Neutrino Processes in Neutron Stars

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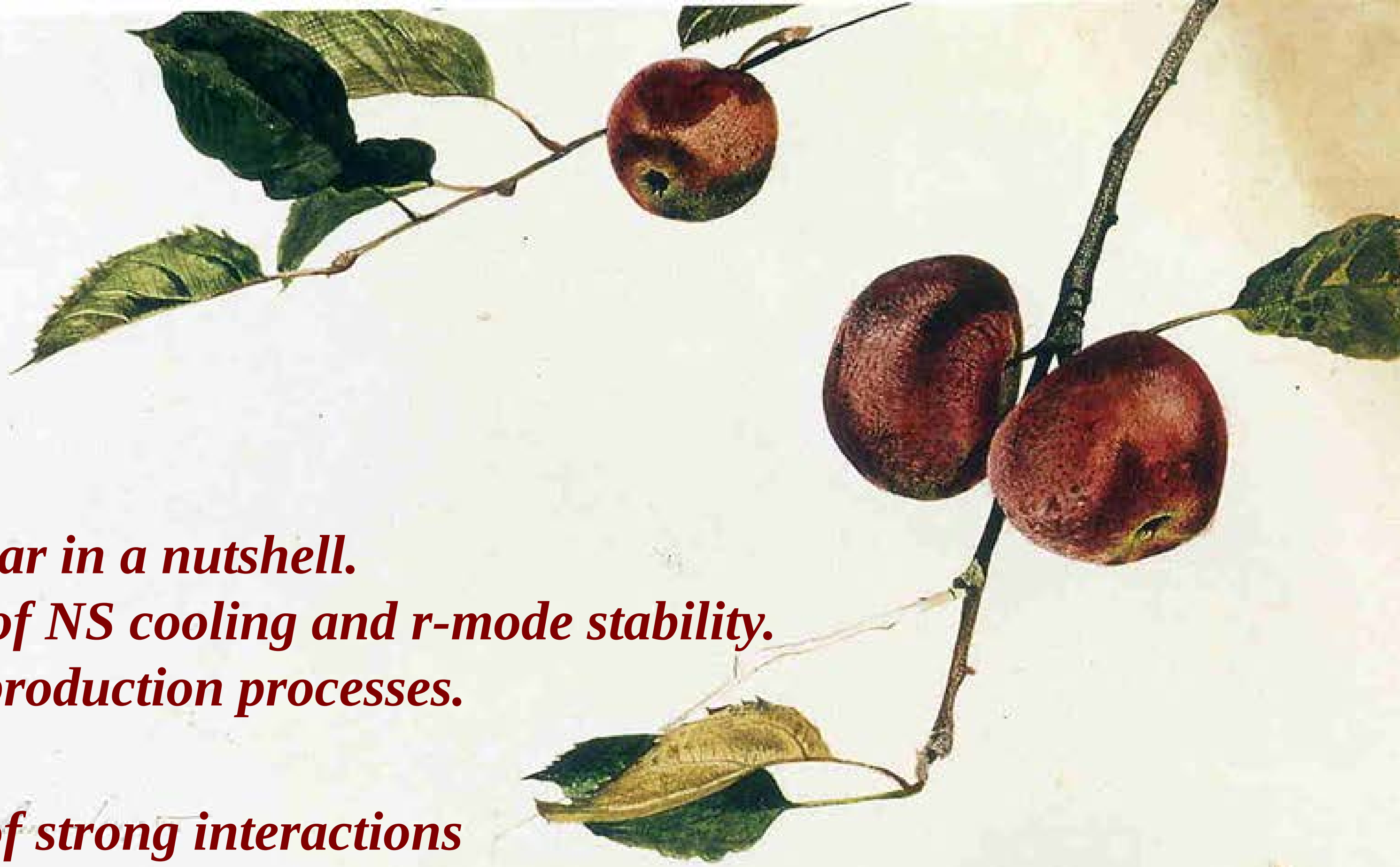
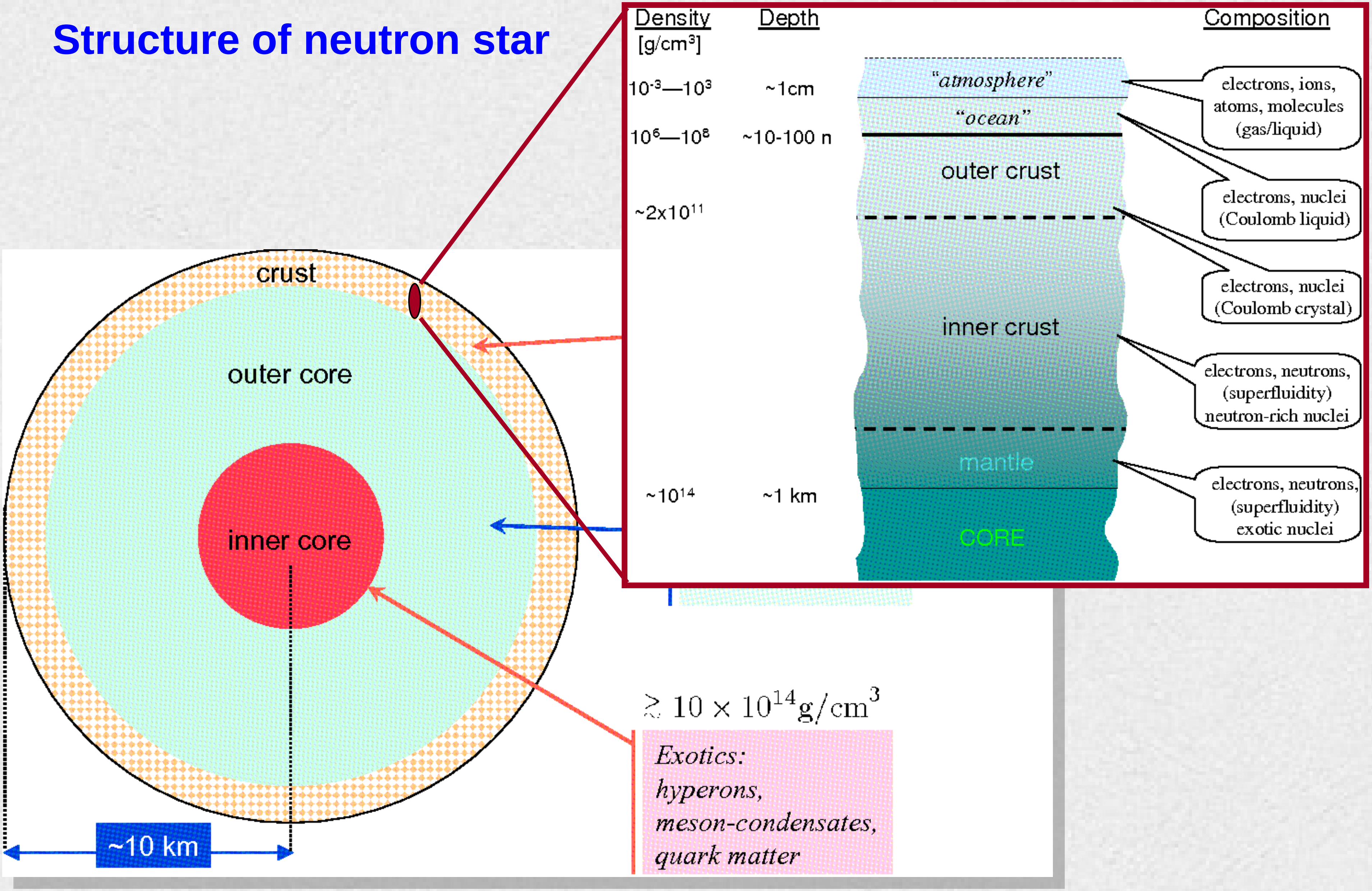


1. ***Neutron star in a nutshell.***
   Problems of NS cooling and r-mode stability.
   Neutrino production processes.

2. ***Inclusion of strong interactions***

3. ***Viscosity of NS matter***

Structure of neutron star



[Potekhin UFN 180]

Two basic physical principles determine the structure of compact stellar objects:

Electroneutrality and **Pauli exclusion principle**.

Macroscopic object held by gravity must be electrically neutral:

Consider a sphere of a radius *R*

with the uniform charge density n_Q and the baryon density *n*

Coulomb energy:

$$E_{\rm C} = e^2\, 2\,\pi\, R^2\, n_Q \; = 1.5\times 10^{36}\ {\rm MeV}\ \left(\frac{n_Q}{n_0}\right)\ \left(\frac{R}{1{\rm km}}\right)^2$$

Gravitational energy:

(n_0=0.16 fm^{-3})

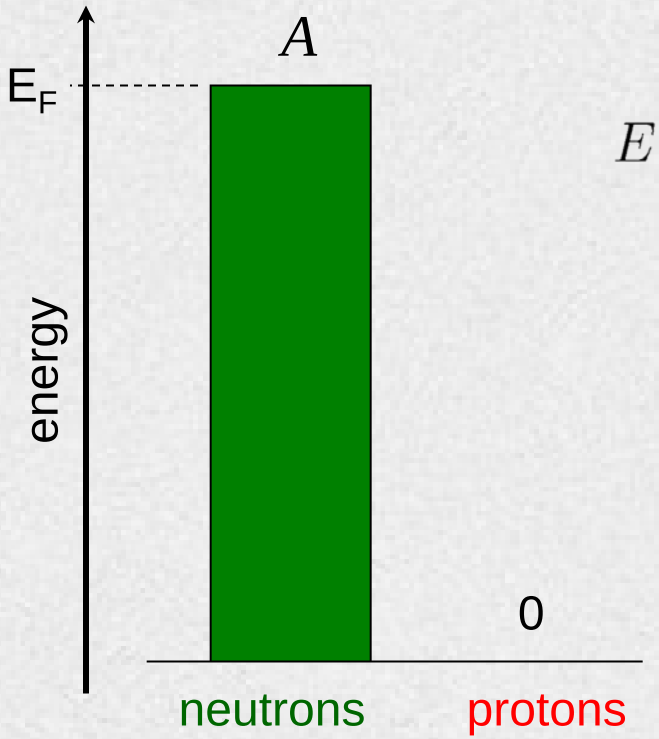
$$E_{\rm G} \;=\; -G\, 2\,\pi\, R^2\, n\, m_N = -0.7\ {\rm MeV}\times\left(\frac{n}{n_0}\right)\ \left(\frac{R}{1{\rm km}}\right)^2$$

$$E_{\rm C} + E_{\rm G} = 0 \qquad \Longrightarrow \qquad \frac{n_Q}{n} \sim 10^{-36}$$

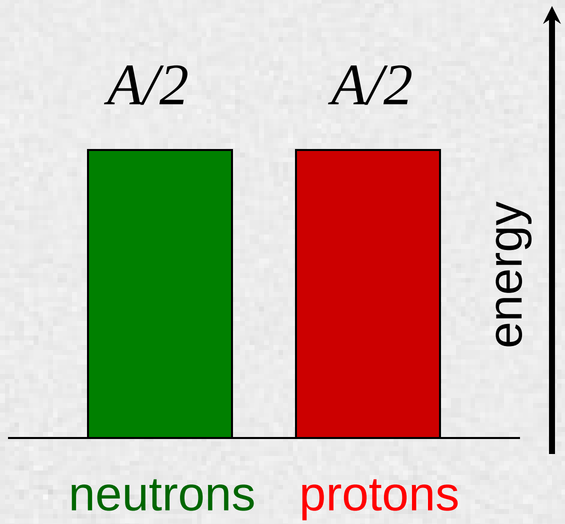
Pauli exclusion principle: nuclear symmetry energy

Neglect electric charge of protons: isospin symmetry.

We want to distribute A nucleons



$$E(A, Z = 0) > E(A, Z = A/2)$$



Two Fermi seas are better than one Fermi sea!

Pauli blocking at work: neutron star composition

Composition is determined by the minimum of energy

Symmetry energy: $= \bar{a}_{\rm Sym}\,(n_n - n_p)^2/n \longrightarrow$ *It is favorable to have some protons.*

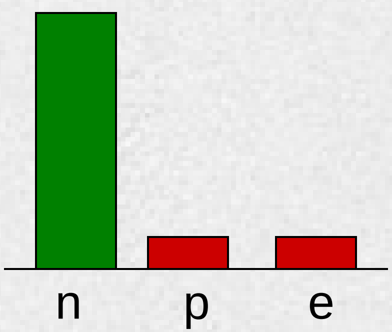
Electroneutrality: $n_e = n_p$ $\longrightarrow$ *There will be also some electrons*

Electron energy:

$$\varepsilon_e = 2\int_0^{p_{\mathrm{F}\,e}} \frac{d^3p}{(2\,\pi)^3}\,E_e(p) = \frac{p_{\mathrm{F}\,e}^4}{4\,\pi^2} = \frac{3}{4}\,(3\,\pi^2)^{1/3}\,n_e^{4/3}$$

$\longrightarrow$ *Electron energy increases fast*

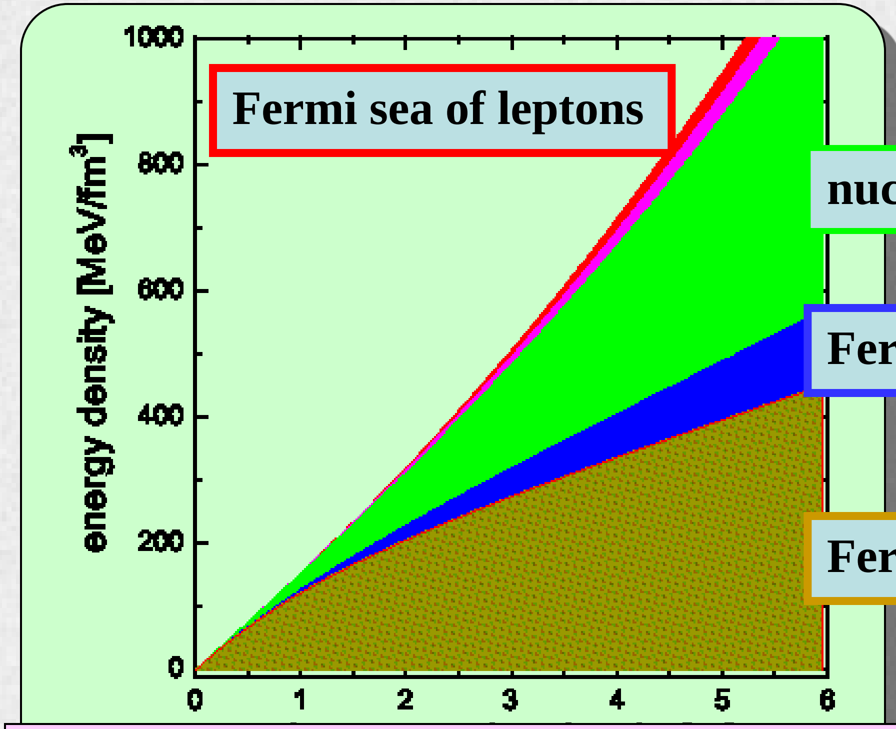
Energy minimization: *Mainly neutrons and small admixture of protons and electrons*



