MODIFYING GRAVITY

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- MAIN MOTIVATION: EVIDENCE FOR ACCELERATED EXPANSION OF THE UNIVERSE.

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3} G\rho$$

Something fishy here?
(vacuum, quintessence, ...)

OR HERE?
NEW GRAVITY IN IR

- DISTANCE / TIME SCALE $\sim H_0^{-1}$

10^{28} cm, 10^{18} s

- THIS TALK: 4 DIMENSIONS
- DIGRESSION: EXTRA DIMENSIONS & BRANE WORLD
  - Possibly BETTER PLACE TO LOOK AT:
    - 4d fields: modes of higher dim. fields which propagate along our brane.

    ⇒ WAVE FUNCTIONS LOCALIZED ON OUR BRANE.

    - 4d graviton is one of these.
    - LOCALIZATION MAY NOT BE EXACT
      QUASI-LOCALIZATION ⇒
      4d graviton IS A RESONANCE, has finite width against escape from our brane to extra dimensions ⇒ gravity on brane not exactly 4-dim. at long distances/times.
  - LONG LIFETIME OF GRAVITON ON BRANE MAY BE DUE TO TUNNELING ⇒ 4d gravity on BRANE UNTIL EXPONENTIALLY LONG DISTANCES/TIMES.

NICE SET OF IDEAS, BUT DOES NOT QUITE WORK...

- BACK TO 4D.

- LORENTZ - PRESERVING THEORIES:

DILEMMA → GHOSTS

OR

→ STRONG COUPLING AT TOO LOW ENERGIES

PROTOTYPE EXAMPLE: MASSIVE GRAVITY

$$S = -M_{Pl}^2 \int d^4x \sqrt{-g} \left[R + m_1^2 h_{\mu\nu} h^{\mu\nu} + m_2^2 h_\mu^\mu h_\nu^\nu \right]$$

WANT $m_1 \sim m_2 \sim H_0$

$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, near MINKOWSKI BACKGROUND

- NEW DEGREES OF FREEDOM:

would-be gauge modes (STÜCKELBERG FIELDS)

$$h_{\mu\nu} = \partial_\mu \pi_\nu + \partial_\nu \pi_\mu + (\partial\pi)^2 + \dots$$

Do not enter Einstein - Hilbert term.

$$\pi_\mu = \pi_\mu^T + \partial_\mu \chi \quad ; \quad \partial_\mu \pi_\mu^T = 0$$

Most dangerous: scalar sector.

DIGRESSION:

HOW TO EXTRACT NEW MODES: EXAMPLE OF MASSIVE VECTOR FIELD.

$$S = \int d^4x \left(-\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + m^2 A^a_\mu A^a_\mu \right)$$

FOR $m=0$: GAUGE SYMMETRY

$$A^a_\mu \to A^a_\mu + \partial_\mu \varepsilon^a + g\, \varepsilon \partial \varepsilon + \dots$$

NON-ABELIAN CASE

FOR $m \neq 0$: "RESTORE" GAUGE SYMMETRY BY INTRODUCING STÜCKELBERG FIELDS φ^a.

CONSIDER ACTION

$$S = \int d^4x \left[-\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + m^2 \left(A^a_\mu + \partial_\mu \varphi^a + g \varphi \partial \varphi \right)^2 \right]$$

GAUGE INVARIANT WITH $\varphi^a \to \varphi^a - \varepsilon^a$.

* GAUGE FIXING $\varphi^a = 0 \Rightarrow$ BACK TO MASSIVE VECTOR.
* GAUGE FIXING $\partial_\mu A^a_\mu = 0 \Rightarrow$ VECTOR FIELD A^a_μ HAS NICE PROPAGATOR $\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{p^2 - m^2}$

STÜCKELBERG FIELD DECOUPLES AT QUADRATIC LEVEL AND HAS QUADRATIC ACTION

$$\int d^4x \; m^2 (\partial_\mu \varphi^a)^2$$

$\Rightarrow$ CANONICALLY NORMALIZED FIELD $\phi^a = m \varphi^a$

SELF-INTERACTION $m^2 g\, \varphi (\partial \varphi)^2 = \frac{g}{m} \phi (\partial \phi)^2$

$\Rightarrow$ UV STRONG COUPLING SCALE $\frac{m}{g}$.

- GENERAL CASE

- EFFECTIVE ACTION FOR χ
(ABOVE ENERGY SCALE m_1, m_2)

$$S_\chi^{eff} = M_{Pl}^2 \int d^4x \cdot 4 \left(m_1^2 + m_2^2\right) \left(\Box\chi\right)^2$$

HIGHER-DERIVATIVE THEORY $\Longrightarrow$ GHOST.

- EXCEPTION $m_2^2 = -m_1^2 = -m_g^2$

FIERZ-PAULI

- WARM UP: TRANSVERSE MODES

$$S_{eff}^{\pi^T} = M_{Pl}^2 \, m_g^2 \int d^4x \left[\left(\partial_\mu \pi_\nu^T\right)^2 + \left(\partial\pi^T\right)^3 + \dots \right]$$

NON-RENORMALIZABLE; MAKES SENSE UP TO UV ENERGY SCALE $\Lambda^T \lesssim \Lambda^T_{strong}$

WHAT IS Λ^T_{Strong}?

unitarity limit at tree level

- CANONICALLY NORMALIZE:

$$\pi_\mu^T = \frac{1}{M_{Pl}\, m_g} \Pi_\mu^T$$

$$\Downarrow$$

$$S_{eff}^{\Pi^T} = \int d^4x \left[\left(\partial\Pi^T\right)^2 + \frac{1}{M_{Pl}\, m_g} \left(\partial\Pi^T\right)^3 + \dots \right]$$

$$\Lambda^T_{Strong} = \sqrt{M_{Pl} \cdot m_g}$$

- IF TRANSVERSE SECTOR ONLY:

SENSIBLE THEORY DOWN TO DISTANCES

$$L^{T}_{strong} = \frac{1}{\Lambda^{T}_{strong}} = \frac{1}{(M_{Pl}\, m_g)^{1/2}}$$

- Something new must happen at

$$\ell \gtrsim L^{T}_{strong}$$

Numerology:

Interesting scale: for $m_g \sim H_0$

$$L^{T}_{strong} \sim 0.01 \text{ mm}$$

Would hint towards possible new gravitational effects at $r \sim 0.01$ mm

NB: CLOSE TO EXPERIMENTAL RANGE.

