BEYOND THE STANDARD MODEL*

(IN SEARCH OF SUPERSYMMETRY)

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Abstract

The present lectures contain an introduction to low energy supersymmetry, a new symmetry that relates bosons and fermions, in particle physics. The Standard Model of fundamental interactions is briefly reviewed, and the motivation to introduce supersymmetry is discussed. The main notions of supersymmetry are introduced. In more detail the supersymmetric extension of the Standard Model - the Minimal Supersymmetric Standard Model - is considered. Phenomenological features of the MSSM as well as possible experimental signatures of SUSY are described. An intriguing situation with the supersymmetric Higgs boson is discussed.

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Contents

1 Introduction. The Standard Model and beyond.

The Standard Model (SM) of fundamental interactions describes *strong, weak* and *electromagnetic* interactions of elementary particles [10]. It is based on a *gauge principle*, according to which all the forces of Nature are mediated by an exchange of the gauge fields of a corresponding local symmetry group. The symmetry group of the SM is

$$SU_{colour}(3) \otimes SU_{left}(2) \otimes U_{hypercharge}(1), \tag{1.1}$$

a

whereas the field content is the following:

Gauge sector : Spin = 1

The gauge bosons are spin 1 vector particles belonging to the adjoint representation of the group (1.1). Their quantum numbers with respect to $SU(3) \otimes SU(2) \otimes U(1)$ are:

$$\begin{array}{cccccc} \textit{gluons} & G^a_\mu : & (\underline{8},\underline{1},0) & SU_c(3) & g_s, \\ \begin{array}{c}\textit{intermediate}\\ \textit{weak bosons}\end{array} & W^i_\mu : & (\underline{1},\underline{3},0) & SU_L(2) & g, \\ \textit{abelian boson} & B_\mu : & (\underline{1},\underline{1},0) & U_Y(1) & g', \end{array} \tag{1.2}$$

where the coupling constants are usually denoted by g_s, g and g', respectively.

Fermion sector : Spin = 1/2

The matter fields are fermions belonging to the fundamental representation of the gauge group. These are believed to be quarks and leptons of at least of three generations. The SM is left-right asymmetric. Left-handed and right-handed fermions have different quantum numbers:

$$\begin{array}{lcclclclcl} \textit{quarks} & & & & & & & & & \\ Q^i_{\alpha L} = & \left(\begin{array}{c} U^i_\alpha \\ D^i_\alpha \end{array}\right)_L & = & \left(\begin{array}{c} u^i \\ d^i \end{array}\right)_L, & \left(\begin{array}{c} c^i \\ s^i \end{array}\right)_L, & \left(\begin{array}{c} t^i \\ b^i \end{array}\right)_L, & \ldots & (\underline{3},\underline{2},1/3) \\ & U^i_{\alpha R} & = & u_{iR}, & c_{iR}, & t_{iR}, & \ldots & (\underline{3}^*,\underline{1},4/3) \\ & D^i_{\alpha R} & = & d_{iR}, & s_{iR}, & b_{iR}, & \ldots & (\underline{3}^*,\underline{1},-2/3) \\ \textit{leptons} & L_{\alpha L} & = & \left(\begin{array}{c} \nu_e \\ e \end{array}\right)_L, & \left(\begin{array}{c} \nu_\mu \\ \mu \end{array}\right)_L, & \left(\begin{array}{c} \nu_\tau \\ \tau \end{array}\right)_L, & \ldots & (\underline{1},\underline{2},-1) \\ & E_{\alpha R} & = & e_R, & \mu_R, & \tau_R, & \ldots & (\underline{1},\underline{1},-2) \end{array} \tag{1.3}$$

$i = 1, 2, 3$ - colour, $\alpha = 1, 2, 3, \ldots$ - generation.

Higgs sector : Spin = 0

In the minimal version of the SM there is one doublet of Higgs scalar fields

$$H = \left(\begin{array}{c} H^0 \\ H^- \end{array}\right) \qquad (\underline{1},\underline{2},-1), \tag{1.4}$$

which is introduced in order to give masses to quarks, leptons and intermediate weak bosons via spontaneous breaking of electroweak symmetry.

In Quantum Field Theory framework the SM is described by the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}, \tag{1.5}$$

SM

Figure 1: Global Fit of the Standard Model smfit

Figure 2: Weak mixing angle and the Higgs boson mass mixang

$$\begin{aligned}\mathcal{L}_{gauge} &= -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} \\ &\quad + i\overline{L}_\alpha\gamma^\mu D_\mu L_\alpha + i\overline{Q}_\alpha\gamma^\mu D_\mu Q_\alpha + i\overline{E}_\alpha\gamma^\mu D_\mu E_\alpha \\ &\quad + i\overline{U}_\alpha\gamma^\mu D_\mu U_\alpha + i\overline{D}_\alpha\gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger(D_\mu H),\end{aligned} \tag{1.6}$$

where

$$\begin{aligned}G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu, \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk}W^j_\mu W^k_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ D_\mu L_\alpha &= (\partial_\mu - i\frac{g}{2}\tau^i W^i_\mu + i\frac{g'}{2}B_\mu)L_\alpha, \\ D_\mu E_\alpha &= (\partial_\mu + ig'B_\mu)E_\alpha, \\ D_\mu Q_\alpha &= (\partial_\mu - i\frac{g}{2}\tau^i W^i_\mu - i\frac{g'}{6}B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu)Q_\alpha, \\ D_\mu U_\alpha &= (\partial_\mu - i\frac{2}{3}g'B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu)U_\alpha, \\ D_\mu D_\alpha &= (\partial_\mu + i\frac{1}{3}g'B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu)D_\alpha.\end{aligned}$$

$$\mathcal{L}_{Yukawa} = y^L_{\alpha\beta}\overline{L}_\alpha E_\beta H + y^D_{\alpha\beta}\overline{Q}_\alpha D_\beta H + y^U_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{H} + h.c., \tag{1.7}$$ yukawa

where $\tilde{H} = i\tau_2 H^\dagger$.

$$\mathcal{L}_{Higgs} = -V = m^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2. \tag{1.8}$$ hig

Here $\{y\}$ are the Yukawa and λ is the Higgs coupling constants, both dimensionless, and m is the only dimensional mass parameter[1].

The Lagrangian of the SM contains the following set of free parameters:

- 3 gauge couplings g_s, g, g';
- 3 Yukawa matrices $y^L_{\alpha\beta}, y^D_{\alpha\beta}, y^U_{\alpha\beta}$;
- Higgs coupling constant λ;
- Higgs mass parameter m^2;
- number of matter fields (generations).

All the particles obtain their masses due to spontaneous breaking of $SU_{left}(2)$ symmetry group via a non-zero vacuum expectation value (v.e.v.) of the Higgs field

$$< H >= \begin{pmatrix} v \\ 0 \end{pmatrix}, \qquad v = m/\sqrt{\lambda}. \tag{1.9}$$ vac

[1]We use the usual for particle physics units $c = \hbar = 1$

As a result the gauge group of the SM is spontaneously broken down to

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \Rightarrow SU_c(3) \otimes U_{EM}(1).$$

The physical weak intermediate bosons are the linear combinations of the gauge ones

$$W^{\pm}_{\mu} = \frac{W^1_{\mu} \mp i W^2_{\mu}}{\sqrt{2}}, \qquad Z_{\mu} = -\sin\theta_W B_{\mu} + \cos\theta_W W^3_{\mu} \tag{1.10}$$

with masses

$$m_W = \frac{1}{\sqrt{2}} g v, \qquad m_Z = m_W / \cos\theta_W, \qquad \tan\theta_W = g'/g, \tag{1.11}$$

while the photon field

$$\gamma_{\mu} = \cos\theta_W B_{\mu} + \sin\theta_W W^3_{\mu} \tag{1.12}$$

remains massless.

The matter fields acquire masses proportional to the corresponding Yukawa couplings:

$$M^u_{\alpha\beta} = y^u_{\alpha\beta} v, \; M^d_{\alpha\beta} = y^d_{\alpha\beta} v, \; M^l_{\alpha\beta} = y^l_{\alpha\beta} v, \; m_H = \sqrt{2} m. \tag{1.13}$$

Explicit mass terms in the Lagrangian are forbidden because they are not $SU_{left}(2)$ symmetrical and would destroy the renormalizability of the Standard Model.

The SM has been constructed as a result of numerous efforts both theoretical and experimental. At present the SM is extraordinary successful, the achieved accuracy of its predictions corresponds to the experimental data within 5 % [11]. The combined result of the Global SM fit are shown in Fig. 1 [11]. All the particles except for the Higgs boson have been discovered experimentally. And the mass of the Higgs boson is severely constrained from precision electroweak data (see Fig. 2 [11]).

However the SM has its natural drawbacks and unsolved problems. Among them are:

- inconsistency of the SM as a QFT (Landau Pole),
- large number of free parameters,
- formal unification of strong and electroweak interactions,
- still unclear mechanism of EW symmetry breaking: the Higgs boson has not yet been observed and it is not clear whether it is fundamental or composite,
- the problem of CP-violation is not well understood including CP-violation in strong interaction,
- flavour mixing and the number of generations are arbitrary,
- the origin of the mass spectrum is unclear.

The answer to these problems lies beyond the SM. There are two possible ways to go beyond the SM:

⇒ To consider the *same* fundamental fields with *new* interactions. This way leads us to supersymmetry, Grand Unification, String Theory, etc. It seems to be favoured by modern experimental data.

⇒ To consider *new* fundamental fields with *new* interactions. This way leads us to compositeness, fermion-antifermion condensates, Technicolour, extended Technicolour, preons, etc. It is not favoured by data at the moment.

There are also possible exotic ways out of the SM: gravity at TeV energies, large extra dimensions, brane world, etc. We do not consider them here. In what follows we go along the lines of the first possibility and describe supersymmetry as a nearest option for the new physics at TeV scale.

2 What is supersymmetry? Motivation in particle physics

Supersymmetry or fermion-boson symmetry has not yet been observed in Nature. This is a purely theoretical invention. Its validity in particle physics follows from the common belief in unification.

2.1 Unification with gravity

The *general idea* is a unification of all forces of Nature. It defines the *strategy* : increasing unification towards smaller distances up to $l_{Pl} \sim 10^{-33}$ cm including quantum gravity. However the graviton has spin 2, while the other gauge bosons (photon, gluons, W and Z weak bosons) have spin 1. Therefore, they correspond to different representations of the Poincaré algebra. Attempts to unify all four forces within the same algebra are faced with the problem. Due to no-go theorems [14] unification of spin 2 and spin 1 gauge forces within unique algebra is forbidden. The only exception from this theorem is supersymmetry algebra. The *uniqueness* of SUSY is due to a strict mathematical statement that algebra of SUSY is the *only* graded (i.e. containing anticommutators as well as commutators) Lie algebra possible within relativistic field theory [14].

If Q is a generator of SUSY algebra, then

$$Q|boson>=|fermion> \quad \text{and} \quad Q|fermion>=|boson>.$$

Hence starting with the graviton state of spin 2 and acting by SUSY generators we get the following chain of states

$$spin\ 2 \quad \rightarrow \quad spin\ 3/2 \quad \rightarrow \quad spin\ 1 \quad \rightarrow \quad spin\ 1/2 \quad \rightarrow \quad spin\ 0.$$

Thus, a partial unification of matter (fermions) with forces (bosons) naturally arises out of an attempt to unify gravity with other interactions.

SUSY algebra appears as a generalization of Poincaré algebra (see next section) and links together various representations with different spin. The key relation is given by the anticommutator

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha,\dot\alpha} P_\mu.$$

Taking an infinitesimal transformations $\delta_\epsilon = \epsilon^\alpha Q_\alpha,\ \bar\delta_{\bar\epsilon} = \bar{Q}_{\dot\alpha}\bar\epsilon^{\dot\alpha}$, one gets

$$\{\delta_\epsilon, \bar\delta_{\bar\epsilon}\} = 2(\epsilon\sigma^\mu\bar\epsilon)P_\mu, \tag{2.1}$$

where ϵ is a transformation parameter. Choosing ϵ to be local, i.e. a function of a space-time point $\epsilon = \epsilon(x)$ one finds from eq.(2.1) that anticommutator of two SUSY transformations is a local coordinate translation. And a theory which is invariant under the general coordinate

Figure 3: Electric Screening and Magnetic Antiscreening fig:run

transformation is General Relativity. Thus, making SUSY local, one obtains General Relativity or a theory of gravity or supergravity [23].

Theoretical attractiveness of SUSY field theories is explained by remarkable properties of SUSY models. This is first of all a cancellation of ultraviolet divergencies in rigid SUSY theories which is the origin of

- possible solution of the hierarchy problem in GUTs;
- vanishing of the cosmological constant;
- integrability allowing for an exact non-perturbative solution.

We believe that along this lines one can also obtain the unification of all forces of Nature including quantum (super)gravity.

What is essential, the standard concepts of QFT allow SUSY without any further assumptions. In recent years supersymmetry became a subject of intensive experimental tests. Its predictions can be verified at modern and future colliders.

2.2 Unification of the gauge couplings

Since the main motivation for SUSY is related with the unification theory, let us briefly recall the main ideas of the Grand Unification [73].

The philosophy of Grand Unification is based on a *hypothesis*: Gauge symmetry increases with energy. Having in mind unification of all forces of Nature on a common basis and neglecting gravity for the time being due to its weakness the idea of GUTs is the following:

All known interactions are different branches of unique interaction associated with a simple gauge group. The unification (or splitting) occurs at high energy

$$\begin{array}{cccccl} & \text{Low energy} & & \Rightarrow & \text{High energy} \\ & & & & \\ SU_c(3)\otimes & SU_L(2)\otimes & U_Y(1) & \Rightarrow & G_{GUT} \ \ (or\ G^n + \text{discrete symmetry}) \\ \text{gluons} & W,Z & \text{photon} & \Rightarrow & \ \text{gauge bosons} \\ \text{quarks} & \text{leptons} & & \Rightarrow & \text{fermions} \\ g_3 & g_2 & g_1 & \Rightarrow & g_{GUT} \end{array} \tag{2.2}$$

At first sight this is impossible due to a big difference in the values of the couplings of strong, weak and electromagnetic interactions. However, this is not so. The crucial point here is the running coupling constants. It is a generic property of quantum field theory which has an analogy in classical physics.

Indeed, consider electric and magnetic phenomena. Let us take some dielectric medium and put a sample electric charge in it. What happens is that the medium is polarized. It contains electric dipoles which are arranged in such a way that to screen the charge (see Fig.3). It is a consequence of a Coulomb law: attraction of the opposite charges and repulsion of the same ones. This is the origin of electric screening.

The opposite situation happens to be in magnetic medium. According to the Biot-Savart law, electric currents of the same direction are attracted to each other, while those of the opposite one are repulsed (see Fig.3). This leads to antiscreening of electric currents in magnetic medium.

In QFT the role of the medium is played by the vacuum. Vacuum is polarized due to the presence of virtual pairs of particles in it. The matter fields and transverse quanta of vector fields in this case behave like dipoles in the dielectric medium and cause screening, while the longitudinal quanta of vector fields behave like currents and cause antiscreening. These two effects compete with each other (see eq.(2.7) below).

Thus, the couplings become the functions of a distance or energy scale

$$\alpha_i = \alpha_i(\frac{Q^2}{\Lambda^2}) = \alpha_i(\text{distance}), \qquad \alpha_i \equiv g_i^2/4\pi.$$

This dependence is described by the renormalization group equations and is confirmed experimentally (see Fig.4).

Figure 4: Summary of running of the strong coupling α_s [?]

In the SM the strong and weak couplings associated with non-abelian gauge groups decrease with energy, while the electromagnetic one associated with the abelian group on the contrary increases. Thus, it becomes possible that at some energy scale they become equal. According to the GUT idea this equality is not occasional but is a manifestation of unique origin of these three interactions. As a result of spontaneous symmetry breaking, the unifying group is broken and unique interaction is splitted into three branches which we call strong, weak and electromagnetic interactions. This happens at a very high energy of the order of $10^{15\div16}$ GeV. Of course, this energy is out of the range of accelerators, however, some crucial predictions follow from the very fact of unification.

After the precise measurement of the $SU(3)\times SU(2)\times U(1)$ coupling constants, is has become possible to check the unification numerically.

The three coupling constants to be compared are:

$$\begin{aligned} \alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2\theta_W), \\ \alpha_2 &= g^2/(4\pi) = \alpha/\sin^2\theta_W, \\ \alpha_3 &= g_s^2/(4\pi) \end{aligned} \tag{2.3}$$

where g' , g and g_s are the usual $U(1)$, $SU(2)$ and $SU(3)$ coupling constants and α is the fine structure constant. The factor of 5/3 in the definition of α_1 has been included for the proper normalization of the generators.

The couplings, when defined as renormalized values including loop corrections require the specification of a renormalization prescription, for which the modified minimal subtraction ($\overline{MS}$) scheme [37] is used.

In this scheme the world averaged values of the couplings at the Z^0 energy are obtained from a fit to the LEP data [12], M_W [38] and [39, 40]:

$$\begin{aligned} \alpha^{-1}(M_Z) &= 128.0 \pm 0.1 \\ \sin^2\theta_{\overline{MS}} &= 0.23149 \pm 0.00017 \\ \alpha_3 &= 0.119 \pm 0.002, \end{aligned} \tag{2.4}$$

that gives

$$\alpha_1(M_Z) = 0.017, \quad \alpha_2(M_Z) = 0.034, \quad \alpha_3(M_Z) = 0.118 \pm 0.005. \tag{2.5}$$

Assuming that the SM is valid up to the unification scale one can then use the known RG equations for the three couplings. They are the following:

$$\frac{d\tilde{\alpha}_i}{dt} = b_i\tilde{\alpha}_i^2, \qquad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi}, \qquad t = log(\frac{Q^2}{\mu^2}), \tag{2.6}$$

where for the SM the coefficients b_i are:

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}. \tag{2.7}$$

Here N_{Fam} is the number of generations of matter multiplets and N_{Higgs} is the number of Higgs doublets. We use $N_{Fam} = 3$ and $N_{Higgs} = 1$ for the minimal SM, which gives $b_i = (41/10, -19/6, -7)$.

Notice a positive contribution (screening) from the matter multiplets and negative one (antiscreening) from the gauge fields. For the abelian group $U(1)$ this contribution is absent due to the absence of a self-interaction of abelian gauge fields.

The solution to eq.(2.6) is very simple

$$\frac{1}{\tilde{\alpha}_i(Q^2)} = \frac{1}{\tilde{\alpha}_i(\mu^2)} - b_i log(\frac{Q^2}{\mu^2}). \tag{2.8}$$

The result is demonstrated in Fig.2.2, which shows the evolution of the inverse of the couplings as function of the logarithm of energy. In this presentation the evolution becomes a straight line in first order. The second order corrections are small and do not cause any visible deviation from a straight line. Fig.2.2 clearly demonstrates that within the SM the coupling constants unification at a single point is impossible. It is excluded by more than 8 standard deviations. This result means that the unification can only be obtained if new physics enters between the electroweak and the Planck scales!

