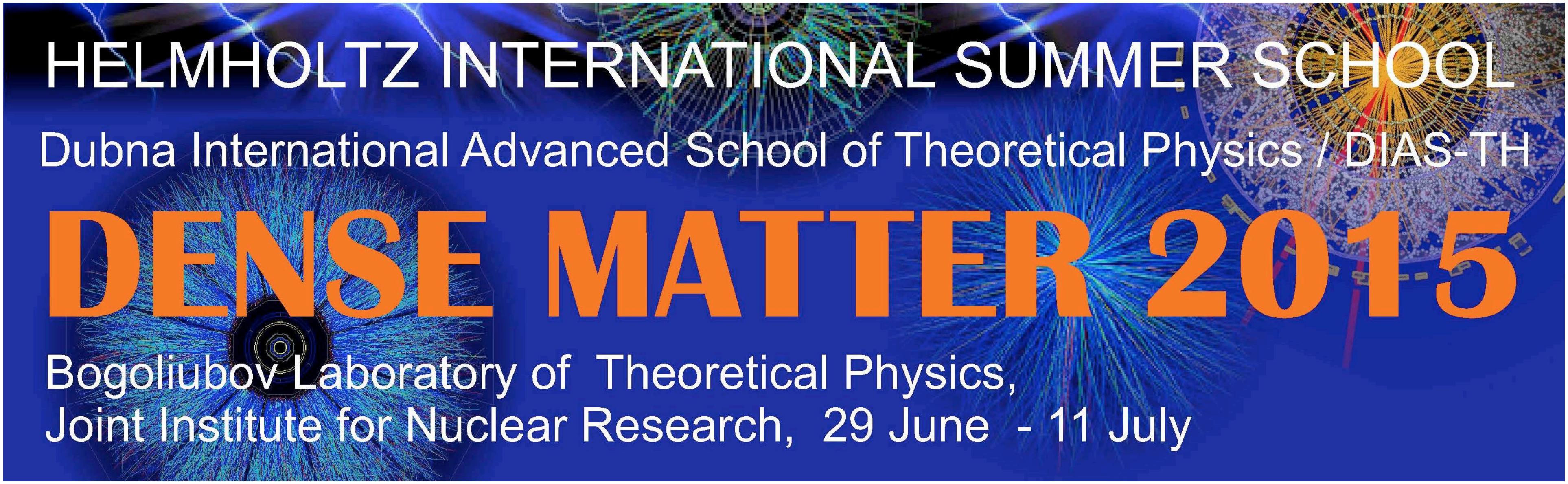
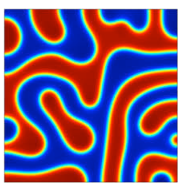


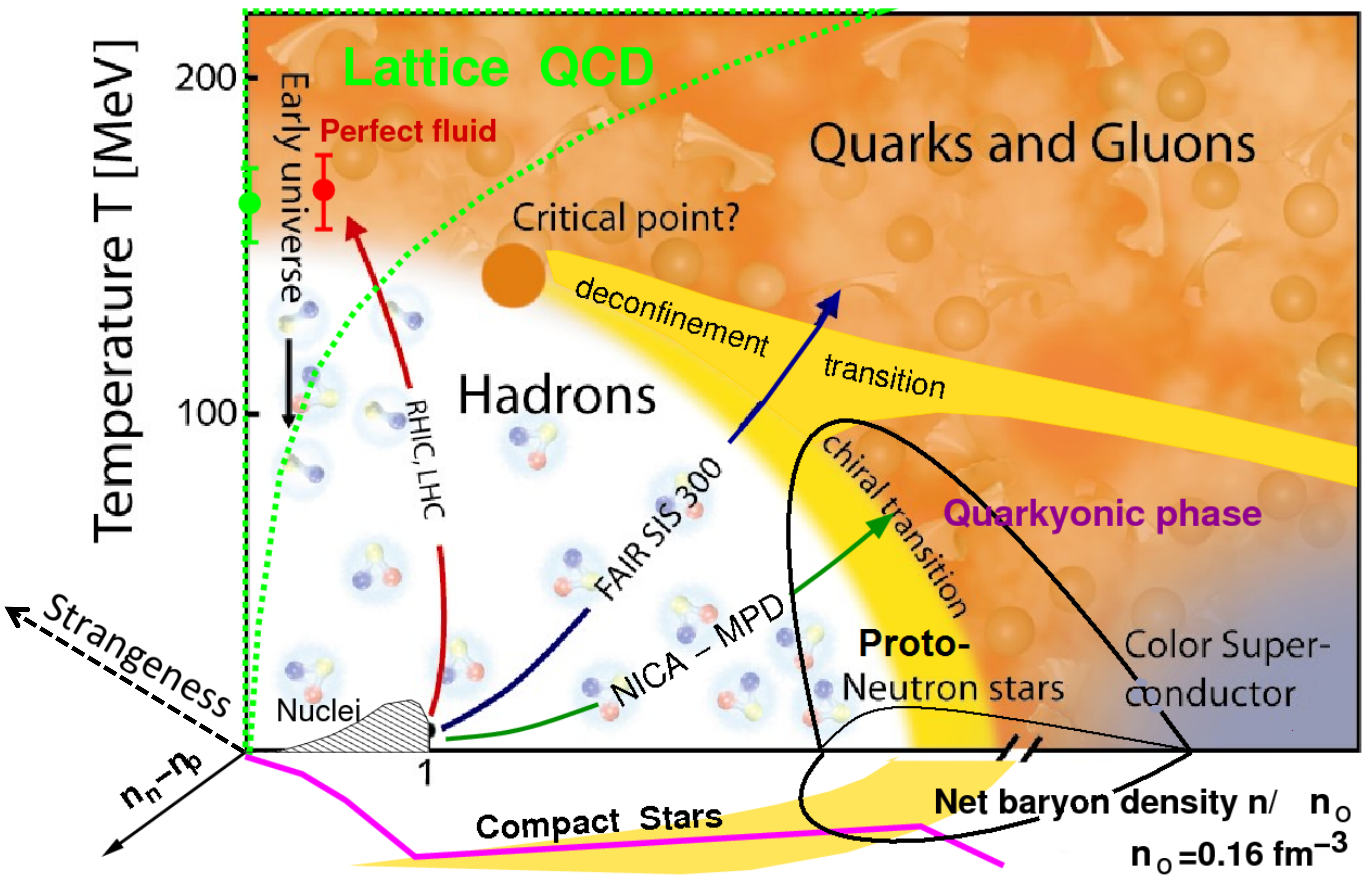
Spinodal Instabilities at the Deconfinement Phase Transition

Jørgen Randrup (Berkeley)



Lecture I:	*Phase coexistence (equilibrium)*	WED 12:00-13:00
Lecture II:	*Phase separation (non-equilibrium)*	THU 16:30-17:30
	Discussion	18:00-19:00
Lecture III:	*Effects on collision dynamics (clumping)*	FRI 11:00-12:00

Possible (ρ, T) phase diagram of strongly interacting matter

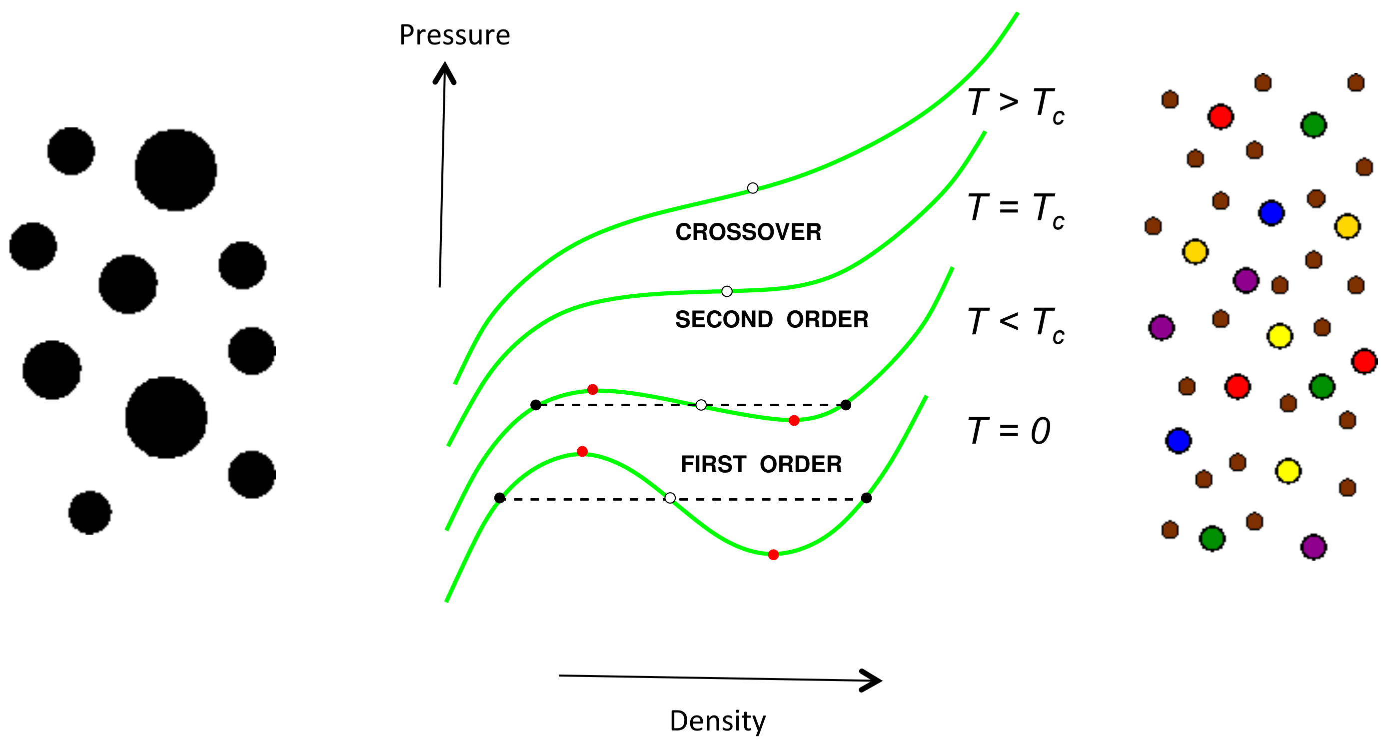


Equation of State

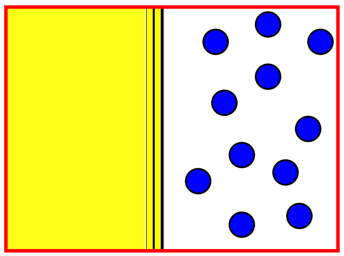
Confined phase:
Hadron gas

?