- BUT SCALAR MODES ARE WORSE.

$$m_1^2 = -m_2^2 = m_g^2$$

$$\Downarrow$$

No kinetic term for $\pi_\mu = \partial_\mu \chi$ ONLY.

HAVE TO INCLUDE ANOTHER SCALAR TERM IN $h_{\mu\nu}$

$$h_{\mu\nu} = \eta_{\mu\nu}\phi + \partial_\mu \partial_\nu \chi$$

$$\Downarrow$$

forget #'s

$$S_{eff}^{\phi,\chi} = M_{Pl}^2 \int d^4x \left[6\, \Box\phi \cdot \phi + m_g^2\, \Box\chi \cdot \phi \right]$$

Kinetic term for χ due to mixing with ϕ, trace part of $h_{\mu\nu}$

Shift

$$\phi = \tilde{\phi} - m_g^2 \chi$$

$$\Downarrow$$

$$S_{eff}^{\chi} = \int d^4x \left[M_{Pl}^2 m_g^4 (\partial\chi)^2 + M_{Pl}^2 m_g^2 (\partial^2\chi)^3 \right]$$

CANONICALLY NORMALIZE:

$$\chi = \frac{1}{M_{Pl} m_g^2} X$$

Fluctuations of scalar degree of freedom enhanced by m_g^{-2} instead of m_g^{-1} for transverse modes π_μ^T.

VAN DAM – VELTMAN – ZAKHAROV DISCONTINUITY

$$h_{\mu\nu} = \phi \cdot \eta_{\mu\nu} + \text{longitudinal} + \ldots$$

$$\phi \simeq m_g^2 \chi$$

$$\chi = \frac{1}{M_{Pl} \cdot m_g^2} X$$

Canonically normalized, propagating

$$\Downarrow$$

$$\phi \simeq \frac{1}{M_{Pl}} X$$, no suppression by m_g

EXTRA SCALAR FIELD (AS COMPARED TO GR)

INTERACTS WITH MATTER THROUGH

$$T^{\mu\nu} h_{\mu\nu} = \eta^{\mu\nu} \phi \cdot T_{\mu\nu} = \frac{1}{M_{Pl}} X \cdot T^{\mu}_{\mu}$$

GRAVITATIONAL STRENGTH EVEN AS $m_g \to 0$.

SCALAR-TENSOR GRAVITY $\boxed{\text{PROBLEM}}$

NAIVELY: CONTRADICTS BENDING OF LIGHT

CLASSICAL THEORY:

MAYBE NOT SO DANGEROUS: NONLINEARITY AT LARGE DISTANCE FROM SOURCE

$$r_V \sim \left[M_{source} \cdot \frac{1}{M_{Pl}^2 m_g^4} \right]^{1/5}$$

VAINSHTEIN

PROBLEM: SHOW THAT AT $r < r_V$ NON-LINEAR TERMS IN EQUATION FOR X ARE LARGER THAN LINEAR.

PROBLEM:

BY SOLVING FOR $h_{\mu\nu}$ THE FIELD EQUATIONS OF FIERZ-PAULI MASSIVE GRAVITY, LINEARIZED ABOUT MINKOWSKI BACKGROUND, FIND GRAVITATIONAL FIELD OF MASSIVE POINT SOURCE (i.e., ALL COMPONENTS OF $h_{\mu\nu}$).

CONSIDER STATIC SOURCE ONLY.

COMPARE WITH GENERAL RELATIVITY.

NB: NEGLECT LONGITUDINAL TERMS in $h_{\mu\nu}$ OF THE FORM $\partial_\mu \pi_\nu + \partial_\nu \pi_\mu$, AS THEY DO NOT INTERACT WITH CONSERVED $T_{\mu\nu}$:

$$\int d^4x \, (\partial_\mu \pi_\nu + \partial_\nu \pi_\mu) T^{\mu\nu} = 0$$

FOR $\partial_\mu T^{\mu\nu} = 0$

PROBLEM: ESTIMATE VAINSHTEIN RADIUS FOR SUN, TAKING $m_g \sim H_0$ (present HUBBLE RATE)

ANSWER: $r_V^{SUN} \sim 100$ kpc.

- AT SHORTER DISTANCES SCALAR SECTOR NON-LINEAR, MAY SUPPRESS ITSELF.

BUT QUANTUM THEORY GETS STRONGLY COUPLED AT LOW ENERGY SCALE

Arkani-Hamed, Georgi, Shwartz

$$S_{eff}^{\chi} = \int d^4x \left[M_{Pl}^2 m_g^4 (\partial \chi)^2 + M_{Pl}^2 m_g^2 (\partial^2 \chi)^3 \right]$$

$$\Downarrow$$

$$S_{eff}^{X} = \int d^4x \left[(\partial X)^2 + \frac{1}{M_{Pl}\, m_g^4} (\partial^2 X)^3 \right]$$

$$\Downarrow$$

$$\Lambda_{STRONG} = \left(m_g^4 \cdot M_{Pl} \right)^{1/5}$$

ADDING TERMS OF HIGHER ORDER IN $h_{\mu\nu}$

$$\Downarrow$$

MAKE

$$\Lambda_{STRONG} = \left(m_g^2 \cdot M_{Pl} \right)^{1/3}$$

STILL TOO LOW:

$$\Lambda_{STRONG}^{-1} \sim 1000 \text{ km.}$$

THEORY NOT TRACTABLE AT SHORTER DISTANCES.

