Universität Bielefeld

Spectral and transport properties of the QGP from Lattice QCD calculations

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Helmholtz International Summer School “Dense Matter“
Dubna
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Lattice calculations of hadronic correlation functions

... and how we try to

extract **transport properties** and **spectral properties** from them

1) Vector meson correlation functions for light quarks

continuum extrapolation — with H-T.Ding, F.Meyer, et al.

comparison to perturbation theory — with J.Ghiglieri, M.Laine, F.Meyer

→ Electrical conductivity

→ Thermal dilepton rates and thermal photon rates

2) Color electric field correlation function — with A.Francis, M. Laine, T.Neuhaus, H.Ohno

Heavy quark momentum diffusion coefficient κ

3) Vector meson correlation functions for heavy quarks — with H-T.Ding, H.Ohno et al.

Heavy quark diffusion coefficients

Charmonium and Bottomonium dissociation patterns

Motivation - Transport Coefficients

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

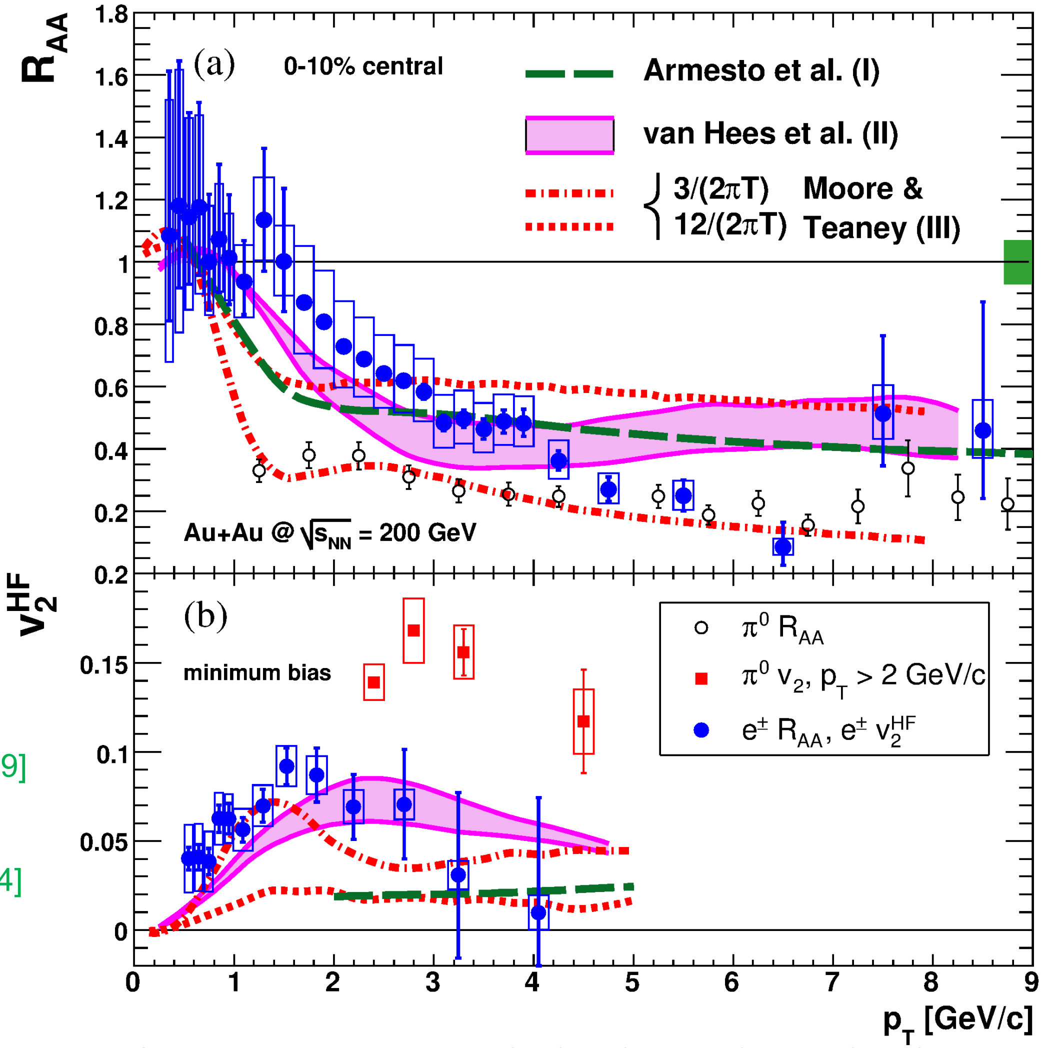
Need to be determined from QCD using first principle lattice calculations!