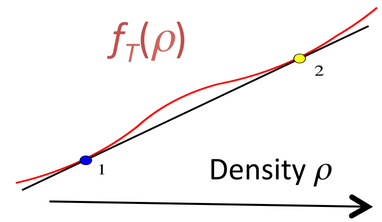
Deconfined phase:
Quark-gluon plasma

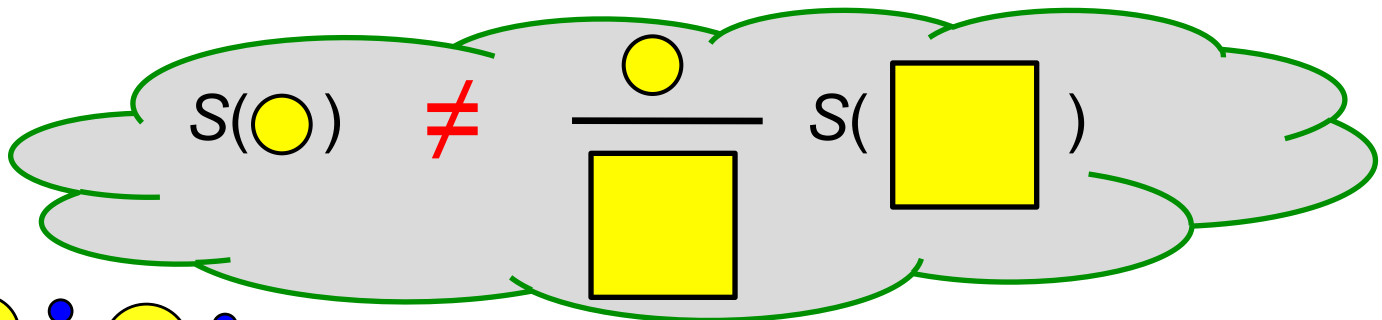


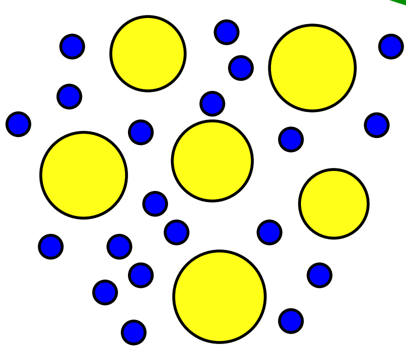
Lecture I: Phase coexistence (equilibrium)



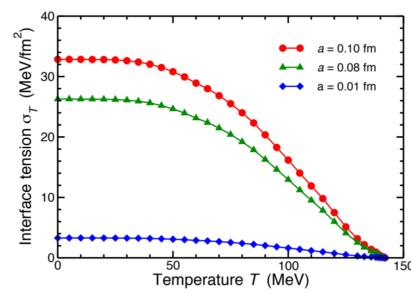
Thermodynamics:
large & uniform systems





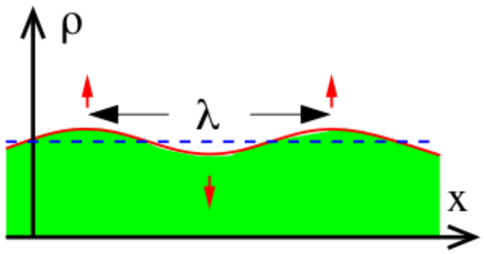


Small & non-uniform:
gradient effects,
interface tension

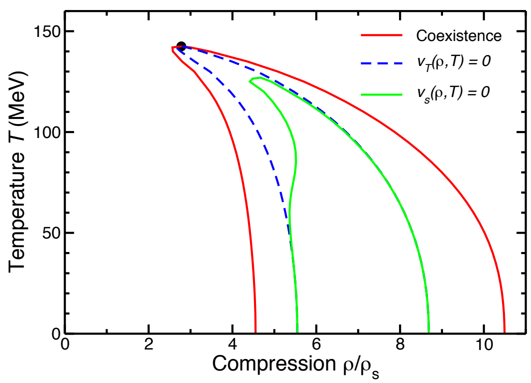


$$\gamma_T = a \int_{\rho_1(T)}^{\rho_2(T)} [2\mathcal{E}_g \Delta f_T(\rho)]^{1/2} \frac{d\rho}{\rho_g}$$

Lecture II: Phase separation (non-equilibrium)



Spinodal instabilities ($v_{sound}^2 < 0$):
Dispersion relation for the amplification of irregularities



Two-phase equation of state $p(\varepsilon,\rho)$:
Interpolate between hadron gas & quark-gluon plasma

$$0 = \partial_\mu T^{\mu\nu}$$
$$\partial_t \rho \doteq -\rho \partial_i v^i$$

Dynamical model for $\varepsilon(t)$ & $\rho(t)$:
Ideal and dissipative fluid dynamics

Hadron Gas versus Quark-Gluon Plasma

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

Free pions, nucleons, and antinucleons:

$$p_\pi(T) = -g_\pi T \int_{m_\pi}^{\infty} \frac{p\varepsilon d\varepsilon}{2\pi^2} \ln[1 - \mathrm{e}^{-\beta\varepsilon}]$$

$$p_N(T) = -g_N T \int_{m_\pi}^{\infty} \frac{p\varepsilon d\varepsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\varepsilon-\mu_0)}]$$

$$p_{\bar{N}}(T) = -g_N T \int_{m_\pi}^{\infty} \frac{p\varepsilon d\varepsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\varepsilon+\mu_0)}]$$

+ compressional energy density:

$$w(\rho) = \left[-A\left(\frac{\rho}{\rho_s}\right)^\alpha + B\left(\frac{\rho}{\rho_s}\right)^\beta\right]\rho$$

$$p_w(\rho) = \rho^2 \partial_\rho (w(\rho)/\rho)$$

$$\mu = \mu_0 + \partial_\rho w = 3\mu_q$$

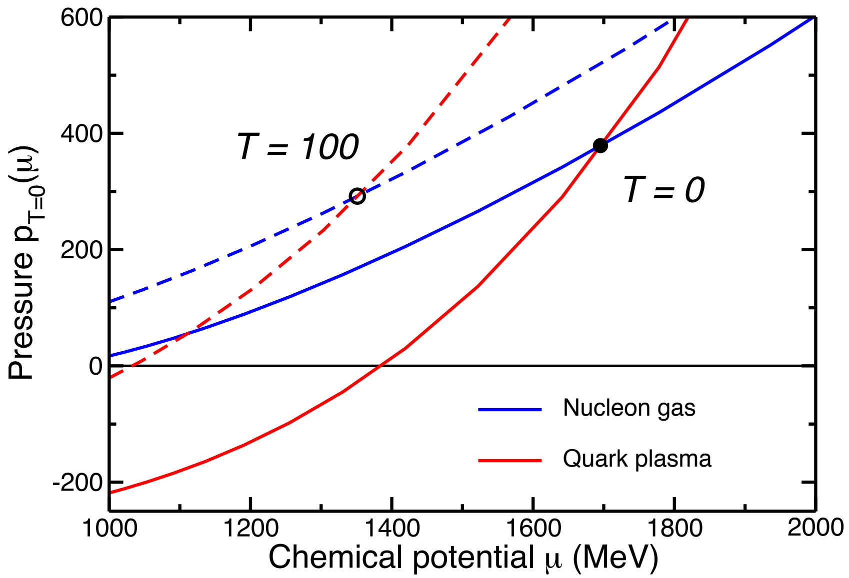
$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

Free gluons, quarks, and antiquarks:

$$p_g = g_g \frac{\pi^2}{90} T^4$$

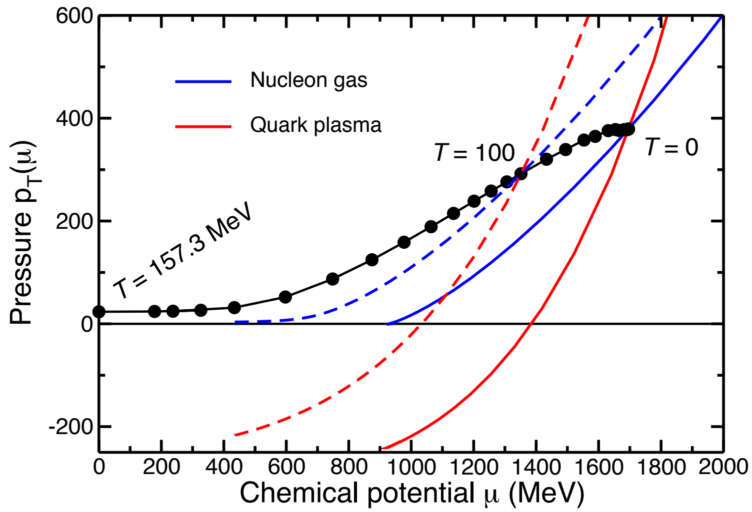
$$p_q + p_{\bar{q}} = g_q \left[\frac{7\pi^2}{360} T^4 + \frac{1}{12}\mu_q^2 T^2 + \frac{1}{24\pi^2}\mu_q^4\right]$$

Phase crossing:

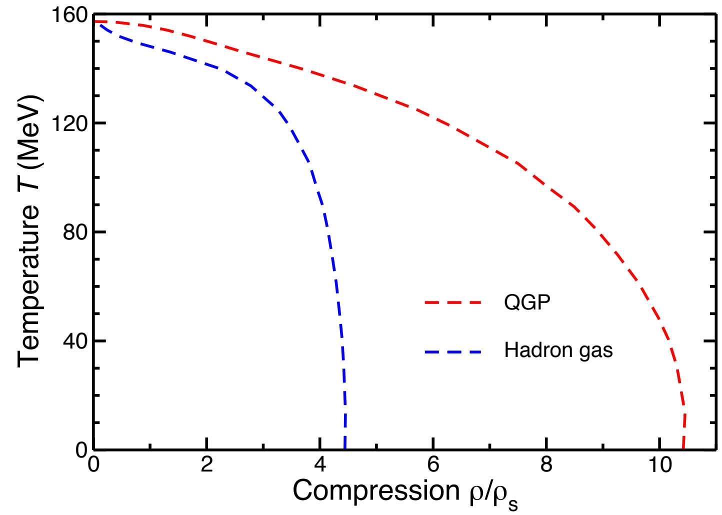


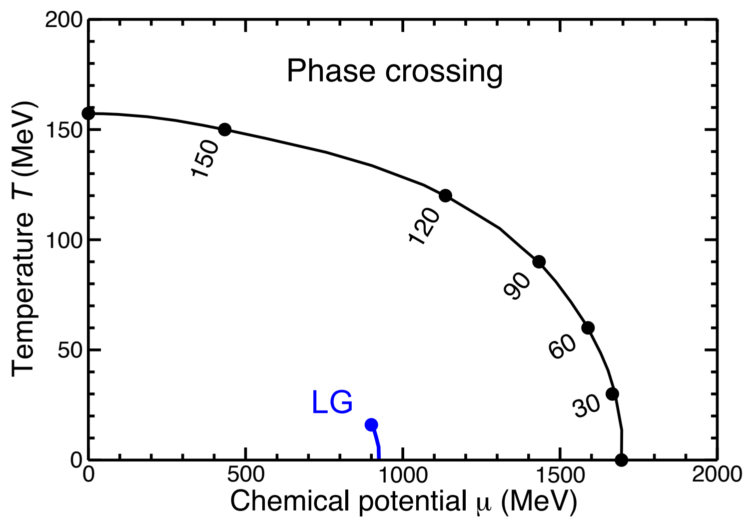
Hadron Gas versus Quark-Gluon Plasma

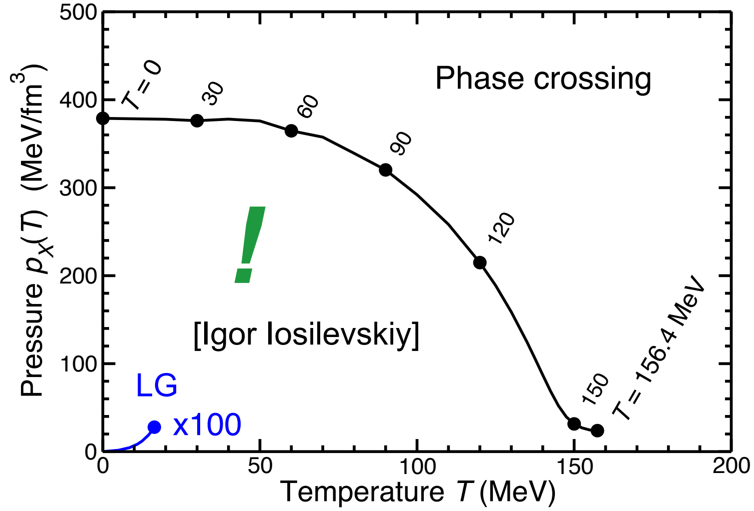
Phase crossing



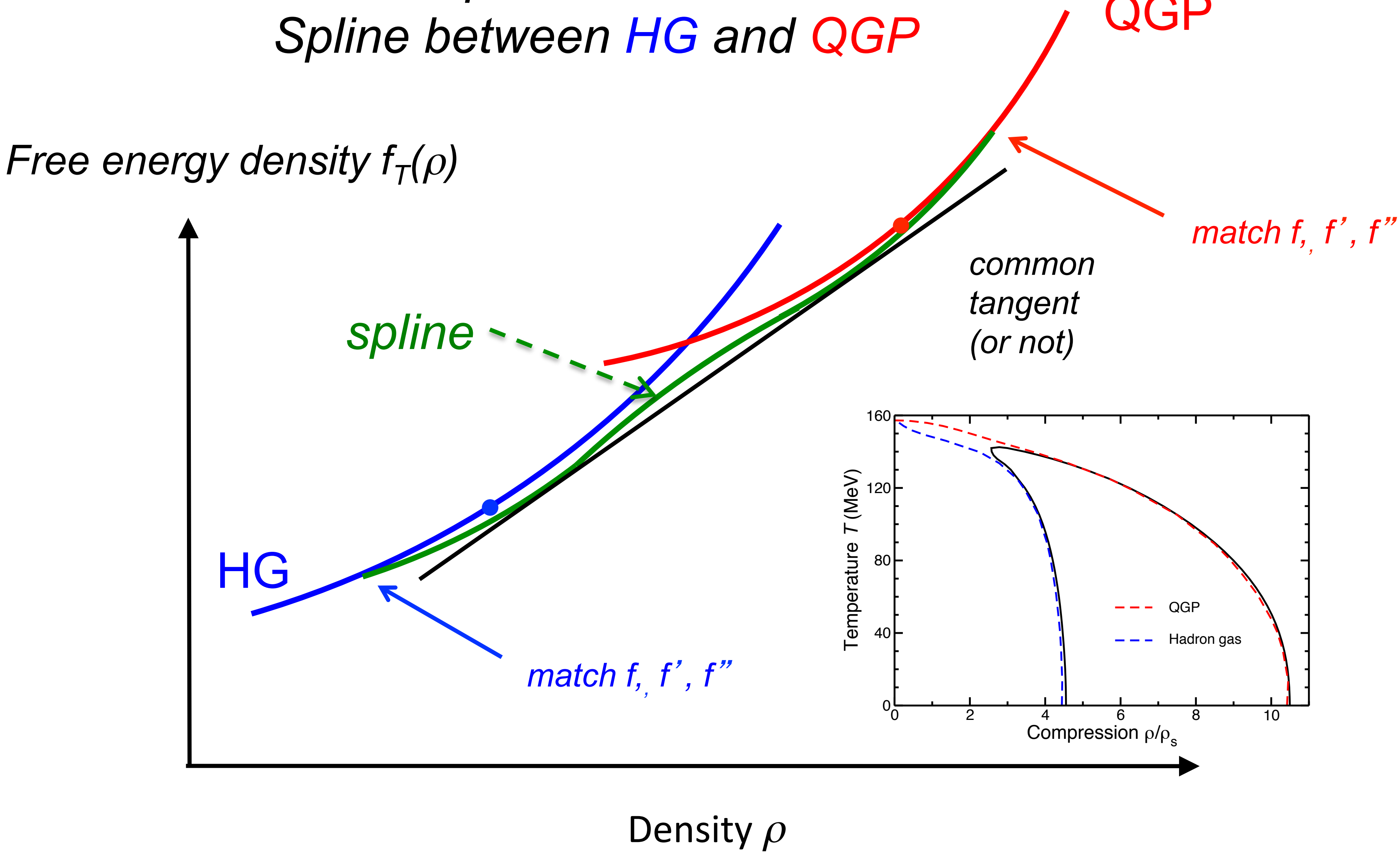
Phase "boundaries"



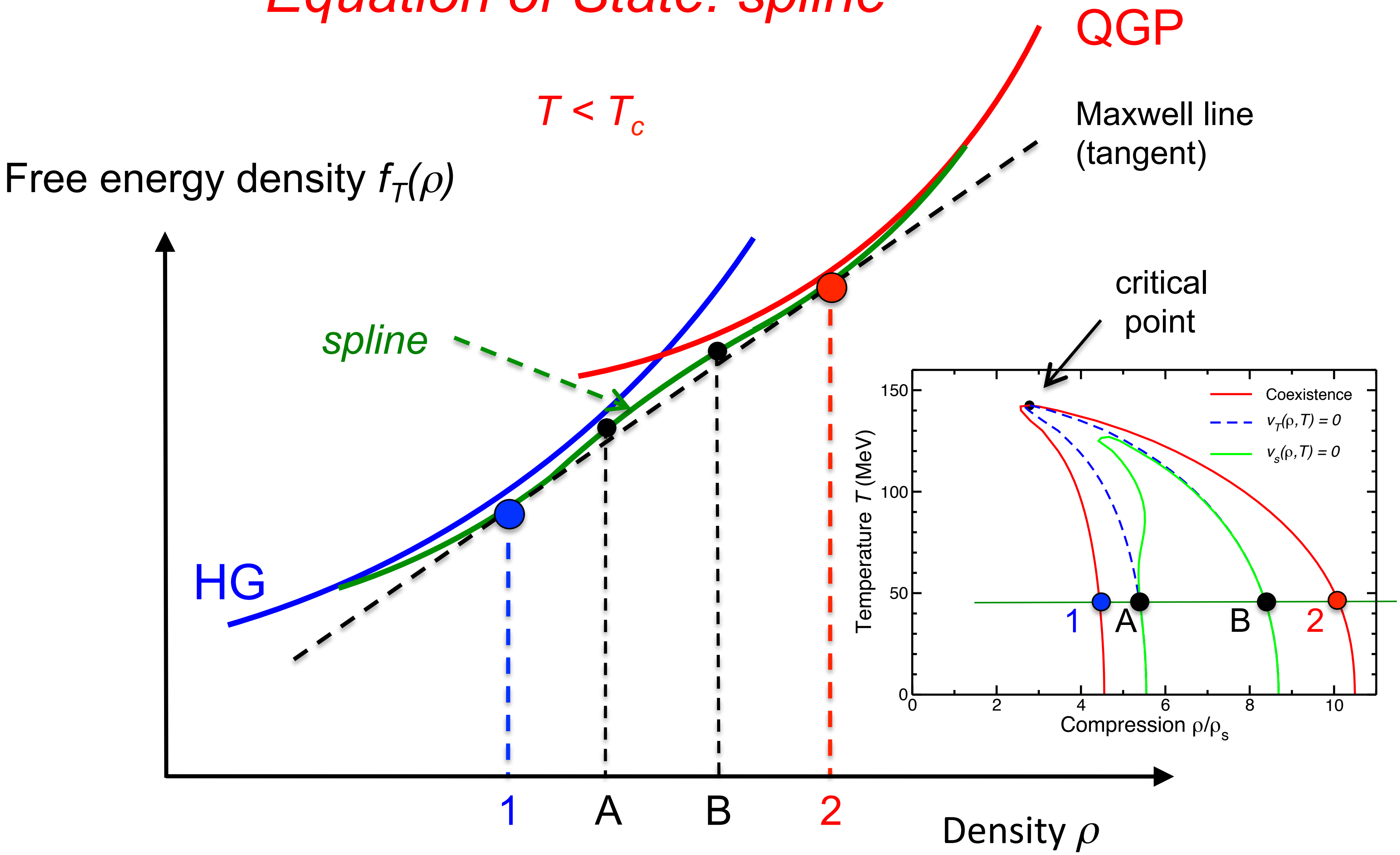




Equation of state:
Spline between HG and QGP



Equation of State: spline



Thermodynamic relations

$$\sigma(\varepsilon,\rho) \qquad \pi = \sigma - \beta\varepsilon - \alpha\rho = p/T$$