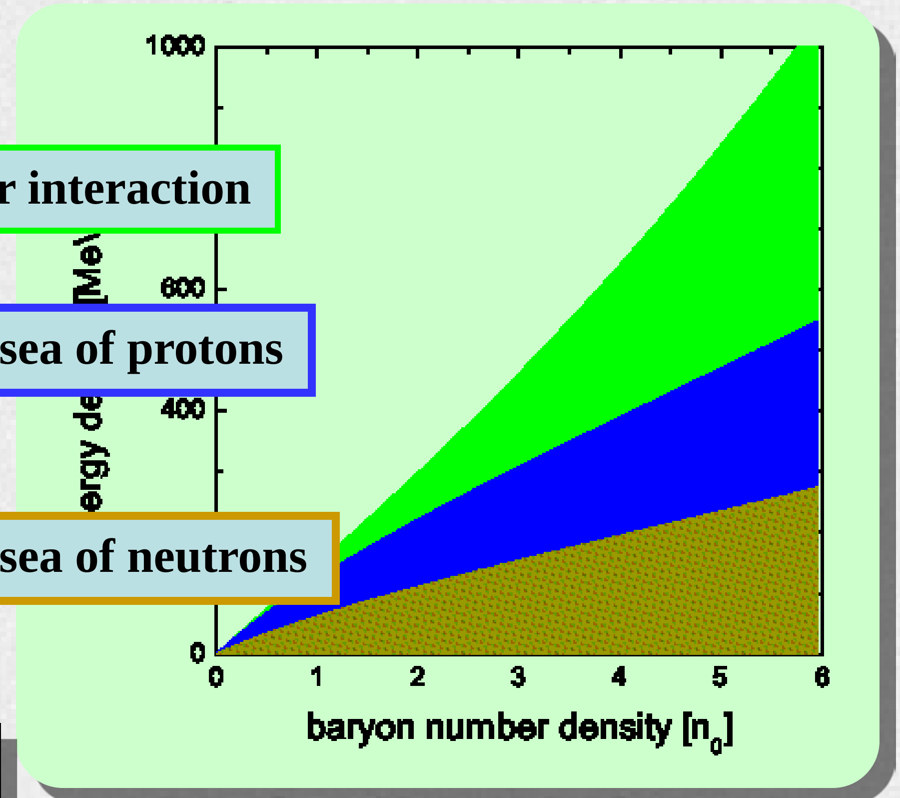
medium in β equilibrium

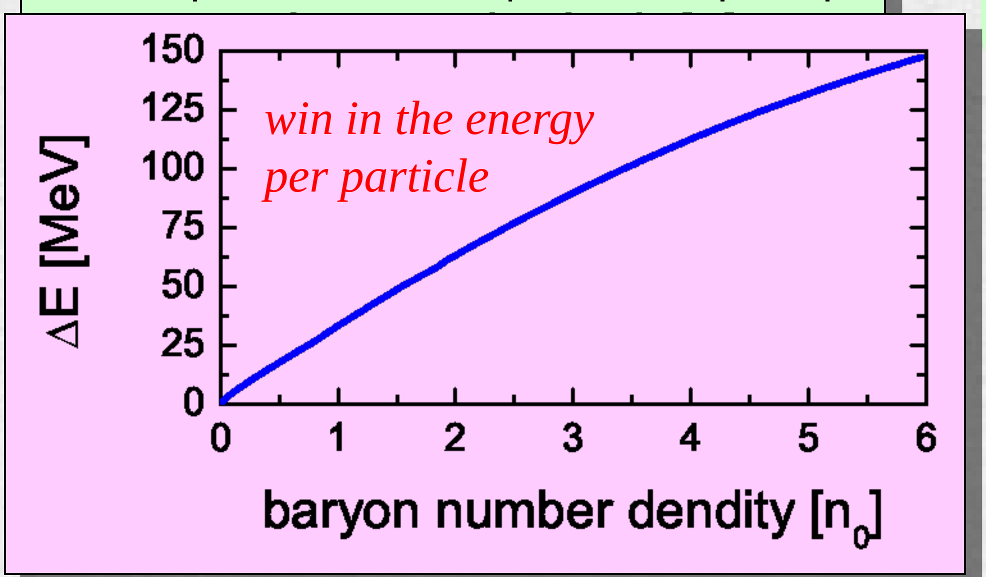
$$n \leftrightarrow p + e^-$$

β equlibrium



isospin symmetric $N_n = N_p$



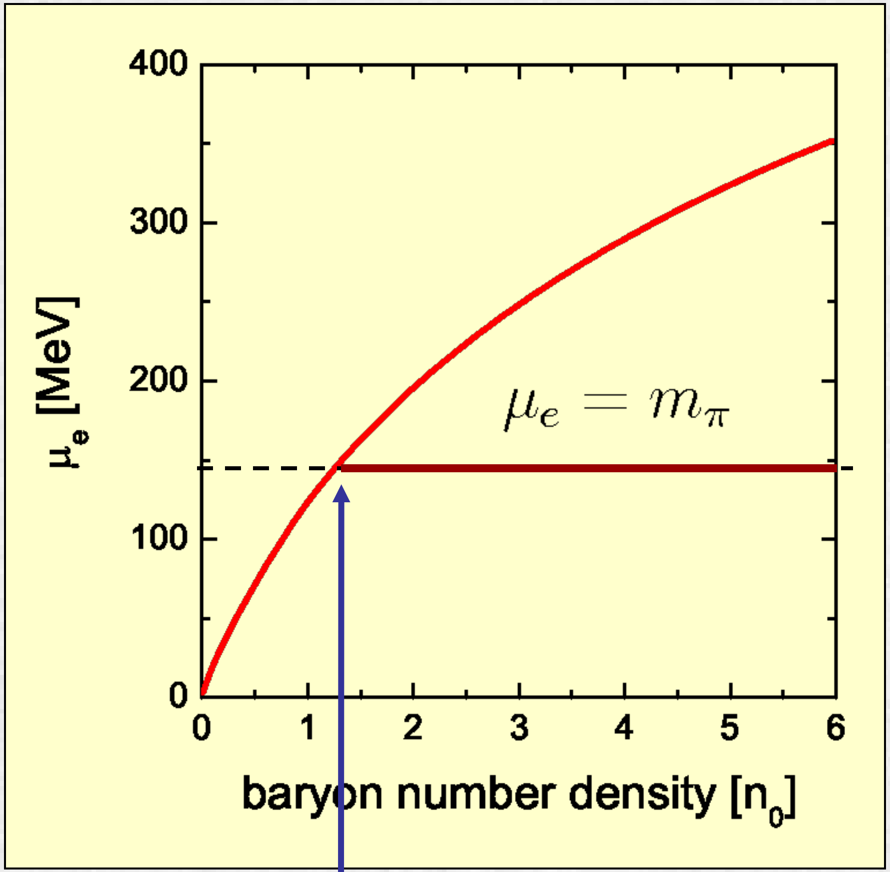


Compensate charge not by fermions but by light bosons

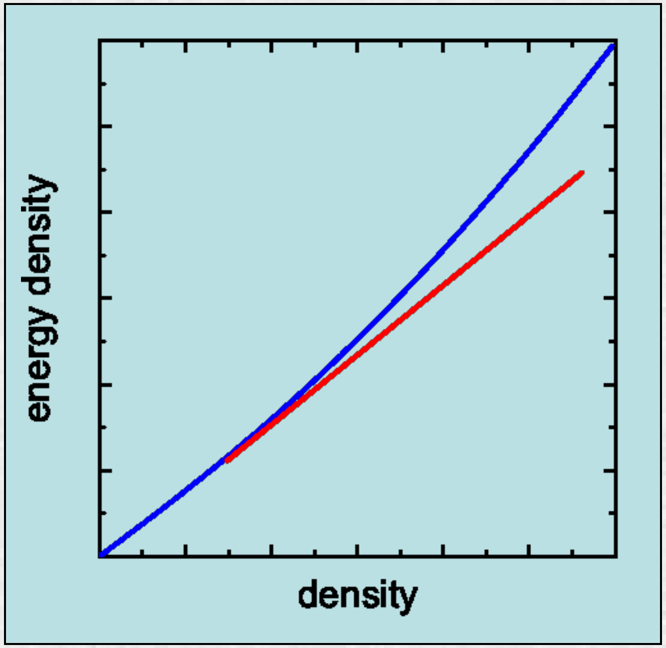
Lightest negatively charged bosons: π^- and K$^-$ mesons:

$$m_\pi = 140 \text{ MeV}$$
$$m_K = 495 \text{ MeV}$$

$\sqrt{m^2 + k^2}$ ⟶ minimum at zero momentum



Weak reactions start
$$e^- \longrightarrow \pi^- + \nu_e$$



second-order phase transition

equation of state of dense matter

integral quantity

mass, size, dynamics of SN explosion

We want to learn about properties of microscopic excitations in dense matter.

How to study response function of the NS?

Changes of strong and weak interactions in medium

Neutrino probe

At temperatures **smaller** than the opacity temperature (T^{opac}~1-few MeV)
mean free path of neutrinos and antineutrinos is **large**r than the neutron star radius

$$\lambda_\nu \gg R \simeq 10\mathrm{km}$$

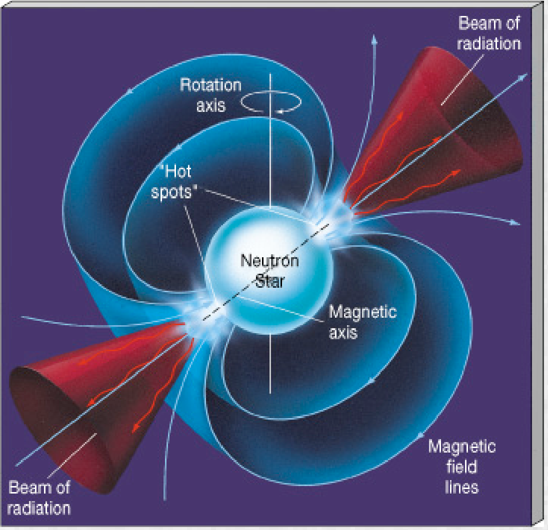
➡ **white body radiation problem**

After >10^5 yr –black body radiation of photons

At temperatures $T > T^{opac}$ $\qquad \lambda_\nu < R$

➡ **neutrino transport problem** ← important for supernova

$$1\,\mathrm{MeV} = 1.16 \cdot 10^{10}\,\mathrm{K}$$



Pulsar age

Pulsar rotation period/frequency changes with time:

$$\dot{\Omega} = -\frac{B^2 R^6}{6\,c^3 I}\sin^2\alpha\,\Omega^3 - G\frac{32\varepsilon^2}{5c^5} I\,\Omega^5$$

e.m. wave emission — **grav. wave emission**

α angle between rotation a and magnetic axes

ε neutron star eccentricity

$$\dot{\Omega} = \varkappa\Omega^n$$

$$P = 2\pi/\Omega$$

→

$$(n-1)\frac{\dot{P}(t)}{P(t)}t = 1 - \left(\frac{P_0}{P(t)}\right)^{n-1}$$

initial period

current period

$$P_0/P \ll 1$$

spin-down age:

$$\tau_{\rm sd} = \frac{1}{n-1}\frac{P}{\dot{P}}$$

$$n = \frac{\ddot{\Omega}\,\Omega}{\dot{\Omega}^2}$$

braking index:

kinematic age:

1) age of the associated SNR

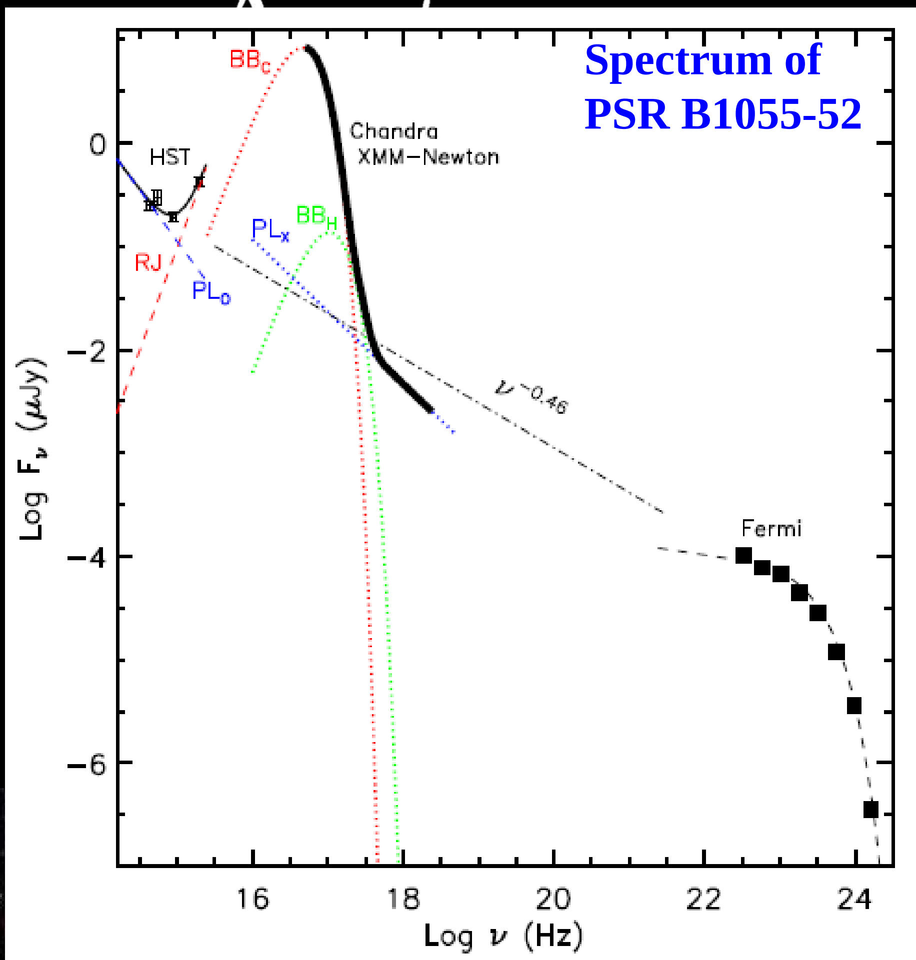
2) pulsar speed and position w.r. to the geometric center of the associated SNR

3) historical events

Crab : 1054 AD
Cassiopeia A: 1680 AD
Tycho’s SN: 1572 AD

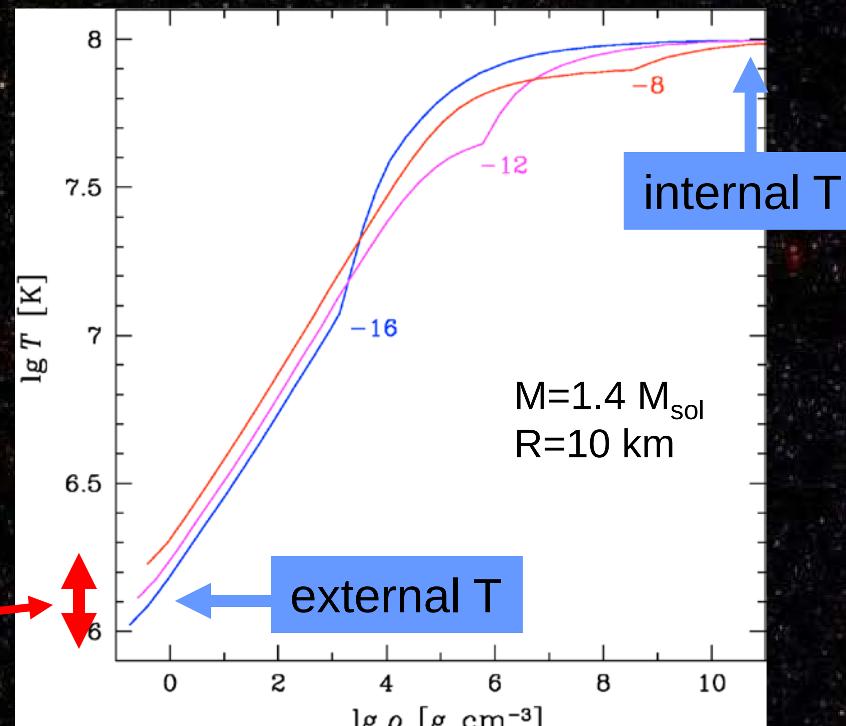
Measuring pulsar temperature

- radiation spectrum and source geometry
- internal vs. external temperature
- heat transfer in envelop



envelop

debris of supernova explosion;
accreted “nuclear trash”



[Yakovlev et al., A&A 417, 169]

