\rangle = 0, \quad \langle 0|\mathcal{W}^\dagger = 0. \tag{53}$$

Since hyperbolic property of the Bogoliubov coefficients we parameterize

$$v(a) = e^{i\theta(a)} \sinh \phi(a), \tag{54}$$

$$u(a) = e^{i\theta(a)} \cosh \phi(a), \tag{55}$$

and by this way the equations (48) are equivalent to

$$\partial_a \theta(a) \;=\; \pm\omega_a = p_a, \tag{56}$$

$$\partial_a \phi(a) \;=\; -2\Delta = -\frac{\partial_a \omega}{\omega} = -\partial_a \ln|\omega_a|\,, \tag{57}$$

with obvious solutions

$$\theta(a) \;=\; \int_{a_0}^{a} p_a da, \tag{58}$$

$$\phi(a) \;=\; -\ln\left|\frac{\omega_a(\eta)}{\omega_a(\eta_0)}\right|. \tag{59}$$

By this we have

$$v(a) = \frac{1}{2}\exp\left\{i\int_{a_0}^{a} p_a da\right\}\left(\frac{\omega_a(\eta_0)}{\omega_a(\eta)} - \frac{\omega_a(\eta)}{\omega_a(\eta_0)}\right), \tag{60}$$

$$u(a) = \frac{1}{2}\exp\left\{i\int_{a_0}^{a} p_a da\right\}\left(\frac{\omega_a(\eta_0)}{\omega_a(\eta)} + \frac{\omega_a(\eta)}{\omega_a(\eta_0)}\right). \tag{61}$$

In the Einstein-Hilbert General Relativity the Space-time creates Gravity, and Gravity creates the Space-time.

Presented formalism describes the Space-time in language of Collective Phenomena. These phenomena have place in gas, which is mixture of quanta of Gravity and the Quintessence, that is approximation of bosons and fermions fields. Our proposition for Quantum Gravity is applying of the Graviton-Matter gas approach. In this language formulation of physics is clear.

Thermodynamics of the Universe

The Graviton-Matter gas is the system in thermodynamical nonequilibrium, particles of the gas go out from the system. As a recept we diagonalized the quantum Hamilton equations of motion, and we have found basis in the Fock space where particle number operator is integral of motion. By this thermodynamical equilibrium of the gas is present.

Density matrix and entropy

Graviton-Matter gas is open quantum system and should be describe by nonequilibrium quantum statistical mechanics methods. In standard approach the oneparticle density operator is particle number operator

$$\varrho_{\mathcal{G}} = \mathcal{G}^{\dagger}\mathcal{G}, \tag{62}$$

and if we rewrite this operator in $(\mathcal{W}, \mathcal{W}^{\dagger})$ basis we have

$$\varrho_{\mathcal{G}} = \mathsf{W}^{\dagger}\rho\mathsf{W}, \tag{63}$$

where $\mathsf{W} = \begin{bmatrix} \mathcal{W} \\ \mathcal{W}^\dagger \end{bmatrix}$ and

$$\rho \;=\; \begin{bmatrix} |u|^2 & -uv \\ -u^* v^* & |v|^2 \end{bmatrix}, \tag{64}$$

is the density matrix for Graviton-Matter gas in thermodynamical equilibrium.

Physical entropy is defined by the Quantum Information Theory

$$\mathrm{S} = -\frac{\mathrm{tr}(\rho \ln \rho)}{\mathrm{tr}(\rho)} \equiv \ln \Omega, \tag{65}$$

where Ω is the partition function that for the Graviton-Quintessence gas is equal to

$$\Omega = \frac{1}{2|u|^2 - 1}. \tag{66}$$

Temperature

If we identify Ω for the gas (66) with the Bose-Einstein type partition function we obtain

$$\Omega = \frac{1}{2|u|^2 - 1} \equiv \frac{1}{\exp \dfrac{\mathrm{E}}{\mathrm{T}} - 1} \quad \Longrightarrow \quad \mathrm{T} = \frac{\mathrm{E}}{\ln 2|u|^2}, \tag{67}$$

where we used the Gibbs state type. This type of identification has a sense if and only if we identify

$$\mathrm{E} \equiv \mathrm{U} - \mu \mathrm{N}, \tag{68}$$

where U is internal energy, μ is chemical potential and N is number of particles of the Graviton-Matter gas, respectively.