$$\beta = \partial_\varepsilon \sigma(\varepsilon,\rho) = \sigma_\varepsilon = 1/T$$

$$\alpha = \partial_\rho \sigma(\varepsilon,\rho) = \sigma_\rho = -\mu/T$$

$$f = \varepsilon - T\sigma$$

$$\mu_T(\rho) = \partial_\rho f_T(\rho) \qquad p_T(\rho) = \rho\partial_\rho f_T(\rho) - f_T(\rho)$$

$$\sigma_T(\rho) = -\partial_T f_T(\rho) \qquad \varepsilon_T(\rho) = f_T(\rho) - T\partial_T f_T(\rho)$$

$$h_T(\rho) = p_T(\rho) + \varepsilon_T(\rho) = \rho\partial_\rho f_T(\rho) - T\partial_T f_T(\rho)$$ *Enthalpy density*

$$v_T^2 = \frac{\rho}{h}\left(\frac{\partial p}{\partial \rho}\right)_T = -\frac{\rho}{h}\frac{\rho T}{\sigma_{\varepsilon\varepsilon}}[\sigma_{\varepsilon\varepsilon}\sigma_{\rho\rho} - \sigma_{\varepsilon\rho}^2]$$ *Isothermal sound speed*

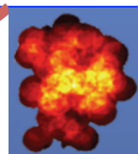
$$v_s^2 = \frac{\rho}{h}\left(\frac{\partial p}{\partial \rho}\right)_s = -\frac{T}{h}[h^2\sigma_{\varepsilon\varepsilon} + 2h\rho\sigma_{\varepsilon\rho} + \rho^2\sigma_{\rho\rho}]$$ *Isentropic sound speed*

Dynamical models for HIC

From the lecture by E. Bratskovskaya:

Macroscopic

hydro-models:
- description of QGP and hadronic phase by hydrodanamical equations for fluid
- assumption of local equilibrium
- EoS with phase transition from QGP to HG
- initial conditions (e-b-e, fluctuating)



ideal
(Jyväskylä,SHASTA, TAMU, …)

viscous
(Romachkke,(2+1)D VISH2+1, (3+1)D MUSIC,…)

fireball models:
- no explicit dynamics: parametrized time evolution (TAMU)

‚Hybrid'
QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

(‚hybrid'-UrQMD, EPOS, …)

Microscopic

Non-equilibrium microscopic transport models – based on many-body theory

Hadron-string models
(UrQMD, IQMD, HSD, QGSM …)

Partonic cascades pQCD based
(Duke, BAMPS, …)

Parton-hadron models:
- QGP: pQCD based cascade
- massless q, g
- hadronization: coalescence

(AMPT, HIJING)

PHSD
- QGP: lQCD EoS
- massive quasi-particles (q and g with spectral functions) in self-generated mean-field
- dynamical hadronization
- HG: off-shell dynamics (applicable for strongly interacting systems)

Ideal fluid dynamics without conserved flavors

$\eta, \zeta, \kappa = 0$

Stress tensor: $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ $\qquad u^\mu = (\gamma, \gamma\mathbf{v})$

$$0 = \partial_\mu T^{\mu\nu} \begin{cases} \nu = 0: \quad 0 = \partial_\mu T^{\mu 0} = \partial_t(\varepsilon + pv^2)\gamma^2 + \partial_i(\varepsilon + p)\gamma^2 v^i & E \\ \nu = i: \quad 0 = \partial_\mu T^{\mu i} = \partial_t(\varepsilon + p)\gamma^2 v^i + \partial_j(\varepsilon + p)\gamma^2 v^j v^i + \partial^i p & P \end{cases}$$

Non-relativistic flow (v << 1):

$$\begin{cases} \nu = 0: \quad \partial_t \varepsilon = -\partial_i(\varepsilon + p)v^i & E \\ \nu = i: \quad \partial_t(\varepsilon + p)v^i = -\partial^i p & P \end{cases}$$

$\partial_t E - \partial_i P_i$:

$$\partial_t^2 \varepsilon(x) = \partial_i \partial^i p(x)$$

Sound equation

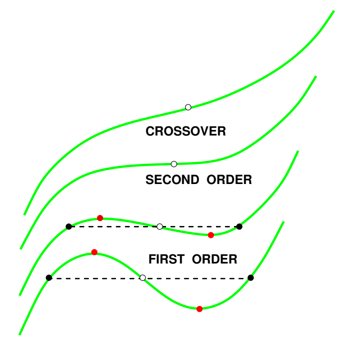
Equation of state: $p_0(\varepsilon)$

$$p(x) = p_0(\varepsilon(x)) \Rightarrow \partial_i \partial^i p(x) = \frac{\partial p_0(\varepsilon)}{\partial \varepsilon} \partial_i \partial^i \varepsilon(x)$$

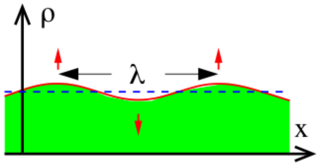
$$\partial_t^2 \varepsilon = v_s^2 \nabla^2 \varepsilon$$

$$v_s^2 \equiv \partial_\varepsilon p_0$$

(sound speed)^2



Evolution of small disturbances



Small disturbance in a uniform stationary fluid

$$\varepsilon(x,t) \;=\; \varepsilon_0 \;+\; \delta\varepsilon(x,t)\;,\;\; \delta\varepsilon \ll \varepsilon_0$$

=> $v << 1$

First order in $\delta\varepsilon$:

$$\begin{cases} \partial_t \delta\varepsilon(x,t) \;\approx\; (\varepsilon_0 + p_0)\partial_x v_x(x,t) \\ (\varepsilon_0 + p_0)\partial_t v_x(x,t) \;\approx\; \partial_x p(x,t) \;\approx\; \dfrac{\partial p_0}{\partial \varepsilon_0}\,\partial_x \delta\varepsilon(x,t) \end{cases}$$

Sound equation:

$$\boxed{\partial_t^2 \delta\varepsilon(x,t) \;=\; \frac{\partial p_0}{\partial \varepsilon_0}\,\partial_x^2 \delta\varepsilon(x,t)}$$

$$v_s^2 \;=\; \frac{\partial p}{\partial \epsilon}$$

Harmonic disturbance: $\delta\varepsilon_k(x,t) \;\sim\; \mathrm{e}^{ikx - i\omega_k t}$

Dispersion relation:

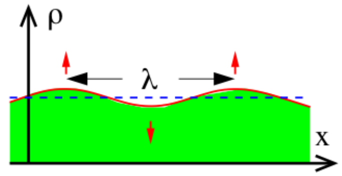
$$\boxed{\omega_k^2 \;=\; v_s^2\, k^2}$$

$$\begin{cases} v_s^2 > 0: \quad \omega_k = \pm v_s k \\ v_s^2 < 0: \quad \omega_k = \pm i\gamma_k = \pm i|v_s|k \end{cases}$$

Diverges for large k!

Ideal fluid dynamics with one conserved flavor

η, ζ, κ = 0



$$\left.\begin{aligned}\varepsilon(x) &= \varepsilon_0 + \delta\varepsilon(x)\\ \rho(x) &= \rho_0 + \delta\rho(x)\end{aligned}\right\} \quad |v(x)| \ll 1$$

$T^{\mu\nu}(x)$: $\quad T^{00} \approx \varepsilon \qquad T^{i0} = T^{0i} \approx (\varepsilon + p)v^i \qquad T^{ij} = T^{ji} \approx \delta_{ij} p$

E $\quad 0 \doteq \partial_\mu T^{\mu 0} = \partial_t T^{00} + \partial_i T^{i0} = \partial_t \varepsilon + (\varepsilon + p)\partial_i v^i \quad \Rightarrow \quad \omega \varepsilon_k \doteq (\varepsilon_0 + p_0) k v_k$

P $\quad 0 \doteq \partial_\mu T^{\mu i} = \partial_t T^{0i} + \partial_j T^{ji} = (\varepsilon + p)\partial_t v^i + \partial_j T^{ji}$

F $\quad \partial_t \rho \doteq -\rho \partial_i v^i \quad \Rightarrow \quad \omega \rho_k \doteq \rho_0 k v_k$ *Continuity equation*

E & F => $\quad (\varepsilon_0 + p_0)\rho_k = \rho_0 \varepsilon_k$ *ρ tracks ε when κ = 0*

$\partial_t E - \partial_i P_i$ => $\quad \partial_t^2 \varepsilon = \partial_i \partial_j T^{ji} = \partial_i \partial^i p$ **=>** $\omega^2 \varepsilon_k = k^2 p_k$ *Sound equation*

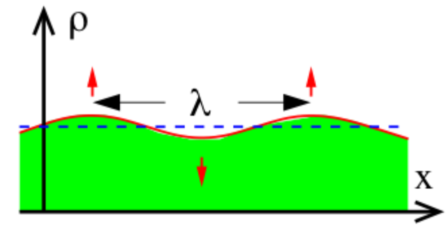
$$p(\varepsilon,\rho) \Rightarrow \quad p_k = \frac{\partial p}{\partial \varepsilon}\varepsilon_k + \frac{\partial p}{\partial \rho}\rho_k = [\frac{\partial p}{\partial \varepsilon} + \frac{\rho_0}{\varepsilon_0 + p_0}\frac{\partial p}{\partial \rho}]\varepsilon_k = v_s^2 \varepsilon_k \qquad v_s^2 \equiv \frac{\rho}{\varepsilon + p}\left(\frac{\partial p}{\partial \rho}\right)_s$$

=> $\quad \omega_k^2 = v_s^2 k^2$ => $\boxed{\gamma_k = |v_s| k}$ *Dispersion relation*

Diverges for large k!