Neutron star cooling data

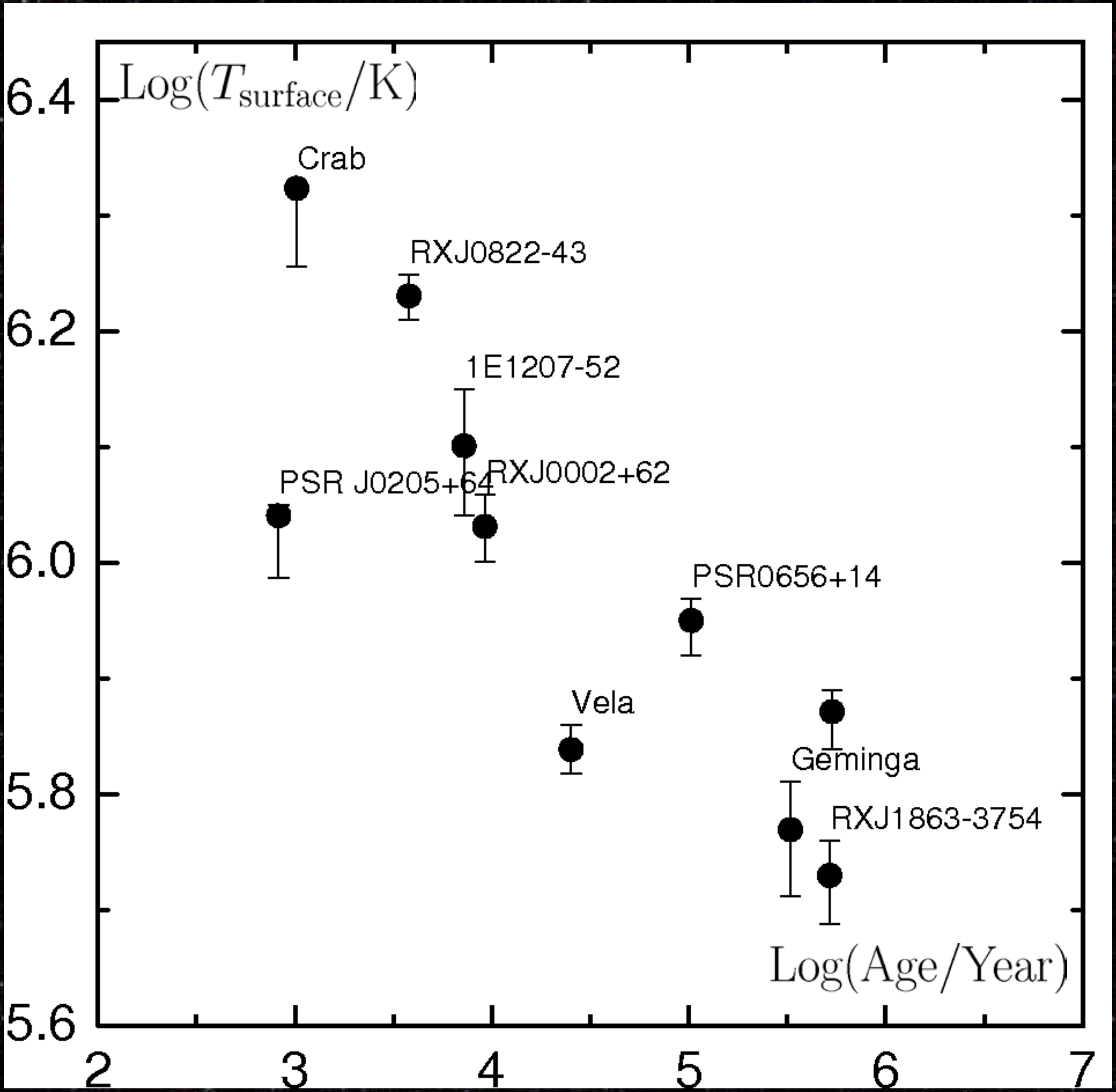
Given:

- **EoS**
- **Cooling scenario**
  [neutrino production]

Mass of NS

↓

Cooling curve



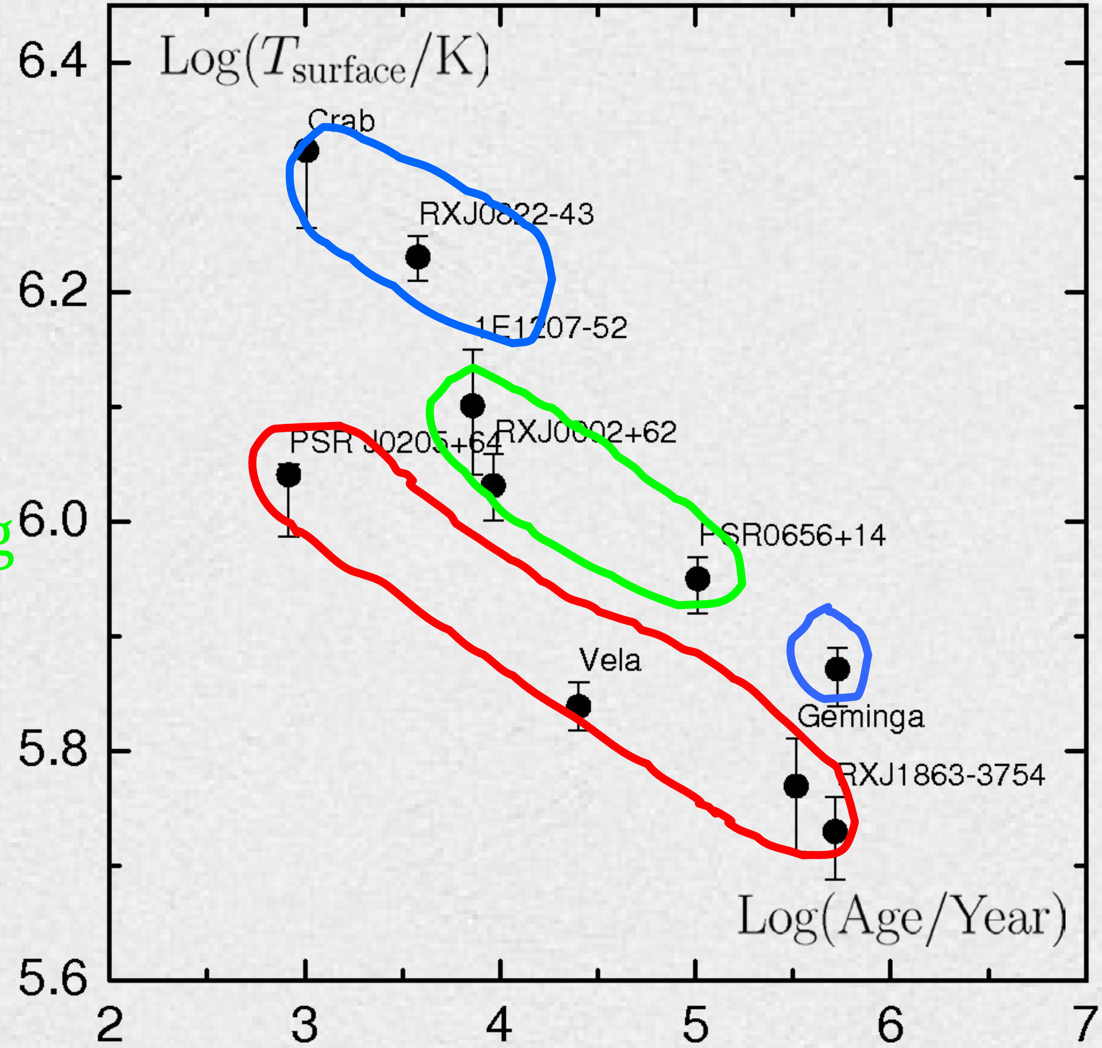
Neutron star cooling data

3 groups:

slow cooling

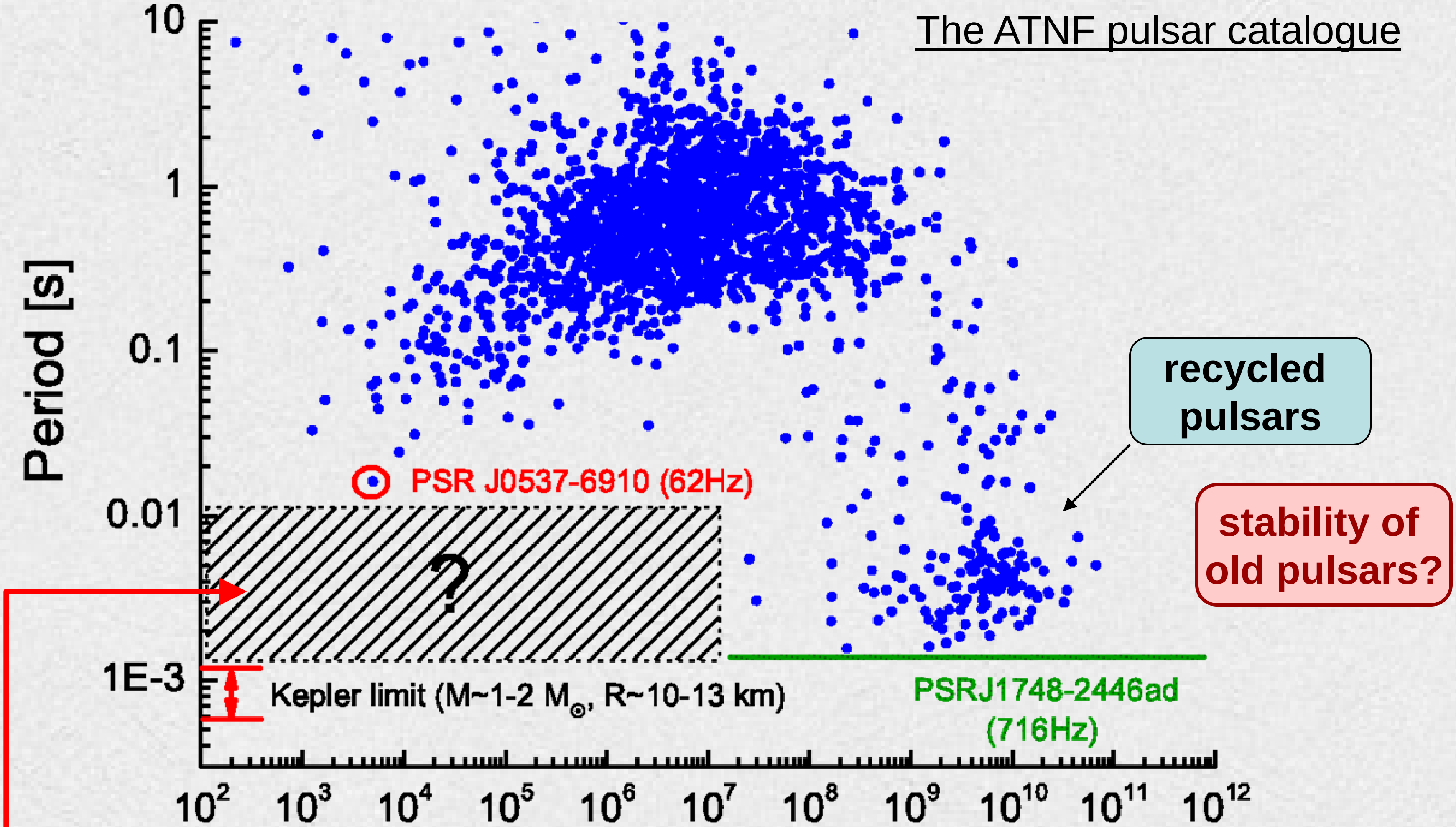
intermediate cooling

rapid cooling



How to describe all groups within one cooling scenario?

Age-period diagram for pulsars



$\tau_{\rm s.d.} = P/(2\dot{P})$

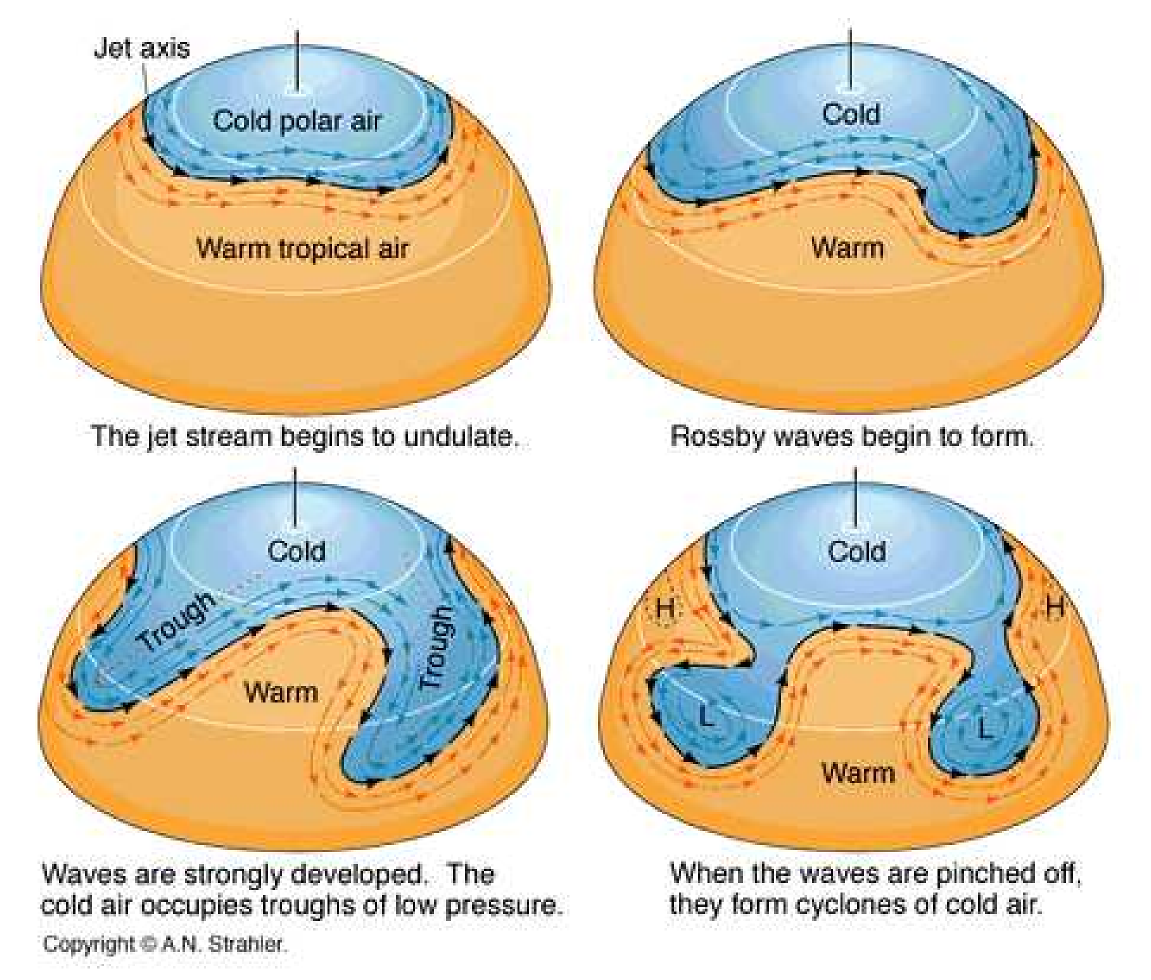
Young pulsars periods are much larger than the Kepler limit.