We have seen that the effective Hamiltonian (41) is

$$\mathcal{H} = \left(\mathcal{G}^\dagger \mathcal{G} + \frac{1}{2}\right) \omega_a + \frac{i}{2} \left(\mathcal{G}^\dagger \mathcal{G}^\dagger - \mathcal{G}\mathcal{G}\right) \Delta. \tag{69}$$

In thermodynamical equilibrium this effective Hamiltonian has a form

$$\mathcal{H} = \mathsf{W}^\dagger \mathbf{H} \mathsf{W}, \tag{70}$$

where

$$\mathbf{H} = \begin{bmatrix} \dfrac{|u|^2+|v|^2}{2}\omega_a + i\dfrac{u^*v-uv^*}{2}\Delta & -uv\omega_a + i\dfrac{uu-vv}{2}\Delta \\ -u^*v^*\omega_a - i\dfrac{u^*u^*-v^*v^*}{2}\Delta & \dfrac{|u|^2+|v|^2}{2}\omega_a + i\dfrac{u^*v-uv^*}{2}\Delta \end{bmatrix}, \quad (71)$$

is matrix of the effective Hamiltonian.

In Quantum Statistical Mechanics internal energy U is defined by quantum average of the effective Hamiltonian

$$\mathrm{U} = \langle \mathbf{H} \rangle = \frac{\mathrm{tr}(\rho\mathbf{H})}{\mathrm{tr}\rho}. \quad (72)$$

After averaging we obtain

$$\mathrm{U} = \left(\frac{1}{2} + 2\mathrm{N} + \frac{\mathrm{N}}{2\mathrm{N}+1}\right)\left(\sqrt{\mathrm{N}+1} - \sqrt{\mathrm{N}}\right)\omega_a(\eta_0), \quad (73)$$

where N is a number of particles of the gas

$$\mathrm{N} = |v|^2. \quad (74)$$

By this the chemical potential for the gas is

$$\mu = \left[2 + \frac{1}{(2\mathrm{N}+1)^2} - \frac{\frac{1}{2} + 2\mathrm{N} + \frac{\mathrm{N}}{2\mathrm{N}+1}}{2\sqrt{\mathrm{N}(\mathrm{N}+1)}}\right]\left(\sqrt{\mathrm{N}+1} - \sqrt{\mathrm{N}}\right)\omega_a(\eta_0), \qquad (75)$$

and temperature T is equal to

$$\frac{\mathrm{T[N]}}{\mathrm{T[0]}} = \frac{\sqrt{\mathrm{N}+1} - \sqrt{\mathrm{N}}}{\ln(2\mathrm{N}+2)/\ln 4}\left[\left(\frac{1}{2} + 2\mathrm{N} + \frac{\mathrm{N}}{2\mathrm{N}+1}\right)\left(1 + \frac{1}{2}\sqrt{\frac{\mathrm{N}}{\mathrm{N}+1}}\right) - 2\mathrm{N} - \frac{\mathrm{N}}{(2\mathrm{N}+1)^2}\right], \qquad (76)$$

where $\mathrm{T[0]} = \dfrac{\omega_a(\eta_0)}{\ln 4}$ is today contribution from the gas temperature. The equation of state for the Graviton-Quintessence gas has a form

$$\frac{\mathrm{U}}{\mathrm{T}} = \frac{\ln(2\mathrm{N}+2)}{1 + \frac{1}{2}\sqrt{\frac{\mathrm{N}}{\mathrm{N}+1}} - \frac{\mathrm{N}}{2\mathrm{N}+1}\frac{1 + 2(2\mathrm{N}+1)^2}{\mathrm{N} - 1 + 3(2\mathrm{N}+1)^2}}. \qquad (77)$$

Graviton-Matter gas as solution for Quantum Gravity

Physical sense of the Graviton-Matter approach to the Cosmic Microwave Background radiation temperature anisotropies arises from the following scenario. From physical viewpoint we can think about our Universe as a gas of gravitons, gauge bosons, and material particles as electrons, quarks, Higgs particles etc. If in our thinking huge volume of the Universe is respected, the conclusion is that during our all observations and measurements of the Universe physical properties, we are on the position of an element of the gas - an observer in the Universe is an element of the Universe. By this way observations of the temperature anisotropies of the CMB radiation, which we understand as an effect of condensation of all particles and fields in the Universe, are natural conceptual consequence of this approach. From the Graviton-Matter gas viewpoint the Quantum Gravity has a sense of effective theory and collective phenomena language seems adequate to description of the Universe physics. For this reason, in our opinion the Graviton-Matter gas approach is interesting for further researches in Quantum Cosmology.

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We are interested in collaboration in Quantum Cosmology with young people from the Helmholtz International Summer School on Modern Mathematical Physics.

Hamiltonian General Relativity and Cosmology at the BLTP JINR are studied by B.M. Barbashov, LAG, V.N. Pervushin and A.F. Zakharov

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Thank you for attention.