NB: DIRECT CALCULATION OF SCATTERING OF LONGITUDINAL GRAVITONS $\Rightarrow A = \frac{E^{10}}{M_{Pl}^2 m_g^8}$

$\Lambda_{Strong}^2 \longrightarrow$

Aubert

• OPTIONS

OPTION 1 • NEW PHYSICS SETS IN AT

$$E \ll \Lambda_{STRONG}$$

THEORY REMAINS WEAKLY COUPLED AT ENERGIES WELL ABOVE Λ_{STRONG}.

CF. ENGLERT - BROUT - HIGGS MECHANISM IN NON-ABELIAN GAUGE THEORIES

MASSIVE GAUGE FIELDS $m_g^2 A_\mu A^\mu \Rightarrow$

PERTURBATIVE STRONG COUPLING SCALE

$$\Lambda_{strong} = \frac{m_g}{g}$$

g ← gauge coupling

WITH EBH MECHANISM, NEW PHYSICS (HIGGS BOSON) AT LOWER ENERGIES

$\Downarrow$

THEORY REMAINS WEAKLY COUPLED AT ALL ENERGIES

Need to make SURE THAT "NEW PHYSICS" INTERACTS WITH MATTER SUFFICIENTLY WEAKLY

NO PROPOSALS OF THIS SORT

OPTION 2. VIOLATION OF LORENTZ - INVARIANCE.

HINT: TRY LORENTZ-VIOLATING GRAVITON MASS TERMS

V.R.

INSTEAD OF $\mathcal{L}_{F.P.} = M_{Pl}^2 m_g^2 \left(h_{\mu\nu} h^{\mu\nu} - h_\mu^\mu h_\nu^\nu \right]$

TRY (ASSUMING UNBROKEN SPATIAL ROTATIONS)

$$\mathcal{L}_{mass} = M_{Pl}^2 \times$$

$$\left[m_0^2 h_{00}^2 + 2 m_1^2 h_{0i}^2 - m_2^2 h_{ij} h_{ij} + m_3^2 h_{ii} h_{jj} - 2 m_4^2 h_{00} h_{ii} \right]$$

- GHOSTS FOR GENERAL $m_0, \dots, m_4$

NO WONDER: LORENTZ - INVARIANT MASSES OF GENERAL FORM ARE SUBCLASS

- A NUMBER OF GHOST - FREE CASES

* $m_0 = 0$, OTHERS $\neq 0$

V.R.

NOT PROTECTED BY ANY RESIDUAL SYMMETRY $\Rightarrow$

FINE TUNING. $\Rightarrow$ MAKES SENSE AS "PROOF BY EXAMPLE" ONLY.

$$h_{\mu\nu} = \partial_\mu \pi_\nu - \partial_\nu \pi_\mu + (\partial\pi)^2$$

$$\Downarrow$$

- 3-VECTOR MODES: $\pi_0 = 0$, $\partial_i \pi_i^T = 0$

$$\mathcal{L}_{mass}^{vec.} = M_{Pl}^2 \left[m_1^2 \left(\partial_0 \pi_i^T\right)^2 - m_2^2 \left(\partial_i \pi_j^T\right)^2 \right]$$

HEALTHY KINETIC TERM, STRONG COUPLING SCALE $\Lambda_{strong} = \sqrt{m_g M_{Pl}}$

- 3-scalar modes; π_0; $\pi_i^L = \partial_i \sigma$

$$\mathcal{L}_{mass}^{scal} = M_{Pl}^2 \left[m_1^2 \left(\partial_0 \pi_i^L\right)^2 - 2\left(m_2^2 - m_3^2\right) \left(\partial_i \pi_j^L\right)^2 + m_1^2 \left(\partial_i \pi_0\right)^2 + \left(2m_4^2 - m_1^2\right) \pi_0 \cdot \partial_0 \partial_i \pi_i^L \right]$$

WOULD-BE GHOST π_0 NON-DYNAMICAL $\Rightarrow$ Eliminate it, OBTAIN

$$\mathcal{L}_{mass}^{scal} = M_{Pl}^2 \left[\tilde{m}_1^2 \left(\partial_0 \pi_i^L\right)^2 - \tilde{m}_2^2 \left(\partial_i \pi_j^L\right)^2 \right]$$

$\tilde{m}_{1,2}$ = combinations of m's.

NO DIFFERENCE WITH VECTOR SECTOR.

PROBLEM.

IN THEORY WITH $m_0=0$, OTHERS $\neq 0$, LINEARIZED ABOUT MINKOWSKI BACKGROUND, FIND GRAVITATIONAL FIELD (ALL COMPONENTS OF $h_{\mu\nu}$) OF STATIC point SOURCE. NEGLECT LONGITUDINAL TERMS OF THE FORM $\partial_\mu \pi_\nu + \partial_\nu \pi_\mu$. SHOW THAT IN THE LIMIT $m_i \to 0$, THE RESULT OF LINEARIZED General Relativity is REPRODUCED.

THUS, LINEARIZED LORENTZ-VIOLATING massive GRAVITY DOES NOT exhibit vDVZ DISCONTINUITY.

PROBLEM.

FIND PARTICLE SPECTRUM OF THIS THEORY, INCLUDING DISPERSION LAWS $\omega(\vec{p})$ OF ALL TYPES OF PARTICLES.

NB: FOR THIS PURPOSE, DECOMPOSE $h_{\mu\nu}(\vec{p})$ INTO TENSORS, SCALARS AND VECTORS IN 3d SENSE.

- STRONG COUPLING SCALE

$$\Lambda_{\text{strong}}^{-1} = \frac{1}{\sqrt{M_{Pl}\, m_g}} \simeq 0.01 \text{ mm}$$

- NO GHOSTS
- No vDVZ discontinuity.

LORENTZ-VIOLATION PROMISING.

QUESTIONS TO ADDRESS:

- WHAT IS BEHIND LORENTZ-VIOLATION? WHOSE CONDENSATES?

- WHAT KIND OF PHENOMENA MAY ONE ENCOUNTER?

- IS $\sqrt{M_{Pl} \cdot m_g}$ INDEED THE RELEVANT UV SCALE?