for heavy flavour:

- Heavy Quark Diffusion Constant D
  [H.T.Ding, OK et al., PRD86(2012)014509]
- Heavy Quark Momentum Diffusion κ
  [OK, arXiv:1409.3724]

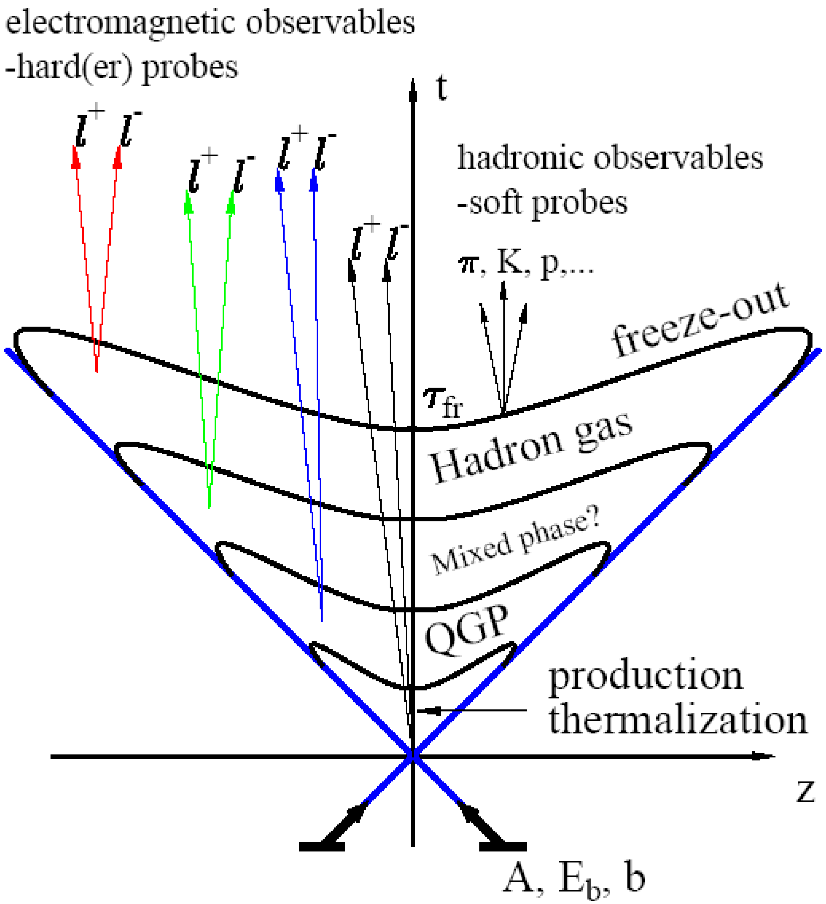
for light quarks:

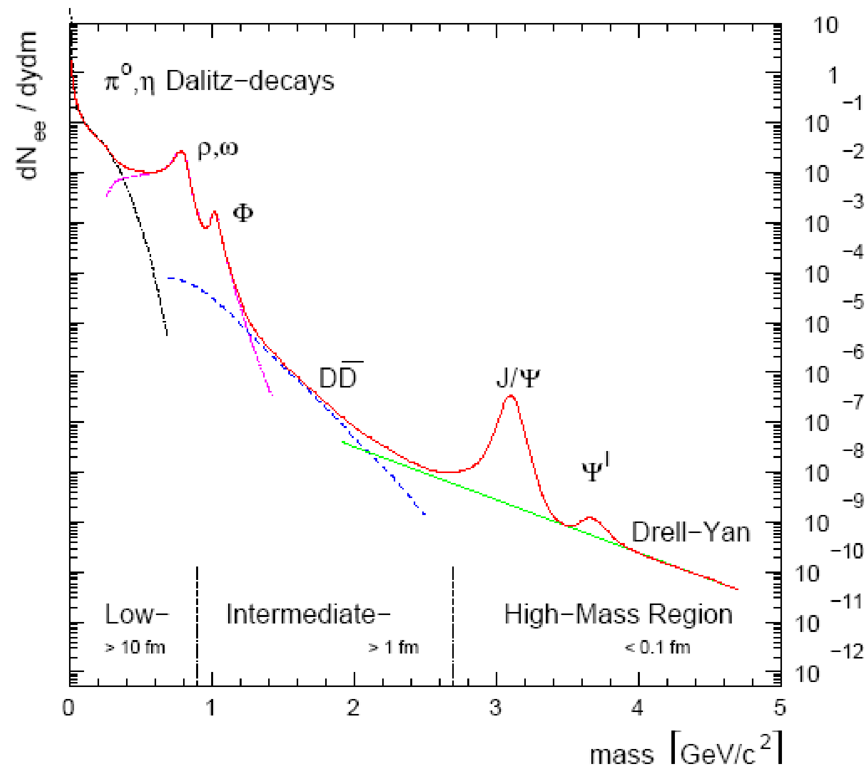
- Light quark flavour diffusion
- Electrical conductivity
  [A.Francis, OK et al., PRD83(2011)034504]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

Hard Probes in Heavy Ion Collisions - Dileptons





Dileptonrate directly related to vector spectral function:

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{(\omega^2-\vec{p}^2)(e^{\omega/T}-1)}\rho_V(\omega,\vec{p},T)$$

Motivation – PHENIX/STAR results for the low-mass dilepton rates

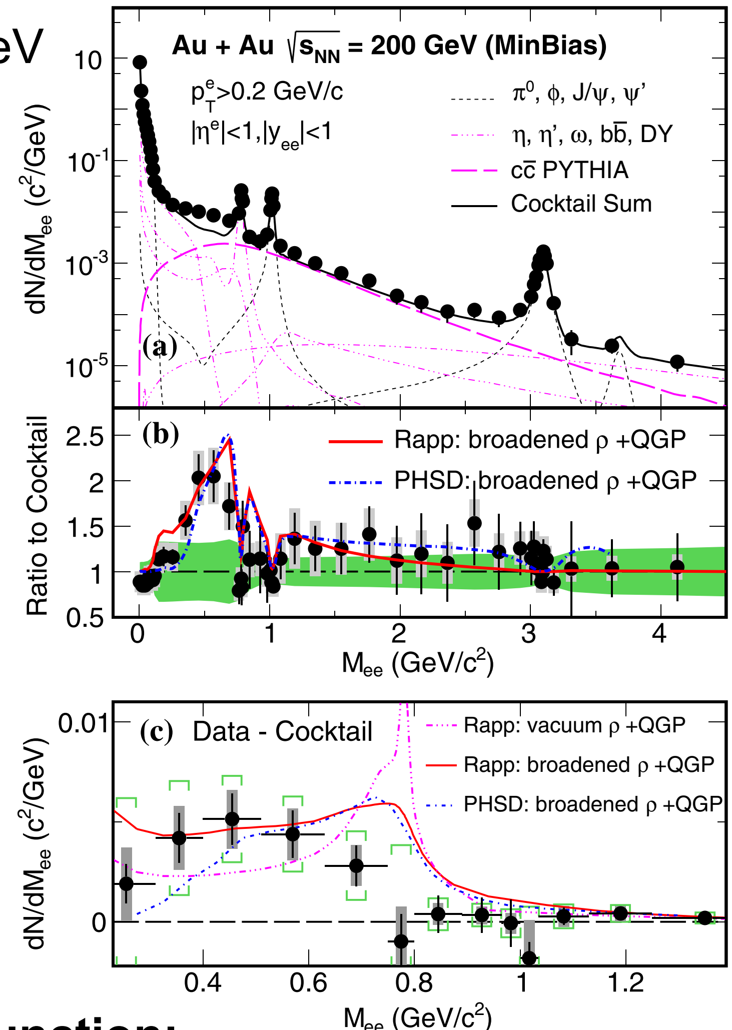
pp-data well understood by hadronic cocktail