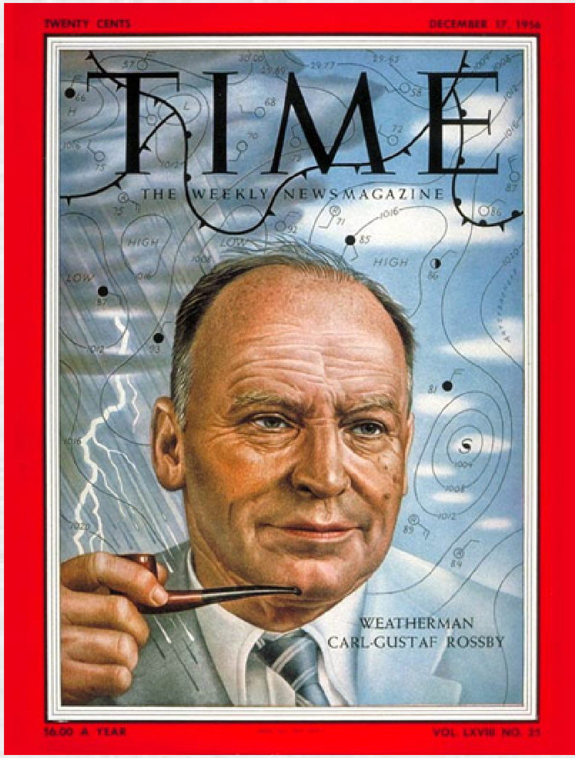
$$\frac{10\mathrm{km}}{c} = 3 \cdot 10^{-5}\,\mathrm{s}$$

$$\nu_{\rm K} \simeq 1.2\ \mathrm{kHz}(M/M_{\odot})^{1/2}(10\mathrm{km}/R)^{3/2}$$

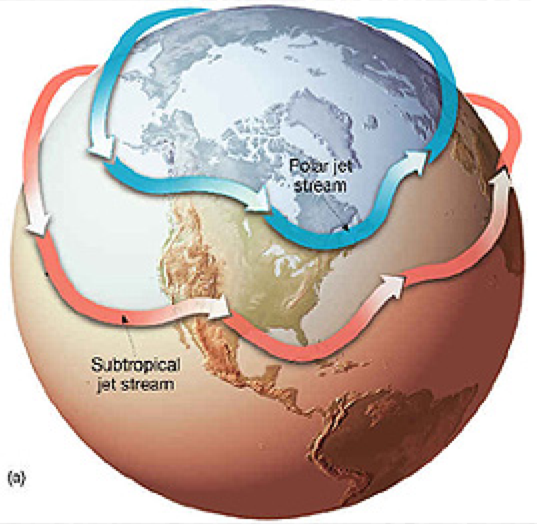
Rossby waves on Earth: in oceans and atmosphere

The returning force for these wave is the Coriolis force





Carl-Gustaf Rossby

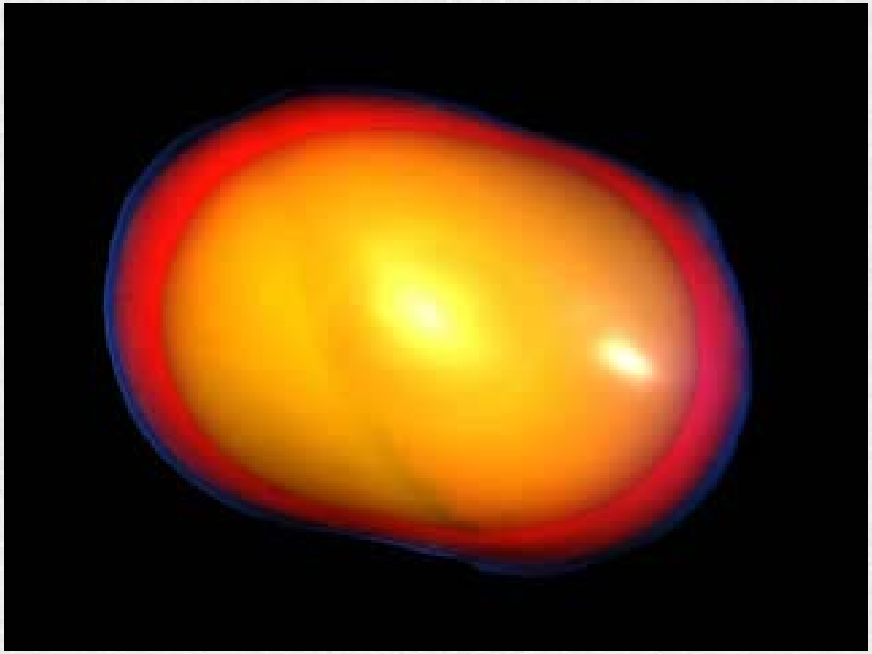


R-mode instability of rotating neutron star and viscosity

In a **superdense** system like a neutron star the Rossby waves are sources of gravitational radiation.

1998 it was shown by Andersson, Friedman and Morsnik showed that this radiation leads to an increase of the amplitude of the mode.

so there is an **instability**



Large R-modes can either destroy the star or the star stop rotating

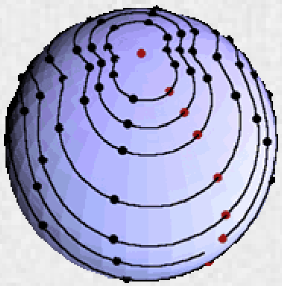
Viscosity of the dense nuclear matter can damp r-modes and save the rotating star

R-modes in rotating neutron star

Consider a neutron star with a radius R rotating with a rotation frequency Ω

and a **perturbation** in the form $$\delta v(\boldsymbol{r},t) = a(t)\, R\Omega \left(\frac{r}{R}\right)^l \boldsymbol{r} \times \boldsymbol{\nabla} Y_{ll} e^{i\omega t - t/\tau}$$

oscillation amplitude



$$\omega = -\Omega \frac{(l-1)(l+2)}{(l+1)}$$ oscillation frequency

(dominant modes for l=m=2)

for $a \ll 1$ amplitude changes $$\dot{a}(t) \approx -\frac{a}{\tau}$$

[Andersson ApJ 502 (98) 708;
Friedman, Morsink, ApJ 502 (98) 714]

$$\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_V}$$

viscous damping

gravitational radiation drives r-mode unstable

[Lindblom, Owen, Morsnik, PRL80 (98) 4843]

$$\frac{1}{\tau_G} = \frac{6.43 \cdot 10^{-2}}{\mathrm{s}} R_6^7\, \Omega_4^6\, \frac{\rho_c}{\rho_0}$$

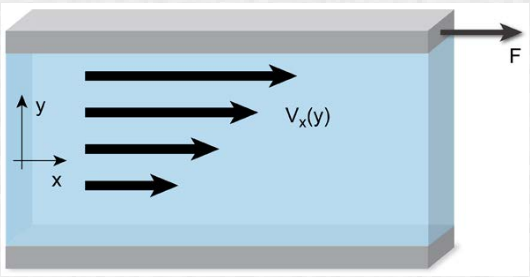
If the r-modes are undamped, the star would lose its angular moment on the time scale of τ_G, because of an enhanced emission of gravitation waves.

Viscosity

Euler equation: $$\rho\partial_t \boldsymbol{v}_i + \boldsymbol{v}_i \mathrm{div}\boldsymbol{v} = -\nabla_i p + \partial_k \sigma_{ki}$$

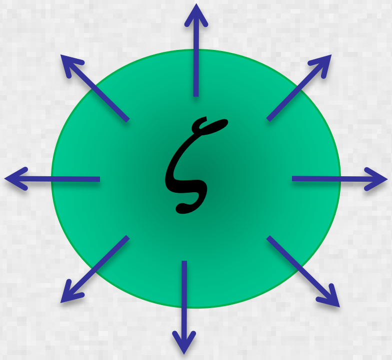
$$\sigma_{ik} = \boldsymbol{\eta}\left(\nabla_k v_i + \nabla_i v_k - \frac{2}{3}\delta_{ik}\mathrm{div}\boldsymbol{v}\right) + \boldsymbol{\zeta}\delta_{ik}\mathrm{div}\boldsymbol{v}$$

shear viscosity



Dissipation when there is a velocity gradient

bulk viscosity



Dissipation after uniform volume change

Maxwell (1860) kinetic theory calculations $\eta \sim \rho\, v_{\mathrm{rms}}\, l$

(density) x (velocity spread) x (length)

Units: $$\frac{\mathrm{g}}{\mathrm{cm}^3}\frac{\mathrm{cm}}{\mathrm{s}}\mathrm{cm} = \frac{\mathrm{g}}{\mathrm{cm}\cdot\mathrm{s}} = 1\,\mathrm{Poise} = 0.1\,\mathrm{Pa}\cdot\mathrm{s}$$

after Jean Léonard Marie Poiseuille

$\eta \sim \rho v_{\rm rms} l$

Viscosity estimates

Air at norm. cond.

$\rho \sim 1.2 \cdot 10^{-3} \mathrm{g/cm^3}$ $\quad l \sim 100\,\mathrm{nm} = 10^{-5}\,\mathrm{cm}$

$v \sim \sqrt{mT} \sim 500\,\mathrm{m/s} = 5 \cdot 10^4\,\mathrm{cm/s}$

➡ $\eta \sim 6 \cdot 10^{-4}\mathrm{Poise}$ Exp. $\eta_{\rm Air} \approx 1.8 \cdot 10^{-4}\mathrm{Poise}$

Water: Exp. $\eta_{\rm water} \approx 1.0 \cdot 10^{-2}\mathrm{Poise}$

Honey: Exp. $\eta_{\rm honey} \sim 140\,\mathrm{Poise}$

Glass: Exp. $\eta_{\rm glass} \sim 10^{18}$—$10^{21}\,\mathrm{Poise}$

Nuclear matter: *(naïve estimate)*

$$\rho_0 = n_0\,(m_N - \mathcal{E}_{\rm bind}/c^2) = 0.16\mathrm{fm}^{-3}(940\mathrm{MeV} - 16\,\mathrm{MeV})/c^2 \simeq 2.6 \cdot 10^{14}\mathrm{g/cm^3}$$

$l \sim 1\,\mathrm{fm} = 10^{-13}\,\mathrm{cm}$ $\quad v \sim c = 3 \cdot 10^8\,\mathrm{cm/s}$

➡ $\eta_{\rm n.m.} \sim 7 \cdot 10^9\mathrm{Poise}$

R-modes stability

R-mode is unstable if the time $\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta} > 0$

gravitational time scale

$$\tau_G{}^{-1} = \frac{4096}{164025} G\Omega^6 R^7 \langle\rho\rangle_6 \approx \frac{6.43 \cdot 10^{-2}}{\mathrm{s}} R_6^7\, \Omega_4^6\, \frac{\rho_{\rm cen}}{\rho_0}$$

damping rate due to the **shear** viscosity

$$\tau_\eta^{-1} = \frac{5}{R^2} \frac{\langle\eta\rangle_4}{\langle\rho\rangle_6} \approx \frac{5.98 \cdot 10^{-5}}{\mathrm{s}} \frac{\langle\eta_{20}\rangle_4}{R_6^2\, \rho_{\rm cen}/\rho_0}$$

damping rate due to the **bulk** viscosity

$$\tau_\zeta^{-1} = \frac{4\pi}{690}\left(\frac{\Omega^2}{\pi G\bar\rho}\right)^2 \frac{\langle\zeta\,(1+0.86(r/R)^2)\rangle_8}{R^2\langle\rho\rangle_6}$$

$$\approx \frac{2.20 \cdot 10^{-7}}{\mathrm{s}} R_6^4\, \Omega_4^4 \frac{\langle\zeta_{20}\,[1+0.86(r/R)^2]\rangle_8}{(M/M_\odot)^2(\rho_{\rm cen}/\rho_0)}$$

$R_6 = R/10^6\,\mathrm{cm}$

$\Omega_4 = \Omega/10^4\,\mathrm{s}$

$\eta_{20} = \eta/(10^{20}\ \frac{\mathrm{g}}{\mathrm{cm\,s}})$

$\zeta_{20} = \zeta/(10^{20}\ \frac{\mathrm{g}}{\mathrm{cm\,s}})$

$$\langle \ldots \rangle_n = \frac{1}{R^{n+1}} \int_0^R (\ldots)\, r^n\, \mathrm{d}r$$

profile averages

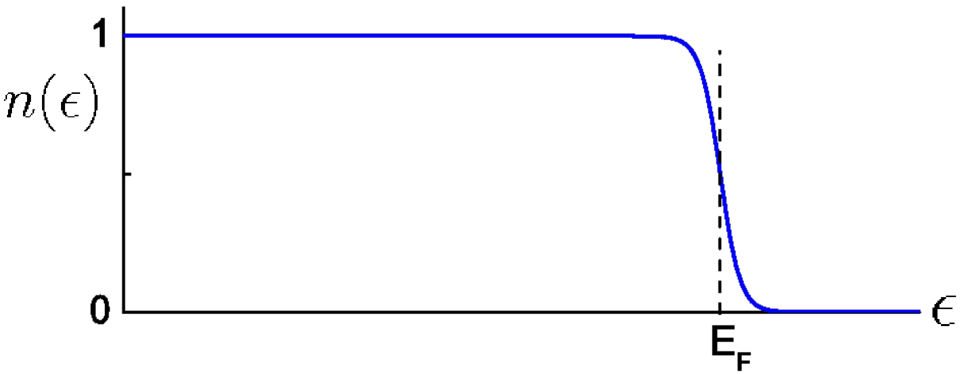
[Lindblom, Owen, Morsnik PRL 80 (98) 4843
Owen, Lindblom, Cutler, Schutz, Vecchio, Andersen PRD58 (98) 084020]

NEUTRINO PRODUCTION PROCESS IN COLD NUCLEAR MATTER

Cold fermion matter

Fermi distribution function : $n(\epsilon) = \dfrac{1}{e^{(\epsilon - E_{\rm F})/T} + 1}$

$$n(\epsilon) \to \theta(E_F - \epsilon) \quad T \to 0$$



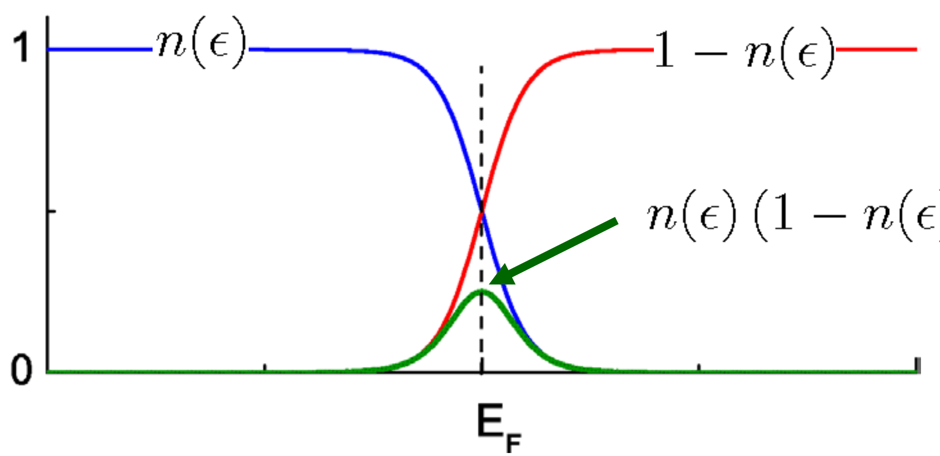
$$p_{\rm F} = (3\pi^2 n)^{1/3} = 331\,{\rm MeV}\left(\frac{n}{n_0}\right)^{1/3}$$

$$E_{\rm F} = \frac{p_{\rm F}^2}{2m_N^*} = 58.5\,{\rm MeV}\left(\frac{m_N}{m_N^*}\right)\left(\frac{n}{n_0}\right)^{2/3}$$

$$T < T_{\rm opac} \sim 10^{-1}\text{–}10^{0}\ {\rm MeV} \ll E_{\rm F}$$

degenerated fermion system

<u>in reactions with small energy transfer</u>



$$n(\epsilon)\,(1 - n(\epsilon)) \approx \frac{1}{4}\left(1 - \frac{(\epsilon - E_{\rm F})^2}{4T^2}\right)$$

Neutrino emission reactions

$T < T_{\rm opac} \sim 10^{-1}$–$10^{0}$ MeV   neutron star is transparent for neutrino

$$C_V \frac{\mathrm{d}T}{\mathrm{d}t} = -L$$

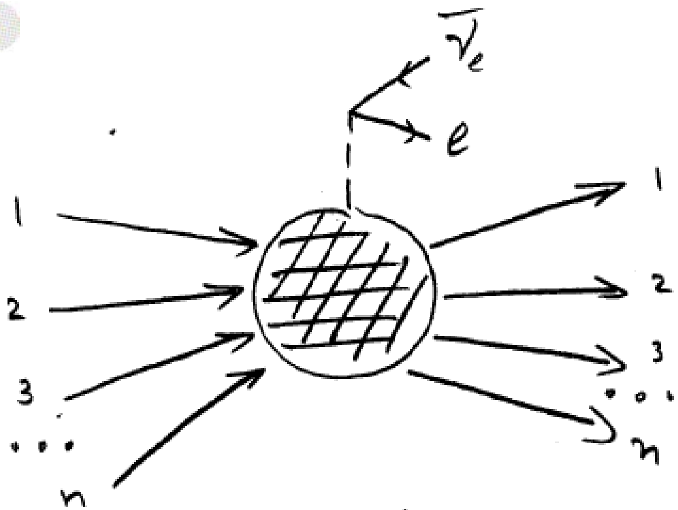
C_V – heat capacity, L - luminosity

$$L = \int \mathrm{d}V \sum_{\text{reaction r}} \epsilon_\nu^{(r)}$$

emissivity

✔ **n-nucleon reaction**

$T \ll \epsilon_{\rm F}$



$\epsilon_\nu \sim T^{2n+4}$

each leg on a Fermi surface / T

neutrino phase space × neutrino energy $\omega_{\bar\nu} \times \delta(\omega_{\bar\nu} - \ldots)\, \omega_{\bar\nu}^2\, d\omega_{\bar\nu} \sim T^3$

Direct reactions

Cooling: role of crust and interior?

most important are reactions in the interior

(The baryon density is $n \gtrsim n_0$ where n_0 is the nuclear saturation density)

one-nucleon reactions:	$n \to p + e + \bar{\nu}$	direct URCA (DU)	$\sim T^6$
two-nucleon reactions:	$n + n \to n + p + e + \bar{\nu}$	modified URCA (MU)	$\sim T^8$
	$n + n \to n + n + \nu + \bar{\nu}$	nucleon bremsstrahlung (NB)	