Inclusion of gradient correction

$$\tilde{f}_T(\boldsymbol{r}) \;=\; f_T(\tilde{\rho}(\boldsymbol{r})) + \tfrac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$$



=> $$p(\boldsymbol{r}) \approx p_0(\varepsilon(\boldsymbol{r}),\rho(\boldsymbol{r})) - C\rho_0\nabla^2\rho(\boldsymbol{r})$$

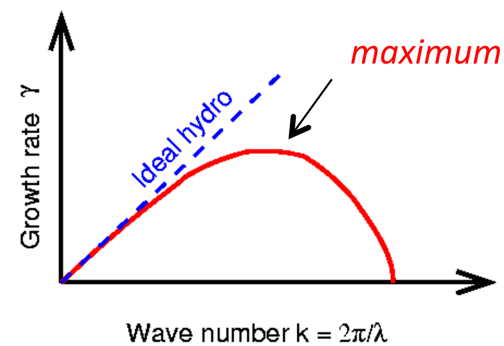
$$\rho(r,t) = \rho_0 + \delta\rho(x,t) \doteq \;\rho_0 + \rho_k e^{ikx-i\omega t}$$

=> $$p_k \;\rightarrow\; p_k + C\rho_0 k^2\rho_k \qquad = [v_s^2 + C\frac{\rho_0^2}{\varepsilon_0+p_0}k^2]\varepsilon_k$$

resists wriggling

=> $$\omega_k^2 = v_s^2 k^2 + C\frac{\rho_0^2}{\varepsilon_0+p_0}k^4$$

=> $$\gamma_k^2 = |v_s^2|k^2 - C\frac{\rho_0^2}{\varepsilon_0+p_0}k^4$$

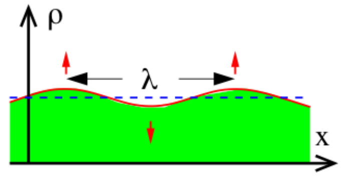


Inclusion of finite range in EoS => favored length scale!

Viscous fluid dynamics with one conserved flavor

$\eta, \zeta > 0$

$\kappa = 0$



$$\left.\begin{array}{l}\varepsilon(x) = \varepsilon_0 + \delta\varepsilon(x) \\ \rho(x) = \rho_0 + \delta\rho(x)\end{array}\right\} \quad |v(x)| \ll 1$$

$T^{\mu\nu}(x)$:
$$\begin{cases} T^{00} \approx \varepsilon \qquad T^{i0} = T^{0i} \approx (\varepsilon + p)v^i \\ T^{ij} = T^{ji} \approx \delta_{ij}p - \eta[\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v}] - \zeta\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v} \end{cases}$$

=> $$\boldsymbol{\nabla}\cdot\boldsymbol{T} \approx \boldsymbol{\nabla}p - \eta\boldsymbol{\Delta}v - [\tfrac{1}{3}\eta + \zeta]\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v})$$

E $$0 \doteq \partial_\mu T^{\mu 0} = \partial_t T^{00} + \partial_i T^{i0} = \partial_t\varepsilon + (\varepsilon + p)\partial_i v^i \quad \Rightarrow \quad \omega\varepsilon_k \doteq (\varepsilon_0 + p_0)kv_k$$

P $$0 \doteq \partial_\mu T^{\mu i} = \partial_t T^{0i} + \partial_j T^{ji} = (\varepsilon + p)\partial_t v^i + \partial_j T^{ji}$$

F $$\partial_t\rho \doteq -\rho\partial_i v^i \quad \Rightarrow \quad \omega\rho_k \doteq \rho_0 k v_k$$
← *Continuity equation*

E & F => $$(\varepsilon_0 + p_0)\rho_k = \rho_0\varepsilon_k$$
← *ρ tracks ε when κ = 0*

$\partial_t E$ - $\partial_i P_i$ => $$\partial_t^2\varepsilon = \partial_i\partial_j T^{ji} = \partial_i\partial^i[p - (\tfrac{\mathbf{4}}{\mathbf{3}}\eta + \zeta)\partial_j v^j]$$
← *Sound equation*

=> $$\omega^2\varepsilon_k = k^2 p_k - i\xi k^3 v_k = v_s^2 k^2\varepsilon_k - i\xi\frac{\omega}{\varepsilon_0 + p_0}k^2\varepsilon_k$$

$$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

=> $$\boxed{\gamma_k^2 = |v_s^2|k^2 - C\frac{\rho_0^2}{\varepsilon_0 + \rho_0}k^4 - \xi\frac{k^2}{\varepsilon_0 + p_0}\gamma_k}$$
← *Dispersion relation*

$\eta, \zeta, \kappa > 0 \;\longrightarrow$ *Dissipative fluid dynamics*

Energy-momentum tensor:

$$T^{00} \approx \varepsilon \quad \& \quad T^{0i} \approx (\varepsilon + p)v^i + q^i \quad \&$$

A Muronga, PRC 76, 014909 (2007)

$$T^{ij} \approx \delta_{ij} p - \eta[\partial_i v^j + \partial_j v^i - \tfrac{2}{3}\delta_{ij}\partial^k v^k] - \zeta \delta_{ij}\partial^k v^k$$

$$|\rho_k| \ll \rho_0 \;\Rightarrow\; |v| \ll 1$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{T} \;\approx\; \boldsymbol{\nabla} p - \eta \Delta \boldsymbol{v} - [\tfrac{1}{3}\eta + \zeta]\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v}) \;\asymp\; \partial_x p - [\tfrac{4}{3}\eta + \zeta]\partial_x^2 v$$

Eckart frame

Equations of motion:

$$C: \;\; \partial_t \rho \;\doteq -\rho_0 \boldsymbol{\nabla}\cdot\boldsymbol{v} \;\Rightarrow\; \omega\rho_k \doteq \rho_0 k v_k \qquad \text{flavor}$$

$$\boldsymbol{M}: \;\; h_0 \partial_t \boldsymbol{v} \;\doteq -\boldsymbol{\nabla}[p - \zeta \boldsymbol{\nabla}\cdot\boldsymbol{v}] - \boldsymbol{\nabla}\cdot\boldsymbol{\pi} - \partial_t \boldsymbol{q} \qquad \text{momentum}$$

$$E: \;\; \partial_t \varepsilon \;\doteq -h_0 \boldsymbol{\nabla}\cdot\boldsymbol{v} - \boldsymbol{\nabla}\cdot\boldsymbol{q} \qquad \text{energy}$$

Sound equation:

$$\partial_t E - \boldsymbol{\nabla}\cdot\boldsymbol{M}: \;\; h_0 \partial_t^2 \varepsilon \;\doteq\; \Delta[p - \zeta \boldsymbol{\nabla}\cdot\boldsymbol{v}] + \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\boldsymbol{\pi})$$

=> $$\omega^2 \varepsilon_k \;\doteq\; k^2 p_k - i[\tfrac{4}{3}\eta + \zeta]\frac{\omega}{\rho_0}k^2 \rho_k$$

$$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

Heat flow:

$$\boldsymbol{q} \approx -\kappa[\boldsymbol{\nabla} T + T_0 \partial_t \boldsymbol{v}]: \;\; q_k = -i\kappa[kT_k - \frac{T_0}{\rho_0}\frac{\omega^2}{k}\rho_k]$$

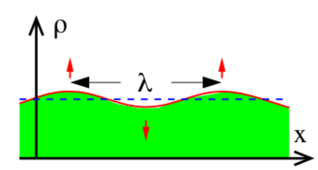
$$T_k \approx \frac{1}{1 + i\kappa k^2/\omega c_v}\frac{T_0}{\rho_0}\left(\frac{\partial p}{\partial \varepsilon}\right)_\rho \rho_k$$

Equation of state:

$$p_T(\rho) \;\Rightarrow\; p_k = \left(\frac{\partial p}{\partial \varepsilon}\right)_\rho c_v T_k + \frac{h_0}{\rho_0} v_T^2 \rho_k$$

Dispersion equation:

$$\omega^2 \;\doteq\; v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2$$



J Randrup, PRC 82, 034902 (2010)

Transport coefficients

$$\eta_0 \geq 1$$
$$\kappa_0 \geq 1$$

1) Bulk viscosity ζ : Ignore $\quad \zeta \ll \eta \Rightarrow \xi \equiv \frac{4}{3}\eta + \zeta \approx \frac{4}{3}\eta$

2) Shear viscosity η :

$$\rho = 0: \quad h \equiv p + \varepsilon = T\sigma$$

*) $$\rho = 0: \quad \eta \geq \frac{\hbar}{4\pi}\sigma = \frac{\hbar}{4\pi}\frac{h}{T}$$

$$\rho > 0,\ T \ll mc^2: \ h \asymp mc^2 n \gg T\sigma$$

$$\rho = 0: \quad n \sim T^3 \ \Rightarrow \ \frac{\hbar c}{T} = 4\pi c_0\, d \qquad d \equiv n^{1/3}$$

$$\eta(\rho,T) = \eta_0 \frac{c_0}{c} d(\rho,T)\, h(\rho,T)$$

$$\lambda_{\rm visc} \equiv \frac{1}{c}\frac{\xi(\rho,T)}{h(\rho,T)/c^2} \approx \tfrac{4}{3}\eta_0\, c_0\, d(\rho,T)$$

3) Heat conductivity κ :

$$\eta \approx \tfrac{1}{3} n\bar{p}\ell \qquad \kappa \approx \tfrac{1}{3}\bar{v}\ell c_v$$

$$\frac{\kappa}{\eta} \approx \frac{c_v}{h/c^2}$$

$$\bar{p} = m\bar{v} \qquad c_v \equiv \partial_T \varepsilon_T(\rho)$$

$$h \asymp mc^2 n \qquad c_v \asymp \tfrac{3}{2}n$$

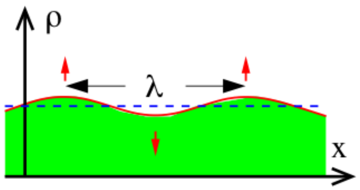
$$\kappa(\rho,T) = \kappa_0\, c_0\, c\, d(\rho,T)\, c_v(\rho,T)$$

$$\lambda_{\rm heat} \equiv \frac{1}{c}\frac{\kappa(\rho,T)}{c_v(\rho,T)} = \kappa_0\, c_0\, d(\rho,T)$$

*) V Koch & J Liao, PRC81, 014902 (2010)