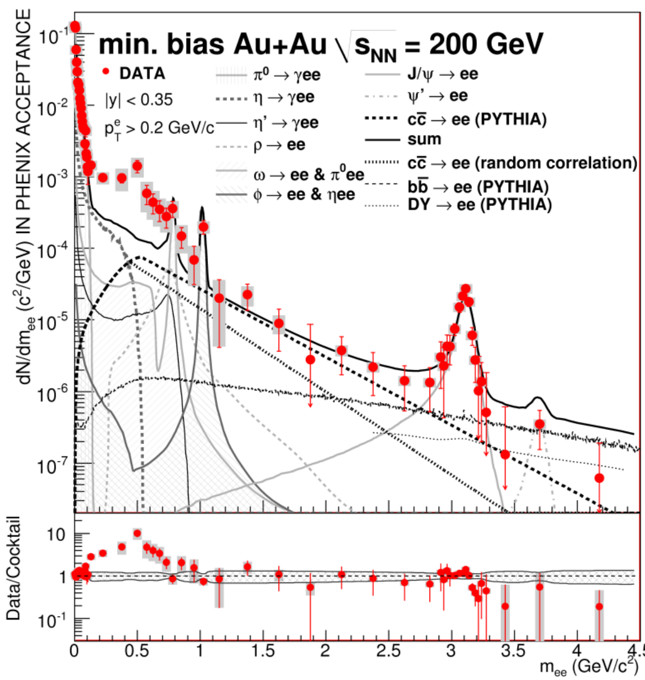
large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP

[STAR collaboration 2014]





[PHENIX PRC81, 034911 (2010)]