Since we do not know what kind of new physics it may be, there is a lot of arbitrariness. In this situation some guiding idea is needed. It is very attempting to try to check whether unification is possible within supersymmetric generalization of the SM. In SUSY case the slopes of the RG evolution curves are modified. The coefficients b_i in eq.(2.6) now are:

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}, \tag{2.9}$$

where use $N_{Fam} = 3$ and $N_{Higgs} = 2$ in the minimal SUSY model which gives $b_i = (33/5, 1, -3)$.

It turns out that within the SUSY model perfect unification can be obtained if the SUSY masses are of the order of 1 TeV. This is shown in Fig.2.2; the SUSY particles are assumed to contribute effectively to the running of the coupling constants only for energies above the typical SUSY mass scale, which causes the change in the slope of the lines near 1 TeV. From the fit requiring unification one finds for the break point M_{SUSY} and the unification point M_{GUT} [26]:

$$\begin{aligned} M_{SUSY} &= 10^{3.4\pm 0.9\pm 0.4}\ GeV, \\ M_{GUT} &= 10^{15.8\pm 0.3\pm 0.1}\ GeV, \\ \alpha^{-1}_{GUT} &= 26.3 \pm 1.9 \pm 1.0, \end{aligned} \tag{2.10}$$

where $\alpha_{GUT} = g_5/4\pi$. The first error originates from the uncertainty in the coupling constant, while the second one is due to the uncertainty in the mass splittings between the SUSY particles. The χ^2 distributions of M_{SUSY} and M_{GUT} are shown in Fig.2.2[26], where

$$\chi^2 = \sum_{i=1}^{3} \frac{(\alpha_i^{-1} - \alpha_{GUT}^{-1})^2}{\sigma_i^2}. \tag{2.11}$$

Figure 5: Evolution of the inverse of the three coupling constants in the Standard Model (left) and in the supersymmetric extension of the SM (MSSM) (right). Only in the latter case unification is obtained. The SUSY particles are assumed to contribute only above the effective SUSY scale M_{SUSY} of about 1 TeV, which causes the change in slope in the evolution of couplings. The thickness of the lines represents the error in the coupling constants [26].

Figure 6: The χ^2 distributions of M_{SUSY} and M_{GUT}

For SUSY models, the dimensional reduction $\overline{DR}$ scheme is a more appropriate renormalization scheme [41]. In this scheme all thresholds are treated by simple step approximations and unification occurs if all three α's meet exactly at one point. This crossing point corresponds to the mass of the heavy gauge bosons. The $\overline{MS}$ and $\overline{DR}$ couplings differ by a small offset

$$\frac{1}{\alpha_i^{\overline{DR}}} = \frac{1}{\alpha_i^{\overline{MS}}} - \frac{C_i}{12\pi}, \tag{2.12}$$

where the C_i are the quadratic Casimir coefficients of the group ($C_i = N$ for SU(N) and 0 for U(1) so α_1 stays the same).

This observation was considered as the first "evidence" for supersymmetry, especially since M_{SUSY} was found in the range preferred by the fine-tuning arguments.

It should be noted, that the unification of the three curves at a single point is not that trivial as it may seem from the existence of three free parameters (M_{SUSY}, M_{GUT} and α_{GUT}). Out of more than thousand models tried, only a handful yielded unification. The reason is simple: introducing new particles one influences all three curves simultaneously, thus giving rise to strong correlations between the slopes of the three lines. For example, adding new generations and/or new Higgs doublets never yield unification! Nevertheless, unification does not prove supersymmetry. The real proof would be the observation of the sparticles.

2.3 Solution of the hierarchy problem

The appearance of two different scales $V \gg v$ in GUT theory, namely, M_W and M_{GUT}, leads to a very serious problem which is called the *hierarchy problem*. There are two aspects of this problem.

The first one is the very existence of the hierarchy. To get the desired spontaneous symmetry breaking pattern, one needs

$$\begin{array}{ccccc} m_H & \sim & v & \sim & 10^2 \;\; GeV \\ m_\Sigma & \sim & V & \sim & 10^{16} \;\; GeV \end{array} \qquad \frac{m_H}{m_\Sigma} \sim 10^{-14} \quad \ll 1, \tag{2.13}$$

where H and Σ are the Higgs fields responsible for the spontaneous breaking of the $SU(2)$ and the GUT groups, respectively.

The question arises how to get so small number in a natural way. One needs some kind of fine tuning in a theory, and we don't know is there anything behind it.

The second aspect of the hierarchy problem is connected with the preservation of a given hierarchy. Even if we choose the hierarchy like eq.(2.13) the radiative corrections will destroy it! To see this, consider the radiative correction to the light Higgs mass. It is given by the Feynman

Figure 7: Radiative correction to the light Higgs boson mass fig:hierar

diagram shown in Fig. 7 and is proportional to the mass squared of the heavy particle. This correction obviously spoils the hierarchy if it is not cancelled. This very accurate cancellation with a precision $\sim 10^{-14}$ needs a fine tuning of the coupling constants.

The only known way to achieve this kind of cancellation of quadratic terms (also known as the cancellation of the quadratic divergencies) is supersymmetry. Moreover, SUSY automatically cancels quadratic corrections in all orders of PT. This is due to the contributions of superpartners of the ordinary particles. The contribution from boson loops cancels those from the fermion ones because of additional factor (-1) coming from Fermi statistics, as shown on Fig. 8. One can see

Figure 8: Cancellation of quadratic terms (divergencies) fig:cancel

here two types of contribution. The first line is the contribution of the heavy Higgs boson and its superpartner. The strength of interaction is given by the Yukawa coupling λ. The second line represents the gauge interaction proportional to the gauge coupling constant g with the contribution from the heavy gauge boson and heavy gaugino.

In both the cases the cancellation of quadratic terms takes place. This cancellation is true in case of unbroken supersymmetry due to the following sum rule relating the masses of superpartners

$$\sum_{bosons} m^2 = \sum_{fermions} m^2 \tag{2.14}$$

and is violated when SUSY is broken. Then the cancellation is true up to the SUSY breaking scale, M_{SUSY}, since

$$\sum_{bosons} m^2 - \sum_{fermions} m^2 = M^2_{SUSY}, \tag{2.15}$$

which should not be very large ($\leq$ 1 TeV) to make the fine-tuning natural. Indeed, let us take the Higgs boson mass. Requiring for consistency of perturbation theory that the radiative corrections to the Higgs boson mass do not exceed the mass itself, gives

$$\delta M_h^2 \sim g^2 M^2_{SUSY} \sim M_h^2. \tag{2.16}$$ del

So, if $M_h \sim 10^2$ GeV and $g \sim 10^{-1}$ one needs $M_{SUSY} \sim 10^3$ GeV in order the relation (2.16) to be valid. Thus, we again get the same rough estimate of $M_{SUSY} \sim 1$ TeV as from the gauge couplings unification above. Two requirements match together.

That is why it is usually said that supersymmetry solves the hierarchy problem. Moreover, sometimes it is said that: "There is no GUT without SUSY". However, this is only the second aspect of the problem, the preservation of the hierarchy. The origin of the hierarchy is the other part of the problem. We show below how SUSY can explain this part as well.

2.4 Beyond GUTs: superstring

Another motivation for supersymmetry follows from even more radical changes of basic ideas related to the ultimate goal of construction of consistent unified theory of everything. At the moment the only viable conception is the superstring theory, which pretends to be a self-consistent quantum field theory in non-perturbative sense allowing exact non-perturbative solutions in quantum case [?]. In superstring theory strings are considered as fundamental objects, closed or open, and are non-local by nature. Ordinary particles are considered as string excitation modes. String's interactions, which are local, generate the proper interactions of usual particles, including gravitational one.

To be consistent the string theory should be conformal invariant in D-dimensional target space and have a stable vacuum. The first requirement is valid in classical theory but may be violated by quantum anomalies. Cancellation of quantum anomalies takes place when space-time dimension of a target space equals to a critical one. For bosonic string the critical dimension is $D = 26$, and for a fermionic one it is $D = 10$.

The second requirement is that the massless sting excitations (the particles of the SM) are stable. This assumes the absence of tachyons, the states with imaginary mass. This can be guaranteed only in supersymmetric string theories!

Thus, the superstring theory proves to be the only known consistent quantum theory. This serves as a justification of research in spite of absence of even a shred of experimental evidence. However, many ingredients of this theory are still unclear.

3 Basics of supersymmetry

Supersymmetry transformations differ from ordinary global transformations as far as they convert bosons into fermions and vice versa. Indeed if we symbolically write SUSY transformation as

$$\delta B = \varepsilon \cdot f,$$

where B and f are boson and fermion fields, respectively, and ε is an infinitesimal transformation parameter, then from the usual (anti)commutation relations for (fermions) bosons

$$\{f, f\} = 0, \qquad [B, B] = 0$$

we immediately find

$$\{\varepsilon, \varepsilon\} = 0.$$

This means that all the generators of SUSY must be *fermionic*, i.e. they must change the spin by a half-odd amount and change the statistics.

3.1 Algebra of SUSY

Combined with the usual Poincaré and internal symmetry algebra the Super-Poincaré Lie algebra contains additional SUSY generators Q^i_α and $\bar{Q}^i_{\dot\alpha}$ super [13]

$$\begin{aligned}
&[P_\mu, P_\nu] = 0,\\
&[P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),\\
&[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),\\
&[B_r, B_s] = iC^t_{rs}B_t,\\
&[B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,\\
&[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,\\
&[Q^i_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta, \qquad [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\tfrac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},\\
&[Q^i_\alpha, B_r] = (b_r)^i_j Q^j_\alpha, \quad [\bar{Q}^i_{\dot\alpha}, B_r] = -\bar{Q}^j_{\dot\alpha}(b_r)^i_j,\\
&\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu,\\
&\{Q^i_\alpha, Q^j_\beta\} = 2\epsilon_{\alpha\beta}Z^{ij}, \quad Z_{ij} = a^r_{ij}b_r, \qquad Z^{ij} = Z^+_{ij},\\
&\{\bar{Q}^i_{\dot\alpha}, \bar{Q}^j_{\dot\beta}\} = -2\epsilon_{\dot\alpha\dot\beta}Z^{ij}, \quad [Z_{ij}, anything] = 0,\\
&\alpha, \dot\alpha = 1, 2 \qquad i, j = 1, 2, \ldots, N.
\end{aligned} \tag{3.1}$$

group

Here P_μ and $M_{\mu\nu}$ are four-momentum and angular momentum operators respectively, B_r are internal symmetry generators, Q^i and $\bar{Q}^i$ are spinorial SUSY generators and Z_{ij} are the so-called central charges. $\alpha, \dot\alpha, \beta, \dot\beta$ are spinorial indices. In the simplest case one has one spinor generator Q_α (and the conjugated one $\bar Q_{\dot\alpha}$) that corresponds to an ordinary or N=1 supersymmetry. When $N>1$ one has an extended sypersymmetry.

A natural question arises: how many SUSY generators are possible, i.e. what is the value of N? To answer this question consider massless states [17]. Let us start with the ground state labeled by energy and helicity, i.e. projection of a spin on the direction of momenta, and let it be annihilated by Q_i

$$\text{Vacuum} = |E,\lambda>, \qquad Q_i|E,\lambda>=0.$$

Then one and more particle states can be constructed with the help of creation operators as

State	Expression	# of States
vacuum	$\lvert E,\lambda>$	1
$1-$ particle state	$\bar Q_i\lvert E,\lambda> = \lvert E,\lambda+1/2>_i$	$\begin{pmatrix} N \\ 1 \end{pmatrix} = N$
$2-$ particle state	$\bar Q_i\bar Q_j\lvert E,\lambda> = \lvert E,\lambda+1>_{ij}$	$\begin{pmatrix} N \\ 2 \end{pmatrix} = \frac{N(N-1)}{2}$
...	...	...
$N-$ particle state	$\bar Q_1\bar Q_2 ... \bar Q_N\lvert E,\lambda> = \lvert E,\lambda+N/2>$	$\begin{pmatrix} N \\ N \end{pmatrix} = 1$

$$\text{Total \# of States} \quad \sum_{k=0}^{N} \begin{pmatrix} N \\ k \end{pmatrix} = 2^N = 2^{N-1} \text{ bosons} + 2^{N-1} \text{ fermions,} \tag{3.2}$$

where the energy E is not changed, since according to (3.1) the operators $\bar Q_i$ commute with the Hamiltonian.

Thus one has a sequence of bosonic and fermionic states and the total number of bosons equals to that of fermions. This is a generic property of any supersymmetric theory. However, in CPT invariant theories the number of states is doubled, since CPT transformation changes the sign of helicity. Hence, in CPT invariant theories, one has to add to the above mentioned states the states with opposite helicity.

Consider some examples. Let us take $N=1$ and $\lambda=0$. Then one has the following set of states

$$N=1 \quad \lambda=0 \qquad \begin{array}{ccc} \text{helicity} & 0 & 1/2 \\ \\ \text{\# of states} & 1 & 1 \end{array} \quad \stackrel{CPT}{\Longrightarrow} \quad \begin{array}{ccc} \text{helicity} & 0 & -1/2 \\ \\ \text{\# of states} & 1 & 1 \end{array} \tag{3.3}$$

Hence, complete $N=1$ multiplet is

$$N=1 \qquad \begin{array}{cccc} \text{helicity} & -1/2 & 0 & 1/2 \\ \text{\# of states} & 1 & 2 & 1 \end{array} \tag{3.4}$$

and contains one complex scalar and one spinor with two helicity states.

This is an example of the so-called self-conjugated multiplet. There are also self-conjugated multiplets with $N>1$ corresponding to extended supersymmetry. Two particular examples are

$N=4$ super Yang-Mills multiplet, and $N=8$ super gravity multiplet

$$
\begin{array}{llccccccccc}
N=4 & \text{SUSY YM} & \text{helicity} & & & -1 & -1/2 & 0 & 1/2 & 1 & & \\
 & \lambda=-1 & \#\text{ of states} & & & 1 & 4 & 6 & 4 & 1 & & \\
 & & & & & & & & & & & \\
N=8 & \text{SUGRA} & \text{helicity} & -2 & -3/2 & -1 & -1/2 & 0 & 1/2 & 1 & 3/2 & 2 \\
 & \lambda=-2 & \#\text{ of states} & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1
\end{array}
\tag{3.5}
$$

One can see that the multiplets of extended supersymmetry are very reach and contain a vast number of particles.

The constraint on the number of SUSY generators comes from a requirement of consistency of the corresponding QFT. The number of supersymmetries and the maximal spin of the particle in the multiplet are related by

$$N \leq 4S,$$

where S is the maximal spin. Since the theories with spin greater than 1 are non-renormalizable and the theories with spin greater than 5/2 have no consistent coupling to gravity, this imposes a constraint on the number of SUSY generators

$$
\begin{array}{ll}
N \leq 4 & \text{for renormalizable theories (YM),} \\
N \leq 8 & \text{for (super)gravity.}
\end{array}
\tag{3.6}
$$

In what follows we shall consider simple supersymmetry, or $N=1$ supersymmetry, contrary to extended supersymmetries with $N>1$. In this case we have two types of supermultiplets, the so-called chiral multiplet with $\lambda=0$, which contains two physical states (ϕ,ψ) with spin 0 and 1/2, respectively, and vector multiplet with $\lambda=1/2$, which also contains two physical states (λ, A_μ) with spin 1/2 and 1.

3.2 Superspace and superfields

An elegant formulation of supersymmetry transformations and invariants can be achieved in the framework of superspace sspace [16]. Superspace differs from the ordinary Euclidean (Minkowski) space by addition of two new coordinates, θ_α and $\bar\theta_{\dot\alpha}$, which are grassmannian, i.e. anticommuting, variables

$$\{\theta_\alpha,\theta_\beta\}=0, \quad \{\bar\theta_{\dot\alpha},\bar\theta_{\dot\beta}\}=0, \quad \theta_\alpha^2=0, \quad \bar\theta_{\dot\alpha}^2=0, \quad \alpha,\beta,\dot\alpha,\dot\beta=1,2.$$

Thus, we go from space to superspace

$$
\begin{array}{ccc}
Space & \Rightarrow & Superspace \\
x_\mu & & x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha}
\end{array}
\tag{3.7}
$$

A SUSY group element can be constructed in superspace in the same way as an ordinary translation in the usual space

$$G(x,\theta,\bar\theta)=e^{i\left(-x^\mu P_\mu+\theta Q+\bar\theta\bar Q\right)}. \tag{3.8}$$ st

It leads to a supertranslation in superspace

$$
\begin{array}{rcl}
x_\mu & \rightarrow & x_\mu + i\theta\sigma_\mu\bar\varepsilon - i\varepsilon\sigma_\mu\bar\theta, \\
\theta & \rightarrow & \theta+\varepsilon, \\
\bar\theta & \rightarrow & \bar\theta+\bar\varepsilon, \boxed{\texttt{sutr}}
\end{array}
\tag{3.9}
$$

where ε and $\bar{\varepsilon}$ are grassmannian transformation parameters. From (3.9) one can easily obtain the representation for the supercharges (3.1) acting on the superspace:

$$Q_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu. \tag{3.10}$$

Taking the grassmannian transformation parameters to be local or space-time dependent one gets local translation. As was already mentioned this leads to a theory of (super) gravity.

To define the fields on a superspace consider representations of the Super-Poincaré group (3.1) [17]. The simplest one is a scalar superfield $F(x,\theta,\bar{\theta})$ which is SUSY invariant. Its Taylor expansion in θ and $\bar{\theta}$ has only several terms due to the nilpotent character of grassmannian parameters. However, this superfield is a reducible representation of SUSY. To get an irreducible one, we define a *chiral* superfield which obeys the equation

$$\bar{D}F = 0, \qquad \text{where } \bar{D} = -\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^\mu\partial_\mu \tag{3.11}$$

is a superspace covariant derivative.

For the chiral superfield grassmannian Taylor expansion looks like ($y = x + i\theta\sigma\bar{\theta}$)

$$\begin{aligned} \Phi(y,\theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ &= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x) \\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x). \end{aligned} \tag{3.12}$$

The coefficients are ordinary functions of x being the usual fields. They are called the *components* of a superfield. In eq.(3.12) one has 2 bosonic (complex scalar field A) and 2 fermionic (Weyl spinor field ψ) degrees of freedom. The component fields A and ψ are called the *superpartners*. The field F is an *auxiliary* field, it has the "wrong" dimension and has no physical meaning. It is needed to close the algebra (3.1). One can get rid of the auxiliary fields with the help of equations of motion.