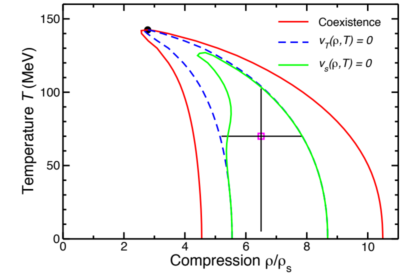
$$c_0 = \frac{1}{4\pi}\left[(g_g + \tfrac{3}{3}g_q)\frac{\zeta(3)}{\pi^2}\right]^{\frac{1}{3}} \approx 0.12779$$

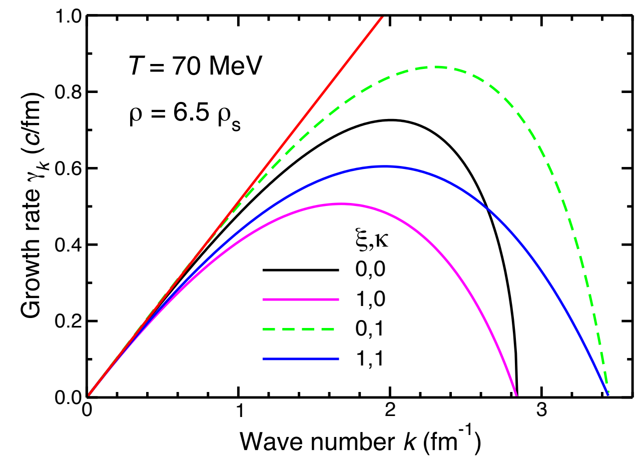
Evolution of density fluctuations with dissipative fluid dynamics



$A_k \sim \exp(-i\omega_k t)$

$$\omega_k = \epsilon_k + i\gamma_k$$





Dispersion equations:

Ideal fluid dynamics: $$\omega^2 \doteq v_s^2 k^2$$

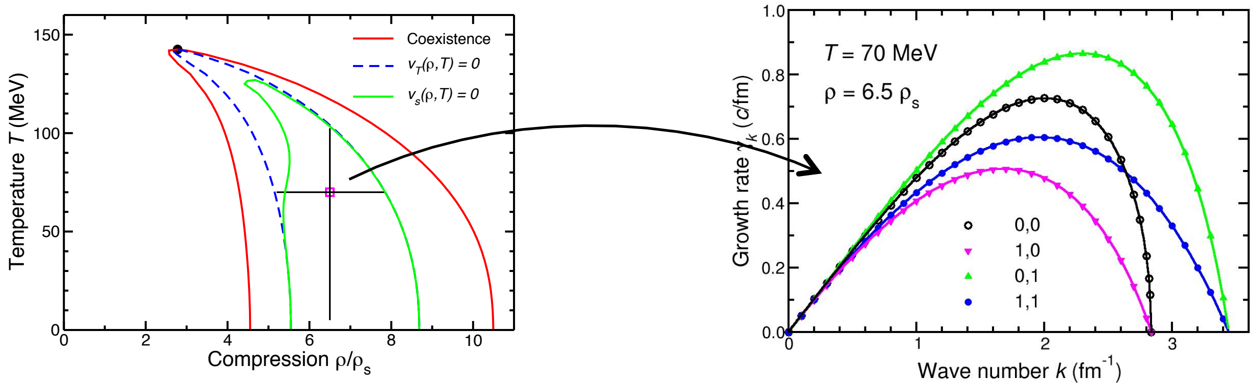
+ gradient term: $$\omega^2 \doteq v_s^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 \quad \Leftarrow \quad p_k \to p_k + C\rho_0 k^2 \rho_k$$

+ shear & bulk viscosity: $$\omega^2 \doteq v_s^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 \quad \Leftarrow \quad \xi \equiv \tfrac{4}{3}\eta + \zeta$$

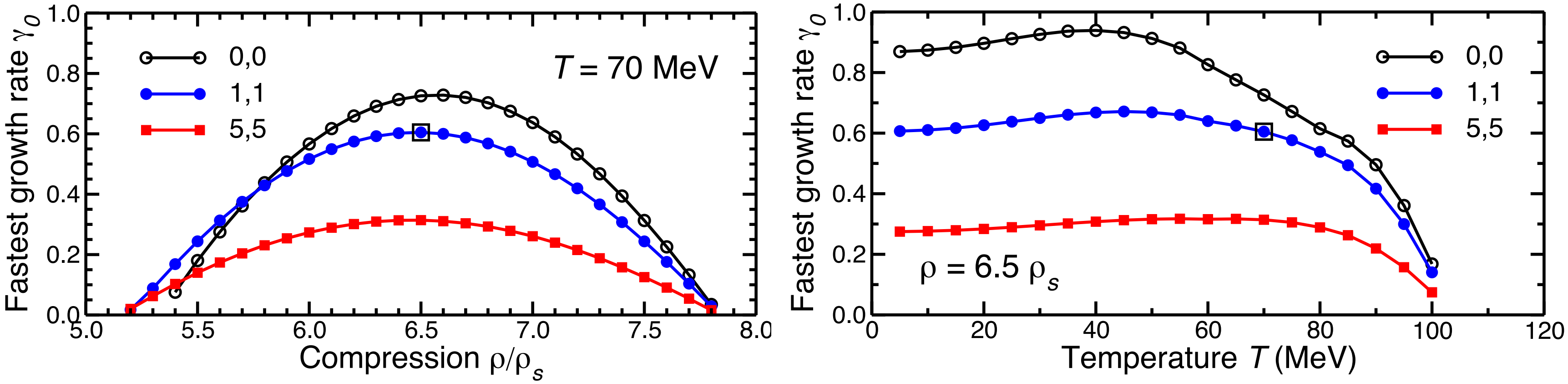
+ heat conduction: $$\omega^2 \doteq v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2 \quad \Leftarrow \quad \kappa$$

Spinodal growth rates

$$(\rho,T):\quad \omega^2 \doteq v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2 \quad \Rightarrow \quad \gamma_k(\rho,T)$$



Fastest growth rates:

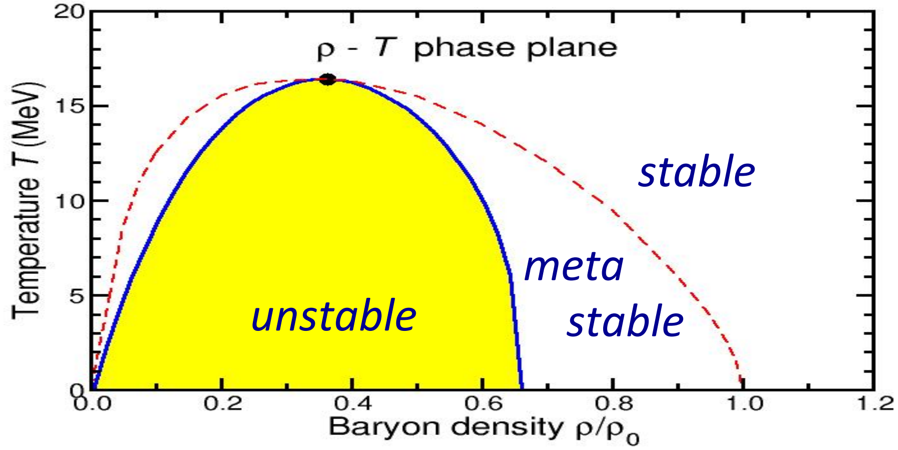


APPLICATION:

Use of spinodal instabilities to reveal
the nuclear liquid-gas phase transition

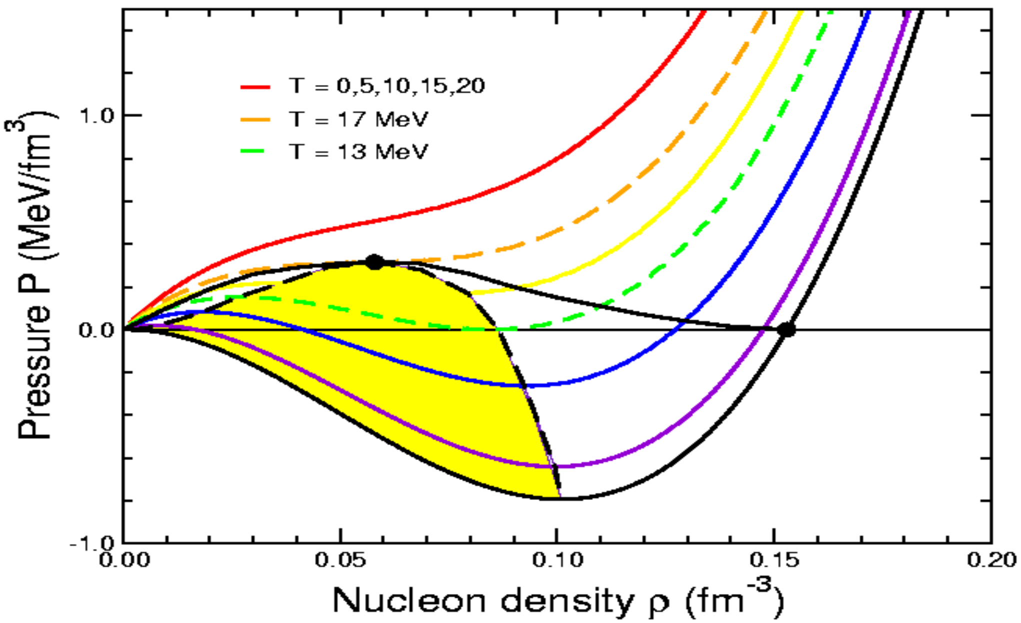
Basic idea:
Spinodal amplification favors a certain length scale,
so all the nuclear fragments should be of similar size