  If so, what is the physics above this scale?

- CAN ACCELERATED EXPANSION OF THE UNIVERSE INDEED COME OUT?

SUMMARY OF LORENTZ-INVARIANT GRAVITON MASSES.

(1) GHOST-FREE THEORY $\Leftrightarrow$ Fierz-PAULI

$$S = -M_{Pl}^2 \int d^4x \left(\sqrt{g}\, R + m_g^2 \left[h_{\mu\nu} h^{\mu\nu} - h_\mu^\mu h_\nu^\nu \right] \right)$$

(2) CLASSICAL THEORY AT LINEARIZED LEVEL:

van DAM - VELTMAN - ZAKHAROV DISCONTINUITY.

$m_g \to 0 \Rightarrow$

$$h_{\mu\nu} = h_{\mu\nu}^{GR} + \frac{1}{6} \eta_{\mu\nu} \Phi$$

$$\Box \Phi = \frac{1}{M_{Pl}} T_\mu^\mu$$

SCALAR-TENSOR GRAVITY.

BECOMES NON-LINEAR FAR AWAY FROM SOURCE,

$$r_V \sim \left(\frac{M_{SOURCE}}{M_{Pl}^2\, m_g^4} \right)^{1/5} \quad \sim 100\ \text{kpc FOR SUN}$$

CF. GR: NON-LINEARITY AT $r \sim r_g \sim \dfrac{M_{SOURCE}}{M_{Pl}^2}$

NON-LINEARITY MAY SUPPRESS Φ AT $r \ll r_V \Rightarrow$

PHENOMENOLOGICALLY ACCEPTABLE.

(3) QUANTUM THEORY:

STRONGLY COUPLED $\equiv$ NOT TRACTABLE

AT

$$r \lesssim \frac{1}{(M_{pl} m_g^2)^{1/3}} \sim 1000 \text{ km}$$

DUE TO SELF-INTERACTION OF SCALAR MODE

$$h_{\mu\nu} = \partial_\mu \partial_\nu \chi$$

PHENOMENOLOGICALLY UNACCEPTABLE.

STORY REPEATS ITSELF IN MORE COMPLICATED MODELS WITH UNBROKEN 4d LORENTZ-INVARIANCE, INCLUDING

- LOGUNOV'S RTG
- BRANE WORLDS / EXTRA DIMENSIONS.

EITHER GHOSTS OR STRONG COUPLING AT TOO LARGE DISTANCES.

POSSIBLE EXCEPTION (STILL NOT QUITE CLEAR):
Dvali - Gabadadze - Porrati model,
see Nicolis, Rattazzi

A WAY OUT: VIOLATION OF LORENTZ-INVARIANCE

GRAVITY ≈ GAUGE THEORY OF LORENTZ GROUP

$\Updownarrow$

GRAVITATIONAL HIGGS MECHANISM LIKELY TO BREAK LORENTZ SYMMETRY.

- EXAMPLE #1:

$$\mathcal{L}_{mass} = M_{Pl}^2 \left[2m_1^2 h_{0i}^2 - m_2^2 h_{ij} h_{ij} + m_3^2 h_{ii} h_{jj} - 2m_4^2 h_{00} h_{ii} \right]$$

NO TERM $m_0^2 h_{00}^2$.

* LINEARIZED ABOUT MINKOWSKI BACKGROUND

$\Downarrow$

- NO GHOST, NO vDVZ
- STRONG COUPLING SCALE $\dfrac{1}{(M_{Pl} m_g)^{1/2}}$

~ 0.01 mm.

PHENOMENOLOGICALLY ACCEPTABLE.

BUT: - NO OBVIOUS SYMMETRY ENSURING $m_0 = 0$
- DANGER OF NEW DEGREES OF FRIEDOM POPPING UP IN CURVED BACKGROUNDS

BOULVARE - DESER IN FIERZ-PAULI CASE

Minimal - AND HEALTHIER - OPTION

$$\mathcal{L}_{mass} = M_{Pl}^2 \, m_0^2 \, h_{00}^2$$

OPPOSITE TO PREVIOUS CASE

PROTECTED BY SUBGROUP OF DIFFEOMORPHISMS:

$$x^i \to x^i + \xi^i(t,x)$$ ← arbitrary

$$t \to t$$

$$\delta h_{00} = 0 \quad , \quad \delta h_{0i} = \partial_0 \xi_i \; ; \quad \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

DECOMPOSITION OF $h_{\mu\nu}$ into 3-TENSORS, 3-VECTORS AND 3-SCALARS

h_{00} = SCALAR

$$h_{0i} = \underset{\text{vector}}{u_i} + \underset{\text{scalar}}{\partial_i v} \qquad \partial_i u_i = 0$$

$$h_{ij} = \underset{\text{TENSOR}}{h_{ij}^{TT}} + (\underset{\text{vector}}{\partial_i w_j + \partial_j w_i}) + \delta_{ij} \cdot y + \partial_i \partial_j z$$

SCALARS

$$\partial_i h_{ij}^{TT} = h_{ii}^{TT} = 0 \qquad \partial_i w_j = 0$$

- Quadratic action separates into

$$S^{(2)}(h_{ij}^{TT}) + S^{(2)}(u_i, w_i) + S^{(2)}(h_{00}, v, y, z)$$

REASON: spatial $O(3)$.

ALWAYS THE CASE FOR UNBROKEN SPATIAL ROTATIONS + space-time translations.

Case $\mathcal{L}_{mass} = M_{Pl}^2 m_0^2 h_{00}^2$

3-Vectors, 3-tensors do not enter $\mathcal{L}_{mass}$

$\Downarrow$

THEY BEHAVE EXACTLY AS IN GR:

tensors ARE MASSLESS (gravitons)

VECTORS DO NOT PROPAGATE.

SOMETHING NEW IN SCALAR SECTOR ONLY

$\mathcal{L}_{mass} = M_{Pl}^2 m_0^2 h_{00}^2 \Leftrightarrow$ GHOST CONDENSATE

ARKANI-HAMED ET. AL.