Thus, a superfield contains an equal number of bosonic and fermionic degrees of freedom. Under SUSY transformation they convert one into the other

$$\begin{aligned} \delta_\varepsilon A &= \sqrt{2}\varepsilon\psi, \\ \delta_\varepsilon\psi &= i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F, \\ \delta_\varepsilon F &= i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi. \end{aligned} \tag{3.13}$$

Notice that the variation of the F-component is a total derivative, i.e. it vanishes when integrated over the space-time.

One can also construct an antichiral superfield Φ^+ obeying the equation

$$D\Phi^+ = 0, \quad \text{with } D = \frac{\partial}{\partial\theta} + i\sigma^\mu\bar{\theta}\partial_\mu.$$

The product of chiral (antichiral) superfields Φ^2, Φ^3, etc is also a chiral (antichiral) superfield, while the product of chiral and antichiral ones $\Phi^+\Phi$ is a general superfield.

For any arbitrary function of chiral superfields one has:

$$\begin{aligned} \mathcal{W}(\Phi_i) &= \mathcal{W}(A_i + \sqrt{2}\theta\psi_i + \theta\theta F) \\ &= \mathcal{W}(A_i) + \frac{\partial\mathcal{W}}{\partial A_i}\sqrt{2}\theta\psi_i + \theta\theta\left(\frac{\partial\mathcal{W}}{\partial A_i}F_i - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j\right). \end{aligned} \tag{3.14}$$

$\mathcal{W}$ is usually referred to as a superpotential, which replaces the usual potential for the scalar fields.

To construct the gauge invariant interactions, one needs a real vector superfield $V = V^+$. It is not chiral but rather a general superfield with the following grassmannian expansion

$$\begin{aligned} V(x,\theta,\bar\theta) &= C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) \\ &+ \frac{i}{2}\theta\theta[M(x)+iN(x)] - \frac{i}{2}\bar\theta\bar\theta[M(x)-iN(x)] \\ &- \theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta[\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)] \\ &- i\bar\theta\bar\theta\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)] + \frac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) + \frac{1}{2}\Box C(x)]. \end{aligned} \tag{3.15}$$

The physical degrees of freedom corresponding to a real vector superfield V are the vector gauge field v_μ and majorana spinor field λ. All other components are unphysical and can be eliminated. Indeed, under the abelian (super)gauge transformation the superfield V is transformed as

$$V \quad \to \quad V + \Phi + \Phi^+,$$

where Φ and Φ^+ are some chiral superfields. In components it looks like

$$\begin{aligned} C &\quad\to\quad C + A + A^*, \\ \chi &\quad\to\quad \chi - i\sqrt{2}\psi, \\ M + iN &\quad\to\quad M + iN - 2iF, \\ v_\mu &\quad\to\quad v_\mu - i\partial_\mu(A - A^*), \\ \lambda &\quad\to\quad \lambda, \\ D &\quad\to\quad D \end{aligned} \tag{3.16}$$

and corresponds to ordinary gauge transformations for physical components. According to eq.(3.16) one can choose a gauge (the Wess-Zumino gauge) where $C = \chi = M = N = 0$, leaving us with only physical degrees of freedom except for the auxiliary field D. In this gauge

$$\begin{aligned} V &= -\theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x), \\ V^2 &= -\frac{1}{2}\theta\theta\bar\theta\bar\theta v_\mu(x)v^\mu(x), \\ V^3 &= 0, \quad etc. \end{aligned} \tag{3.17}$$

One can define also a field strength tensor (as analog of $F_{\mu\nu}$ in gauge theories)

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar D^2 e^V D_\alpha e^{-V}, \\ \bar W_{\dot\alpha} &= -\frac{1}{4} D^2 e^V \bar D_\alpha e^{-V}, \end{aligned} \tag{3.18}$$

which is a polynomial in the Wess-Zumino gauge. (Here Ds are the supercovariant derivatives.)

The strength tensor is a chiral superfield

$$\bar D_{\dot\beta} W_\alpha = 0, \qquad D_\beta \bar W_{\dot\alpha} = 0.$$

In the Wess-Zumino gauge it is a polynomial over component fields:

$$W_\alpha = T^a\left(-i\lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F^a_{\mu\nu} + \theta^2\sigma^\mu D_\mu\bar\lambda^a\right), \tag{3.19}$$

where

$$F^a_{\mu\nu} = \partial_\mu v^a_\nu - \partial_\nu v^a_\mu + f^{abc} v^b_\mu v^c_\nu, \quad D_\mu \bar{\lambda}^a = \partial \bar{\lambda}^a + f^{abc} v^b_\mu \bar{\lambda}^c.$$

In abelian case eqs.(3.18) are simplified and take form

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4} D^2 \bar{D}_\alpha V.$$

3.3 Construction of SUSY Lagrangians

Let us start with the Lagrangian which has no local gauge invariance. In the superfield notation SUSY invariant Lagrangians are the polynomials of superfields. Having in mind that for component fields we should have the ordinary terms, and the above mentioned property of SUSY invariance of the highest dimension components of a superfield, the general SUSY invariant Lagrangian has the form

$$\mathcal{L} = \Phi^+_i \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} + [(\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k)|_{\theta\theta} + h.c.]. \tag{3.20}$$

Hereafter the vertical line means the corresponding term of a Taylor expansion.

The first term is a kinetic term. It contains both the chiral and antichiral superfields Φ_i and Φ^+_i, respectively, and is a function of grassmannian parameters θ and $\bar{\theta}$. Being expanded over θ and $\bar{\theta}$ it leads to the usual kinetic terms for the corresponding component fields.

The terms in the bracket form the superpotential. It is composed of the chiral fields only (plus the hermitian conjugated counterpart composed of antichiral superfields) and is a chiral superfield. Since the products of a chiral superfield and antichiral one produce a general superfield they are not allowed in a superpotential. The last coefficient of its expansion over parameter θ is supersymmetrically invariant and gives the usual potential after getting rid of the auxiliary fields, as it will be clear later.

The Lagrangian (3.20) can be written in much more elegant way in superspace. The same way as an ordinary action is an integral over space-time of Lagrangian density, in supersymmetric case the action is an integral over the superspace. The space-time Lagrangian density then is[17, 15, 16]

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\ \Phi^+_i \Phi_i + \int d^2\theta\ [\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k] + h.c. \tag{3.21}$$

where the first part is a kinetic term and the second one is a superpotential $\mathcal{W}$. Here instead of taking the proper components we use an integration over the superspace according to the rules of grassmannian integration [18]

$$\int d\theta_\alpha = 0, \qquad \int \theta_\alpha\ d\theta_\beta = \delta_{\alpha\beta}.$$

Performing explicit integration over the grassmannian parameters we get from eq.(3.21)

$$\begin{aligned} \mathcal{L} \;&=\; i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A^*_i \Box A_i + F^*_i F_i \qquad (3.22) \\ &+\; [\lambda_i F_i + m_{ij}(A_i F_j - \frac{1}{2}\psi_i \psi_j) + y_{ijk}(A_i A_j F_k - \psi_i \psi_j A_k) + h.c.]. \end{aligned}$$

The last two terms are the interaction ones. To obtain a familiar form of the Lagrangian, we have to solve the constraints:

$$\frac{\partial \mathcal{L}}{\partial F^*_k} = F_k + \lambda^*_k + m^*_{ik} A^*_i + y^*_{ijk} A^*_i A^*_j = 0, \tag{3.23}$$

$$\frac{\partial \mathcal{L}}{\partial F_k} = F^*_k + \lambda_k + m_{ik} A_i + y_{ijk} A_i A_j = 0. \tag{3.24}$$

Expressing the auxiliary fields F and F^* from these equations we finally get

$$\begin{aligned} \mathcal{L} &= i\partial_\mu\bar{\psi}_i\bar{\sigma}^\mu\psi_i + A_i^*\Box A_i - \frac{1}{2}m_{ij}\psi_i\psi_j - \frac{1}{2}m_{ij}^*\bar{\psi}_i\bar{\psi}_j \\ &\quad -y_{ijk}\psi_i\psi_j A_k - y_{ijk}^*\bar{\psi}_i\bar{\psi}_j A_k^* - V(A_i, A_j), \end{aligned} \tag{3.25}$$ m

where the scalar potential $V = F_k^* F_k$. We will return to the discussion of the form of the scalar potential in SUSY theories later.

Consider now the gauge invariant SUSY Lagrangians. They should contain gauge invariant interaction of the matter fields with the gauge ones and the kinetic term and the self interaction of the gauge fields.

Let's start with the gauge fields kinetic terms. In the Wess-Zumino gauge one has

$$W^\alpha W_\alpha|_{\theta\theta} = -2i\lambda\sigma^\mu D_\mu\bar{\lambda} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D^2 + i\frac{1}{4}F^{\mu\nu}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma}, \tag{3.26}$$

where $D_\mu = \partial_\mu + ig[v_\mu,]$ is the usual covariant derivative and the last, the so-called topological θ term,[2] is the total derivative.

The gauge invariant Lagrangian now has familiar form

$$\begin{aligned} \mathcal{L} &= \frac{1}{4}\int d^2\theta\ W^\alpha W_\alpha + \frac{1}{4}\int d^2\bar{\theta}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \\ &= \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\bar{\lambda}. \end{aligned} \tag{3.27}$$ 29

To obtain a gauge-invariant interaction with matter chiral superfields, consider their gauge transformation (abelian)

$$\Phi \quad\rightarrow\quad e^{-ig\Lambda}\Phi, \quad \Phi^+ \quad\rightarrow\quad \Phi^+ e^{ig\Lambda^+}, \quad V \quad\rightarrow\quad V + i(\Lambda - \Lambda^+),$$

where Λ is a gauge parameter (chiral superfield).

It is clear now how to construct both the SUSY and gauge invariant kinetic term (compare with the covariant derivative in a usual gauge theory):

$$\Phi_i^+\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \Rightarrow \Phi_i^+ e^{gV}\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \tag{3.28}$$

A complete SUSY and gauge invariant Lagrangian then looks like:

$$\begin{aligned} \mathcal{L}_{inv} &= \frac{1}{4}\int d^2\theta\ W^\alpha W_\alpha + \frac{1}{4}\int d^2\bar{\theta}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} + \int d^2\theta d^2\bar{\theta}\ \Phi_i^+ e^{gV}\Phi_i \\ &+ \int d^2\theta\ (\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k) + h.c. \end{aligned} \tag{3.29}$$ 30

In particular the SUSY generalization of QED looks as follows

$$\begin{aligned} \mathcal{L}_{SUSY\ QED} &= \frac{1}{4}\int d^2\theta\ W^\alpha W_\alpha + \frac{1}{4}\int d^2\bar{\theta}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \\ &+ \int d^4\theta\ (\Phi_+^+ e^{gV}\Phi_+ + \Phi_-^+ e^{-gV}\Phi_-) \\ &+ \int d^2\theta\ m\ \Phi_+\Phi_- + \int d^2\bar{\theta}\ m\ \Phi_+^+\Phi_-^+, \end{aligned} \tag{3.30}$$

[2]Terminology comes from the θ term of QCD and has nothing to do with the grassmannian parameter θ

where two superfields Φ_+ and Φ_- have been introduced in order to have both left and right handed fermions.

The non-abelian generalization is straightforward

$$\begin{aligned}\mathcal{L}_{SUSY\ YM} &= \frac{1}{4}\int d^2\theta\ Tr(W^\alpha W_\alpha) + \frac{1}{4}\int d^2\bar{\theta}\ Tr(\bar{W}^\alpha \bar{W}_\alpha) \\ &+ \int d^2\theta d^2\bar{\theta}\ \bar{\Phi}_{ia}(e^{gV})^a_b \Phi^b_i + \int d^2\theta\ \mathcal{W}(\Phi_i) + \int d^2\bar{\theta}\ \bar{\mathcal{W}}(\bar{\Phi}_i),\end{aligned} \tag{3.31} \quad \boxed{\texttt{nonab}}$$

where $\mathcal{W}$ is a superpotential, which should be invariant under the group of symmetry of a particular model.

In terms of component fields the above Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{SUSY\ YM} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^a\sigma^\mu D_\mu\bar{\lambda}^a + \frac{1}{2}D^a D^a \\ &+ (\partial_\mu A_i - igv^a_\mu T^a A_i)^\dagger(\partial_\mu A_i - igv^{a\mu}T^a A_i) - i\bar{\psi}_i\bar{\sigma}^\mu(\partial_\mu\psi_i - igv^{a\mu}T^a\psi_i) \\ &- D^a A_i^\dagger T^a A_i - i\sqrt{2}A_i^\dagger T^a\lambda^a\psi_i + i\sqrt{2}\bar{\psi}_i T^a A_i\bar{\lambda}^a + F_i^\dagger F_i \\ &+ \frac{\partial\mathcal{W}}{\partial A_i}F_i + \frac{\partial\bar{\mathcal{W}}}{\partial A_i^\dagger}F_i^\dagger - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i \partial A_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\bar{\mathcal{W}}}{\partial A_i^\dagger\partial A_j^\dagger}\bar{\psi}_i\bar{\psi}_j.\end{aligned} \tag{3.32} \quad \boxed{\texttt{sulag}}$$

Integrating out the auxiliary fields D^a and F_i, one reproduces the usual Lagrangian.

3.4 The scalar potential

Contrary to the SM, where the scalar potential is arbitrary and is defined only by the requirement of the gauge invariance, in supersymmetric theories it is completely defined by the superpotential. It consists of the contributions from D-terms and F-terms. The kinetic energy of the gauge fields (recall eq.(3.27) yields $1/2D^aD^a$ term, and the matter-gauge interaction (recall eq.(3.29) yields $gD^aT^a_{ij}A_i^*A_j$. Together they give

$$\mathcal{L}_D = \frac{1}{2}D^aD^a + gD^aT^a_{ij}A^*_iA_j. \tag{3.33} \quad \boxed{\texttt{d}}$$

The equation of motion reads

$$D^a = -gT^a_{ij}A^*_iA_j. \tag{3.34} \quad \boxed{\texttt{sol}}$$

Substituting it back to eq.(3.33) yields the D-term part of the potential

$$\mathcal{L}_D = -\frac{1}{2}D^aD^a \quad \Longrightarrow V_D = \frac{1}{2}D^aD^a, \tag{3.35}$$

where D is given by eq.(3.34).

The F-term contribution can be derived from the matter fields self-interaction eq.(3.22). For a general type superpotential W one has

$$\mathcal{L}_F = F^*_iF_i + (\frac{\partial W}{\partial A_i}F_i + h.c.). \tag{3.36}$$

Using equations of motion for the auxiliary field F_i

$$F^*_i = -\frac{\partial W}{\partial A_i} \tag{3.37} \quad \boxed{\texttt{solf}}$$

yields

$$\mathcal{L}_F = -F_i^* F_i \qquad \Longrightarrow V_F = F_i^* F_i, \tag{3.38}$$

where F is given by eq.(3.37). The full potential is the sum of the two contributions:

$$V = V_D + V_F. \tag{3.39}$$

Thus, the form of the Lagrangian is practically fixed by symmetry requirements. The only freedom is the field content, the value of the gauge coupling g , Yukawa couplings y_{ijk} and the masses. Because of the renormalizability constraint $V \leq A^4$ the superpotential should be limited by $\mathcal{W} \leq \Phi^3$ as in eq.(3.21). All members of a supermultiplet have the same masses, i.e. bosons and fermions are degenerate in masses. This property of SUSY theories contradicts the phenomenology and requires supersymmetry breaking.

3.5 Spontaneous breaking of SUSY

Since supersymmetric algebra leads to mass degeneracy in a supermultiplet it should be broken to explain the absence of superpartners at modern energies. There are several ways of supersymmetry breaking. It can be broken either explicitly or spontaneously. Performing SUSY breaking one has to be careful not to spoil the cancellation of quadratic divergencies which allows to solve the hierarchy problem. This is achieved by spontaneous breaking of SUSY.

Apart from non-supersymmetric theories in SUSY models the energy is always nonnegative definite. Indeed, according to quantum mechanics

$$E =< 0|\ H\ |0>$$

and due to SUSY algebra eq.(3.1)

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu,$$

taking into account that $tr(\sigma^\mu P_\mu) = 2P_0$, we get

$$E = \frac{1}{4}\sum_{\alpha=1,2} < 0|\{Q_\alpha, \bar{Q}_\alpha\}|0 >= \frac{1}{4}\sum_\alpha |Q_\alpha|0>|^2 \geq 0.$$

Hence

$$E =< 0|\ H\ |0 >\neq 0 \quad if\ and\ only\ if \quad Q_\alpha|0>\neq 0.$$

Therefore a supersymmetry is spontaneously broken, i.e. vacuum is not invariant ($Q_\alpha|0>\neq 0$) , *if and only if* the minimum of the potential is positive ($i.e. E > 0$) .

The situation is illustrated in Fig.9. The SUSY ground state has $E = 0$, while a non-SUSY one has $E > 0$. On the right hand side a non-SUSY potential is shown. It does not appear even in spontaneously broken SUSY theories. However, just this type of the potential is used for spontaneous breaking of the gauge invariance via the Higgs mechanism. This property has crucial consequences for the spontaneous breaking of the gauge invariance. Indeed, as will be seen later, in the MSSM spontaneous breaking of $SU(2)$ invariance takes place only after SUSY is broken.

Figure 9: Scalar potential in supersymmetric and non-supersymmetric theories poten

Spontaneous breaking of supersymmetry is achieved in the same way we break electroweak symmetry. One introduces the field whose vacuum expectation value is non-zero and breaks the symmetry. However, due to a special character of SUSY, this should be a superfield whose auxiliary F and D components acquire non-zero v.e.v.'s. Thus, among possible spontaneous SUSY breaking mechanisms one distinguishes F and D ones.

i) Fayet-Iliopoulos (D-term) mechanism [?].

In this case the linear D-term is added to the Lagrangian

$$\Delta\mathcal{L} = \xi V|_{\theta\theta\bar{\theta}\bar{\theta}} = \int d^4\theta \; V. \tag{3.40}$$

It is gauge and SUSY invariant by itself, however may lead to spontaneous breaking of both of them depending on the value of ξ. We show in Fig.10a the sample spectrum for two chiral matter multiplets. The drawback of this mechanism is the necessity of $U(1)$ gauge invariance.

Figure 10: Spectrum of spontaneously broken SUSY theories FI

It can be used in SUSY generalizations of the SM but not in GUTs.

The mass spectrum also causes some troubles since the following sum rule is always valid

$$\sum_{boson\ states} m_i^2 = \sum_{fermion\ states} m_i^2, \tag{3.41}$$ sumrule

which is bad for a phenomenology.

ii) O'Raifeartaigh (F-term) mechanism [?].