URCA=Gamow's acronym for "Un-Recordable Coolant Agent"

NS cooling in a nutshell

black body radiation $$L_\gamma = 4\pi R^2 \sigma_{\rm SB} T_{\rm ext}^4 = 7.8\cdot 10^{43}\, T_{\rm ext,9}^{4}\, \frac{\rm erg}{s}$$

external temperature $$T_{\rm ext}/K \simeq \begin{cases} \alpha\, T_{\rm int}/K & T > T_{\rm crust} \\ 45(T_{\rm int}/K)^{0.65} & T < T_{\rm crust} \end{cases}$$

star is too hot; crust is not formed

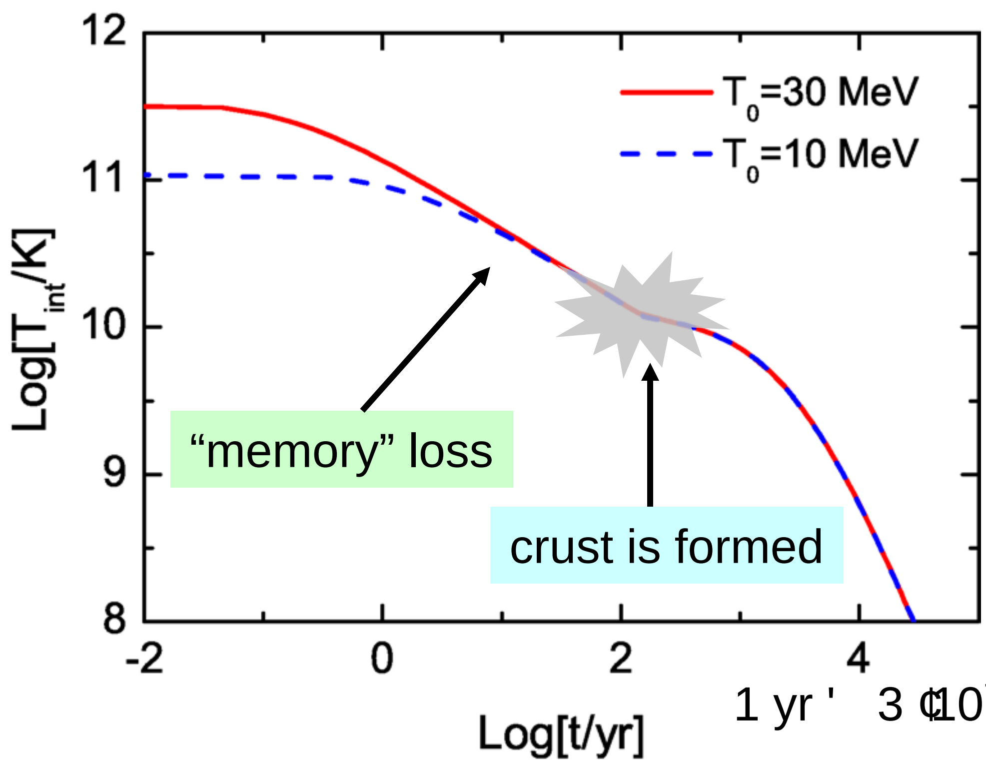
heat transport thru envelop

$$C_V = C\cdot T$$

$$C = 10^{30}\frac{\rm erg}{\rm K^2} = 10^{48}\,\frac{\rm erg}{(10^9\,{\rm K})^2}$$

$$C\int_{T_0}^{T(t)} \frac{T\,dT}{L_\gamma(T)} = -t$$

$$T_9 = T/(10^9{\rm K})$$



$1\ {\rm yr} \simeq 3\cdot 10^7 {\rm s}$

NS cooling in a nutshell

volume neutrino radiation $T < T_{\rm opac} = 1\ {\rm MeV}$

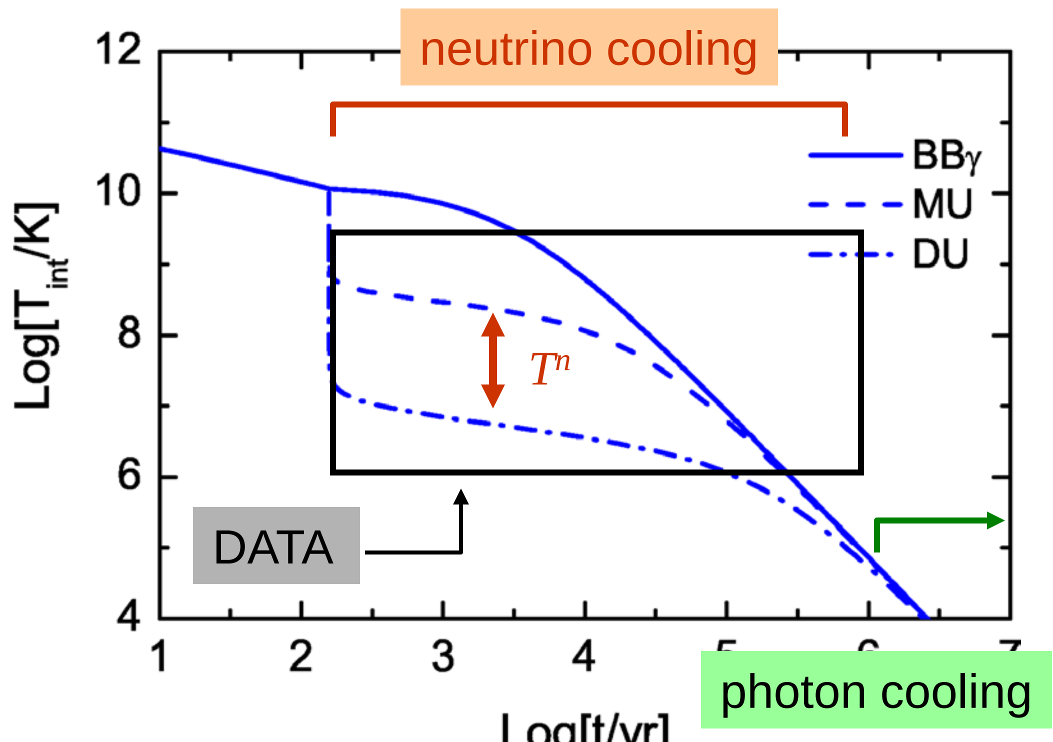
$$\int_{T(t)}^{T_0} \frac{C\,T\,dT}{L_\gamma(T) + L_\nu(T)\,\theta(T_{\rm opac} - T)} = t$$

DU: $\epsilon_{\rm DU} = 10^{27}\, T_9^6\ \frac{\rm erg}{\rm s\ cm^3}$

$L_{\rm DU} = V\,\epsilon_{\rm DU} = 4 \cdot 10^{45}\, T_9^6\ \frac{\rm erg}{\rm s}$

MU: $\epsilon_{\rm MU} = 2 \cdot 10^{21}\, T_9^8\ \frac{\rm erg}{\rm s\ cm^3}$

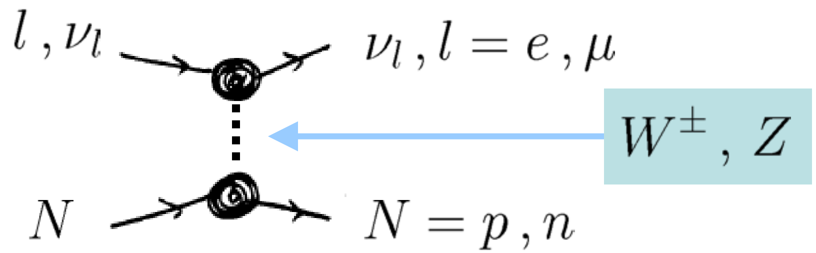
$L_{\rm MU} = V\,\epsilon_{\rm MU} = 8 \cdot 10^{39}\, T_9^8\ \frac{\rm erg}{\rm s}$



$V = \frac{4\pi}{3}\,R^3 = \frac{4\pi}{3}\,(10\,{\rm km})^3 = 4 \cdot 10^{18}\,{\rm cm}^3$

Weak interactions

Low-energy weak interaction of nucleons



effective weak coupling constant

$$L_{\text{weak}} = \frac{G}{\sqrt{2}}\, j_\mu \times l^\mu$$

nucleon current **lepton current**

$$G_{\text{F}} = 1.436 \cdot 10^{-49}\,\frac{\text{cm}^5 \cdot \text{g}}{\text{s}^2}$$

$$\frac{G_{\text{F}}}{(\hbar\, c)^3} = \frac{1.16 \cdot 10^{-5}}{\text{GeV}^2}$$

$G = G_{\text{F}}\, \cos\theta_{\text{C}}$ Cabibbo angle

$$l_\mu = \bar{u}(q_1)\, \gamma_\mu (1 - \gamma_5)\, u(q_2)$$

$$j_\mu = V_\mu - A_\mu = \#\,(\bar{N}\,\gamma_\mu N) - \#\,(\bar{N}\,\gamma_\mu\,\gamma_5 N)$$

$$\sin\theta_{\text{C}} = 0.225$$

Weak interactions

Weinberg's Lagrangian:

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^- \right) + \frac{g}{\cos\theta_W} \left(J_\mu^{(3)} - \sin^2\theta_W \, J_\mu^{\text{e.m.}} \right) Z_\mu + g \, \sin\theta_W \, J_\mu^{\text{e.m.}} A_\mu$$

<u>lepton current</u> $\quad l_\mu = \bar{u}(q_1)\, \gamma_\mu (1-\gamma_5)\, u(q_2)$

<u>nucleon current</u> $\quad < N|j_\mu|N > \; = \; V_\mu^{NN} - A_\mu^{NN} = g_V\, (\bar{N}\, \gamma_\mu N) - g_A (\bar{N}\, \gamma_\mu\, \gamma_5 N)$

$$V_\mu^{np} \approx g_V\, \chi_p^\dagger(p') \big(1, \boldsymbol{v}\big) \chi_n(p)$$

$$V_\mu^{nn} \approx -\frac{g_V}{2}\, \chi_n^\dagger(p') \big(1, \boldsymbol{v}\big) \chi_n(p)$$

$$V_\mu^{pp} \approx +\frac{g_V}{2}\, \mathbf{c_V} \chi_p^\dagger(p') \big(1, \boldsymbol{v}\big) \chi_p(p)$$

$$A_\mu^{np} = -2\, A_\mu^{pp} = -2\, A_\mu^{nn} \approx g_A\, \chi_p^\dagger(p') \big(\boldsymbol{\sigma}\cdot\boldsymbol{v}, \boldsymbol{\sigma}\big) \chi_n(p)$$

$$g_A = 1.26$$

$$g_V = 1 \qquad \boldsymbol{v} = \frac{\boldsymbol{p} + \boldsymbol{p}'}{2\, m_N}$$

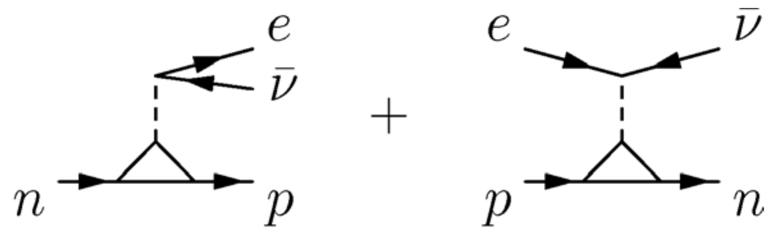
$$\mathbf{c_V} = 1 - 4\sin^2\theta_W \simeq \mathbf{0.08}$$

Note 1/2 in neutral channel,
since *Z* boson is neutral and *W* is charged!

ONE-NUCLEON PROCESS

DIRECT URCA

One-nucleon processes (DU)



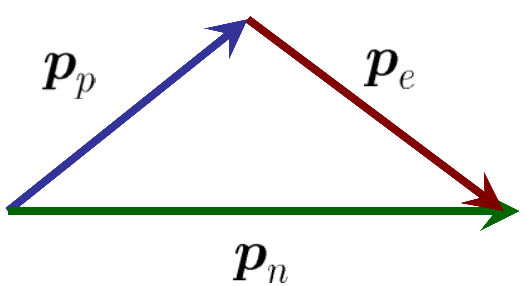
emissivity:

$$\epsilon_\nu^{\rm DU} = 2\int \frac{d^3p_n}{(2\pi)^3} f_n \int \frac{d^3p_p}{(2\pi)^3}(1-f_p)\int \frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)$$

$$\times \int \frac{d^3q_{\bar\nu}\,\omega_{\bar\nu}}{2\omega_{\bar\nu}(2\pi)^3}(2\pi)^4\delta^{(4)}(P_f-P_i)\sum_{\rm spins}|M|^2$$

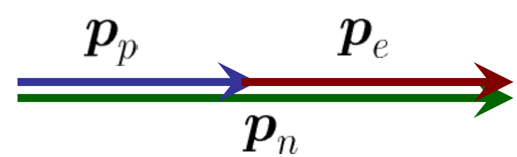
simplifications for $T \ll \epsilon_{Fn}, \epsilon_{Fp} \qquad \omega_{\bar\nu} = q_{\bar\nu} \sim T$

triangle inequality



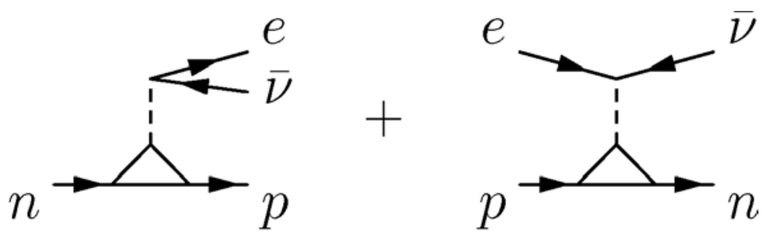
$|\boldsymbol{p}_p| = |\boldsymbol{p}_e|$

critical condition



$$\epsilon_\nu^{\rm DU} = \frac{457\,\pi}{10080}\,G^2\,(1+3\,g_A^2)\,m_n\,m_p\,p_{{\rm F},e}\,T^6\,\Theta(2\,p_{{\rm F}p}-p_{{\rm F}n})$$

One-nucleon processes (DU)



allowed if $|p_{F,n} - p_{F,p}| < p_{F,e}$ → proton concentration > 11-14%

$$\epsilon_\nu^{DU} \simeq 4 \cdot 10^{27} \left(\frac{n_e}{n_0}\right)^{1/3} T_9^6 \, \Theta(2p_{F,p} - p_{F,n}) \, \frac{\text{erg}}{\text{cm}^3 \cdot \text{s}}$$

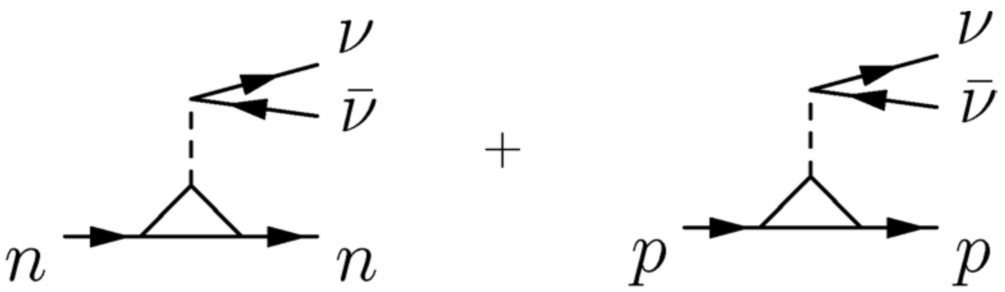
- large prefactor $\sim 10^{27} - 10^{28}$
- weak temperature dependence T^6

✔ very efficient reaction

BUT does not always occur

$n > n_c^{DU}$ $(M > M_c^{DU})$

One-nucleon processes on neutral currents



energy-momentum conservation

$$\delta(\boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{q}_1 - \boldsymbol{q}_2)\,\delta(E_1 - E_2 - \omega_1 - \omega_2)$$

➡ $$E_1 = E(p_1) = \frac{p_1^2}{2m_N} = \frac{(\boldsymbol{p}_1 - \boldsymbol{q}_1 - \boldsymbol{q}_2)^2}{2m_N} + \omega_1 + \omega_2$$
$$\approx \frac{p_1^2}{2m_N} - v_F\,|\boldsymbol{q}_1 + \boldsymbol{q}_2|\,\cos\theta + |\boldsymbol{q}_1| + |\boldsymbol{q}_2|$$

➡ requires $v_{\mathrm{F}} \geq 1$

processes on neutral currents are forbidden!