Nuclear matter



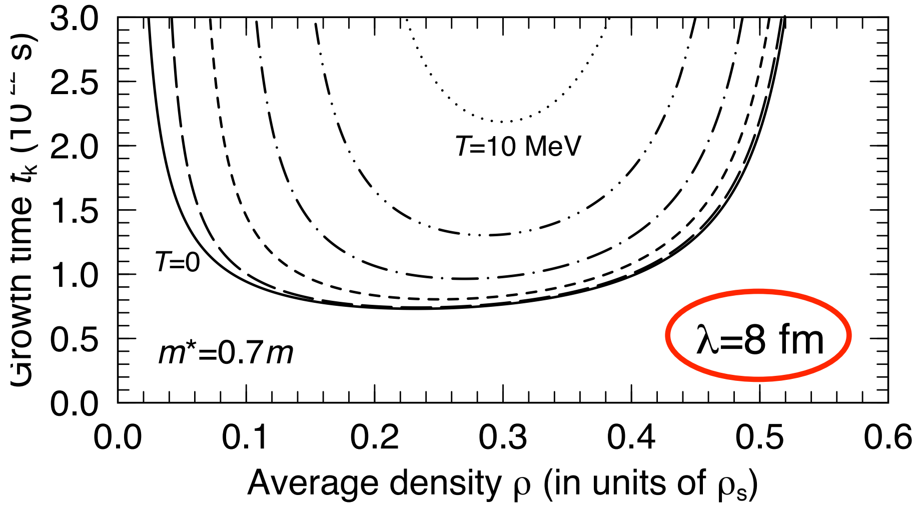
(ρ,T) phase diagram

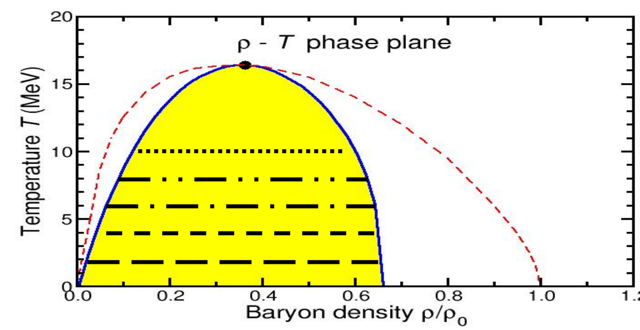
$$\varepsilon(T;\rho) = \varepsilon_{\rm FG}(T;\rho) + w(\rho)$$



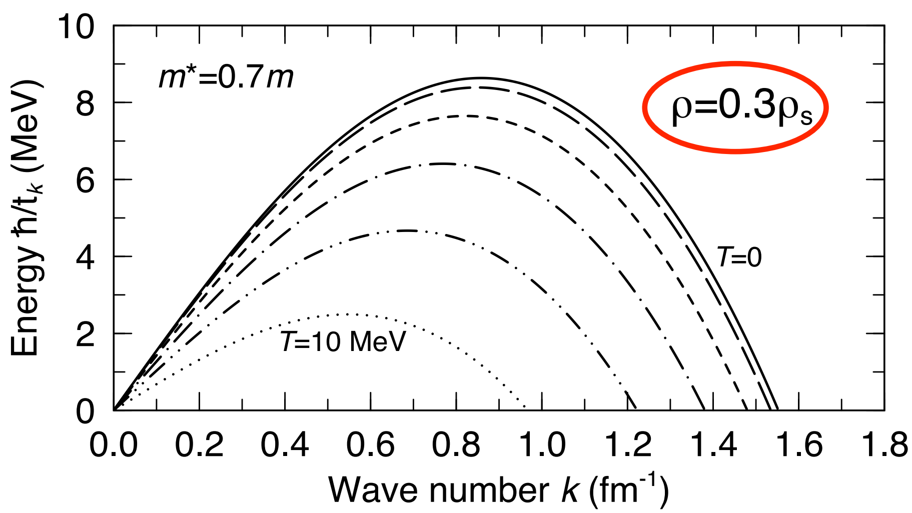
Equation of state:
$p_T(\rho)$

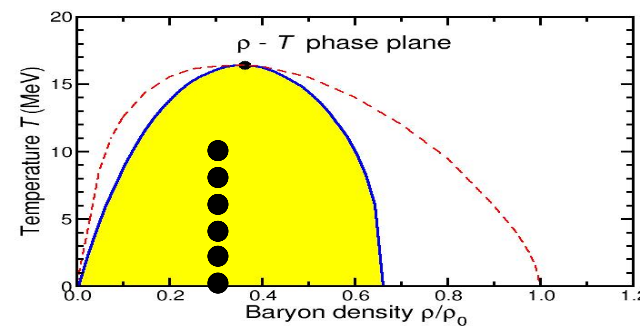
Dependence of growth rates on density, temperature, and wave length:



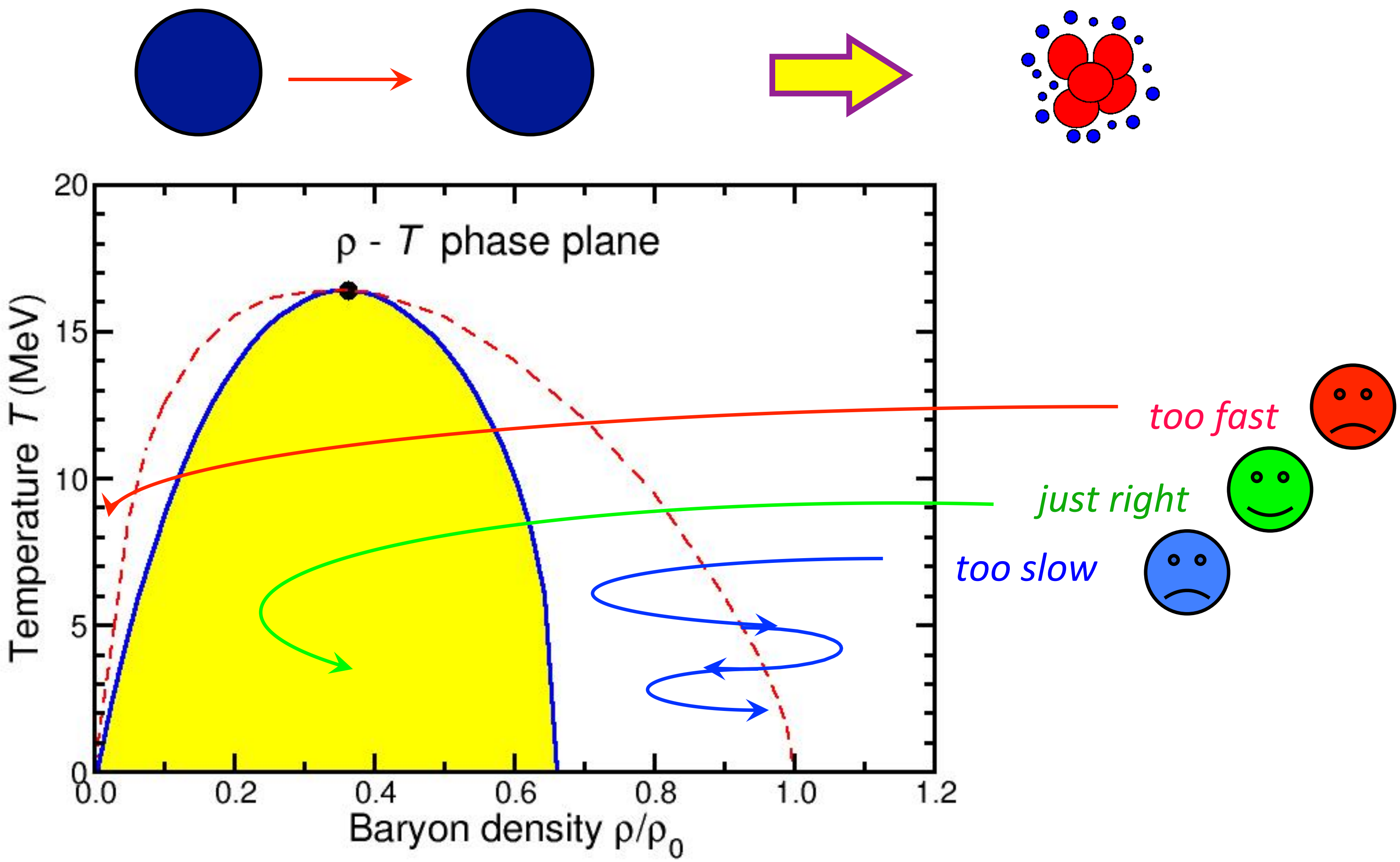


$2\pi/8 \approx 0.8$



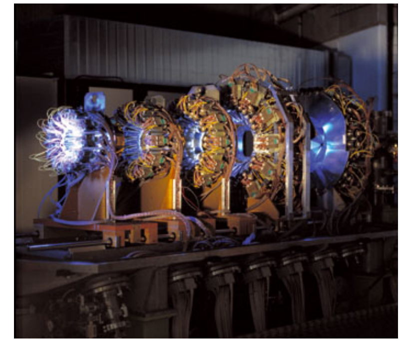


Optimal collision energy



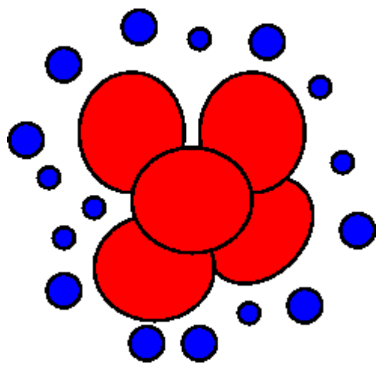
Experiment: *INDRA @ GANIL*

B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252



INDRA

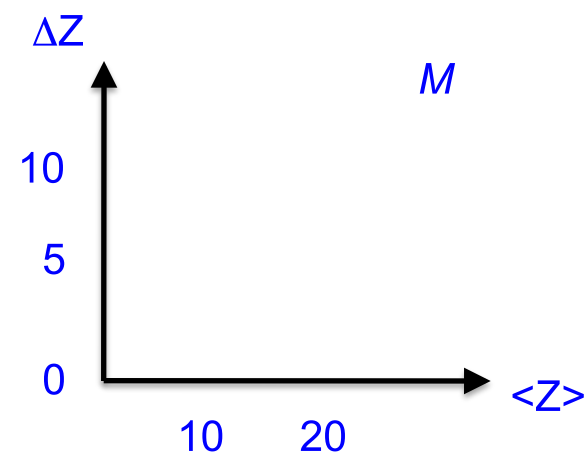
32 MeV/A Xe + Sn (b=0)



Analysis:

For each event having M IMFs, calculate mean IMF charge <Z> and IMF charge dispersion ΔZ.
(suggested by L.G. Moretto)

Make a LEGO plot of (<Z>, ΔZ):