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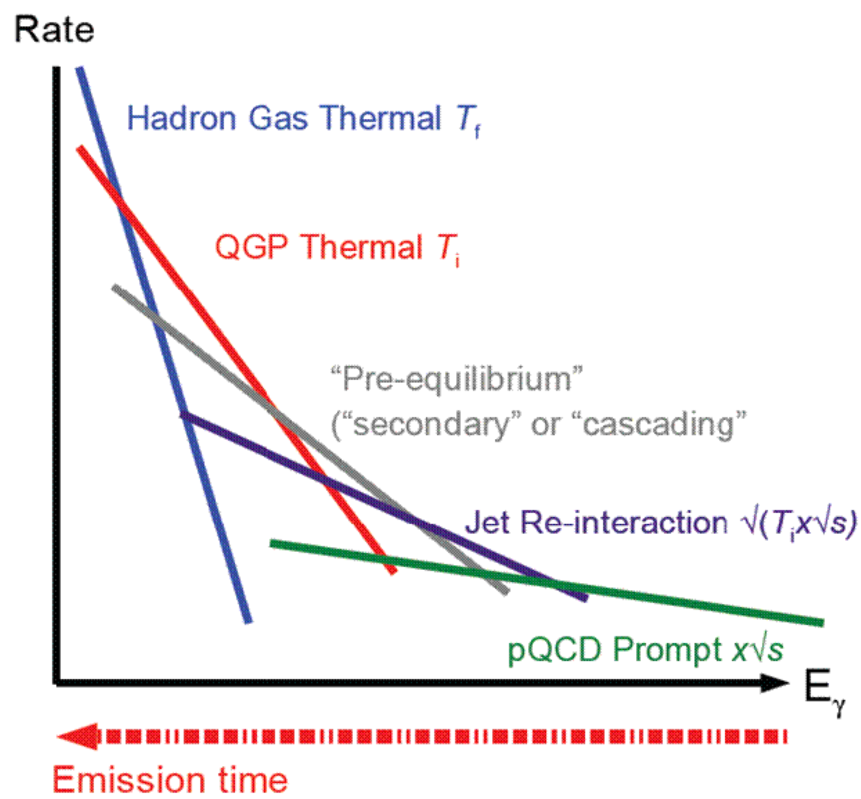
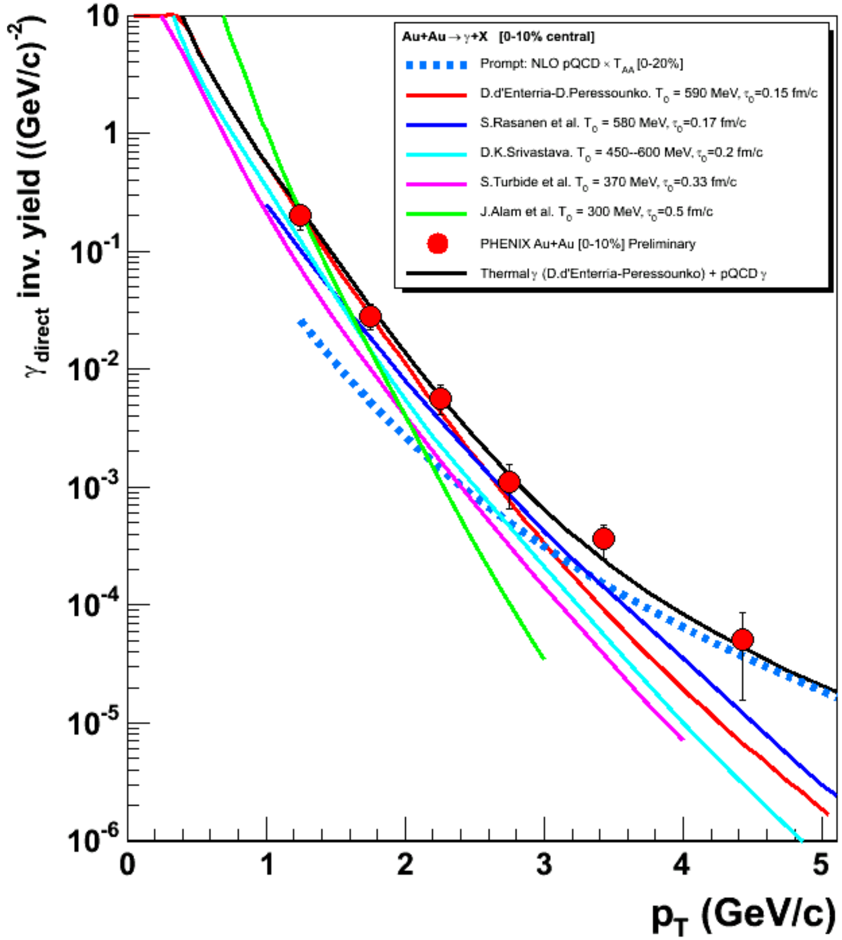
Direct and fragmentation photon relative contribution



[Fleuret 2009]



Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V(\omega = |\vec{p}|, T)$$

Vector meson correlation function from Lattice QCD

Transport coefficients usually calculated using correlation function of conserved currents

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\mathbf{p},T)K(\tau,\omega,T)$$

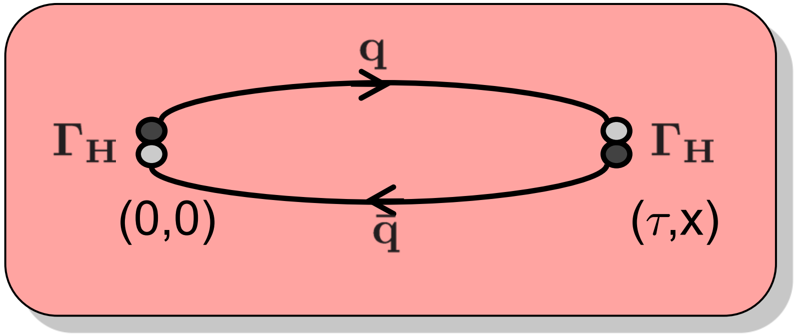
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})e^{i\vec{p}\vec{x}}$$