In this case several chiral fields are needed and the superpotential should be chosen in a way that trivial zero v.e.v.s for the auxiliary F-fields are absent. For instance, choosing the superpotential to be

$$\mathcal{W}(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2,$$

one gets the equations for the auxiliary fields

$$\begin{aligned} F_1^* &= mA_2 + 2gA_1A_3, \\ F_2^* &= mA_1, \\ F_3^* &= \lambda + gA_1^2, \end{aligned}$$

which have no solutions with $< F_i >= 0$ and SUSY is spontaneously broken. The sample spectrum is shown in Fig.10b.

The drawbacks of this mechanism is a lot of arbitrariness in the choice of potential. The sum rule (3.41) is also valid here.

Unfortunately none of these mechanisms explicitly works in SUSY generalizations of the SM. None of the fields of the SM can develop non-zero v.e.v.s for their F or D components without breaking of $SU(3)$ or $U(1)$ gauge invariance, since they are not singlets with respect to these groups. This requires the presence of extra sources of spontaneous SUSY breaking, which we consider below. They are based, however, on the same F and D mechanisms.

4 SUSY generalization of the Standard Model. The MSSM

As has been already mentioned, in SUSY theories the number of bosonic degrees of freedom equals that of fermionic. At the same time, in the SM one has 28 bosonic and 90 fermionic degrees of freedom (with massless neutrino, otherwise 96). So the SM is in great deal non-supersymmetric. Trying to add some new particles to supersymmetrize the SM, one should take into account the following observations:

Figure 11: The shadow world of SUSY particles [?]

1. There are no fermions with quantum numbers of the gauge bosons;
2. Higgs fields have a non-zero v.e.v.s, hence they cannot be superpartners of quarks and leptons since this would induce a spontaneous violation of baryon and lepton numbers;
3. One needs at least two complex chiral Higgs multiplets to give masses to Up and Down quarks.

The latter is due to the form of a superpotential and chirality of matter superfields. Indeed, the superpotential should be invariant under $SU(3)\times SU(2)\times U(1)$ gauge group. If one looks at the Yukawa interaction in the Standard Model, eq.(1.7), one finds that it is indeed $U(1)$ invariant since the sum of hypercharges in each vertex equals zero. In the last term this is achieved by taking the conjugated Higgs doublet $\tilde{H}=i\tau_2 H^\dagger$ instead of H. However, in SUSY H is a chiral superfield and hence a superpotential, which is constructed out of chiral fields, can contain only H but not $\tilde{H}$, which is an antichiral superfield.

Another reason for the second Higgs doublet is related to chiral anomalies. It is known that chiral anomalies spoil the gauge invariance and, hence, the renormalizability of the theory. They are canceled in the SM between quarks and leptons in each generation.

Indeed, chiral (or triangle anomaly) is proportional to the trace of three hypercharges. In the SM one has

$$\begin{array}{cccccccccccc} TrY^3 & = & 3 & \Big(\frac{1}{27} & +\frac{1}{27} & -\frac{64}{27} & +\frac{8}{27}\Big) & -1 & -1 & +8 & =0. \\ & & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ & & colour & u_L & d_L & u_R & d_R & \nu_L & e_L & e_R & \end{array} \tag{4.1}$$

However, if one introduces a chiral Higgs superfield, it contains higgsinos, which are chiral fermions, and contain anomalies. To cancel them one has to add the second Higgs doublet with the opposite hypercharge.

Therefore the Higgs sector in SUSY models is inevitably enlarged, it contains an even number of doublets.

Conclusion: In SUSY models supersymmetry associates *known* bosons with *new* fermions and *known* fermions with *new* bosons.

4.1 The field content

Consider the particle content of the Minimal Supersymmetric Standard Model [19]. According to the previous discussion in the minimal version we double the number of particles (introducing a superpartner to each particle) and add another Higgs doublet (with its superpartner). The particle content of the MSSM then looks as follows [42]:

Particle Content of the MSSM

Superfield	Bosons		Fermions		$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
Gauge							
$\mathbf{G^a}$	gluon	g^a	gluino	$\tilde{g}^a$	8	0	0
$\mathbf{V^k}$	Weak	W^k $(W^\pm, Z)$	wino, zino	$\tilde{w}^k$ $(\tilde{w}^\pm, \tilde{z})$	1	3	0
$\mathbf{V'}$	Hypercharge	B (γ)	bino	$\tilde{b}(\tilde{\gamma})$	1	1	0
Matter							
$\mathbf{L_i}$	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	1	2	-1
$\mathbf{E_i}$		$\tilde{E}_i = \tilde{e}_R$		$E_i = e_R$	1	1	2
$\mathbf{Q_i}$	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	3	2	$1/3$
$\mathbf{U_i}$		$\tilde{U}_i = \tilde{u}_R$		$U_i = u_R^c$	3^*	1	$-4/3$
$\mathbf{D_i}$		$\tilde{D}_i = \tilde{d}_R$		$D_i = d_R^c$	3^*	1	$2/3$
Higgs							
$\mathbf{H_1}$	Higgses	H_1	higgsinos	$\tilde{H}_1$	1	2	-1
$\mathbf{H_2}$		H_2		$\tilde{H}_2$	1	2	1

where $a = 1, 2, ..., 8$ and $k = 1, 2, 3$ are $SU(3)$ and $SU(2)$ indices, respectively, and $i = 1, 2, 3$ is the generation index. Hereafter tilde denotes a superpartner of an ordinary particle.

Thus, the characteristic feature of any supersymmetric generalization of the SM is the presence of superpartners (see Fig. fig:shadow 4). If supersymmetry is exact, superpartners of ordinary particles should have the same masses and have to be observed. The absence of them at modern energies is believed to be explained by the fact that their masses are very heavy, that means that supersymmetry should be broken. Hence, if the energy of accelerators is high enough, the superpartners will be created.

The presence of an extra Higgs doublet in SUSY model is a novel feature of the theory. In the MSSM one has two doublets with the quantum numbers (1,2,-1) and (1,2,1), respectively:

$$H_1 = \left(\begin{array}{c} H_1^0 \\ H_1^- \end{array} \right) = \left(\begin{array}{c} v_1 + \dfrac{S_1 + iP_1}{\sqrt{2}} \\ H_1^- \end{array} \right), \quad H_2 = \left(\begin{array}{c} H_2^+ \\ H_2^0 \end{array} \right) = \left(\begin{array}{c} H_2^+ \\ v_2 + \dfrac{S_2 + iP_2}{\sqrt{2}} \end{array} \right), \tag{4.2}$$

where v_i are the vacuum expectation values of the neutral components.

Hence, one has 8=4+4=5+3 degrees of freedom. As in the case of the SM, 3 degrees of freedom can be gauged away, and one is left with 5 physical states compared to 1 state in the SM.

Thus, in the MSSM, as actually in any two Higgs doublet model, one has five physical Higgs bosons: two CP-even neutral, one CP-odd neutral and two charged. We consider the mass eigenstates below.

4.2 Lagrangian of the MSSM

The Lagrangian of the MSSM consists of two parts; the first part is SUSY generalization of the Standard Model, while the second one represents the SUSY breaking as mentioned above.

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{Breaking}, \tag{4.3}$$

where

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Gauge} + \mathcal{L}_{Yukawa} \tag{4.4}$$

and

$$\mathcal{L}_{Gauge} = \sum_{SU(3),SU(2),U(1)} \frac{1}{4}\left(\int d^2\theta\ TrW^{\alpha}W_{\alpha} + \int d^2\bar\theta\ Tr\bar W^{\dot\alpha}\bar W_{\dot\alpha}\right)$$
$$+ \sum_{Matter} \int d^2\theta d^2\bar\theta\ \Phi_i^{\dagger} e^{g_3\hat V_3 + g_2\hat V_2 + g_1\hat V_1}\Phi_i, \qquad (4.5)$$
$$\mathcal{L}_{Yukawa} = \int d^2\theta\ (\mathcal{W}_R + \mathcal{W}_{NR}) + h.c. \qquad (4.6)$$

The index R in a superpotential refers to the so-called R-parity which adjusts a "+" charge to all the ordinary particles and a "−" charge to their superpartners [45]. The first part of $\mathcal{W}$ is R-symmetric

$$W_R = \epsilon_{ij}(y^U_{ab}Q^j_aU^c_bH^i_2 + y^D_{ab}Q^j_aD^c_bH^i_1 + y^L_{ab}L^j_aE^c_bH^i_1 + \mu H^i_1H^j_2), \qquad (4.7)$$

where $i,j = 1,2,3$ are $SU(2)$ and $a,b = 1,2,3$ are the generation indices; colour indices are suppressed. This part of the Lagrangian almost exactly repeats that of the SM except that the fields are now the superfields rather than the ordinary fields of the SM. The only difference is the last term which describes the Higgs mixing. It is absent in the SM since we have only one Higgs field there.

The second part is R-nonsymmetric

$$W_{NR} = \epsilon_{ij}(\lambda^L_{abd}L^i_aL^j_bE^c_d + \lambda^{L\prime}_{abd}L^i_aQ^j_bD^c_d + \mu'_aL^i_aH^j_2)$$
$$+ \lambda^B_{abd}U^c_aD^c_bD^c_d. \qquad (4.8)$$

These terms are absent in the SM. The reason is very simple: one can not replace the superfields in eq.(4.8) by the ordinary fields like in eq.(4.7) because of the Lorentz invariance. These terms have the other property, they violate either lepton (the first line in eq.(4.8)) or baryon number (the second line). Since both effects are not observed in Nature, these terms must be suppressed or be excluded. One can avoid such terms if one introduces the special symmetry called R-symmetry. This is the global $U(1)_R$ invariance:

$$U(1)_R: \quad \theta \to e^{i\alpha}\theta, \Phi \to e^{in\alpha}\Phi, \qquad (4.9)$$

i.e. the superfield has $R = n$. To preserve $U(1)_R$ invariance the superpotential W must have $R = 2$. Thus, to get $W_{NR} = 0$ one must choose $R = 1$ for all the Higgs superfields and $R = 1/2$ for quark and lepton ones. However, this property happens to be too restrictive. Indeed, the gaugino mass term, which is Lorentz and gauge invariant and is introduced while supersymmetry breaking, happen to be R-invariant only for $\alpha = \pm\pi$. This reduces the R-symmetry to the discrete group Z_2, called R-parity. The R-parity quantum number is given by

$$R = (-1)^{3(B-L)+2S} \qquad (4.10)$$

for particles with spin S. Thus, all the ordinary particles have R-parity quantum number equal to $R = +1$, while all the superpartners have R-parity quantum number equal to $R = -1$. R-parity obviously forbids the W_{NR} terms. It is usually assumed that they are absent in the MSSM, i.e. R-parity is preserved. However, there is no physical principle behind it. It may well be that these terms are present, though experimental limits on the couplings are very severe:

$$\lambda^L_{abc}, \quad \lambda^{L\prime}_{abc} < 10^{-4}, \qquad \lambda^B_{abc} < 10^{-9}.$$

4.3 Properties of interactions

If one assumes that the R-parity is preserved, then the interactions of superpartners are essentially the same as in the SM, but two of three particles involved into an interaction at any vertex are replaced by superpartners. The reason for it, as we discussed earlier, is R-parity. According to eq.(4.10) all the ordinary particles are R-even, while all the superpartners are R-odd.

A conservation of R-parity has two consequences:

- the superpartners are created in pairs;
- the lightest superparticle (LSP) is stable.

Usually it is photino $\tilde{\gamma}$, the superpartner of a photon with some admixture of neutral higgsino.

A typical vertices are shown in Figs.12-14. Tilde above a letter denotes a corresponding superpartner. Note that the coupling is the same in all the vertices involving superpartners.

In case of R-parity violation one has additional vertices with new types of interaction. As has been already mentioned they violate either lepton or baryon number. The typical ones are

$$\mathcal{L}_{LLE} = \lambda' \{\tilde{\nu}_L e_L e^c_R - \tilde{e}_L \nu_L e^c_R + \tilde{e}^*_R \nu_L e_R + \ldots\}, \tag{4.11}$$

$$\mathcal{L}_{LQD} = \lambda \Big\{\tilde{\nu}_L d_L \bar{d}_R - \tilde{e}_L u_L \bar{d}_R + \tilde{d}_L \nu_L \bar{d}_R - \tilde{u}_L e_L \bar{d}_R + \ldots\Big\}. \tag{4.12}$$

Figure 12: Gauge-Matter Interaction

Figure 13: Gauge Self-Interaction

Figure 15: Proton decay in R-parity violating models

Figure 14: Yukawa-Type Interaction

There are also UDD terms which violate baryon number. These term together lead to a fast proton decay via the process shown in Fig.15. To avoid it one usually leaves either L or B violating interactions.

The limits on R-parity violating couplings come from non-observation of various processes, like proton decay, $\nu_\mu e$ scattering, etc and also from the charged current universality: $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu), \Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$, etc.

4.4 Creation and decay of superpartners

The above-mentioned rule together with the Feynman rules for the SM enables us to draw diagrams describing creation of superpartners. One of the most promising processes is e^+e^- annihilation (see Fig.16). The usual kinematic restriction is given by the central of mass energy

$$m^{max}_{sparticle} \leq \frac{\sqrt{s}}{2}.$$

Similar processes take place at hadron colliders with electrons and positrons being replaced by quarks and gluons.

Creation of superpartners can be accompanied by creation of the ordinary particles as well. We consider various signatures for e^+e^- and hadron colliders below. They crucially depend on SUSY breaking pattern and on the mass spectrum of superpartners.

The decay properties of superpartners also depend on their masses. For the quark and lepton superpartners the main processes are shown in Fig.17.

Since R-parity is conserved, new particles will eventually end up giving neutralinos (the lightest superparticle) whose interactions are comparable to those of neutrinos and they leave undetected. Therefore, their signature would be missing energy and transverse momentum.

Figure 16: Creation of Superpartners creation

Figure 17: Decay of Superpartners decay

Examples. Consider some explicit examples of superpartners decays.

$$
\begin{array}{rclll}
\text{squarks}: \ \tilde{q}_{L,R} & \to & q+\tilde{\chi}^0_i & \text{(quark + photino)} & \\
\tilde{q}_L & \to & q'+\tilde{\chi}^\pm_i & \text{(quark + chargino)} & \\
\tilde{q} & \to & q+\tilde{g} & \text{(quark + gluino)} \ for \ m_{\tilde{q}} > m_{\tilde{g}} & \\
\tilde{t}_1 & \to & c+\tilde{\chi}^0_1 & \text{(main decay)} & \text{signal: 2 acollinear jets} + \not{E}_T \\
\tilde{t}_1 & \to & b+\tilde{\chi}^+_1 & & \text{signal: 2 b jets + 2 leptons} + \not{E}_T \\
 & & \hookrightarrow \tilde{\chi}^0_1 f\bar{f}' & (f\bar{f}' = l\bar{\nu}, q\bar{q}) & \text{(4 jets)} + \not{E}_T
\end{array}
$$

$$
\begin{array}{rlcll}
\text{sleptons}: & \tilde{l} & \to & l+\tilde{\chi}^0_i & \text{(lepton + photino)} \\
 & \tilde{l}_L & \to & \nu_l+\tilde{\chi}^\pm_i & \text{(neutrino + chargino)} \\
\text{gluino}: & \tilde{g} & \to & q+\bar{q}+\tilde{\gamma} & \text{(quark + antiquark + photino)} \\
 & \tilde{g} & \to & g+\tilde{\gamma} & \text{(gluon + photino)} \\
\text{chargino}: & \tilde{\chi}^\pm_i & \to & e+\nu_e+\tilde{\chi}^0_i & \text{(electron + neutrino + photino)} \\
 & \tilde{\chi}^\pm_i & \to & q+\bar{q}'+\tilde{\chi}^0_i & \text{(quark + antiquark + photino)} \\
\text{neutralino}: & \tilde{\chi}^0_2 & \to & \tilde{\chi}^0_1+X &
\end{array}
$$

In the last case there are many possible channels both visible and invisible.

$$
\begin{array}{lll}
 & \underline{\text{Visible Channels}} & \underline{\text{Final States}} \\
\tilde{\chi}^0_2 \to & \tilde{\chi}^0_1 l^+l^- \quad (l=e,\mu,\tau) & \\
\to & \tilde{\chi}^\pm_1 l^\mp \nu_l & l^+l^- + \not{E}_T \\
 & \hookrightarrow \tilde{\chi}^0_1 l^\pm \nu_l & \\
\to & \tilde{\chi}^0_1 q\bar{q} & \text{2 jets} + \not{E}_T \\
\to & \tilde{\chi}^0_\gamma & \gamma + \not{E}_T \\
\to & \tilde{\chi}^\pm_1 q\bar{q}' & \\
 & \hookrightarrow \tilde{\chi}^0_1 l^\pm q\bar{q}' & \text{2 jets} + \not{E}_T \\
\to & \tilde{\chi}^\pm_1 l^\mp \nu_l & \\
 & \hookrightarrow \tilde{\chi}^0_1 q\bar{q}' & l^\pm + \text{2 jets} + \not{E}_T \\
\to & \tilde{\chi}^\pm_1 q\bar{q}' & \\
 & \hookrightarrow \tilde{\chi}^0_1 l^\pm \nu_l & l^\pm + \text{2 jets} + \not{E}_T \\
 & \underline{\text{Invisible Channel}} & \underline{\text{Final State}} \\
\to & \tilde{\chi}^0_1 \nu_l \bar{\nu}_l & \not{E}_T
\end{array}
$$

Thus, if supersymmetry exists in Nature and if it is broken somewhere below 1 TeV, then it will be possible to detect it in the nearest future.

5 Breaking of SUSY in the MSSM

Since none of the fields of the MSSM can develop non-zero v.e.v. to break SUSY without spoiling the gauge invariance, it is supposed that spontaneous supersymmetry breaking takes place via some other fields. The most common scenario for producing low-energy supersymmetry breaking is called *hidden sector* one. According to this scenario, there exist two sectors: the usual matter belongs to the "visible" one, while the second, "hidden" sector, contains fields which lead to breaking of supersymmetry. These two sectors interact with each other by exchange of some fields called *messengers*, which mediate SUSY breaking from the hidden to the visible sector (see Fig. 18). There might be various types of messenger fields: gravity, gauge, etc. Below we consider four possible scenarios.

The hidden sector is the weakest part of the MSSM. It contains a lot of ambiguities and leads to uncertainties of the MSSM predictions considered below.

Figure 18: Hidden Sector Scenario

5.1 The hidden sector: four scenarios

So far there are known four main mechanisms to mediate SUSY breaking from a hidden to a visible sector:

- Gravity mediation (SUGRA);
- Gauge mediation;
- Anomaly mediation;
- Gaugino mediation.

Consider them in more detail.