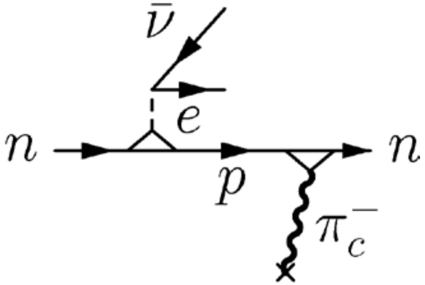
Process on π^- condensate (PU)

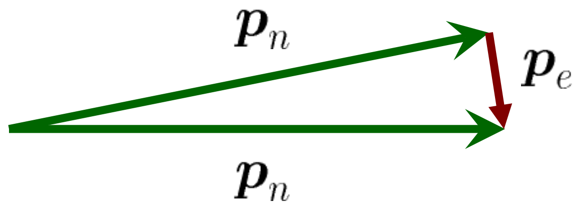
assume μ_e reaches m_π

$$e^- \to \pi^-$$
$$n \to p + \pi^-$$

Bose condensate of pions

$k_\mu=(m_\pi,0)$





neutrons in both initial and final states

energy-momentum conservation is easily fulfilled

$$\epsilon_\nu^{\pi_c^-} = \frac{457\,\pi}{5040} G^2 \,(1+3\,g_A^2)\, m_n^2\, p_{\mathrm{F},e}\, a_\pi^2\, T^6\, \Theta(n-n_c) \qquad a_\pi^2 = \frac{<|\pi|^2>}{4\,f_\pi^2}$$

condensate amplitude

$$a^2 \sim (0.01\text{–}0.1) \qquad \Rightarrow \qquad \epsilon_\nu^{\pi_c^-} \sim (0.1\text{–}1)\,\epsilon_\nu^{\mathrm{DU}}$$

Migdal's pion condensate $k=(\mu_\pi,k_c)$: $\mu_\pi<m_\pi$, $k_c \gg p_{F,e}$ *p-wave condensate*

$$\epsilon_\nu \sim 10^{26}\; T_9^6 (n/n_0)^{1/3} \;\frac{\mathrm{erg}}{\mathrm{cm^3\ sec}}$$

Kaon condensate processes yield a smaller contribution $[\propto \sin^2\theta_{\mathrm{C}} \simeq (0.23)^2]$

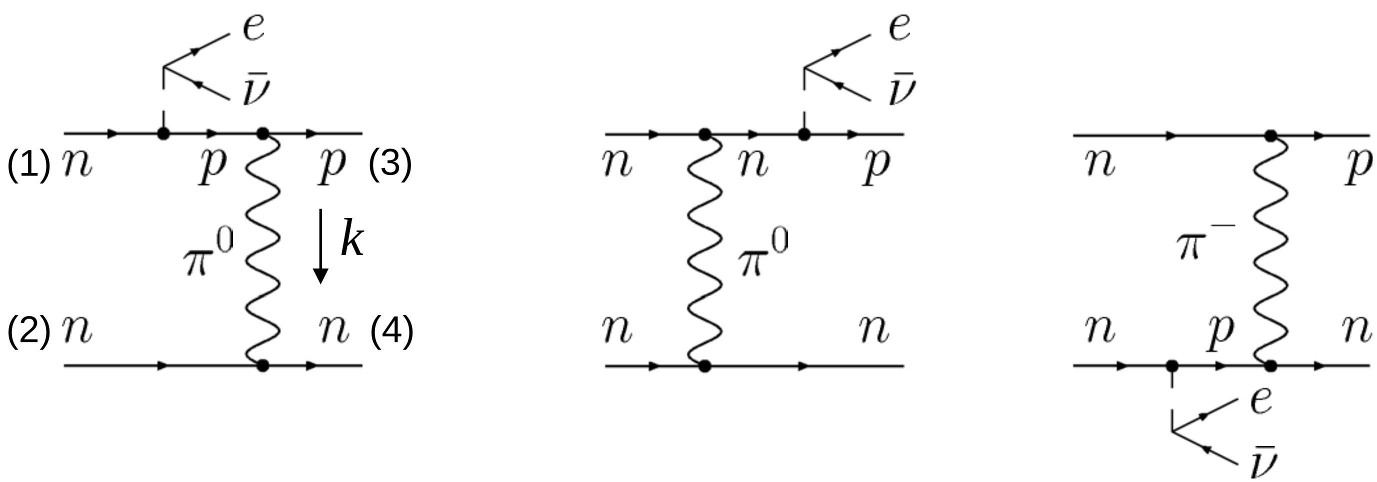
All "exotic" processes start only when the density exceeds some <u>critical density</u>

TWO-NUCLEON PROCESSES

MODIFIED URCA

Two-nucleon process (Modified Urca)

$n + p \to p + p + e + \bar{\nu}$ $\qquad$ $n + n \to n + p + e + \bar{\nu}$



Additionally one should take into account exchange reactions (identical nucleons)

- ✔ no critical density
- ✔ 5 fermions ➡ suppressed phase-space volume (compared to one-nucleon processes)
- ✔ T^8 dependence of the emissivity (5 fermions ➡ $\sim T^5$, $\omega_{\bar{\nu}}\, \delta(\omega_{\bar{\nu}} + \ldots)\, \omega_{\bar{\nu}}^2\, d\omega_{\bar{\nu}}$ ➡ T^3)

Two-nucleon process (Modified Urca)

Emissivity:

$$\epsilon_\nu^{\rm MU} = \prod_{i=1}^{4} \int \left[\frac{d^3 p_i}{(2\pi)^3}\right] f_1\, f_2\,(1-f_3)\,(1-f_4)\,\frac{d^3 q_e\,(1-f_e)}{2\,\omega_e\,(2\pi)^3}$$
$$\times\ \frac{d^3 q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\pi)^3}\,\omega_{\bar\nu}\,(2\,\pi)^4\,\delta^{(4)}(P_f - P_i)\,\frac{1}{s}\sum_{spins}|M|^2,$$

$s=2$ is symmetry factor. Reactions with the electron in an initial state yield extra factor 2.

Finally

$$\epsilon_\nu^{\rm MU} = \frac{11513}{60480\,\pi}\,G^2\,g_A^2\,f_{\pi NN}^4\,m_n^3\,m_p\,p_{{\rm F},e}\,T^8\,1.3 \simeq 8\cdot 10^{21}\,(n_p/n_0)^{1/3}\,T_9^8\ \times\ \frac{\rm erg}{{\rm cm}^3\cdot{\rm s}}$$

↑ due to exchange reactions (1.3)

Coherence: only axial-vector term contributes (!) whereas for PU processes both vector and axial-vector terms contribute

Standard scenario. DU scenario

standard scenario (MU)

only part of the data can be described

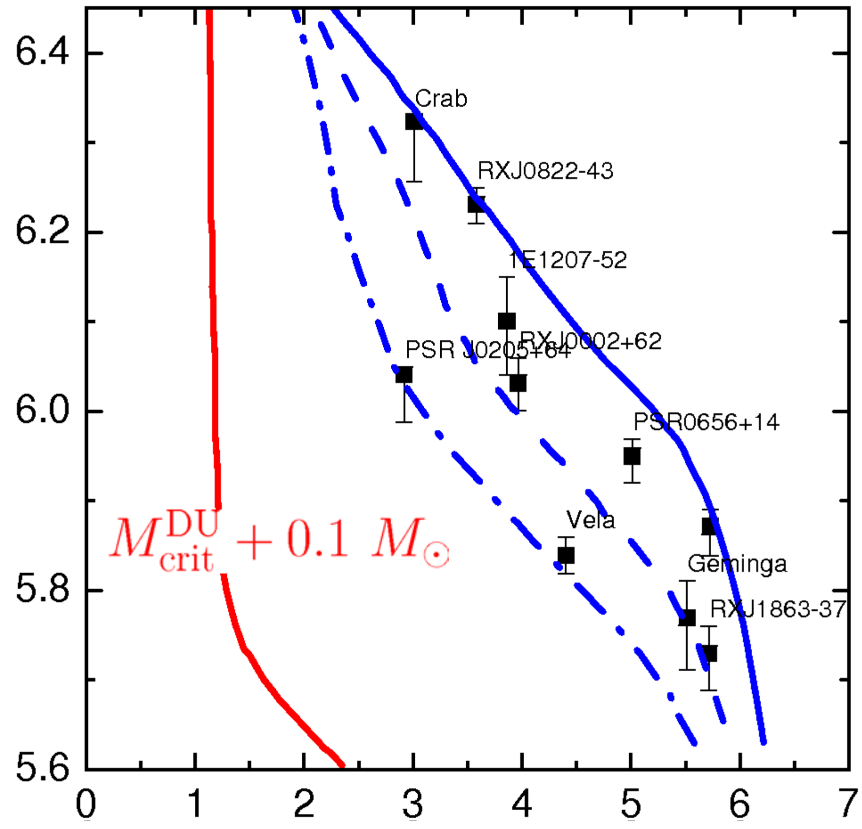
Direct-Urca scenario

NS masses close to M_{crit}^{DU}

Neutron stars with $M > M_{crit}^{DU}$

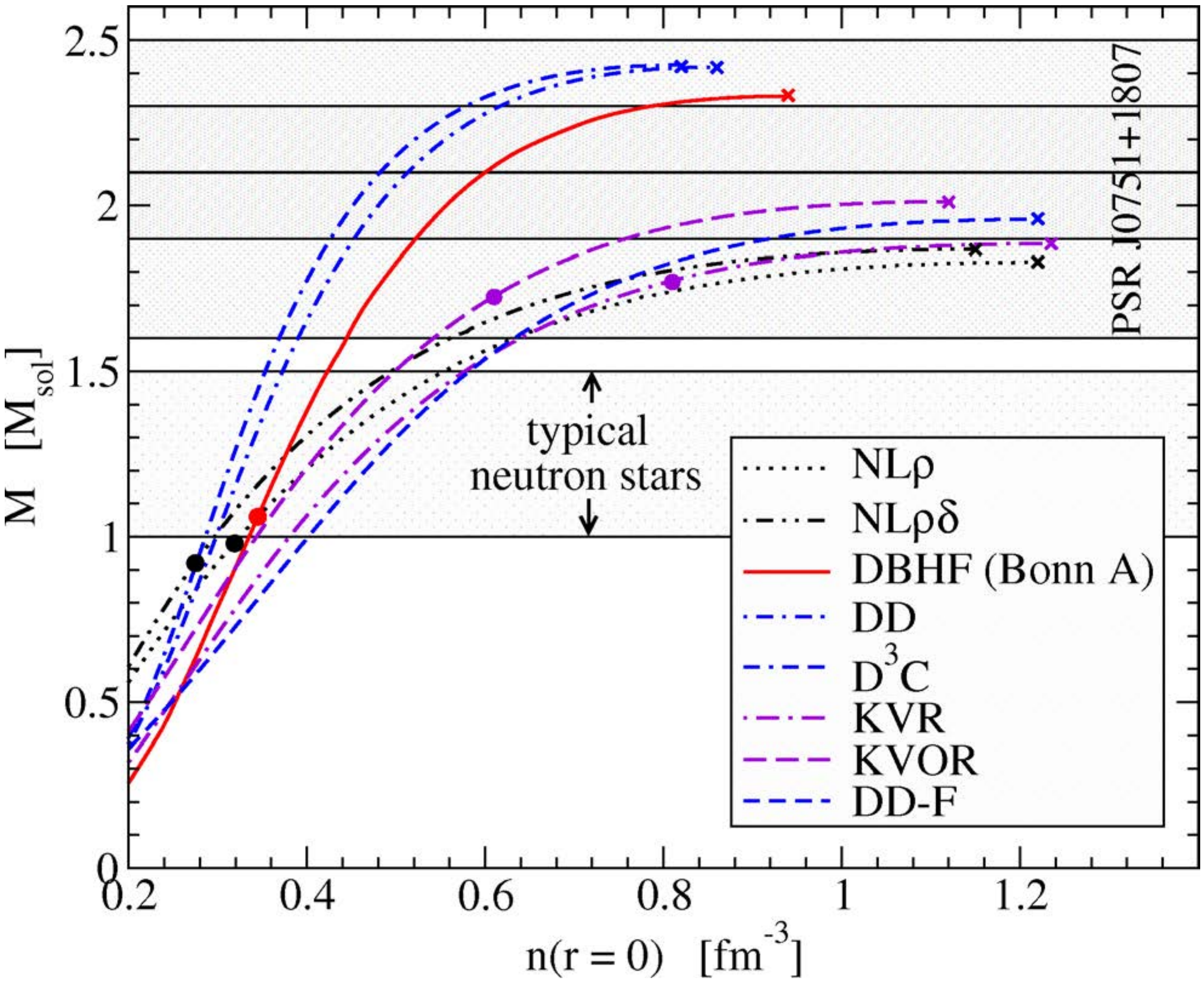
will be too cold

But masses of NS
are not close to each others



[Blaschke, Grigorian, Voskresensky A&A 424 (2004) 979]

DU thresholds



Klähn et al. PRC 74, 035802 (2006)

SUPERFLUID MATTER

Pairing in nuclei

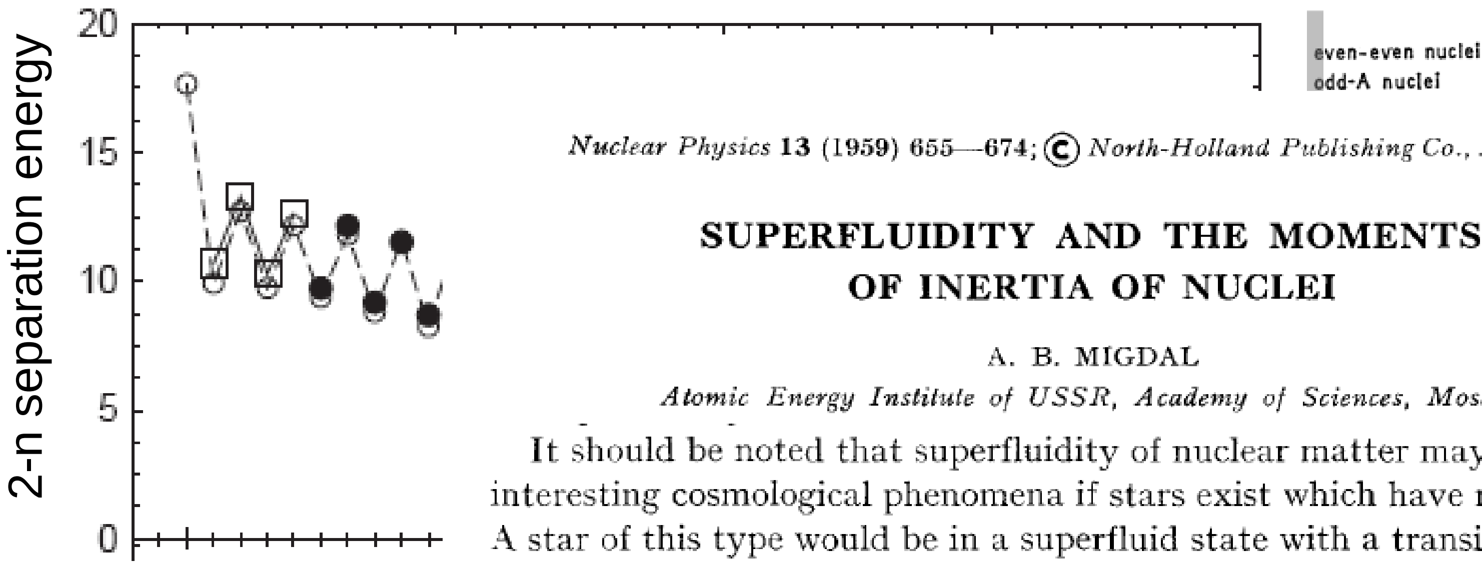
PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.



