Exercises I: Computation of (tensor) quasinormal modes

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(Dated: July 1, 2015)

These Exercises accompany the lectures given at the HISS “Dense Matter 2015”. This first set of exercises deals with the computation of hydrodynamic retarded Green’s functions in field theory on one hand, and with the computation of tensor (spin 2) quasi-normal modes of the (asymptotically AdS_5) Reissner-Nordström black brane on the other hand.

Overview Quasinormal modes (QNM) of black hole and black brane spacetimes can serve as a model for late-time behavior of heavy-ion-collisions or cold atomic gas cloud collisions. On a more technical level, QNMs should match the late-time behavior of fully time depedent holographic thermalization codes in the spirit of Chesler/Yaffe [8], for more detail see [9]. (Note that working out the fully time-dependent solution to a given system would be the natural continuation of the present QNM project.)

Heavy ion collisions are believed to produce a charged plasma in presence of magnetic fields. Off-center heavy ion collisions also have a nonzero angular momentum. When holographically modeling heavy ion collisions by off-center collisions of gravitational shocks, those setups should approach a late time state which has nonzero angular momentum. Some of those holographic thermalization codes (depending on the details of the model) should approach the Kerr black brane or black hole spacetime. But for simplicity, we will consider only a charged (non-rotating, non-magnetic) plasma here.

The goal here is the computation of the quasinormal modes of a charged in asymptotically AdS_5 spacetime. We are first interested in the case of vanishing spatial momentum $\vec{k} = 0$. Metric perturbations $h_{\mu\nu}$ around the black brane metric can be classified into scalars, vectors, and tensors under the $SO(3)$ rotation group in the x, y, and z directions. For this exercise we focus on the tensor fluctuation h_{xy}, which is associated with the shear viscosity within the field theory dual to this gravitational theory. The field theory state we are examining here is a thermal plasma that is electrically charged.

In order to get an intuition for poles in Green's functions and their physical interpretation, we first work in the hydrodynamic approximation, using only field theory (no gravitational model, no holography), in exercise I.1.

References A good but slightly outdated general introduction to holography applied to QCD is given in [4]. Thermalization and far-from-equilibrium dynamics is also an important topic in condensed matter physics. Relations between holographic models and condensed matter physics are nicely worked out in this set of lectures: [5] (which is also a great general introduction), [7], [6]. Very good lectures on hydrodynamics as an effective field theory from a modern perspective are provided in [2], a more involved example (including chiral transport) is considered in [10], and the holographically dual description is discussed in [3]. In exercise I.2, we then compute the higher poles, lying deeper in the complex frequency plane than the hydrodynamic ones.

Exercise I. 1. Hydrodynamic Green's functions Compute the two-point correlation function $\langle j^x\, j^x \rangle$ from the hydrodynamic constitutive equation (for simplicity in 2+1 dimensions)

$$\langle j^\mu \rangle = n u^\mu - \sigma\, T \left[(g^{\mu\nu} + u^\mu u^\nu) \partial_\nu \left(\frac{\mu}{T} \right) - \frac{E^\mu}{T} \right] , \tag{1}$$

where $E^\mu = u_\nu F^{\mu\nu}$, by varying with respect to the appropriate source ($A_x(t, x, y)$). Assume that only the chemical potential μ (or eqivalently the charge density n) is allowed to fluctuate, i.e. we keep the temperature T and fluid velocity u^μ fixed. Further, we pick a frame where the fluid velocity is given by $u^\mu = (1, 0, 0)$. Note that the susceptibility $\chi = \partial n/\partial\mu$ can be used to translate a gradient in chemical potential to a gradient in charge density. Without restricting the generality, we also rotate to a coordinate system in which the momentum of our fluctuations points along the z-direction. The fluctuations of the charge density can be expanded in Fourier modes.

Exercise I. 2) Compute the tensor metric fluctuations $h_{xy} \propto e^{-i\omega t + i\vec{k}\cdot\vec{x}}\, \phi(u)$ (u is the radial AdS coordinate) at vanishing spatial momentum $\vec{k} = 0$ around asymptotically AdS_5 charged black brane, i.e. Reissner-Nordström black branes. The equation of motion for the fluctuations $\phi = {h_x}^y$ is given by

$$0 = \phi''(u) - \frac{f(u) - u f'(u)}{u f(u)} \phi'(u) + \frac{\tilde{\omega}^2 - f(u)\tilde{k}^2}{4 u f(u)^2} \phi(u) , \tag{2}$$

where the blackening factor $f(u) = 1 - (1 + \tilde{q}^2)u^2 + \tilde{q}^2 u^3$ stems from the black brane metric background shown during the lecture, the dimensionless momentum $\tilde{k} = k/r_H$ and frequency $\tilde{\omega} = \omega/r_H$. Quasinormal modes are the frequencies for which two boundary conditions are satisfied: (i) at the horizon, $u = 1$, the field ϕ behaves like $\phi = (1-u)^\alpha(\phi^{(0)} + \phi^{(1)}(1 - u) + \phi^{(2)}(1-u)^2 + \ldots)$; and (ii) vanishes at the AdS boundary, $\lim_{u\to 0} \phi(u; \omega) = 0$. First, compute α using the equation of motion, then use the equation of motion in order to relate the coefficients $\phi^{(1)}$, $\phi^{(2)}$ to $\phi^{(0)}$. Then pick a $\phi^{(0)} = 1$ and find a solution that satisfies condition (ii). A very useful reference may be [1].

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