<u>SUGRA</u>

This mechanism is based on effective non-renormalizable interactions arising as a low-energy limit of supergravity theories [?]. In this case two sectors interact with each other via gravity. There are two types of scalar fields that develop non-zero v.e.v.s, namely moduli fields T, which appear as a result of compactification from higher dimensions, and the dilaton field S, part of SUGRA supermultiplet. These fields obtain non-zero v.e.v.s for their F components: $< F_T > \neq 0, < F_S > \neq 0$, which leads to spontaneous SUSY breaking. Since in SUGRA supersymmetry is local, spontaneous breaking leads to Goldstone particle which is a Goldstone fermion in this case. With the help of a super Higgs effect this particle may be absorbed into the additional component of a spin 3/2 particle, called *gravitino*, which becomes massive.

SUSY breaking is then mediated to a visible sector via gravitational interaction leading to the following SUSY breaking scale

$$M_{SU\!\!\!/SY} \sim \frac{< F_T >}{M_{PL}} + \frac{< F_S >}{M_{PL}} \sim m_{3/2},$$

where $m_{3/2}$ is the gravitino mass.

The effective low-energy theory, which emerges, contains explicit soft supersymmetry breaking terms

$$\mathcal{L}_{soft} = -\sum_i m_i^2 |A_i|^2 - \sum_i M_i(\lambda_i\lambda_i + \bar\lambda_i\bar\lambda_i) - B\ \mathcal{W}^{(2)}(A) - A\ \mathcal{W}^{(3)}(A), \tag{5.1}$$

where $\mathcal{W}^{(2)}$ and $\mathcal{W}^{(3)}$ are the quadratic and cubic terms of a superpotential, respectively. The mass parameters are

$$\begin{aligned} m_i^2 &\sim \frac{< F_S >}{M_{PL}} \sim m_{3/2}^2, \quad M_i \sim \frac{< F_S >}{M_{PL}} \sim m_{3/2}, \\ B &\sim \frac{< F_T >}{M_{PL}} \sim m_{3/2}^2, \quad A \sim \frac{< F_{T,S} >}{M_{PL}} \sim m_{3/2}. \end{aligned}$$

To have SUSY masses of the order of 1 TeV one needs $\sqrt{< F_{T,S} >} \sim 10^{11}$ GeV.

In spite of attractiveness of these mechanism in general, since we know that gravity exists anyway, it is not truly substantiated due to the lack of a consistent theory of quantum (super)gravity. Among the problems of supergravity mechanism are also the large freedom of parameters and the absence of automatic suppression of flavour violation.

Gauge Mediation

In this version of a hidden sector scenario the SUSY breaking effects are mediated to the observable world not via gravity but via gauge interactions [67]. The messengers are the gauge bosons and matter fields of the SM and of some GUT theory. The hidden sector is necessary since the dynamical SUSY breaking requires the fields with quantum numbers not compatible with the SM. The advantage of this scenario is that one can construct a renormalizable model with dynamical SUSY breaking, where in principle all the parameters can be calculated.

Consider some simplest possibility where in a hidden sector one has a singlet scalar superfield S with non-zero v.e.v. $< F_S > \neq 0$. The messenger sector consists of some superfield Φ, for instance, $\bar{5}$ of SU(5), that couples to S and to the SM fields with a superpotential

$$\mathcal{W} \sim S\Phi^\dagger\Phi, \qquad < S >= M \neq 0. \tag{5.2}$$

Integrating out the messenger fields gives mass to gauginos at the one loop level (see Fig. 19) and to the scalar fields (squarks and sleptons) at the two loop one (see Fig. 20). So, in gauge

Figure 19: Gaugino mass generation

gaugino

Figure 20: Squark mass generation

squark

mediated scenario all the soft masses are correlated to the gauge couplings and in this sense this scenario is more restrictive than the SUGRA one. There is no problem with flavour violating processes as well, since the soft terms automatically repeat the rigid sector.

It is remarkable that in this scenario the LSP happens to be the gravitino. The mass of the gravitino is given by

$$m_{\tilde{G}} \sim \frac{< F_S >}{M} \cdot \frac{M}{M_{PL}} \sim 10^{-14} \frac{M}{[GeV]}, \tag{5.3}$$

that leads to a very light gravitino field.

The problem of the gauge mediated SUSY breaking scenario emerge in the Higgs sector, since the Higgs mass mixing parameters which break an unwanted Peccei-Quin symmetry can not be generated by gauge interactions only. In order to parameterize some new unknown interactions, two new inputs have to be introduced (μ and B in SUGRA conventions).

Anomaly Mediation

Anomaly mediation mechanism assumes no SUSY breaking at the tree level. SUSY breaking is generated due to conformal anomaly. This mechanism refers to a hidden sector of a multidimensional theory with the couplings being dynamical fields which may acquire v.e.v.s. for their F components [?]. The external field or scale dependence of the couplings emerges as a result of conformal anomaly and that is why is proportional to the corresponding β functions. In the leading order one has

$$\begin{aligned} M_i(\Lambda) &\sim b_i\alpha_i(\Lambda)\,\frac{< F_{T,S} >}{M_{PL}} \sim b_i\ \alpha_i\ m_{3/2}, \\ m^2(\Lambda) &\sim b_i^2\ \alpha_i^2(\Lambda)\ m_{3/2}^2, \end{aligned} \tag{5.4}$$

where b_i are one-loop RG coefficients (see eq.(2.9)).

This reminds supergravity mediation mechanism but with fixed coefficients. It leads to two main differences:
i) the inverted relation between the gaugino masses at high energy scale

$$M_1 : M_2 : M_3 = b_1 : b_2 : b_3,$$

ii) negative slepton mass squared (tachyons!) at the tree level.
This problem has to be cured.

Gaugino Mediation

At last we would like to mention the gaugino mediation mechanism of SUSY breaking [?]. This is less developed scenario so far. It is based on a paradigm of a brane world. According to this paradigm there exists multidimensional world where our four dimensional space-time represents a brane of 4 dimensions. The fields of the SM live on the brane, while gravity and some other fields can propagate in the bulk. There also exists another brane where supersymmetry is broken. SUSY breaking is mediated to our brane via the fields propagating in the bulk. It is assumed that the gaugino field plays essential role in this mechanism (see Fig.21)

Figure 21: Gaugino mediated SUSY breaking

All four mechanisms of soft SUSY breaking are different in details but are common in results. They generate gauge invariant soft SUSY breaking operators of dimension ≤ 4 of the form

$$\begin{aligned} \mathcal{L}_{soft} &= -\sum_i m_i^2 |A_i|^2 - \sum_i M_i(\lambda_i\lambda_i + \bar{\lambda}_i\bar{\lambda}_i) \\ &\quad -\sum_{ij} B_{ij}A_iA_j - \sum_{ijk} A_{ijk}A_iA_jA_k + h.c., \end{aligned} \tag{5.5}$$

where the bilinear and trilinear couplings B_{ij} and A_{ijk} are such that not to break the gauge invariance. These are the only possible soft terms that do not break renormalizability of a theory and preserve SUSY Ward identities for the rigid terms.

Predictions for the sparticle spectrum depend on the mechanism of SUSY breaking. For comparison of four above mentioned mechanisms we show in Fig.5.1 the sample spectra as the ratio to the gaugino mass M_2.

Figure 22: Superparticle spectra for various mediation mechanisms

In what follows to calculate the mass spectrum of superpartners we need explicit form of SUSY breaking terms. Applying eq.(5.5) to the MSSM and avoiding R-parity violation gives

$$\begin{aligned} -\mathcal{L}_{Breaking} &= \sum_i m_{0i}^2|\varphi_i|^2 + \Bigg(\frac{1}{2}\sum_\alpha M_\alpha\tilde{\lambda}_\alpha\tilde{\lambda}_\alpha + BH_1H_2 \\ &+ A^U_{ab}\tilde{Q}_a\tilde{U}^c_bH_2 + A^D_{ab}\tilde{Q}_a\tilde{D}^c_bH_1 + A^L_{ab}\tilde{L}_a\tilde{E}^c_bH_1 + h.c.\Bigg), \end{aligned} \tag{5.6}$$

where we have suppressed $SU(2)$ indices. Here φ_i are all scalar fields, $\tilde{\lambda}_\alpha$ are the gaugino fields, $\tilde{Q}, \tilde{U}, \tilde{D}$ and $\tilde{L}, \tilde{E}$ are the squark and slepton fields, respectively, and $H_{1,2}$ are the SU(2) doublet Higgs fields.

The eq.(5.6) contains a vast number of free parameters which spoils the prediction power of the model. To reduce their number we adopt the so-called universality hypothesis, i.e. we assume the universality or equality of various soft parameters at high energy scale, namely we put all the spin 0 particle masses to be equal to the universal value m_0, all the spin 1/2 particle (gaugino) masses to be equal to $m_{1/2}$ and all the cubic and quadratic terms, proportional to A and B, to repeat the structure of the Yukawa superpotential (4.7). This is an additional requirement motivated by the supergravity mechanism of SUSY breaking. Universality is not a necessary requirement and one may consider non-universal soft terms as well. However, it will not change the qualitative picture presented below, so for simplicity in what follows we consider the universal boundary conditions. In this case eq.(5.6) takes the form

$$
\begin{aligned}
-\mathcal{L}_{Breaking} \quad &= \quad m_0^2 \sum_i |\varphi_i|^2 + \left(\frac{1}{2} m_{1/2} \sum_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha \right. \\
&+ \quad \left. A[y^U_{ab} \tilde{Q}_a \tilde{U}^c_b H_2 + y^D_{ab} \tilde{Q}_a \tilde{D}^c_b H_1 + y^L_{ab} \tilde{L}_a \tilde{E}^c_b H_1] + B[\mu H_1 H_2] + h.c. \right),
\end{aligned}
\tag{5.7}
$$

It should be noted that supergravity induced universality of the soft terms is more likely to be valid at the Planck scale, rather than at the GUT one. This is because a natural scale for gravity is M_{Planck}, while M_{GUT} is the scale for the gauge interactions. However, due to a small difference between these two scales, it is usually ignored in the first approximation resulting in minor uncertainties in the low-energy predictions [47].

The soft terms explicitly break supersymmetry. As will be shown later they lead to the mass spectrum of superpartners different from that of the ordinary particles. Remind that the masses of quarks and leptons remain zero until $SU(2)$ invariance is spontaneously broken.

5.2 The soft terms and the mass formulas

There are two main sources of the mass terms in the Lagrangian: the D terms and soft ones. With given values of $m_0, m_{1/2}, \mu, Y_t, Y_b, Y_\tau, A$, and B one can construct the mass matrices for all the particles. Knowing them at the GUT scale, one can solve the corresponding RG equations thus linking the values at the GUT and electroweak scales. Substituting these parameters into the mass matrices one can predict the mass spectrum of superpartners [19, 25].

5.2.1 Gaugino-higgsino mass terms

The mass matrix for gauginos, the superpartners of the gauge bosons, and for higgsinos, the superpartners of the Higgs bosons, is non-diagonal, thus leading to their mixing. The mass terms look like

$$
\mathcal{L}_{Gaugino-Higgsino} = -\frac{1}{2} M_3 \bar{\lambda}_a \lambda_a - \frac{1}{2} \bar{\chi} M^{(0)} \chi - (\bar{\psi} M^{(c)} \psi + h.c.), \tag{5.8}
$$

where $\lambda_a, a = 1, 2, \ldots, 8$, are the Majorana gluino fields and

$$
\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}^0_1 \\ \tilde{H}^0_2 \end{pmatrix}, \quad \psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} \tag{5.9}
$$

are, respectively, the Majorana neutralino and Dirac chargino fields. The neutralino mass matrix is:

$$
M^{(0)} = \begin{pmatrix}
M_1 & 0 & -M_Z \cos\beta \sin_W & M_Z \sin\beta \sin_W \\
0 & M_2 & M_Z \cos\beta \cos_W & -M_Z \sin\beta \cos_W \\
-M_Z \cos\beta \sin_W & M_Z \cos\beta \cos_W & 0 & -\mu \\
M_Z \sin\beta \sin_W & -M_Z \sin\beta \cos_W & -\mu & 0
\end{pmatrix}, \tag{5.10}
$$

where $\tan\beta = v_2/v_1$ is the ratio of two Higgs v.e.v.s and $\sin_W = \sin\theta_W$ is the usual sinus of the weak mixing angle.

The physical neutralino masses $M_{\tilde{\chi}_i^0}$ are obtained as eigenvalues of this matrix after diagonalization. For charginos one has:

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu \end{pmatrix}. \qquad (5.11) \quad \boxed{\texttt{char}}$$

This matrix has two chargino eigenstates $\tilde{\chi}^{\pm}_{1,2}$ with mass eigenvalues

$$M^2_{1,2} = \frac{1}{2}\left[M_2^2 + \mu^2 + 2M_W^2 \mp \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2(M_2^2 + \mu^2 + 2M_2\mu \sin 2\beta)}\right]. \qquad (5.12)$$

5.2.2 Squark and slepton masses

The non-negligible Yukawa couplings cause a mixing between the electroweak eigenstates and the mass eigenstates of the third generation particles. The mixing matrices for the $\tilde{m}_t^2, \tilde{m}_b^2$ and $\tilde{m}_\tau^2$ are:

$$\begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & \tilde{m}_{tR}^2 \end{pmatrix}, \qquad (5.13) \quad \boxed{\texttt{stopmat}}$$

$$\begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & \tilde{m}_{bR}^2 \end{pmatrix}, \qquad (5.14) \quad \boxed{\texttt{sbotmat}}$$

$$\begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & \tilde{m}_{\tau R}^2 \end{pmatrix} \qquad (5.15) \quad \boxed{\texttt{staumat}}$$

with

$$\begin{aligned}
\tilde{m}_{tL}^2 &= \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta, \\
\tilde{m}_{tR}^2 &= \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta, \\
\tilde{m}_{bL}^2 &= \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta, \\
\tilde{m}_{bR}^2 &= \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta, \\
\tilde{m}_{\tau L}^2 &= \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta, \\
\tilde{m}_{\tau R}^2 &= \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta
\end{aligned}$$

and the mass eigenstates are the eigenvalues of these mass matrices. For the light generations the mixing is negligible.

The first terms here ($\tilde{m}^2$) are the soft ones, which are calculated using the RG equations starting from their values at the GUT (Planck) scale. The second ones are the usual masses of quarks and leptons, and the last ones are the D terms of the potential.

5.3 The Higgs potential

As has been already mentioned, the Higgs potential in MSSM is totally defined by superpotential (and the soft terms). Due to the structure of $\mathcal{W}$ the Higgs self-interaction is given by the D-terms, while the F-terms contribute only to the mass matrix. The tree level potential is:

$$\begin{aligned}
V_{tree}(H_1, H_2) &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + h.c.) \\
&+ \frac{g^2 + g^{'2}}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^+ H_2|^2,
\end{aligned} \qquad (5.16) \quad \boxed{\texttt{Higpot}}$$

where $m_1^2 = m_{H_1}^2 + \mu^2, m_2^2 = m_{H_2}^2 + \mu^2$. At the GUT scale $m_1^2 = m_2^2 = m_0^2 + \mu_0^2,\ m_3^2 = -B\mu_0$. Notice, that the Higgs self-interaction coupling in eq.(5.16) is fixed and is defined by the gauge interactions as opposite to the SM.

The potential (5.16), in accordance with supersymmetry, is positively definite and stable. It has no non-trivial minimum different from zero. Indeed, let us write the minimization condition for the potential (5.16)

$$\frac{1}{2}\frac{\delta V}{\delta H_1} = m_1^2 v_1 - m_3^2 v_2 + \frac{g^2+g'^2}{4}(v_1^2 - v_2^2)v_1 = 0, \tag{5.17}$$

$$\frac{1}{2}\frac{\delta V}{\delta H_2} = m_2^2 v_2 - m_3^2 v_1 + \frac{g^2+g'^2}{4}(v_1^2 - v_2^2)v_2 = 0, \tag{5.18}$$

where we have introduced the notation

$$< H_1 > \equiv v_1 = v\cos\beta, \quad < H_2 > \equiv v_2 = v\sin\beta, \qquad v^2 = v_1^2 + v_2^2, \quad \tan\beta \equiv \frac{v_2}{v_1}.$$

Solution of eqs.(5.17),(5.18) can be expressed in terms of v^2 and $\sin\beta$:

$$v^2 = \frac{4(m_1^2 - m_2^2\tan^2\beta)}{(g^2+g'^2)(\tan^2\beta - 1)}, \quad \sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}. \tag{5.19}$$

One can easily see from eq.(5.19) that if $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, v^2 happens to be negative, i.e. the minimum does not exist. In fact, real positive solutions to eqs.(5.17),(5.18) exist only if the following conditions are satisfied [42]:

$$m_1^2 + m_2^2 > 2m_3^2, \qquad m_1^2 m_2^2 < m_3^4, \tag{5.20}$$

which is not the case at the GUT scale. This means that spontaneous breaking of the $SU(2)$ gauge invariance, which is needed in the SM to give masses for all the particles, does not take place in the MSSM.

This strong statement is valid, however, only at the GUT scale. Indeed, going down with energy the parameters of the potential (5.16) are renormalized. They become the "running" parameters with the energy scale dependence given by the RG equations. The running of the parameters leads to a remarkable phenomenon known as a *radiative spontaneous symmetry breaking* which we discuss below.