- CONSIDER A THEORY OF ONE REAL SCALAR FIELD Φ WITH ACTION [MINKOWSKI SPACE, FORGET ABOUT GRAVITY FOR A WHILE]

$$S = \int d^4x \; P\left(\partial_\mu \Phi, \frac{1}{M}\partial_\mu\partial_\nu \Phi, \frac{1}{M^2}\partial_\mu\partial_\nu\partial_\lambda \Phi, \dots\right)$$

NB: protected by shift symmetry $\Phi \to \Phi + \text{const.}$

Field EQUATION

$$-\partial_\mu\left(\frac{\partial P}{\partial(\partial_\mu \phi)}\right) + \partial_\mu\partial_\nu\left(\frac{\partial P}{\partial(\partial_\mu\partial_\nu\phi)}\right) + \dots = 0$$

$\propto \partial_\mu \phi \;, \; \partial_\mu \Box \phi, \dots$

SOLUTION

$$\boxed{\phi = ct \quad , \quad c = \text{arbitrary constant}}$$

Indeed, $\dfrac{\partial P}{\partial(\partial_0 \phi)} = \text{const}$, $\dfrac{\partial P}{\partial(\partial_0^2 \phi)} = \text{const}, \dots$

OTHER $\dfrac{\partial P}{\partial(\partial_i \phi)} = \dots = 0.$

TIME-DEPENDENT BACKGROUND; LINEAR IN TIME.

Lorentz-invariance spontaneously broken

Quadratic action for perturbations at low momenta and frequencies,

$\omega, |\vec{p}| \ll M$

- For general c, forget about higher-derivative terms

$$S = \int d^4x \; P(X) \quad , \quad X = \partial_\mu \Phi \partial^\mu \Phi$$

Perturbations:

$$\Phi = ct + \pi(\vec{x}, t)$$

$$\partial_0 \Phi = c + \dot{\pi}$$

$$\partial_i \Phi = \partial_i \pi$$

$$X = c^2 + 2c \cdot \dot{\pi} + \dot{\pi}^2 - (\partial_i \pi)^2$$

$$S = \int d^4x \left[P_X'(c^2) \cdot (\dot{\pi}^2 - (\partial_i \pi)^2) + 2c^2 P''(c^2) \dot{\pi}^2 \right]$$

$$= \int d^4x \Big[\left(P'(c^2) + 2c^2 P''(c^2)\right) \dot{\pi}^2$$

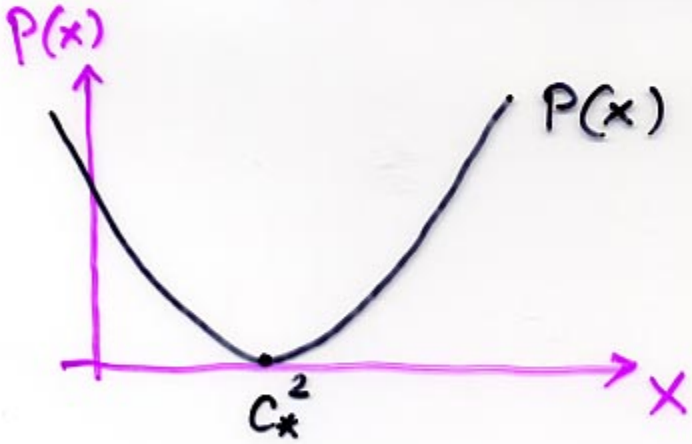
$$- P'(c^2) \cdot (\partial_i \pi)^2 \Big]$$

Healthy theory below intrinsic mass scale M for

$$P'(c^2) \geq 0$$

$$P'(c^2) + 2c^2 P''(c^2) > 0$$

$$S = \int d^4x \left[\left(P'(c^2) + 2c^2 P''(c^2) \right) \dot{\pi}^2 - P'(c^2) \left(\partial_i \pi \right)^2 \right]$$



Unbroken Lorentz-invariance at $c=0 \Rightarrow X=0$
$\pi \equiv \Phi$ is a ghost. Hence the name.

HEALTHY THEORY AT $c > c_*$: NEITHER GHOST NOR TACHYON.

AT $\underline{c = c_*}$: SPATIAL GRADIENT TERM FROM HIGHER DERIVATIVE TERMS ONLY

$$S = \int d^4x \left[2c^2 P''(c^2) \cdot \dot{\pi}^2 + \frac{(\Delta \pi)^2}{M^2} + \mathcal{O}\left(\frac{\dot{\pi} \cdot \Delta \pi}{M} \right) \right]$$

Dispersion LAW:

$$\omega^2 = \text{const} \cdot \frac{\vec{p}^{\,4}}{M^2}, \text{ i.e.}$$

$$\omega = \text{const} \cdot \frac{\vec{p}^{\,2}}{M}$$

NON-RELATIVISTIC MATTER WITHOUT REST MASS.

- GHOST CONDENSATE IN EXPANDING UNIVERSE (LATE UNIVERSE)

FORGET ABOUT HIGHER-DERIVATIVE TERMS

$$S = \int d^4x \sqrt{-g}\; P(X) \qquad X = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$$ds^2 = dt^2 - a^2(t)\,d\vec{x}^2$$

Field equation

$$\partial_\mu\left(\sqrt{-g}\cdot g^{\mu\nu} P'(X)\cdot \partial_\nu\phi\right) = 0$$

Covariant conservation of $J_\mu = P'(X)\,\partial_\mu\Phi$

TAKE $\phi = \phi(t)$; $c(t) \equiv \dot\phi(t) \Rightarrow X = \dot\phi^2 = c^2(t)$

$$\Downarrow$$

$$\partial_0\left(a^3(t)\cdot P'(c^2(t))\cdot c(t)\right) = 0$$

$$\Downarrow$$

$$P'(c^2)\cdot c \propto \frac{1}{a^3}$$

- GHOST condensate is driven to $c = c_*$ where $P'(c_*^2) = 0$!