Nuclear Physics **13** (1959) 655—674; © *North-Holland Publishing Co., Amsterdam*

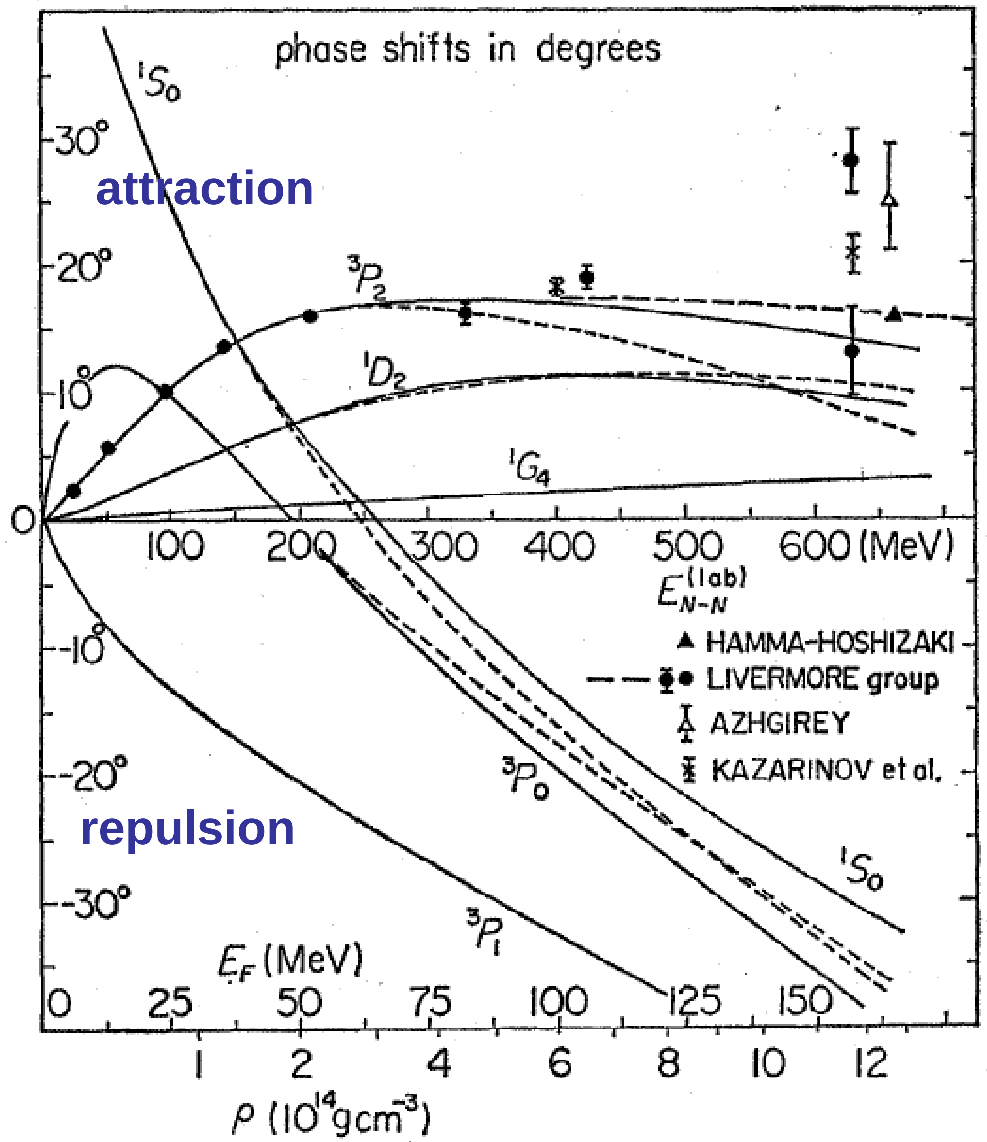
SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL
Atomic Energy Institute of USSR, Academy of Sciences, Moscow

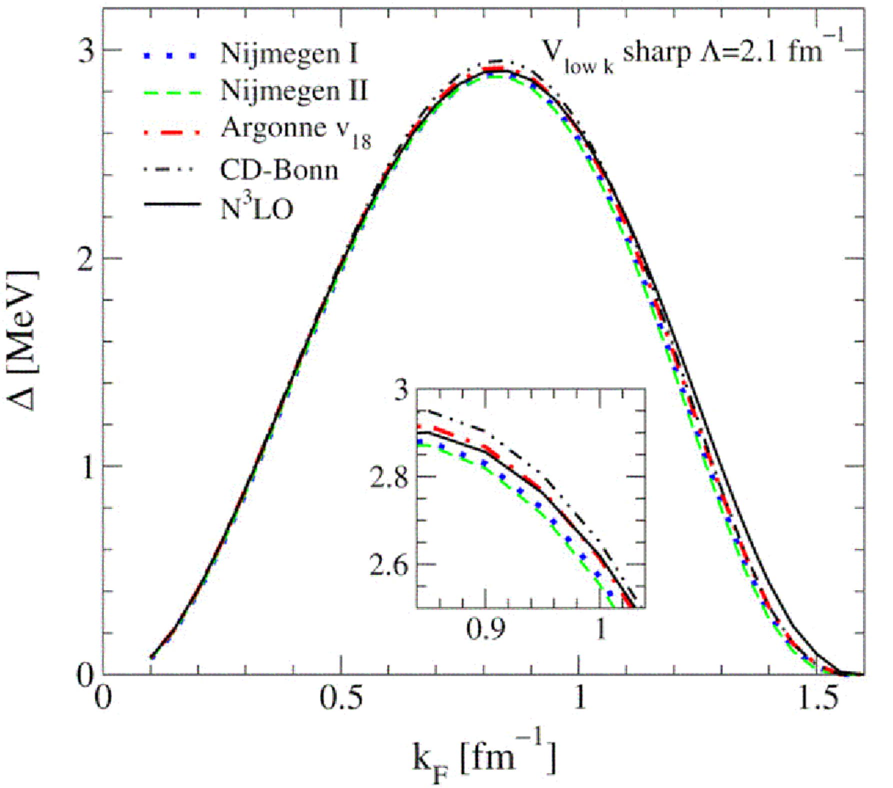
It should be noted that superfluidity of nuclear matter may lead to some interesting cosmological phenomena if stars exist which have neutron cores. A star of this type would be in a superfluid state with a transition temperature corresponding to 1 MeV.

Pairing gaps in nuclear matter

✔ nucleon-nucleon interaction



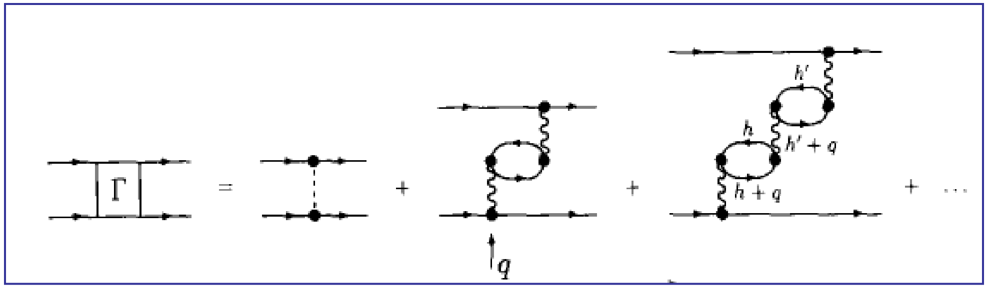
✔ $1S_0$ pairing gaps

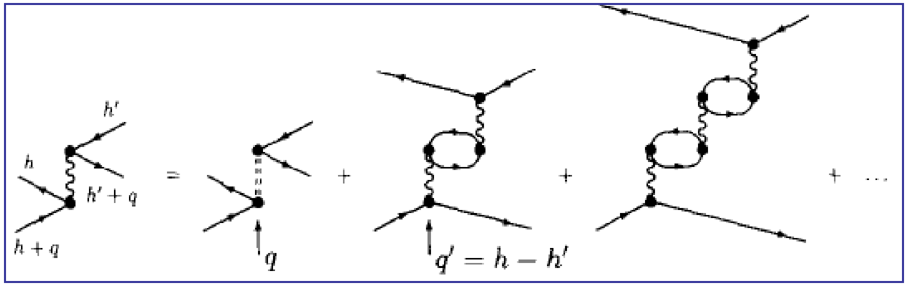


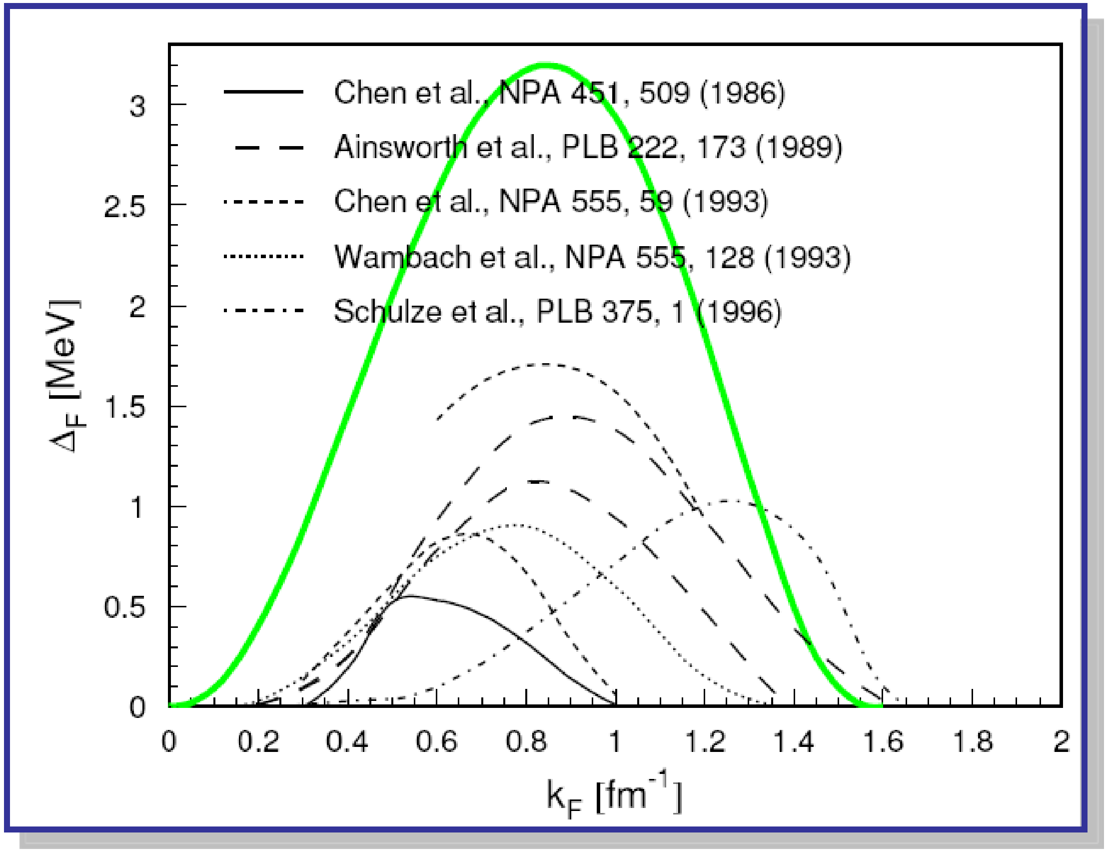
Hebeler, Schwenk, and Friman, PLB 648 (2007) 176

✔ medium polarization effects on superfluidity

U. Lombardo and H.-J. Schulze, astro-ph/0012209







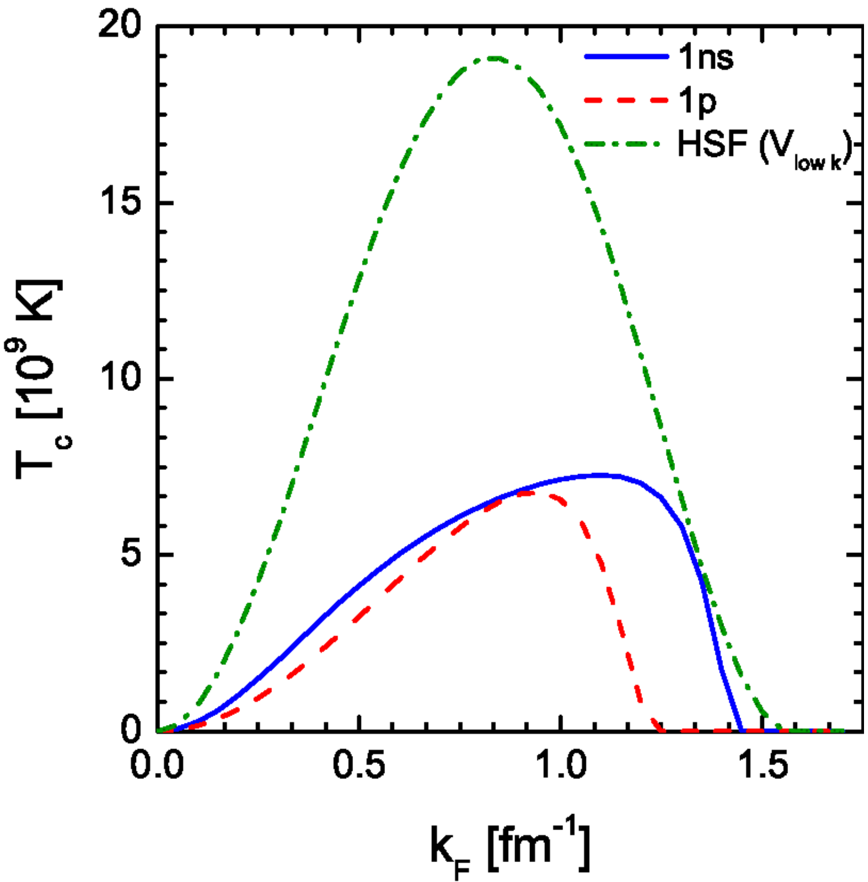
$$T_c(k_\mathrm{F}) = T_0 \frac{(k_\mathrm{F}-k_0)^2}{(k_\mathrm{F}-k_0)^2+k_1^2} \frac{(k_\mathrm{F}-k_2)^2}{(k_\mathrm{F}-k_2)^2+k_3^2}$$

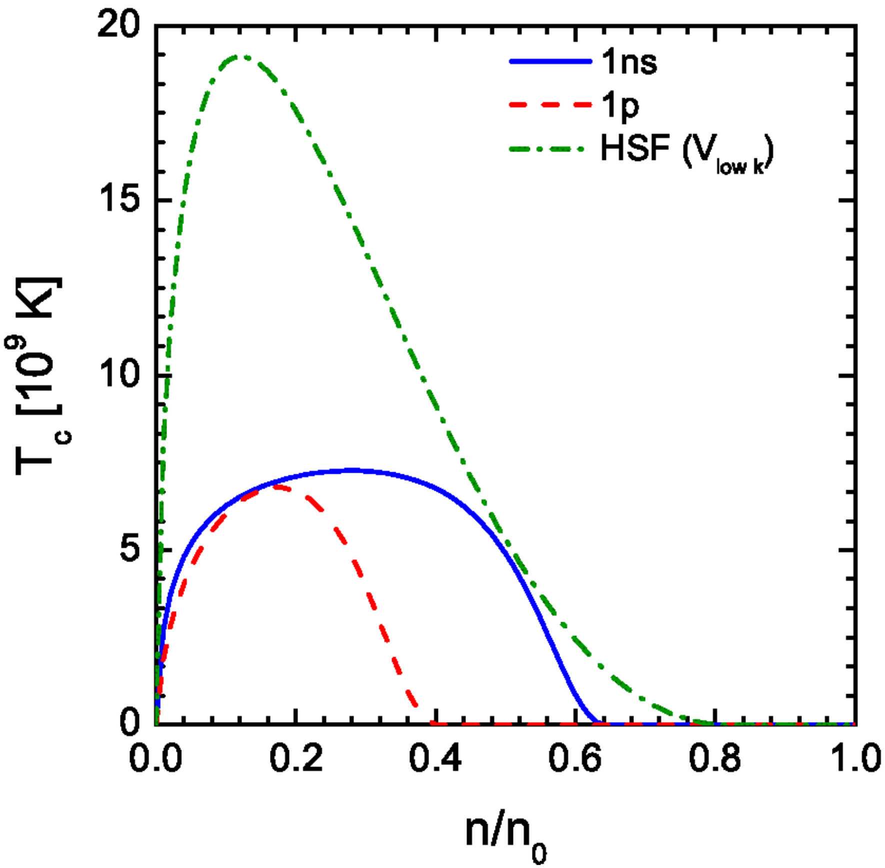
[Kaminker, Yakovlev, Gnedin, A&A 383 (2002) 1076]