Provided conditions (5.20) are satisfied the mass matrices at the tree level are
CP-odd components P_1 and P_2 :

$$\mathcal{M}^{odd} = \left.\frac{\partial^2 V}{\partial P_i \partial P_j}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} m_3^2, \tag{5.21}$$

CP-even neutral components S_1 and S_2:

$$\mathcal{M}^{even} = \left.\frac{\partial^2 V}{\partial S_i \partial S_j}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} m_3^2 + \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} M_Z \cos\beta\sin\beta, \tag{5.22}$$

Charged components H^- and H^+:

$$\mathcal{M}^{charged} = \left.\frac{\partial^2 V}{\partial H_i^+ \partial H_j^-}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} (m_3^2 + M_W\cos\beta\sin\beta). \tag{5.23}$$

Diagonalising the mass matrices one gets the mass eigenstates haber [42]:

$$\left\{\begin{array}{lcll} G^0 &=& -\cos\beta P_1 + \sin\beta P_2, & \textit{Goldstone boson } \to Z_0, \\ A &=& \sin\beta P_1 + \cos\beta P_2, & \textit{Neutral } CP = -1 \textit{ Higgs}, \end{array}\right.$$

$$\left\{\begin{array}{lcll} G^+ &=& -\cos\beta (H_1^-)^* + \sin\beta H_2^+, & \textit{Goldstone boson } \to W^+, \\ H^+ &=& \sin\beta (H_1^-)^* + \cos\beta H_2^+, & \textit{Charged Higgs}, \end{array}\right. \tag{5.24}$$

$$\left\{\begin{array}{lcll} h &=& -\sin\alpha S_1 + \cos\alpha S_2, & \textit{SM Higgs boson } CP = 1, \\ H &=& \cos\alpha S_1 + \sin\alpha S_2, & \textit{Extra heavy Higgs boson}, \end{array}\right.$$

where the mixing angle α is given by

$$\tan 2\alpha = -\tan 2\beta \left(\frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2}\right).$$

The physical Higgs bosons acquire the following masses MSSM [19]:

$$\begin{array}{rl} \text{CP-odd neutral Higgs} \quad A: & m_A^2 = m_1^2 + m_2^2, \\ \text{Charge Higgses} \quad H^\pm: & m_{H^\pm}^2 = m_A^2 + M_W^2, \end{array} \tag{5.25}$$

CP-even neutral Higgses H,h:

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}\right], \tag{5.26}$$

where as usual

$$M_W^2 = \frac{g^2}{2}v^2, \quad M_Z^2 = \frac{g^2 + g'^2}{2}v^2.$$

This leads to the once celebrated SUSY mass relations:

$$\begin{aligned} m_{H^\pm} &\geq M_W, \\ m_h &\leq m_A \leq M_H, \\ m_h &\leq M_Z|\cos 2\beta| \leq M_Z, \\ m_h^2 + m_H^2 &= m_A^2 + M_Z^2. \end{aligned} \tag{5.27}$$

bound

Thus, the lightest neutral Higgs boson happens to be lighter than Z boson, that clearly distinguishes it from the SM one. Though we do not know the mass of the Higgs boson in the SM, there are several indirect constraints leading to the lower boundary of $m_h^{SM} \geq 135$ GeV bound1 [27]. After including the radiative corrections the mass of the lightest Higgs boson in the MSSM, m_h, increases. We consider it in more detail below.

5.4 Renormalization group analysis

To calculate the low energy values of the soft terms we use the corresponding RG equations. The one-loop RG equations for the rigid MSSM couplings are Ibanez [29]:

$$\begin{aligned} \frac{d\tilde{\alpha}_i}{dt} &= b_i \tilde{\alpha}_i^2, \qquad t \equiv \log Q^2/M_{GUT}^2 \\ \frac{dY_U}{dt} &= -Y_L\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{15}\tilde{\alpha}_1 - 6Y_U - Y_D\right), \end{aligned}$$

$$\frac{dY_D}{dt} = -Y_D\left(\frac{16}{3}\tilde\alpha_3 + 3\tilde\alpha_2 + \frac{7}{15}\tilde\alpha_1 - Y_U - 6Y_D - Y_L\right),$$
$$\frac{dY_L}{dt} = -Y_L\left(3\tilde\alpha_2 + \frac{9}{5}\tilde\alpha_1 - 3Y_D - 4Y_L\right), \qquad (5.28)$$

where we use the notation $\tilde\alpha = \alpha/4\pi = g^2/16\pi^2,\ Y = y^2/16\pi^2$.

For the soft terms one finds:

$$\begin{aligned}
\frac{dM_i}{dt} &= b_i\tilde\alpha_i M_i. \\
\frac{dA_U}{dt} &= \frac{16}{3}\tilde\alpha_3 M_3 + 3\tilde\alpha_2 M_2 + \frac{13}{15}\tilde\alpha_1 M_1 + 6Y_U A_U + Y_D A_D, \\
\frac{dA_D}{dt} &= \frac{16}{3}\tilde\alpha_3 M_3 + 3\tilde\alpha_2 M_2 + \frac{7}{15}\tilde\alpha_1 M_1 + 6Y_D A_D + Y_U A_U + Y_L A_L, \\
\frac{dA_L}{dt} &= 3\tilde\alpha_2 M_2 + \frac{9}{5}\tilde\alpha_1 M_1 + 3Y_D A_D + 4Y_L A_L, \\
\frac{dB}{dt} &= 3\tilde\alpha_2 M_2 + \frac{3}{5}\tilde\alpha_1 M_1 + 3Y_U A_U + 3Y_D A_D + Y_L A_L. \\
\frac{d\tilde m_Q^2}{dt} &= -\Big[(\frac{16}{3}\tilde\alpha_3 M_3^2 + 3\tilde\alpha_2 M_2^2 + \frac{1}{15}\tilde\alpha_1 M_1^2) - Y_U(\tilde m_Q^2 + \tilde m_U^2 + m_{H_2}^2 + A_U^2) \\
&\quad - Y_D(\tilde m_Q^2 + \tilde m_D^2 + m_{H_1}^2 + A_D^2)\Big], \\
\frac{d\tilde m_U^2}{dt} &= -\Big[(\frac{16}{3}\tilde\alpha_3 M_3^2 + \frac{16}{15}\tilde\alpha_1 M_1^2) - 2Y_U(\tilde m_Q^2 + \tilde m_U^2 + m_{H_2}^2 + A_U^2)\Big], \\
\frac{d\tilde m_D^2}{dt} &= -\Big[(\frac{16}{3}\tilde\alpha_3 M_3^2 + \frac{4}{15}\tilde\alpha_1 M_1^2) - 2Y_D(\tilde m_Q^2 + \tilde m_D^2 + m_{H_1}^2 + A_D^2)\Big], \\
\frac{d\tilde m_L^2}{dt} &= -\Big[3(\tilde\alpha_2 M_2^2 + \frac{1}{5}\tilde\alpha_1 M_1^2) - Y_L(\tilde m_L^2 + \tilde m_E^2 + m_{H_1}^2 + A_L^2)\Big], \\
\frac{d\tilde m_E^2}{dt} &= -\Big[(\frac{12}{5}\tilde\alpha_1 M_1^2) - 2Y_L(\tilde m_L^2 + \tilde m_E^2 + m_{H_1}^2 + A_L^2)\Big], \\
\frac{d\mu^2}{dt} &= -\mu^2\Big[3(\tilde\alpha_2 + \frac{1}{5}\tilde\alpha_1) - (3Y_U + 3Y_D + Y_L)\Big], \qquad (5.29) \\
\frac{dm_{H_1}^2}{dt} &= -\Big[3(\tilde\alpha_2 M_2^2 + \frac{1}{5}\tilde\alpha_1 M_1^2) - 3Y_D(\tilde m_Q^2 + \tilde m_D^2 + m_{H_1}^2 + A_D^2) \\
&\quad - Y_L(\tilde m_L^2 + \tilde m_E^2 + m_{H_1}^2 + A_L^2)\Big], \\
\frac{dm_{H_2}^2}{dt} &= -\Big[3(\tilde\alpha_2 M_2^2 + \frac{1}{5}\tilde\alpha_1 M_1^2) - 3Y_U(\tilde m_Q^2 + \tilde m_U^2 + m_{H_2}^2 + A_U^2)\Big],
\end{aligned}$$

Having all the RG equations, one can find now the RG flow for the soft terms. To see what happens at lower scales one has to run the RG equations for the mass parameters in the opposite direction from GUT to the EW scale. Let us take some initial values of the soft masses at the GUT scale in the interval between $10^2 \div 10^3$ GeV consistent with SUSY scale suggested by unification of the gauge couplings (2.10). This leads to the following RG flow of the soft terms shown in Fig.23. [25]

Figure 23: An example of evolution of sparticle masses and soft supersymmetry breaking parameters $m_1^2 = m_{H_1}^2 + \mu^2$ and $m_2^2 = m_{H_2}^2 + \mu^2$ for low (left) and high (right) values of $\tan\beta$

One should mention the following general features common to any choice of initial conditions:

i) The gaugino masses follow the running of the gauge couplings and split at low energies. The gluino mass is running faster than the others and is usually the heaviest due to the strong interaction.

ii) The squark and slepton masses also split at low energies, the stops (and sbottoms) being the lightest due to relatively big Yukawa couplings of the third generation.

iii) The Higgs masses (or at least one of them) are running down very quickly and may even become negative.

To calculate the masses one has also to take into account the mixing between various states (see eqs.(5.10,5.11, 5.13-5.15)).

Numerical solutions allow one to understand the significance of different initial conditions for the evolution down to low energies. As an example we present below the results of numerical solution of the RG equations for the soft terms in case of low values of $\tan\beta$. In this case one can ignore the bottom and tau Yukawa couplings and keep only the top one. Taking $M_{GUT} = 2.0\cdot 10^{16}$ GeV, $\alpha(M_{GUT}) \approx 1/24.3,\ Y_t(M_{GUT}) \approx \tilde\alpha(M_{GUT}),\ tan\beta = 1.65$ one gets the following numerical results[74]:

$$M_3(M_Z) = 2.7\ m_{1/2},$$

$$
\begin{aligned}
M_2(M_Z) &= 0.8\ m_{1/2}, \\
M_1(M_Z) &= 0.4\ m_{1/2}, \\
\mu(M_Z) &= 0.63\ \mu_0, \\
A_t(M_Z) &= 0.009\ A_t(0) - 1.7\ m_{1/2}, \\
\tilde{m}^2_{E_L}(M_Z) &= m_0^2 + 0.52\ m_{1/2}^2 - 0.27\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{\nu_L}(M_Z) &= m_0^2 + 0.52\ m_{1/2}^2 + 0.5\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{E_R}(M_Z) &= m_0^2 + 0.15\ m_{1/2}^2 - 0.23\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{U_L}(M_Z) &= m_0^2 + 6.6\ m_{1/2}^2 + 0.35\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{D_L}(M_Z) &= m_0^2 + 6.6\ m_{1/2}^2 - 0.42\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{U_R}(M_Z) &= m_0^2 + 6.2\ m_{1/2}^2 + 0.15\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{D_R}(M_Z) &= m_0^2 + 6.1\ m_{1/2}^2 - 0.07\ \cos(2\beta) M_Z^2, \\
\tilde{m}^2_{b_R}(M_Z) &= \tilde{m}^2_{D_R}, \\
\tilde{m}^2_{b_L}(M_Z) &= \tilde{m}^2_{D_L} - 0.48\ m_0^2 - 1.21\ m_{1/2}^2, \\
\tilde{m}^2_{t_R}(M_Z) &= \tilde{m}^2_{U_R} - 0.96\ m_0^2 - 2.42\ m_{1/2}^2, \\
\tilde{m}^2_{t_L}(M_Z) &= \tilde{m}^2_{U_L} - 0.48\ m_0^2 - 1.21\ m_{1/2}^2, \\
m_1^2(M_Z) &= m_0^2 + 0.40\ \mu_0^2 + 0.52\ m_{1/2}^2, \\
m_2^2(M_Z) &= -0.44\ m_0^2 + 0.40\ \mu_0^2 - 3.11\ m_{1/2}^2 - 0.09\ A_0 m_{1/2} - 0/02\ A_0^2.
\end{aligned}
$$

Typical dependence of the mass spectra on the initial conditions (m_0) is also shown in Fig. 24 [?]. For a given value of $m_{1/2}$ the masses of the lightest particles are practically independent of m_0, while the heavier ones increase with it monotonically as it follows also from the numerical solutions given above. One can see that the lightest neutralinos and charginos as well as the stop squark may be rather light.

Figure 24: The masses of sparticles as functions of the initial value m_0

5.5 Radiative electroweak symmetry breaking

The running of the Higgs masses leads to the phenomenon known as a *radiative electroweak symmetry breaking*. By this we mean the following: at the GUT energy scale both the Higgs mass parameters m_1^2 and m_2^2 are positive and the Higgs potential has no non-trivial minima. However, when running down to the EW scale due to the radiative corrections they may change sign so that the potential develops a non-trivial minimum. At this minimum the electroweak symmetry happens to be spontaneously broken. Thus, contrary to the SM where one has to choose the negative sign of the Higgs mass squared "by hand", in the MSSM the effect of spontaneous symmetry breaking is triggered by the radiative corrections.

Indeed, one can see in Fig. 23 is that m_2^2 (or both m_1^2 and m_2^2) decreases when going down from the GUT scale to the M_Z scale and can even become negative. This is the effect of the large top (and bottom) Yukawa couplings in the RG equations. As a result, at some value of Q^2 the conditions (5.20) are satisfied, so that the non-trivial minimum appears. This triggers spontaneous breaking of the $SU(2)$ gauge invariance. The vacuum expectations of the Higgs fields acquire non-zero values and provide masses to quarks, leptons and $SU(2)$ gauge bosons, and additional masses to their superpartners.

This way one obtains also the explanation of why the two scales are so much different. Due to the logarithmic running of the parameters one needs a long "running time" to get m_2^2 (or both m_1^2 and m_2^2) to be negative when starting from a positive value of the order of $M_{SUSY} \sim 10^2 \div 10^3$ GeV at the GUT scale.

6 Constrained MSSM

6.1 Parameter space of the MSSM

The Minimal Supersymmetric Standard Model has the following free parameters:

- Three gauge couplings α_i.
- The matrices of the Yukawa couplings y^i_{ab}, where $i = L, U, D$.
- The Higgs field mixing parameter μ.
- The soft supersymmetry breaking parameters.

Compared to the SM there is an additional Higgs mixing parameter, but the Higgs self-coupling, which is arbitrary in the SM, is fixed by supersymmetry. The main uncertainty comes from the unknown soft terms.

With universality hypothesis one is left with the following set of 5 free parameters defining the mass scales

$$\mu,\ m_0,\ m_{1/2},\ A \text{ and } B.$$

Parameter B is usually traded for $\tan\beta$, the ratio of the v.e.v.s of the two Higgs fields.

In particular models, like in SUGRA or gauge and anomaly mediation, some of soft parameters may be related to each other. However, since the mechanism of SUSY breaking is unknown, in what follows we consider them as free phenomenological parameters to be fitted by experiment. The experimental constraints are sufficient to determine these parameters, albeit with large uncertainties. The statistical analysis yields the probability for every point in the SUSY parameter space, which allows one to calculate the cross sections for the expected new physics of the MSSM at the existing or future accelerators (LEP II, Tevatron, LHC).

While choosing parameters and making predictions, one has two possible ways to proceed:

i) take the low-energy parameters as input, impose the constraints, define the allowed parameter space and calculate the spectrum and cross-sections as functions of these parameters. They might be the superparticle masses $\tilde m_{t1}, \tilde m_{t2}, m_A$, $\tan\beta$, mixings X_{stop}, μ, etc.

ii) take the high-energy parameters as input, run the RG equations, find the low-energy values, then impose the constrains and define the allowed parameter space for initial values. Now the calculations can be done in terms of initial parameters. They might be, for example, the above mentioned 5 soft parameters.

Both the ways are used in a phenomenological analysis. We show below how it works in practice.

6.2 The choice of constraints

Among the constraints that we are going to impose on the MSSM model are those which follow from the comparison of the SM with the experimental data, from the experimental limits on the masses of as yet unobserved particles, etc, and also those that follow from the ideas of unification and SUSY GUT models. Some of them look very obvious while the others depend on a choice.

Perhaps the most remarkable fact is, that all of them can be fulfilled simultaneously. The only model where one can do it is proved to be the MSSM.

In our analysis we impose the following constraints on the parameter space of the MSSM:

• Gauge coupling constant unification;

This is one of the most restrictive constraints, which we discussed in Sect 2. It fixes the scale of SUSY breaking of the order of 1 TeV.

• M_Z from electroweak symmetry breaking;

Radiative corrections trigger spontaneous symmetry breaking in the electroweak sector. In this case the Higgs potential does not have its minimum for all fields equal zero, but the minimum is obtained for non-zero vacuum expectation values of the fields. Solving M_Z from eq.(5.19) yields:

$$M_Z^2 = 2\frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1}. \tag{6.1}$$

To get the right value of M_Z requires proper adjustment of parameters. This condition determines the value of μ for a given values of m_0 and $m_{1/2}$.

• Yukawa coupling constant unification;

The masses of top, bottom and τ can be obtained from the low energy values of the running Yukawa couplings

$$m_t = y_t\ v\sin\beta,\quad m_b = y_b\ v\cos\beta,\quad m_\tau = y_\tau\ v\cos\beta. \tag{6.2}$$

Eq.(6.2) is written for the so-called running masses. They can be translated to the pole masses with account of the radiative corrections. For the pole masses of the third generation the following values are taken:

$$\begin{aligned} M_t &= 179 \pm 12\ \mathrm{GeV}/c^2\ [39, 40], \\ M_b &= 4.94 \pm 0.15\ \mathrm{GeV}/c^2\ [55], \\ M_\tau &= 1.7771 \pm 0.0005\ \mathrm{GeV}/c^2\ [56, 38]. \end{aligned} \tag{6.3}$$

The requirement of bottom-tau Yukawa coupling unification strongly restricts the possible solutions in the m_t versus $\tan\beta$ plane [?, 49, 50, 51, 54, 52, 53].

Figure 25: The upper part shows the top quark mass as function of $\tan\beta$ for $m_0 = 600$ GeV, $m_{1/2} = 400$ GeV. The middle part shows the corresponding values of the Yukawa couplings at the GUT scale and the lower part the χ^2 values.

• Branching ratio $BR(b \to s\gamma)$;

The branching ratio $BR(b \to s\gamma)$ has been measured by the CLEO collaboration and later by ALEPH and yields the world average of $BR(b \to s\gamma) = (3.14 \pm 0.48)\cdot 10^{-4}$. The Standard Model contribution to this process comes from the $W - t$ loop and gives a prediction which is very close to the experimental value leaving few space for SUSY. In the MSSM this flavour changing neutral current (FCNC) receives additional contributions from $H^\pm - t$, $\tilde\chi^\pm - \tilde t$ and $\tilde g - \tilde q$ loops. The $\tilde\chi^0 - \tilde t$ loops, which are much smaller [57, 58]. In leading order SUSY contribution may be rather big exceeding the experimental value by several standard deviations. However, the NLO corrections are essential.

This requirement imposes severe restrictions on the parameter space, specially for the case of large $\tan\beta$.

• Experimental lower limits on SUSY masses;

SUSY particles have not been found so far and from the searches at LEP one knows the lower limit on the charged lepton and chargino masses of about a half of a central of mass energy. The lower limit on the neutralino masses is lower. The lower limit on the Higgs mass is roughly given by the c.m.e. minus the Z-boson mass. These limits restrict the minimal values for the SUSY mass parameters. There exist also limits on squark and gluino masses from the hadron colliders [38], but these limits depend on the assumed decay modes. Furthermore, if one takes the limits given above into account, the constraints from the limits on all other particles are usually fulfilled, so they do not provide additional reductions of the parameter space in case of the minimal SUSY model.

• Dark Matter constraint;

Abundant evidence for the existence of non-relativistic, neutral, non-baryonic dark matter exists in our universe[59, 60]. The lightest supersymmetric particle (LSP) is supposedly stable and would be an ideal candidate for dark matter.