$c = c_*$ is COSMOLOGICAL ATTRACTOR

• ENERGY - momentum

$$T_{\mu\nu} = P'(X)\, \partial_\mu \Phi\, \partial_\nu \Phi + g_{\mu\nu} P(X)$$

$P'(X) \propto \frac{1}{a^3}$

$\partial_0 \phi \to c_*$; $P(X) = P(c_*^2) + P''(c_*^2)\left(\frac{c^2 - c_*^2}{2}\right)^2$

faster than $\frac{1}{a}^3$

$$\Downarrow$$

$$T_{00} = g_{00} P(c_*^2) + \frac{\text{const}}{a^3}$$

$$T_{ij} = g_{ij} P(c_*) + \mathcal{O}\left(\frac{1}{a^4}\right)$$

• GHOST CONDENSATE IN COSMOLOGICAL CONTEXT

= COSMOLOGICAL CONSTANT + NON-RELATIVISTIC (PRESSURELESS) FLUID

Interesting phenomenology; NOT SO interesting cosmology.

- HOW IS ALL THIS RELATED TO GRAVITON MASS TERMS?

$$S = \int d^4x \sqrt{-g} \cdot \left(-M_{Pl}^2 R + P(X, \dots)\right)$$

$$X = g^{\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi$$

ASSUME $P(c_*^2) = 0$ $\Leftrightarrow$ NO COSMOLOGICAL CONSTANT

$$\Downarrow$$

$T_{\mu\nu}(c = c_*) = 0$, Minkowski vacuum at $c = c_*$

COSMOLOGICAL EXPANSION DRIVES THERE.

PERTURBATIONS ABOUT Minkowski VACUUM

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Phi = c_* t + \pi$$

- Unitary gauge $\pi = 0$. $\Leftrightarrow$ SLICING $\phi = \text{const}$ $\Updownarrow$ $t = \text{const}$

$$X = c_*^2 (1 - h_{00} + \dots)$$

$$\Downarrow$$

M^4, ghost condensate scale

$$S = \int d^4x \sqrt{-g} \left[-M_{Pl}^2 \cdot R + \frac{1}{2} c_*^4 P''(c_*) \cdot h_{00}^2 \right]$$

m_0 - term

NB: $m_0^2 M_{Pl}^2 = M^4 \Rightarrow M = \sqrt{m_0 M_{Pl}}$ is THE UV scale

"GHOST CONDENSATE"

NON-RELATIVISTIC matter WITHOUT REST MASS,

$$\Rightarrow \omega(\vec{p}) = \frac{\vec{p}^{\,2}}{M}$$

Jeans instability with large time scale

YES

Unknown: Cannot trust ghost condensate theory above this momentum/energy

NO!

GHOST CONDENSATE BEHAVES AS PRESSURELESS FLUID

ANOTHER EXAMPLE

DUBOVSKY, TINYAKOV, TKACHEV

$$\mathcal{L}_{mass} = M_{Pl}^2 \left[-3\gamma \mu^2 h_{00}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2\mu^2 h_{00} h_{ii} \right]$$

WITH $\quad m_2^2 - 3m_3^2 = \frac{1}{\gamma}\mu^2$

- PROTECTED BY SUBGROUP OF DIFFEOMORPHISMS

time-dep. shift (a) $x^i \to x^i + \xi^i(t)$

dilatation (b) $t \to \lambda t, \quad x^i \to \lambda^{-\gamma} x^i$

Possible only if Lorentz is violated.

- TENSOR MODES massive — GRAVITON = DARK MATTER candidate
- COSMOLOGICAL ATTRACTOR
- Longitudinal gravitons BEHAVE like QUINTESSENCE
- PRESENT VALUE OF COSMOLOGICAL CONSTANT DETERMINED by initial conditions.
- Newton's LAW VALID FOR <u>ALL</u> DISTANCES.

Interesting, BUT NOT EXACTLY WHAT WE WANT

Version of quintessence + cosmological constant

- WHAT IS BEHIND LORENTZ-VIOLATION? WHOSE CONDENSATES?

  - OPTION: VECTOR FIELDS.

* KEEP ROTATIONAL SYMMETRY OF 3-SPACE

$\Downarrow$

EITHER B_0 FROM B_μ — Kostelecky, Samuel; Jacobson, Mattingly

OR: A^a_μ, internal SYMMETRY $\supset O(3)_{INT}$

and condensate of $A^a_i = \mu\, \delta^a_i$

$SO(3)_{SPACE} \times SO(3)_{int} \to SO(3)$ — Bento et. al.; Armendariz-Picon

OR $\boxed{\text{BOTH}}$ ← WHY? LATER

* VECTOR FIELDS WITHOUT GAUGE SYMMETRY: DANGER OF GHOSTS.

NOT NECESSARILY, ONCE THERE ARE LORENTZ-VIOLATING CONDENSATES

Gripaios

OMITTING COUPLINGS

$$\mathcal{L}^{V,2} = -\ldots B_{\mu\nu}B^{\mu\nu} + \ldots(D_\mu B^\mu)^2$$ ← danger of ghosts

$$\left\{\begin{array}{l} +\ldots D_\mu B^{\underline{\nu}} \cdot D^\mu B^{\underline{\lambda}}\, B_{\underline{\nu}} B_{\underline{\lambda}} \\ +\ldots D_{\underline{\mu}} B^{\underline{\nu}}\, D_{\underline{\lambda}} B^{\underline{\sigma}}\, B^\mu B^\lambda B_{\underline{\nu}} B_{\underline{\sigma}} + \ldots\ldots \end{array}\right.$$

HELP OUT FOR $B_0 = \mu$

$$\Downarrow$$

$$\ldots D_\mu B^0 \cdot D^\mu B^0$$

$$\ldots D_0 B^0 \; D_0 B^0$$

HEALTHY KINETIC TERMS FOR δB^0.

PLAY WITH PARAMETERS $\Rightarrow$ NO GHOSTS NOR TACHYONS. IN $L^{V,2}$.