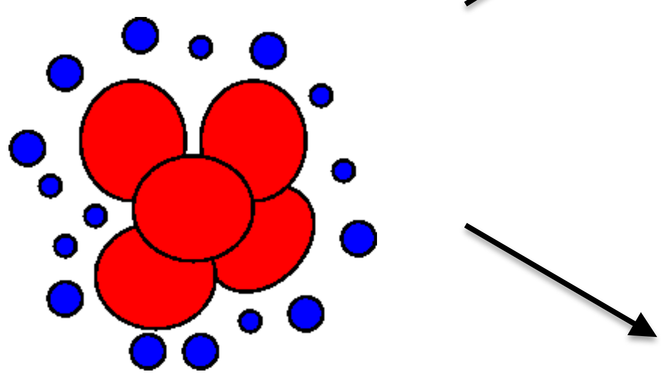
Transport calculations

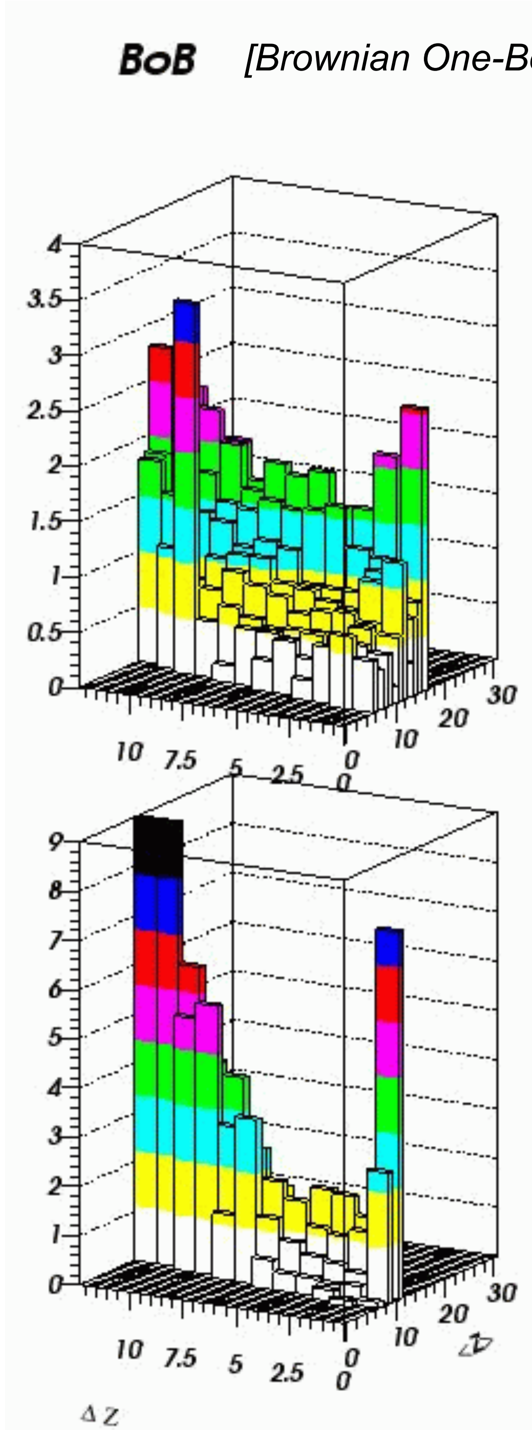
… suggest a visible spinodal signal:

Brownian One-Body dynamics *)
≈ Boltzmann-Langevin

$$\delta K[f] \;\rightarrow\; -\delta \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}}$$

32 MeV/A Xe + Sn (b=0):





*) Ph. Chomaz, M. Colonna, A. Guarnera, J. Randrup, Physical Review Letters 73 (1994) 3512

Experiment: *INDRA @ GANIL*

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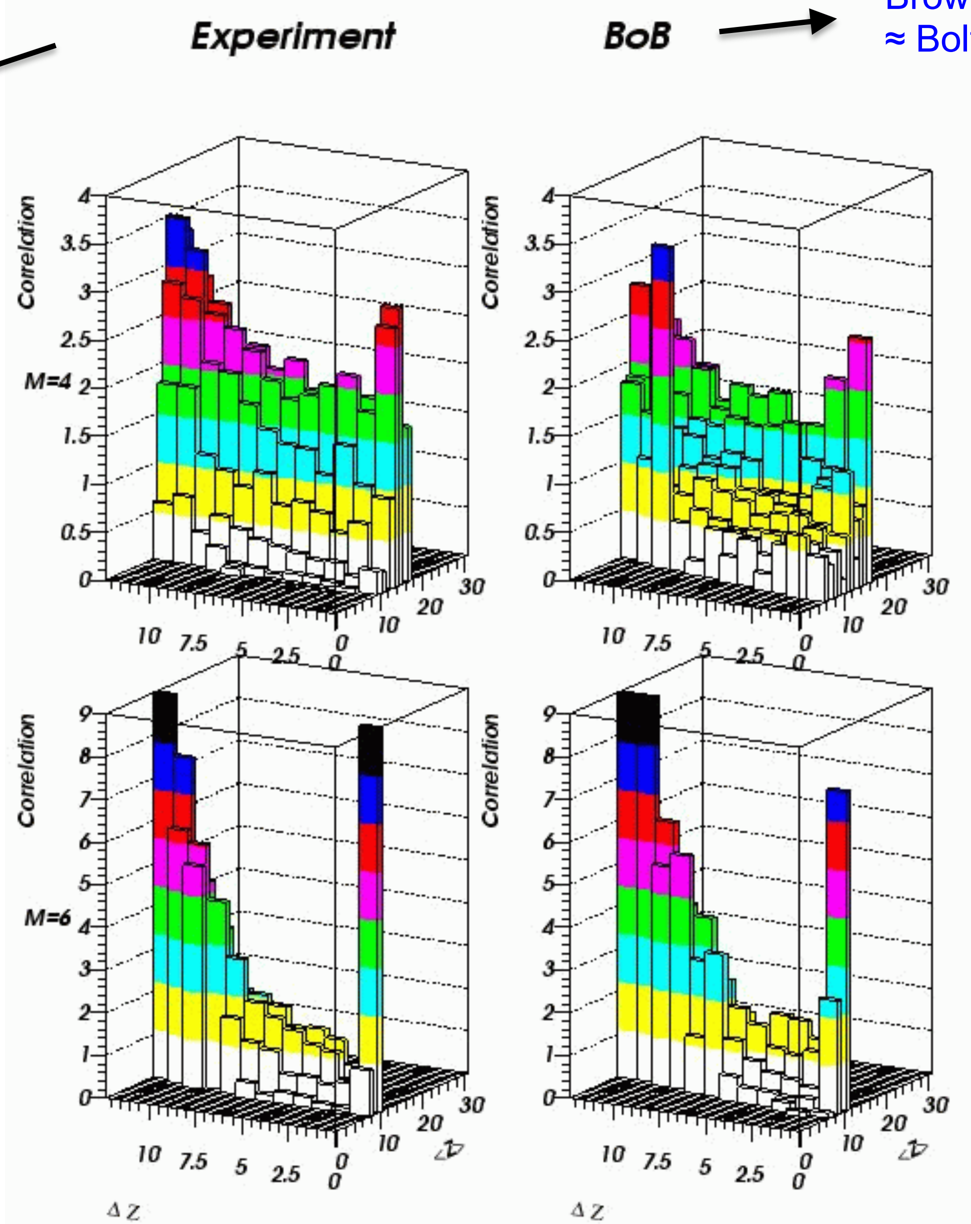


Ph. Chomaz *et al*, Phys. Rev. Lett. 73 (1994) 3512

Use of spinodal instabilities to reveal the nuclear liquid-gas phase transition

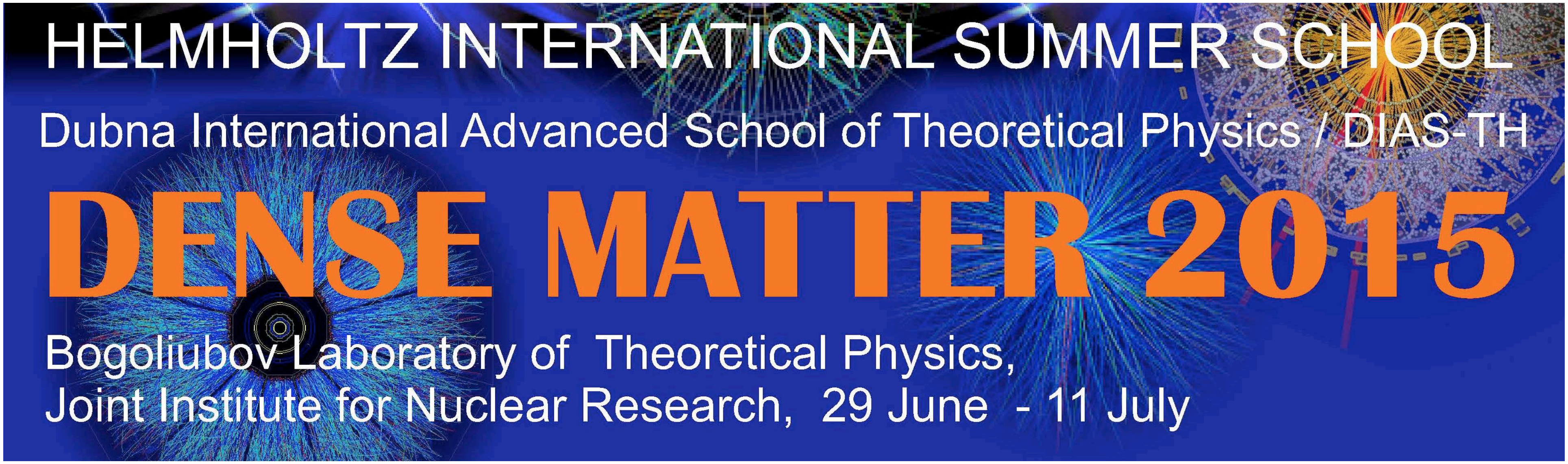
LESSONS:

A first-order phase transition
implies existence of instabilities

Such instabilities can have large dynamical effects
- provided that the conditions are suitably chosen

These effect may be seen experimentally
- If the data is analyzed appropriately

Ph. Chomaz, M. Colonna, J. Randrup
Physics Reports 389 (2004) 263-440



Spinodal Instabilities at the Deconfinement Phase Transition

Jørgen Randrup (Berkeley)

Lecture I:	*Phase coexistence (equilibrium)*	WED 12:00-13:00
Lecture II:	*Phase separation (non-equilibrium)*	THU 16:30-17:30
	Discussion	18:00-19:00
Lecture III:	*Effects on collision dynamics (clumping)*	FRI 11:00-12:00