The present lifetime of the universe is at least 10^{10} years, which implies an upper limit on the expansion rate and correspondingly on the total relic abundance. Assuming $h_0 > 0.4$ one finds that the contribution of each relic particle species χ has to obey [60]:

$$\Omega_\chi h_0^2 < 1,$$

where $\Omega_\chi h^2$ is the ratio of the relic particle density of particle χ and the critical density, which overcloses the Universe. This bound can only be met, if most of the LSP's annihilated into fermion-antifermion pairs, which in turn would annihilate into photons again.

Since the neutralinos are mixtures of gauginos and higgsinos, the annihilation can occur both, via s-channel exchange of the Z^0 and Higgs bosons and t-channel exchange of a scalar particle, like a selectron [61]. This constrains the parameter space, as discussed by many groups[62, 64, 63, 65].

• Proton life time constraint;

There are two sources of proton decay in SUSY GUTs. The first one is the same as in non-SUSY theories and is related to the s-channel exchange of heavy gauge bosons. To avoid contradiction with experiment the unification scale has to be above 10^{15} GeV that is usually satisfied in any SUSY GUT.

The second source is more specific to SUSY models. The proton decay in this case takes place due to the loop diagrams with the exchange of heavy higgsino triplets. The preferable decay mode in this case is $p \to \bar{\nu} K$ or $p \to \mu^+ K$ instead of $p \to e^+ \pi$ in non-SUSY GUTs. The decay rate in this case depends on a particular GUT model and it is not so easy to satisfy the experimental requirements.

Having in mind the above mentioned constraints one can try to fix the arbitrariness in the parameters. In a kind of a statistical analysis, in which all the constraints are implemented in a χ^2 definition, one can find the most probable region of the parameter space by minimizing the χ^2 function. For the purpose of this analysis the following χ^2 definition is used [74, ?]:

$$\begin{aligned} \chi^2 \quad = \quad & \sum_{i=1}^{3} \frac{(\alpha_i^{-1}(M_Z) - \alpha_{MSSM_i}^{-1}(M_Z))^2}{\sigma_i^2} \\ & + \frac{(M_Z - 91.18)^2}{\sigma_Z^2} + \frac{(m_t - 174)^2}{\sigma_t^2} \\ & + \frac{(m_b - 4.98)^2}{\sigma_b^2} + \frac{(m_\tau - 1.7771)^2}{\sigma_\tau^2} \\ & + \frac{(Br(b \to s\gamma) - 3.15 \times 10^{-4})^2}{\sigma(b \to s\gamma)^2} \end{aligned} \tag{6.4}$$

$$
\begin{aligned}
&+\frac{(\Omega h^2-1)^2}{\sigma_\Omega^2} \qquad (for\ \Omega h^2>1)\\
&+\frac{(\tilde{M}-\tilde{M}_{exp})^2}{\sigma_{\tilde{M}}^2} \qquad (for\ \tilde{M}<\tilde{M}_{exp})\\
&+\frac{(\tilde{m}_{LSP}-\tilde{m}_\chi)^2}{\sigma_{LSP}^2} \qquad (for\ \tilde{m}_{LSP}\ charged).
\end{aligned}
$$

The first six terms are used to enforce gauge coupling unification, electroweak symmetry breaking and $b-\tau$ Yukawa coupling unification, respectively. The following two terms impose the constraints from $b\to s\gamma$ and the relic density, while the last terms require the SUSY masses to be above the experimental lower limits and the lightest supersymmetric particle (LSP) to be a neutralino, since a charged stable LSP would have been observed. The input and fitted output variables have been summarized in table 1.

exp. input data	$\Rightarrow$	Fit parameters: low $\tan\beta$	Fit parameters: high $\tan\beta$
$\alpha_1,\alpha_2,\alpha_3$		$M_{GUT},\ \alpha_{GUT}$	$M_{GUT},\ \alpha_{GUT}$
m_t		$Y_t^0,\ Y_b^0=Y_\tau^0$	$Y_t^0=Y_b^0=Y_\tau^0$
m_b	minimize	$m_0, m_{1/2}$	$m_0, m_{1/2}$
m_τ	χ^2	$\tan\beta$	$\tan\beta$
M_Z		μ	μ
$b\to s\gamma$		(A_0)	A_0
$\tau_{universe}$			

Table 1: Summary of fit input and output variables.

The five dimensional parameter space of the MSSM is big enough to be presented on a two or three dimensional picture. To make our analysis more clear we consider various low dimensional projections.

We first choose the value of the Higgs mixing parameter μ from the requirement of radiative EW symmetry breaking, then we take the values of $\tan\beta$ from the requirement of Yukawa coupling unification (see Fig.25). One finds two possible solutions: low $\tan\beta$ solution corresponding to $\tan\beta\approx 1.7$ and high $\tan\beta$ solution corresponding to $\tan\beta\approx 30\div 60$. In what follows we refer to this two solutions as low and high $\tan\beta$ scenarios, respectively.

What is left are the values of the soft parameters A, m_0 and $m_{1/2}$. However, the role of the trilinear coupling A is not essential, since at low energies it runs to the infra-red fixed point and is almost independent on initial conditions. Therefore, imposing the above mentioned constraints, the parameter space of the MSSM is reduced to a two dimensional one. In what follows we consider the plane $m_0, m_{1/2}$ and find the allowed region in this plane. Each point at this plane corresponds to a fixed set of parameters and allows one to calculate the spectrum, the cross-sections and the other quantities of interest.

We present the allowed regions of the parameter space for low and high $\tan\beta$ scenarios in Fig.26. In case when the requirement of $b\to s\gamma$ decay rate is not taken into account (due to

Figure 26: The χ^2-distribution for low and high $\tan\beta$ solutions. The different shades in the projections indicate steps of $\Delta\chi^2=4$, so basically only the light shaded region is allowed. The stars indicate the optimum solution. Contours enclose domains by the particular constraints used in the analysis [?].

uncertainties of the high order contributions), the allowed region of parameter space becomes much wider as it is illustrated in Fig. 27. Now much lower values of m_0 and $m_{1/2}$ are allowed leading to lower values of sparticle masses. This plot demonstrates the role of various constraints in the χ^2 analysis.

Figure 27: The same as Fig. 26 but with the $b \to s\gamma$ constraint released with account of the higher order corrections [?].

6.3 The mass spectrum of superpartners

When the parameter set is fixed one can calculate the mass spectrum of superpartners. We show below the set of parameters and predicted mass spectrum corresponding to the best fit values indicated by stars in Fig. 26.

Fitted SUSY parameters		
Symbol	**low** $\tan\beta$	**high** $\tan\beta$
$\tan\beta$	1.71	35.7
m_0	200	800
$m_{1/2}$	500	900
$\mu(0)$	1084	-938
$A(0)$	0	1200
$1/\alpha_{GUT}$	24.8	24.3
M_{GUT}	1.6 10^{16}	2.5 10^{16}

Table 2: Values of the fitted SUSY parameters for low and high $\tan\beta$ (in GeV, when applicable).

SUSY masses in [GeV]		
Symbol	**low** $\tan\beta$	**high** $\tan\beta$
$\tilde\chi^0_1(\tilde B)$, $\tilde\chi^0_2(\tilde W^3)$	214, 413	397, 722
$\tilde\chi^0_3(\tilde H_1)$,$\tilde\chi^0_4(\tilde H_2)$	1028, 1016	834, 791
$\tilde\chi^\pm_1(\tilde W^\pm)$, $\tilde\chi^\pm_2(\tilde H^\pm)$	413, 1026	721, 834
$\tilde g$	1155	1994
$\tilde e_L$, $\tilde e_R$	303, 270	902, 802
$\tilde\nu_L$	290	889
$\tilde q_L$, $\tilde q_R$	1028, 936	1825, 1723
$\tilde\tau_1$, $\tilde\tau_2$	279, 403	888, 1107
$\tilde b_1$, $\tilde b_2$	953, 1010	1734, 1782
$\tilde t_1$, $\tilde t_2$	727, 1017	1537, 1765
h, H	95, 1344	115, 1092
A, $H^\pm$	1340, 1344	1092, 1096

Table 3: Values of the SUSY mass spectra for the low and high $\tan\beta$ solutions, given in table 2.

To demonstrate the dependence of masses of the lightest particles on the choice of parameters, we show below their values in the whole $m_0, m_{1/2}$ plane for the case of low and high $tan\beta$

solutions, respectively. One can see that the masses of gauginos (charginos and neutralinos) and Higgses basically depend on $m_{1/2}$, while those of squarks and sleptons on m_0.

Figure 28: The masses of the lightest particles in the CMSSM. The contours show the fixed mass values of the corresponding particles. fig:susyall

6.4 Experimental signatures at e^+e^- colliders

Experiments are finally beginning to push into a significant region of supersymmetry parameter space. We know the sparticles and their couplings. We do not know their masses and mixings. Given the mass spectrum one can calculate the cross-sections and consider the possibilities of observation of new particles at modern accelerators. Otherwise one can get the restrictions on unknown parameters.

We start with e^+e^- colliders and, first of all with LEP II. In the leading order creation of superpartners is given by the diagrams shown in Fig.16 above. For a given center of mass energy the cross-sections depend on the mass of created particles and vanish at the kinematical boundary. For a sample example of c.e.m. of LEP II equal to 183 GeV they are shown at Fig.30.

Experimental signatures are defined by the decay modes which vary with the mass spectrum. The main ones are summarized below.

$$
\begin{array}{llll}
 & \underline{\text{Production}} & \underline{\text{Key Decay Modes}} & \underline{\text{Signatures}} \\
\bullet & \tilde{l}_{L,R}\tilde{l}_{L,R} & \tilde{l}^\pm_R \to l^\pm\tilde{\chi}^0_i \searrow \text{cascade} & \text{acomplanar pair of} \\
 & & \tilde{l}^\pm_L \to l^\pm\tilde{\chi}^0_i \nearrow \text{decays} & \text{charged leptons} + \not{E}_T \\
\bullet & \tilde{\nu}\tilde{\nu} & \tilde{\nu} \to l^\pm\tilde{\chi}^0_1 & \not{E}_T \\
\bullet & \tilde{\chi}^\pm_1\tilde{\chi}^\pm_1 & \tilde{\chi}^\pm_1 \to \tilde{\chi}^0_1 l^\pm\nu,\ \tilde{\chi}^0_1 q\bar{q}' & \text{isolated lepton} + 2\text{ jets} + \not{E}_T \\
 & & \tilde{\chi}^\pm_1 \to \tilde{\chi}^0_2 f\bar{f}' & \text{pair of acomplanar} \\
 & & \tilde{\chi}^\pm_1 \to l\tilde{\nu}_l \to l\nu_l\tilde{\chi}^0_1 & \text{leptons} + \not{E}_T \\
 & & \tilde{\chi}^\pm_1 \to \nu_l\tilde{l} \to \nu_l l\tilde{\chi}^0_1 & 4\text{ jets} + \not{E}_T
\end{array}
\qquad (6.5)
$$

$$
\begin{array}{llll}
\bullet & \tilde{\chi}^0_i\tilde{\chi}^0_j & \tilde{\chi}^0_i \to \tilde{\chi}^0_1 X, \tilde{\chi}^0_j \to \tilde{\chi}^0_1 X' & X = \nu_l\bar{\nu}_l \text{ invisible} \\
 & & & \quad = \gamma, 2l, 2\text{ jets} \\
 & & & 2l + \not{E}_T, l + 2j + \not{E}_T \\
\bullet & \tilde{t}_i\tilde{t}_j & \tilde{t}_1 \to c\tilde{\chi}^0_1 & 2\text{ jets} + \not{E}_T \\
 & & \tilde{t}_1 \to b\tilde{\chi}^\pm_1 \to bf\bar{f}'\tilde{\chi}^0_1 & 2\text{ b jets} + 2\text{ leptons} + \not{E}_T \\
 & & & 2\text{ b jets} + 2\text{ jets} + \text{lepton} + \not{E}_T \\
\bullet & \tilde{b}_i\tilde{b}_j & \tilde{b}_i \to b\tilde{\chi}^0_1 & 2\text{ b jets} + \not{E}_T \\
 & & \tilde{b}_i \to b\tilde{\chi}^0_2 \to bf\bar{f}'\tilde{\chi}^0_1 & 2\text{ b jets} + 2\text{ leptons} + \not{E}_T \\
 & & & 2\text{ b jets} + 2\text{ jets} + \not{E}_T
\end{array}
\qquad (6.6)
$$

Characteristic feature of all possible signatures is the missing energy and transverse momenta, which is a trade mark of new physics.

Numerous attempts to find out the superpartners at LEP II gave no positive result thus imposing the lower bounds on their masses. They are shown on parameter plane in Figs.31-32

In case of stop masses the result depends on the stop mixing angle $\Theta_{\tilde{t}}$ calculated from the stop mixing matrix. It defines the mass eigenstate basis $\tilde{t}_1$ and $\tilde{t}_2$

$$
\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\Theta_{\tilde{t}} & \sin\Theta_{\tilde{t}} \\ -\sin\Theta_{\tilde{t}} & \cos\Theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \qquad (6.7)
$$

$$\bullet \quad \tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{g}\tilde{q} \qquad \left.\begin{array}{r} \tilde{g} \to q\bar{q}\tilde{\chi}_1^0 \\ q\bar{q}'\tilde{\chi}_1^{\pm} \\ g\tilde{\chi}_1^0 \end{array}\right\} m_{\tilde{q}} > m_{\tilde{g}} \qquad \begin{array}{c} \not{E}_T + \text{multijets} \\ (+\text{leptons}) \end{array} \tag{6.9}$$

$$\left.\begin{array}{r} \tilde{q} \to q\tilde{\chi}_i^0 \\ \tilde{q} \to q'\tilde{\chi}_i^{\pm} \end{array}\right\} m_{\tilde{g}} > m_{\tilde{q}}$$

$$\begin{array}{llll}
\bullet & \tilde{\chi}_1^{\pm}\tilde{\chi}_2^0 & \tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 l^{\pm}\nu, \ \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 ll & \text{Trilepton} + \not{E}_T \\
 & & \tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 q\bar{q}', \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 ll, & \text{Dilepton + jet} + \not{E}_T \\
\bullet & \tilde{\chi}_1^{+}\tilde{\chi}_1^{-} & \tilde{\chi}_1^{+} \to l\tilde{\chi}_1^0 l^{\pm}\nu & \text{Dilepton} + \not{E}_T \\
\bullet & \tilde{\chi}_i^0\tilde{\chi}_i^0 & \tilde{\chi}_i^0 \to \tilde{\chi}_1^0 X, \tilde{\chi}_i^0 \to \tilde{\chi}_1^0 X' & \not{E}_T + \text{Dilepton + (jets) + (leptons)} \\
\bullet & \tilde{t}_1\tilde{t}_1 & \tilde{t}_1 \to c\tilde{\chi}_1^0 & \text{2 acollinear jets} + \not{E}_T \\
 & & \tilde{t}_1 \to b\tilde{\chi}_1^{\pm}, \tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 l^{\pm}\nu, \tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 q\bar{q}' & \text{single lepton} + \not{E}_T + b's \\
 & & \tilde{t}_1 \to b\tilde{\chi}_1^{\pm}, \tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 l^{\pm}\nu, \tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 l^{\pm}\nu & \text{Dilepton} + \not{E}_T + b's \\
\bullet & \tilde{l}\tilde{l}, \tilde{l}\tilde{\nu}, \tilde{nu}\tilde{\nu} & \tilde{l}^{\pm} \to l \pm \tilde{\chi}_i^0, \tilde{l}^{\pm} \to \nu_l\tilde{\chi}_i^{\pm} & \text{Dilepton} + \not{E}_T \\
 & & \tilde{\nu} \to \nu\tilde{\chi}_1^0 & \text{Single lepton} + \not{E}_T + (jets) \\
 & & & \not{E}_T
\end{array} \tag{6.10}$$

Note again the characteristic missing energy and transverse momenta events.

Unless e^+e^- colliders at hadron machines the background is extremely rich and essential.

6.6 The Lightest Superparticle

One of the crucial questions is the properties of the lightest superparticle. Different SUSY breaking scenarios lead to different experimental signatures and different LSP.

• Gravity mediation:

In this case the LSP is the lightest neutralino $\tilde{\chi}_1^0$, which is almost 90% photino for low $\tan\beta$ solution and contains more higgsino admixture for high $\tan\beta$. The usual signature for LSP is missing energy. $\tilde{\chi}_1^0$ is stable and is the best candidate for the cold dark matter in the Universe. Typical processes where the LSP is created end up with jets + $\not{E}_T$, or leptons + $\not{E}_T$, or both jest + leptons + $\not{E}_T$.

• Gauge mediation:

In this case the LSP is the gravitino $\tilde{G}$ which also leads to missing energy. The actual question here is what is the NLSP, the next lightest particle. There are two possibilities:

i) $\tilde{\chi}_1^0$ is the NLSP. Then the decay modes are

$$\tilde{\chi}_1^0 \to \gamma\tilde{G}, \ h\tilde{G}, \ Z\tilde{G}.$$

As a result one has two hard photons + $\not{E}_T$, or jets + $\not{E}_T$.

ii) $\tilde{l}_R$ is the NLSP. Then the decay mode is $\tilde{l}_R \to \tau\tilde{G}$ and the signature is a charged lepton and the missing energy.

• Anomaly mediation:

In this case one also has two possibilities:

i) $\tilde{\chi}_1^0$ is the LSP and wino-like. It is almost degenerate with the NLSP.

ii) $\tilde{\nu}_L$ is the LSP. Then it appears in the decay of chargino $\tilde{\chi}^+ \to \tilde{\nu}l$ and the signature is the charged lepton and the missing energy.

• R-parity violation:

In this case the LSP is no longer stable and decays into the SM particles. It may be charged (or even colored) and may lead to rare decays like neutrinoless double β-decay, etc.

Experimental limits on the LSP mass follow from non-observation of the corresponding events. Modern low limit is around 40 GeV (see Fig. fig:lsp 35).

The last unobserved particle from the Standard Model is the Higgs boson. Its discovery would allow one to complete the SM paradigm and confirm the mechanism of spontaneous symmetry breaking. On the contrary, the absence of the Higgs boson would awake doubts about the whole picture and would require new concepts.

Figure 34: Gluon fusion, $q\bar{q}$ scattering, quark-gluon scattering fusion

Figure 35: Annihilation Channel annihil

Experimental limits on the Higgs boson mass come from a direct search at LEP II and Tevatron and from indirect fits of electroweak precision data, first of all from the radiative corrections to the W and top quark masses. A combined fit of modern experimental data gives [30]

$$m_h = 78^{+86}_{-47}\ \text{GeV}, \tag{7.1}$$

which at the 95% confidence level leads to the upper bound of 200 GeV (see Fig. 36). At the same time, recent direct searches at LEP II for the c.m. energy of 209 GeV give the lower limit of 113.4 GeV [30]. From theoretical point of view low Higgs mass could be a limit for physics beyond the SM, in particular for the supersymmetric extension of the SM.