- LIGHT FIELDS REMAIN EVEN FOR

WITH GRAVITY

$$B_0 = \ldots \mu$$

$$A_i^a = \ldots \mu \quad , \quad \mu \text{ \underline{large}, BUT } \boxed{\mu \ll M_{Pl}}$$

DUE TO SOME POTENTIAL $V(g^{\mu\nu}A_\mu^a A_\nu^a, g^{\mu\nu}B_\mu B_\nu)$

GAUGE SYMMETRIES OF GR + unbroken O(3)

$$\Downarrow$$

MASS TERMS FROM POTENTIAL

$$(M \cdot h_{00} + \delta B_0)^2, \quad (\mu h_{0i} - \delta A_0^i + \delta B_i)^2$$

$$(\mu h_{ij} - \delta A_i^j - \delta A_j^i)^2, \quad (\mu h_i^i - 2\delta A_i^i)^2$$

OTHER COMBINATIONS DO NOT ENTER δV.

* LIGHT MODES PARAMETRIZED BY

$$\underline{\underline{h_{\mu\nu}}}, \quad \frac{(\delta A_i^j - \delta A_j^i)}{2\mu}, \quad \frac{2}{\mu}\,\delta A_0^i$$

ALL REMAIN LIGHT

* LORENTZ - VIOLATION DOES NOT GUARANTEE UNCONVENTIONAL DISPERSION RELATION.
INDEED, FOR

$$L = L_{GR} + L^{V,2}$$

ALL DISPERSION RELATIONS ARE

$$\omega^2 = v^2 \vec{p}^{\,2}$$

NB: GRAVITON VELOCITY DIFFERENT FROM SPEED OF LIGHT.

$$v^2 = 1 + \ldots \frac{\mu^2}{M_{Pl}^2}$$ FOR GRAVITON

TECHNICAL REASON FOR ALMOST CONVENTIONAL DISPERSION RELATION: GAUGE FIELD FOR LORENTZ GROUP IS CONNECTION, $\omega_{\mu\beta}^{\alpha}$, or $\Gamma_{\mu\nu}^{\lambda}$, not metric.

GAUGE FIELDS: $(D_\mu \varphi)^2 \xrightarrow{\langle\varphi\rangle = \mu} \mu^2 A_\mu A_\mu$

GRAVITY: $(D_\mu A_\nu)^2 \xrightarrow{\langle A\rangle = \mu} \mu^2\, \Gamma \cdot \Gamma \sim \mu^2 (\partial h)^2$

ADDITION TO KINETIC TERM OF $h_{\mu\nu}$, NOT MASS

- New degrees of freedom introduced
- Gravity gets modified BUT
  - Not so dramatically
  - No scale of modification: same dispersion law at all $\vec{p}$; Newton's law always the same (at distances above strong coupling scale intrinsic in vector theory itself).

- Introducing extra mass scale: add 1-derivative terms into vector Lagrangian

Libanov, V.R.

$$\mathcal{L}^{V,1} = \lambda D_\mu A_\nu^a A^{b\mu} A^{c\nu} \cdot \varepsilon^{abc} + \dots$$

a la Chern-Simons term in 3d.

with $A_i^a = \mu \delta_i^a$

$$\boxed{\lambda \ll 1}$$

$$\mathcal{L}^{V,1} = \lambda\mu \cdot DA \cdot A$$

Reintroduces ghost in vector sector. even without gravity

EVEN WITHOUT GRAVITY, NEW SCALE $\lambda\mu$

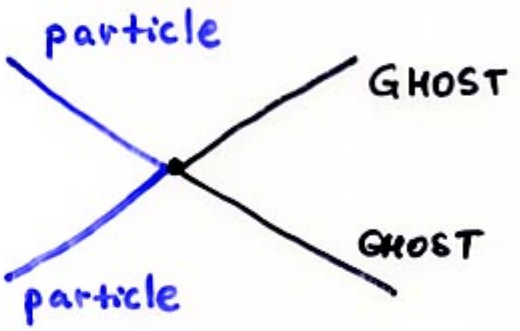
DISPERSION RELATIONS

- IN 3-VECTOR SECTOR

$\omega^2 = \text{const} \cdot p^2$, NO GHOSTS, $p \gg \lambda\mu$

$$\omega^2 = \begin{matrix} (\lambda\mu)^2 \\ \text{const} \cdot p^2 \Leftarrow \text{GHOST}, \end{matrix} \left. \vphantom{\begin{matrix} a \\ b \end{matrix}} \right\} p^2 \ll (\lambda\mu)^2$$

IS GHOST A PROBLEM?



EVEN WITHOUT COUPLING CONSTANT SUPPRESSION

$$\frac{d\varepsilon}{dt} \sim (\lambda\mu)^5$$

REQUIRE

Lifetime of the Universe.

$$\frac{d\varepsilon}{dt} \cdot \frac{1}{H_0} \ll \rho_c \sim H_0^2 M_{Pl}^2$$

$$\Downarrow$$

$$(\lambda\mu)^{-1} \gtrsim 10^4 \text{ cm}$$

MAY even be useful!
Holdom

HARMLESS, IF VECTORS DO NOT DIRECTLY INTERACT WITH OUR MATTER.

NB: AVOID TACHYONS AT THIS SCALE!