<u>1ns for neutrons</u> $T_0 = 10.2 \cdot 10^9\mathrm{K}\ k_0 = 0,\ k_1 = 0.6,\ k_2 = 1.45,\ k_3 = 0.1$ [in fm^{-1}]

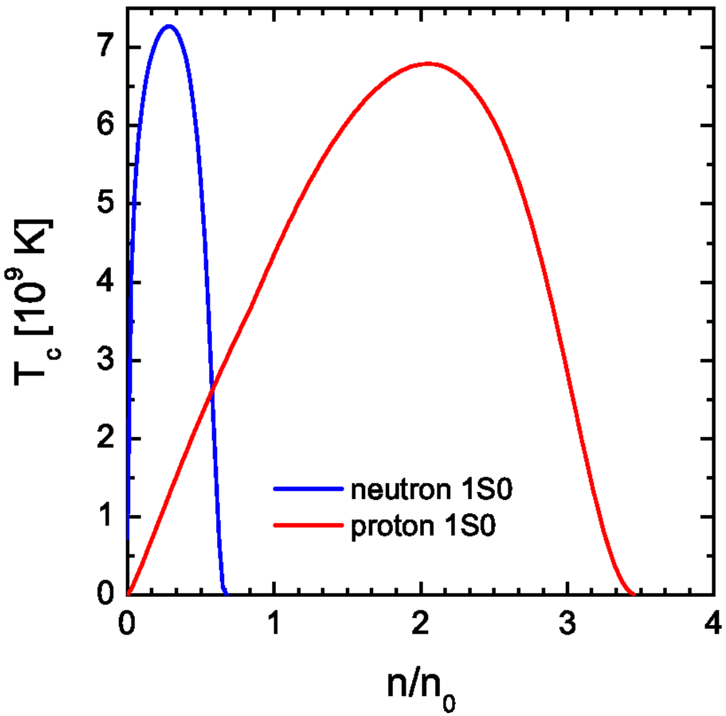
<u>1p for protons</u> $T_0 = 20.29 \cdot 10^9\mathrm{K}\ k_0 = 0,\ k_1 = 1.117,\ k_2 = 1.241,\ k_3 = 0.1473$ [in fm^{-1}]

<u>HSF</u> $T_0 = 98.81 \cdot 10^9\mathrm{K}\ k_0 = 0,\ k_1 = 1.069,\ k_2 = 1.571,\ k_3 = 0.830$ [in fm^{-1}]





✔ T_c in the neutron star matter

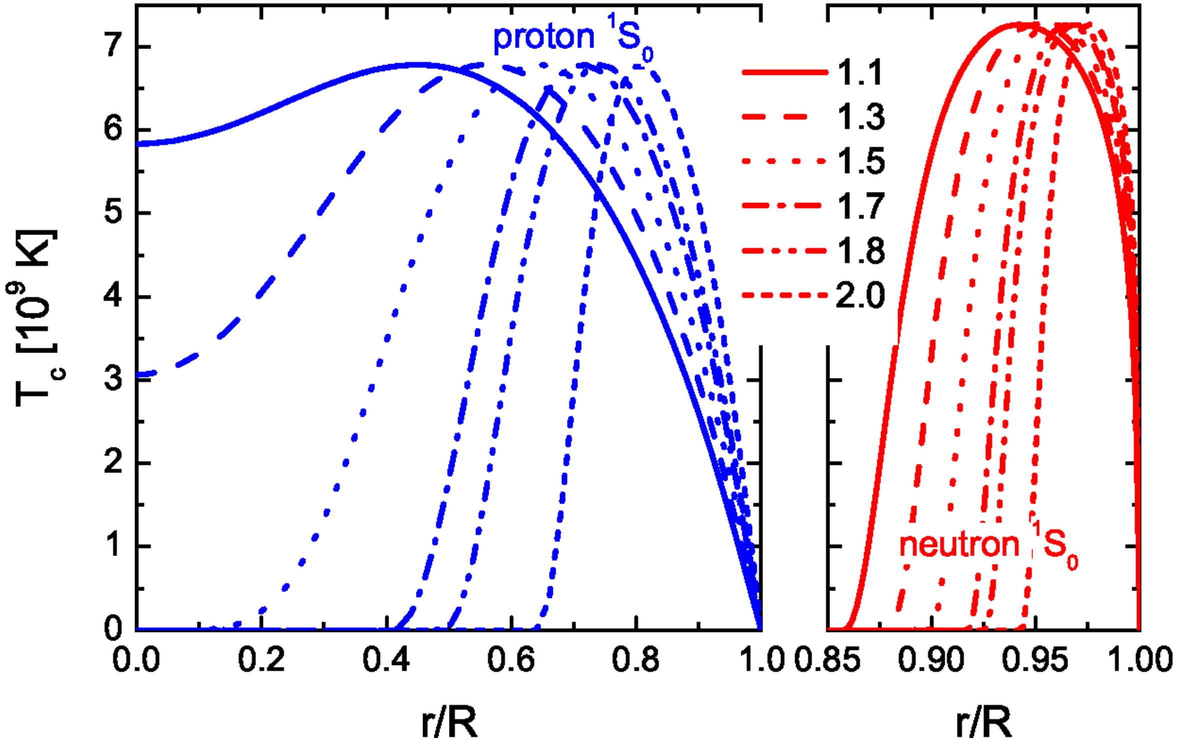


for HDD EoS from
[Blaschke, Grigorian, Voskresnesky PRC 88, 065805(2013)]

For the s-wave paring

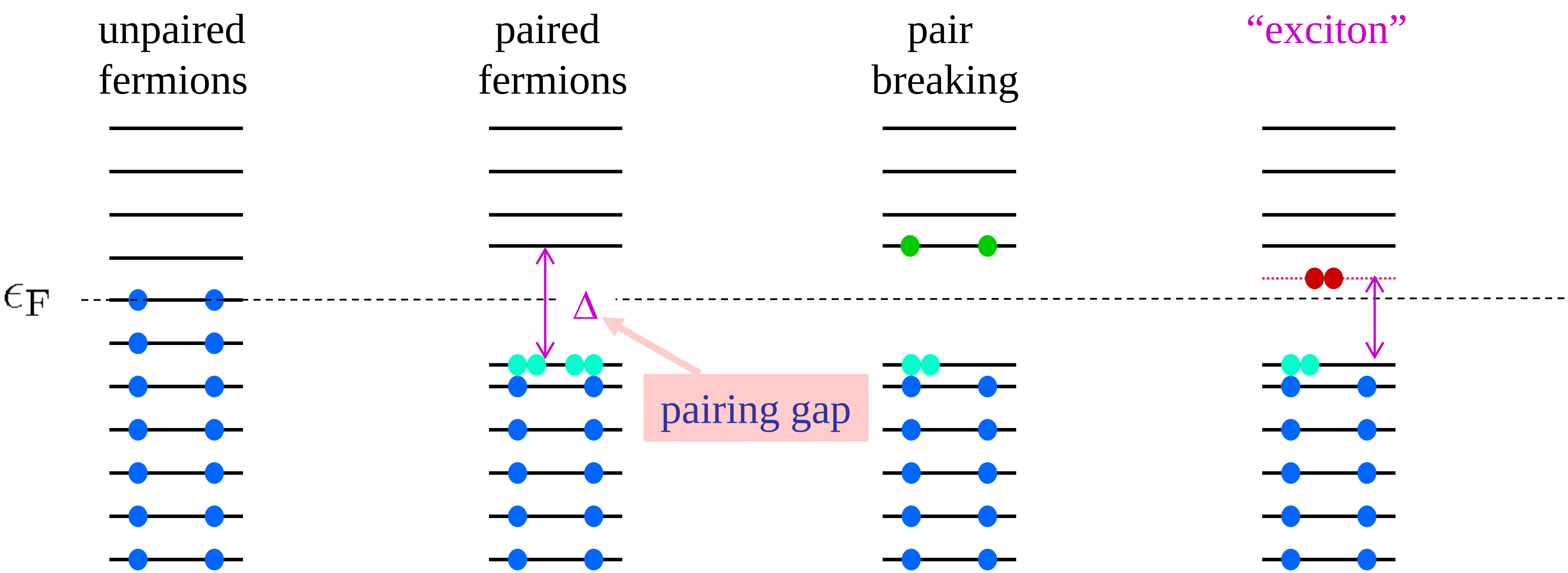
$$T_c = 0.5669\,\Delta(T=0)$$

$$\Delta(T=0) = 1.764\,T_c$$



Fermi system with pairing

Ground state | Excited state



- **quasiparticle spectrum**

$$\xi_p = \frac{p^2}{2\,m^*} - \epsilon_F \simeq v_F\,(p - p_F)$$

$$E_p = \pm\sqrt{\xi_p^2 + \Delta^2}$$

- **excitation spectrum**

$$\epsilon > 2\,\Delta$$

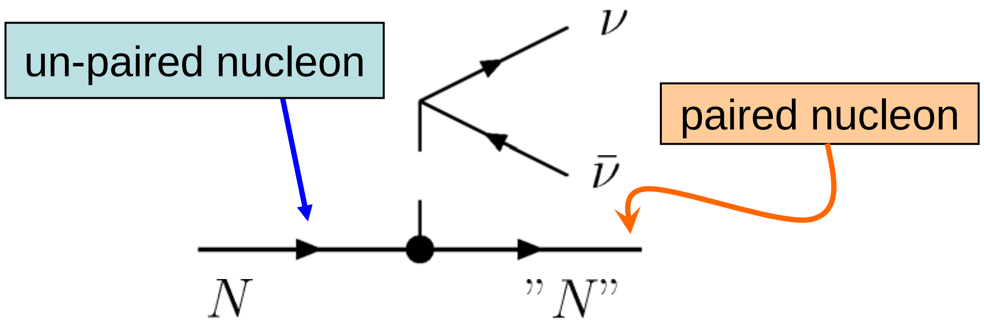
$$\epsilon \sim \omega_c < 2\,\Delta$$

excitation spectrum → emission spectrum

Breaking and Formation of Cooper pairs (PBF)

In <u>superfluid</u> ($T<T_c<0.1$-1 MeV) all two-nucleon processes are suppressed by factor $exp(-2\Delta/T)$

new "quasi"-one-nucleon-like processes (one-nucleon phase space volume) become permitted



[Flowers, Ruderman, Sutherland, AJ 205 (1976), Voskresensky& Senatorov, Sov. J. Nucl. Phys. 45 (1987)]

$$\epsilon_\nu \sim 10^{29} \left[\frac{\Delta_{nn}}{\text{MeV}}\right]^7 \left[\frac{T}{\Delta_{nn}}\right]^{1/2} (n/n_0)^{1/3}\, e^{-2\Delta_{nn}/T}, \quad \frac{\text{erg}}{\text{cm}^3\ \text{sec}}$$

Δ_{nn} is neutron gap

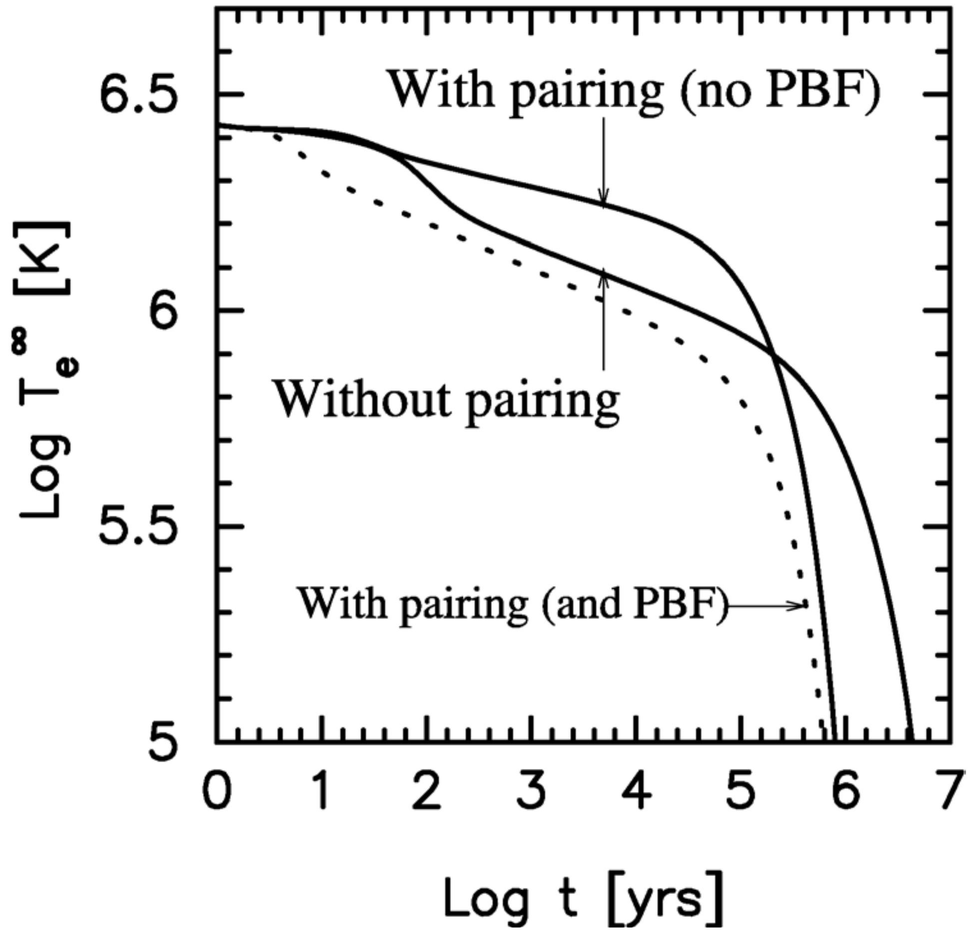
[Voskresensky, Senatorov, Sov. J. Nucl. Phys. 45 (1987); Senatorov, Voskresensky, Phys. Lett. B184 (1987); Voskresensky astro-ph/0101514]

not $\epsilon_\nu \sim 10^{20} T_9^7\ \xi_{nn}^2$ as in Flowers et al. (1976)

Naively one expect the emissivity of $p \to p\,\overline{\nu\nu}$ to be suppressed by extra $c_V^2 \sim 0.006$ factor.

Effects of pairing on the neutron star cooling

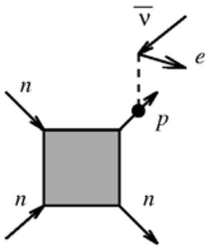
- ✔ enhanced cooling
- ✔ mass dependence

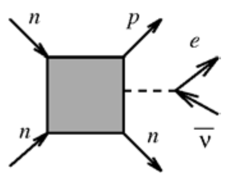


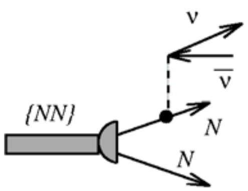
pair breaking and formation (PBF) processes are important!

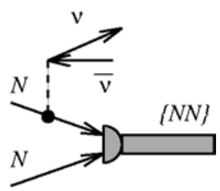
[Page, Geppert, Weber , NPA 777, 497 (2006)]

Neutrino emission reactions









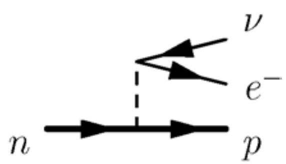
standard

$T < T_{\rm opac} \sim 10^{-1}$–$10^{0}$ MeV

$$10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\rm erg}{\rm cm^3\, s} \times e^{-2\Delta/T}$$

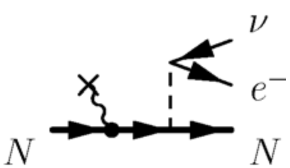
$$10^{29} \times \left(\frac{m_N^*}{m_N}\right) \left(\frac{\Delta}{\rm MeV}\right)^7 \left(\frac{T}{\Delta}\right)^{\frac{1}{2}} e^{-2\Delta/T} \frac{\rm erg}{\rm cm^3\, s}$$

exotic



$$10^{27} \times \left(\frac{m_N^*}{m_N}\right)^2 T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\rm erg}{\rm cm^3\, s} \times e^{-\Delta/T}$$

allowed if $|p_{{\rm F},n} - p_{{\rm F},p}| < p_{{\rm F},e}$



$$7 \cdot 10^{26} \times \left(\frac{m_N^*}{m_N}\right)^2 T_9^6 \frac{|\varphi_c|^2}{m_\pi^2} \frac{\rm erg}{\rm cm^3\, s} \times e^{-\Delta/T}$$