Figure 36: The $\Delta\chi^2$ distribution as function of the Higgs mass from the SM fit to the electroweak precision observables and the top mass. The shaded area is excluded by the direct searches. fig:1

Within the Standard Model the value of the Higgs mass m_h is not predicted. However, one can get the bounds on the Higgs mass [27, 28]. They follow from the behaviour of the quartic coupling which is related to the Higgs mass by eqs.(1.9,1.13) $m_h^2 = 2\lambda v$ and obeys the following renormalization group equation describing the change of λ with a scale:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(6\lambda^2 + 6\lambda y_t^2 - 6y_t^4 + gauge\ terms\right) \tag{7.2}$$ betalambda

with $t = \ln(Q^2/\mu^2)$. Here y_t is the top-quark Yukawa coupling.

Since the quartic coupling grows with rising energy infinitely and reaches the Landau pole, an upper bound on m_h follows from the requirement that the theory be valid up to the scale M_{Planck} or up to a given cut-off scale Λ below M_{Planck} [27]. The scale Λ could be identified with the scale at which the Landau pole develops. The upper bound on m_h depends mildly on the top-quark mass through the impact of the top-quark Yukawa coupling on the running of the quartic coupling λ in eq.(7.2).

On the other hand, the requirement of vacuum stability in the SM (positivity of λ) imposes a lower bound on the Higgs boson mass, which crucially depends on the top-quark mass as well as on the cut-off Λ [27, 28]. Again, the dependence of this lower bound on m_t is due to the effect of the top-quark Yukawa coupling on the quartic coupling in eq.(7.2), which drives λ to negative values at large scales, thus destabilizing the standard electroweak vacuum (see Figs. 37).

From the point of view of LEP and Tevatron physics, the upper bound on the SM Higgs boson mass does not pose any relevant restriction. The lower bound on m_h, instead, is particularly important in view of search for the Higgs boson at LEPII and Tevatron. For $m_t \sim 174$ GeV and $\alpha_s(M_Z) = 0.118$ the running of the Higgs quartic coupling is shown in Fig. 38. The results at $\Lambda = 10^{19}$ GeV or at $\Lambda = 1$ TeV can be given by the approximate formulae [28]

$$m_h > 135 + 2.1[m_t - 174] - 4.5\left[\frac{\alpha_s(M_Z) - 0.118}{0.006}\right], \quad \Lambda = 10^{19}\ GeV, \tag{7.3}$$ 19G

$$m_h > 72 + 0.9[m_t - 174] - 1.0\left[\frac{\alpha_s(M_Z) - 0.118}{0.006}\right], \quad \Lambda = 1\ TeV, \tag{7.4}$$ 1T

where the masses are in units of GeV.

Fig. 39 [68] shows the perturbativity and stability bounds on the Higgs boson mass of the SM for different values of the cut-off Λ at which new physics is expected. We see from Fig. 39 and eqs.(7.3, 7.4) that indeed for $m_t \sim 174$ GeV the discovery of a Higgs particle at LEPII would imply that the Standard Model breaks down at a scale Λ well below M_{GUT} or M_{Planck}, smaller for lighter Higgs. Actually, if the SM is valid up to $\Lambda \sim M_{GUT}$ or M_{Planck}, for $m_t \sim 174$ GeV only a small range of values is allowed: $134 < m_h <\sim 200$ GeV. For $m_t = 174$ GeV and $m_h < 100$ GeV [i.e. in the LEPII range] new physics should appear below the scale $\Lambda \sim$ a few to 100 TeV. The dependence on the top-quark mass however is noticeable. A lower value, $m_t \simeq 170$ GeV, would relax the previous requirement to $\Lambda \sim 10^3$ TeV, while a heavier value $m_t \simeq 180$ GeV would demand new physics at an energy scale as low as 10 TeV.

Figure 37: The shape of the Higgs potential higpot

Figure 38: The running of the Higgs quartic coupling. Numbers shown above the lines indicate the value of the Higgs mass in GeV. fig:stab

Figure 39: Strong interaction and stability bounds on the SM Higgs boson mass. Λ denotes the energy scale up to which the SM is valid. mariano

7.2 SM Higgs production at LEP

The dominant mechanism for Higgs boson production at LEP is the Higgsstrahlung. The Higgs boson is produced together with the Z^0 boson. Small contribution to the cross section comes also from the WW- and ZZ- fusion processes (see Fig. 40). The cross section depends on the Higgs boson mass and decreases with increase of the latter. On the other hand it grows with the central of mass energy as shown in Fig. 41. Kinematical limit on the Higgs production is given by the c.m. energy minus the Z-boson mass.

Figure 40: SM Higgs production at LEP: Higgsstrahlung (above) and WW- and ZZ- fusion (below) higprod

However, one of the main problems is to distinguish the final products of the Higgs boson decay from the background, mainly the ZZ pair production. The branching ratios for the Higgs boson decay are shown in Fig. 42. Z boson has the same decay modes with different branchings. In final states one has either four hadronic jets, or two jets and two leptons, or for leptons. The most probable is the four jet configuration, which is the most difficult from the point of view of unwanted background. Two jet and two lepton final state is more clean though less probable.

Attempts to find the Higgs boson have not meet success so far. All the data are consistent with the background. An interesting four jest event is shown in Fig. 43 and is most likely a ZZ candidate. A reconstructed invariant mass of two jets does not show noticeable deviation from background expectation. For 68.1 background events expected there are 70 events observed. The reconstructed Higgs mass for four jet events is shown in Fig. 44. At this kind of plots the real Higgs boson should give a peak above the background as is shown for a would be Higgs mass of 110 GeV in Fig. 44.

Combined results from four LEP collaborations (ALEPH, DELPHI, L3 and OPAL) in the energy interval $\sqrt{s} = 200 - 210$ GeV allow one to get a lower limit on the Higgs mass. As it follows from Fig. 45 at the 95% confidence level it is [12]

$$m_h > 113.3 \text{ GeV/c}^2 \quad @\ 95\%\ C.L.$$

Recent hot news from LEP accelerator show slight excess of events in hadronic channels. For the hard cuts keeping only "really good" events one can achieve the signal/background ratio of 2 with few signal events indicating on 114 GeV Higgs boson (see Fig. 46). Deviation form the background achieves 2.9 standard deviations and is better seen in the confidence level plots. However, statistics is not enough to make definite conclusions.

7.3 The Higgs boson mass in the MSSM

It has been already mentioned that in the MSSM the mass of the lightest Higgs boson is predicted to be less than the Z-boson mass. This is, however, the tree level result and the masses acquire the radiative corrections.

Figure 41: The cross section of the Higgs production at LEP II higxsec

Figure 42: The final states of Higgs boson decay with the branching ratios. final

Figure 43: Typical four jet event event

Figure 44: Reconstructed Higgs mass for four jet events. The peak shown in red corresponds to a would be Higgs boson with mass of 110 GeV. massdis

With account of the radiative corrections the effective Higgs bosons potential is

$$V^{eff}_{Higgs} = V_{tree} + \Delta V, \tag{7.5}$$

where V_{tree} is given by eq.(5.16) and in one loop order

$$\Delta V_{1loop} = \sum_k \frac{1}{64\pi^2}(-1)^{J_k}(2J_k+1)c_k m_k^4 \left(\log\frac{m_k^2}{Q^2} - \frac{3}{2}\right). \tag{7.6}$$

Here the sum is taken over all the particles in the loop, J_k is the spin and m_k is the field dependent mass of a particle at the scale Q.

The main contribution comes from the diagrams shown in Fig. 47. These radiative corrections vanish when supersymmetry is not broken and are positive in softly broken case. They are proportional to the mass squared of top (stop) quarks and depend on the values of the soft breaking parameters. Contributions from the other particles are much smaller [48, 69, 35]. The leading contribution comes from (s)top loops

$$\Delta V^{stop}_{1loop} = \frac{3}{32\pi^2}\left[\tilde m^4_{t_1}(\log\frac{\tilde m^2_{t_1}}{Q^2} - \frac{3}{2}) + \tilde m^4_{t_2}(\log\frac{\tilde m^2_{t_2}}{Q^2} - \frac{3}{2}) - 2m_t^4(\log\frac{m_t^2}{Q^2} - \frac{3}{2})\right]. \tag{7.7}$$

These corrections lead to the following modification of the tree level relation for the lightest Higgs mass

$$m_h^2 \approx M_Z^2\cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2}\log\frac{\tilde m^2_{t_1}\tilde m^2_{t_2}}{m_t^4}. \tag{7.8}$$ rad

One finds that the one loop correction is positive and increases the mass value. Two loop corrections have the opposite effect but are smaller and result in slightly lower value of the Higgs mass.

To find out numerical values of these corrections one has to determine the masses of all superpartners. Within the Constrained MSSM, imposing various constraints, one can define the allowed region in the parameter space and calculate the spectrum of superpartners and, hence, the radiative corrections to the Higgs boson mass (see Figs. 48, 49).

The Higgs mass depends mainly on the following parameters: the top mass, the squark masses, the mixing in the stop sector, the pseudoscalar Higgs mass and $\tan\beta$. As will be shown below, the maximum Higgs mass is obtained for large $\tan\beta$, for a maximum value of the top and squark masses and a minimum value of the stop mixing.

Note that in the CMSSM the Higgs mixing parameter μ is determined by the requirement of EWSB, which yields large values for μ [?]. Given that the pseudoscalar Higgs mass increases plots show demonstrate the saturation of the mass at high values of mass parameters.

rapidly with μ, this mass is always much larger than the lightest Higgs mass and thus decouples. This decoupling is effective for all regions of the CMSSM parameter space, i.e. the lightest Higgs has the couplings of the SM Higgs within a few percent.

We present below the value of the lightest Higgs mass in the whole $m_0, m_{1/2}$ plane for low and high $\tan\beta$ solutions, respectively. One can see that it is practically constant in the whole plane and saturates for high value of m_0 and $m_{1/2}$.

The lightest Higgs boson mass m_h is shown as function of $\tan\beta$ in Fig. 50 [?]. The shaded band corresponds to the uncertainty from the stop mass and stop mixing for $m_t = 175$ GeV. The upper and lower lines correspond to m_t=170 and 180 GeV, respectively.

The parameters used for the calculation of the upper limit were: $m_t = 180$ GeV, $A_0 = -3m_0$ and $m_0 = m_{1/2} = 1000$ GeV. The lowest line of the same figure gives the minimal values of m_h. For high $\tan\beta$ the values of m_h range from 105 GeV 125 GeV. There is at present no preference for any of the values in this range, but it can be seen, that the 95% C.L. lower limit on the Higgs mass [?] of 113.3 GeV excludes $\tan\beta < 3.3$.

In order to understand better the Higgs mass uncertainties, the relevant parameters were varied one by one. The largest uncertainty on the light Higgs mass originates from the stop masses. The Higgs mass varies between 110 and 120 GeV, if m_0 and $m_{1/2}$ are varied between 200 and 1000 GeV, which implies stop masses varying between 400 and 2000 GeV. Since at present there is no preference for any of the values between 110 and 120 GeV, the variance for a flat probability distribution is $10/\sqrt{12}$=3 GeV, which we take as an error estimate.

The remaining uncertainty on the Higgs mass originates from the mixing in the stop sector when one leaves A_0 a free parameter. The mixing is determined by the off-diagonal element in the stop mass matrix $X_t = A_t - \mu/\tan\beta$. Its influence on the Higgs mass is quite small in the CMSSM, since the low energy value A_t tends to a fixed point, so that the stop mixing parameter $X_t = A_t - \mu/\tan\beta$ is not strongly dependent on A_0. Furthermore, the μ term is not important at large $\tan\beta$. If we vary A_0 between $\pm 3m_0$, the error from the stop mixing in the Higgs boson mass is estimated to be ± 1.5 GeV. The values of $m_0 = m_{1/2} = 370$ GeV yield the central value of $m_h = 115$ GeV.

The uncertainty from the top mass at large $\tan\beta$ is $\pm$ 5 GeV [?], given the uncertainty on the top mass of 5.2 GeV.

The uncertainties from the higher order calculations (HO) is estimated to be 2 GeV from a comparison of the full diagrammatic method [?] and the effective potential approach [?]. So combining all the uncertainties discussed before the results for the Higgs mass in the CMSSM can be summarized as follows:

- The low $\tan\beta$ scenario ($\tan\beta < 3.3$) of the CMSSM is excluded by the lower limit on the Higgs mass of 113.3 GeV [?].

- For the high $\tan\beta$ scenario the Higgs mass is found to be in the range from 110 to 120 GeV for $m_t = 175$ GeV. The central value is found to be [?]:

$$m_h = 115 \pm 3\ (stopmass)\ \pm 1.5\ (stopmixing)\ \pm 2\ (theory)\ \pm 5\ (topmass)\ \text{GeV}, \quad (7.9)$$

  where the errors are the estimated standard deviations around the central value. This prediction is independent of $\tan\beta$ for $\tan\beta > 20$ and decreases for lower $\tan\beta$.

However, these SUSY limits on the Higgs mass may not be so restricting if non-minimal SUSY models are considered. In a SUSY model extended by a singlet, the so-called Next-to-Minimal model, eq.(5.27) is modified and at the tree level the bound looks like [?]

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta, \tag{7.10}$$

where λ is an additional singlet Yukawa coupling. This coupling being unknown brings us back to the SM situation, though its influence is reduced by $\sin 2\beta$. As a result, for low $\tan\beta$ the upper bound on the Higgs mass is slightly modified (see Fig.51).

In case of supersymmetry, contrary to the SM, there are two competing processes for neutral Higgs production. Besides the usual Higgsstrahlung diagram there is also the pair production one when two Higgs bosons (the usual one and the pseudoscalar boson A) are produced. The cross-section of these two processes are complimentary and related to the SM one by a simple formula (see Fig. 52). Thus, the cross-section for Higgs production in the MSSM is usually lower

Figure 49: The same as in Fig. 48 but for the high $\tan\beta$ solution $\tan\beta = 35$.

Figure 52: MSSM Higgs production at LEP: complimentary diagrams

Figure 50: The mass of the lightest Higgs boson as function of $\tan\beta$

than that of the SM. Therefore, searches for pair production are limited by low cross-section rather than by a threshold. Non-observation of the Higgs boson at LEP in general gives lower bound on the Higgs boson mass than that in the SM. Modern experimental limits on the MSSM Higgs bosons are

$$m_h > 90.5 \text{ GeV/c}^2, \quad m_A > 90.5 \text{ GeV/c}^2 \quad @ \ 95\% \ C.L. \tag{7.11}$$

Even more dramatic changes are possible in models containing non-standard fields at intermediate scales. These fields appear in scenarios with gauge mediated supersymmetry breaking. In this case the upper bound on the Higgs mass may increase up to 155 GeV [?] (the upper curve in Fig. 51), though it is not necessarily saturated. One should notice, however, that these more sophisticated models do not change the generic feature of SUSY theories, the presence of the light Higgs boson.

However, for a heavy pseudoscalar boson A the second process is decoupled and one basically has the same production rate as in the SM. Therefore in this case the SM experimental limit is applicable also to the MSSM.

To present the result for the Higgs search in the MSSM various variables can be used. The most popular ones are (m_h, m_A), $(m_h, \tan\beta)$ and $(m_A, \tan\beta)$ planes. They are shown below in Figs. 53-54 for two particular cases: no-mixing and maximal mixing in the stop sector. For comparison the theoretically allowed regions are shown. One can see that

a) low $\tan\beta$ solution $(0.5 < \tan\beta < 3.3)$ is already excluded;

b) very small region for the lightest neutral Higgs boson mass is left (specially for the no-mixing case).

As it has been explained, in the MSSM one has also the charged Higgs bosons. The searches for the charged Higgs bosons are the attempts to look beyond the Standard Model. It is basically the same in the MSSM and in any two Higgs doublet model. The charged Higgs bosons are produced in pairs in annihilation process like any charged particles. The couplings are the standard EW couplings and the only unknown quantity is the charged Higgs mass. However, the branching ratios for the decay channels depend on the mass and the model. Large background comes from the W-pair production. Non-observation of charged Higgs bosons at LEP gives the lower limit on their masses. The combined exclusion plot for various channels is shown in Fig.55. This imposes the absolute lower limit on the charged Higgs boson mass

$$m_{H^\pm} > 77.5 \text{ GeV/c}^2 \quad @ \;\; 95\% \;\; C.L. \tag{7.12}$$

<u>Tevatron and LHC</u>

With shut down of LEP next attempts to discover the Higgs boson are connected with the Tevatron and LHC hadron colliders.

Tevatron will start the Run II next year and will reach the c.m. energy of 2 TeV with almost 10 times greater luminosity. However, since it is hadron collider, not the full energy goes into collision taken away by those quarks in a proton that do not take part in the interaction. Having very severe background this collider needs long time of running to reach the integrated luminosity required for the Higgs discovery. A combined CDF/D0 plot shows the integrated luminosity at Tevatron as function of the Higgs mass (see Fig.56). The three curves correspond to 2σ (95% confidence level), 3σ and 5σ signal necessary for exclusion, evidence and discovery of the Higgs boson, respectively. One can see that the integrated luminosity of $2fb^{-1}$, which is planned to be achieved at the end of 2001, will allow to exclude the Higgs boson with the mass of the order of 115 GeV, i.e. just the limit reached by LEP. One will need RUN III to reach $10fb^{-1}$ to cover the most interesting interval, even at the level of exclusion (2σ). To find the Higgs boson one will need still greater integrated luminosity. The signatures of the Higgs boson are related to the dominant decay modes which depend on the mass of the Higgs boson. In the Tevatron region they are

$$\begin{array}{ll} H \to b\bar{b}, & 100 < m_H < 140 \text{ GeV}, \\ H \to WW^*, & 140 < m_H < 175 \text{ GeV}, \\ H \to ZZ^*, & 175 < m_H < 190 \text{ GeV}. \end{array} \tag{7.13}$$

The LHC hadron collider is the ultimate machine for a new physics at the TeV scale. Its c.m. energy is planned to be 14 TeV with very high luminosity up to a few hundred fb^{-1}. It is supposed to start operating in 2006. In principle LHC will be able to cover the whole interval of SUSY and Higgs masses up to a few TeV. It will either discover the SM or the MSSM Higgs boson, or prove their absence. In terms of exclusion plots shown in Figs.53,54 the LHC collider will cover the whole region. Various decay modes allow to probe different areas as shown in Fig.57, though the background will be very essential.

8 Conclusion

LEP II has neither discovered the new physics, nor has proven the existence of the Higgs boson. However, it gave us some indication that both of them exist. Supersymmetry now is the most

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Figure 57: Exclusion plots for LHC hadron collider for different Higgs decay modes lhc

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