Fine-tuning in this example

DISPERSION RELATIONS CONT'D
(NO GRAVITY YET):

- 3-SCALAR SECTOR (NO GHOSTS, TACHYONS).

$$\omega^2 = \text{const} \cdot p^2, \quad \underline{p \gg \lambda\mu}$$

$$\omega^2 = \begin{cases} \lambda^2\mu^2 \\ \dfrac{p^4}{\lambda^2\mu^2} \end{cases} \leftarrow \text{similar to ghost condensate} \qquad p \ll \lambda\mu$$

WITH GRAVITY:

TWO NEW SCALES APPEAR.

$$(1)\ \Lambda_1 = \lambda\mu \cdot \frac{\mu}{M_{Pl}}$$

- <u>DISTANCE</u> SCALE AT WHICH GRAVITY OF STATIC SOURCES GETS MODIFIED

GR $\rightarrow$ SCALAR-TENSOR

- 3-momentum scale in scalar sector,

$$\omega^2 = \frac{p^4}{\lambda^2\mu^2} \xRightarrow{p \sim \Lambda_1} \omega^2 = p^2 \frac{\mu^2}{M_{Pl}^2}$$

(2) $\Lambda_2 = \lambda\mu \cdot \frac{\mu^2}{M_{Pl}^2} \ll \Lambda_1$ assuming $\mu \ll M_{Pl}$

- TIME SCALE OF MODIFICATION OF GRAVITY BY STATIC SOURCES

Similar to GHOST CONDENSATE!

- TACHYONS IN TENSOR AND VECTOR SECTORS.

e.g. h_{ij}^{TT} $\quad (\partial_i h_{ij}^{TT} = h_{ii}^{TT} = 0)$

$$\omega^2 = v^2 p^2 \pm \Lambda_2 \cdot p$$

SMELLS GOOD : CHOOSE $\Lambda_2 \sim H_0 \Rightarrow$

GRAVITY UNSTABLE AT COSMOLOGICAL DISTANCE/TIME SCALE.

BUT WAIT...

Numerology

(a) $(\lambda\mu)^5 < H_0^3 M_{Pl}^2$

GHOST NOT DANGEROUS

(b) $\lambda\mu \cdot \frac{\mu^2}{M_{Pl}^2} \sim H_0$

Instability at time scale $\sim H_0^{-1}$

$$\Downarrow$$

$\mu > 10^7$ GeV

$\lambda\mu < 10^{-18}$ GeV

WHAT DO WE LEARN FROM THIS EXAMPLE?

- WHAT KIND OF PHENOMENA MAY ONE ENCOUNTER (IN LORENTZ-VIOLATING THEORIES)?

  - GHOSTS, BUT ONLY AT LOW ENOUGH 3-momenta $\Longleftrightarrow$ HARMLESS

NOT PHANTOM CANDIDATE YET, BUT FAIRLY SIMILAR

TRANSVERSE VECTOR b_i^T, $\partial_i b_i^T = 0$

PHANTOM IS SCALAR

  - SEPARATION OF SPACE AND TIME SCALES OF MODIFICATION OF GR

SEEN IN GHOST CONDENSATE MODEL, APPEARS TO HAVE MORE GENERAL CHARACTER

  - TACHYONS AT VERY LOW MOMENTA (WELL BELOW GHOSTS). TENSORS (GRAVITONS) INCLUDED.

GRAVITY UNSTABLE AT VERY LARGE DISTANCE/TIME SCALES.

- IS $\sqrt{M_{Pl} \cdot m_g}$ INDEED THE RELEVANT UV SCALE?

NO. GRAVITY MODIFIED AT

$$\Lambda_1 = \lambda\mu \cdot \frac{\mu}{M_{Pl}}$$

$$\Downarrow$$

$$\sqrt{M_{Pl} \cdot m_g} = \sqrt{M_{Pl} \cdot \Lambda_1} = \sqrt{\lambda} \cdot \mu \gg \lambda\mu$$

LOWEST INTERESTING SCALE: ENERGY SCALE IN VECTOR SECTOR

"NEW PHYSICS" (OF VECTOR FIELDS B_μ, A_μ^a)

DECOUPLES AT $E = \lambda\mu \ll \sqrt{M_{Pl} \cdot m_g}$

GRAVITY IS ALWAYS WEAK.

• CAN ACCELERATED EXPANSION OF THE UNIVERSE INDEED COME OUT?

NO!

QUALIFICATION: WITH VECTOR FIELDS PINNED DOWN TO THEIR VACUUM VALUES (IN LOCALLY LORENTZ REFERENCE FRAME).

OTHERWISE: VECTOR FIELD QUINTESSENCE a la KISELEV; ARMENDARIZ-PICON

TECHICALLY:

$$ds^2 = N^2(t)\,dt^2 - a^2(t)\,d\vec{x}^{\,2}$$

$$B_0 = \mu\, N(t)$$

$$A_i^a = \mu\, a(t) \cdot \delta_i^a$$

$\Downarrow$ ALL TERMS IN $L_{GR} + L^V$ have same form

$$S(N,a) = \tilde{M}_{Pl}^2 \int dt \cdot \frac{1}{N}\left(\frac{\dot{a}}{a}\right)^2$$

combination of M_{Pl} and μ; "renormalized" COSMOLOGICAL M_{Pl}

cf. Carroll and Lim, DUBOVSKY, Tinyakov, Tkachev

$\Downarrow$

FRIEDMANN EQS. OF GR WITH $M_{Pl} \to \tilde{M}_{Pl}$.

FRIEDMANN EQS. MORE ROBUST THAN GR!

REASON: SYMMETRIES OF HOMOGENEOUS, ISOTROPIC, SPATIALLY FLAT METRIC

- TIME REPARAMETRIZATION
- SPACE DILATION

$$\Downarrow$$

IF NO EVOLVING MATTER FIELDS, THE ONLY INVARIANTS WITHOUT HIGHER DERIVATIVES ARE (assuming derivative expansion)

$$\underbrace{\int N(t)\,dt}_{\text{COSMOLOGICAL CONSTANT}} \quad \text{AND} \quad \underbrace{\int dt \cdot N(t) \cdot \left(\frac{\dot a}{Na}\right)^2}_{\text{EINSTEIN - HILBERT}}$$

ONCE ONE DESTROYS LORENTZ-INVARIANCE,
A LOT IS ALLOWED

GHOSTS, TACHYONS, ...

BUT NOT EVERYTHING

MODIFICATION OF FRIEDMANN EQS.

CHANCE FOR SUCCESSFULL (COSMOLOGICALLY) MODIFICATION OF GRAVITY:

INFINITE EXTRA DIMENSIONS.

WE ARE SLOWLY LEARNING...