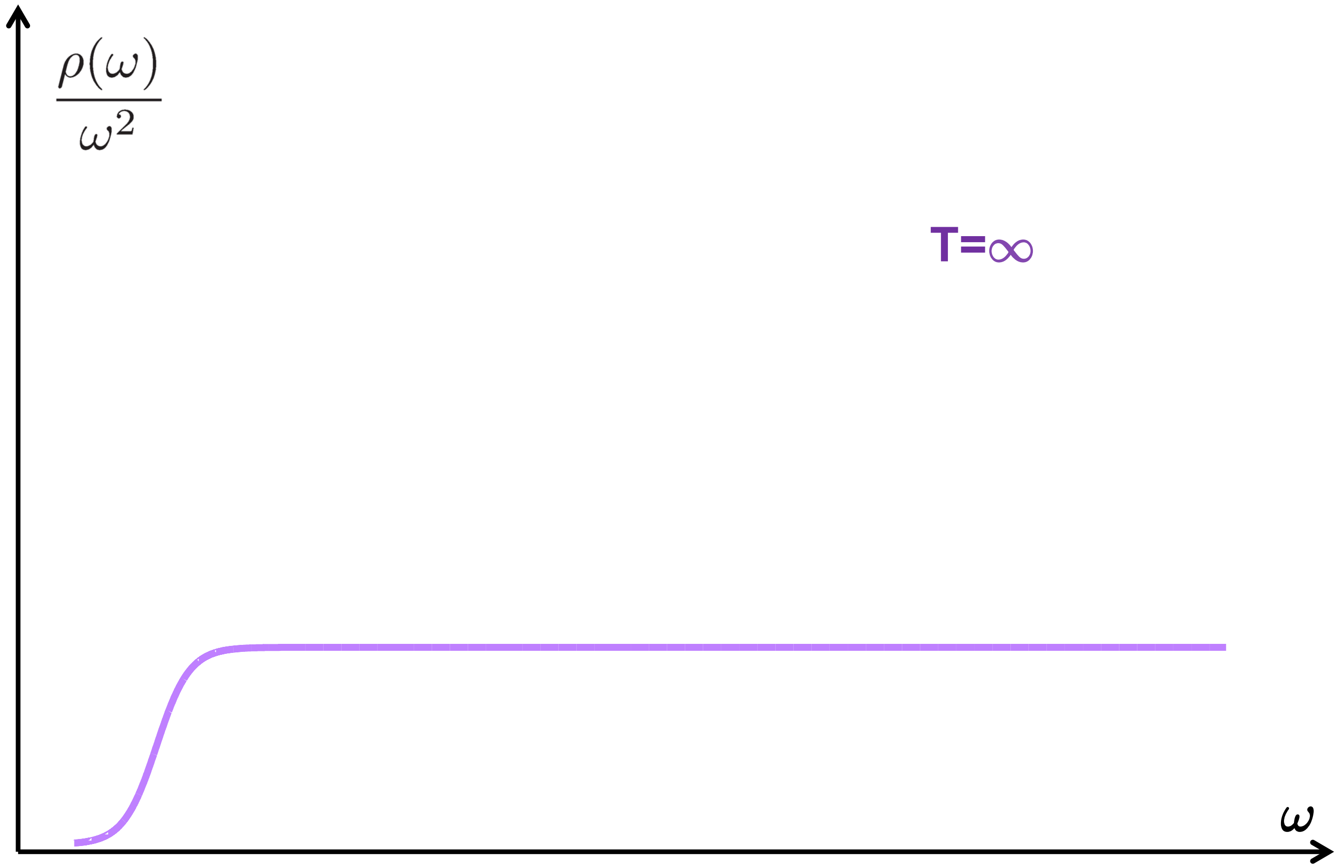
related to a conserved current

only correlation functions calculable on lattice but

Transport coefficient determined by slope of spectral function at ω=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

Vector spectral function – What do we expect!?



+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

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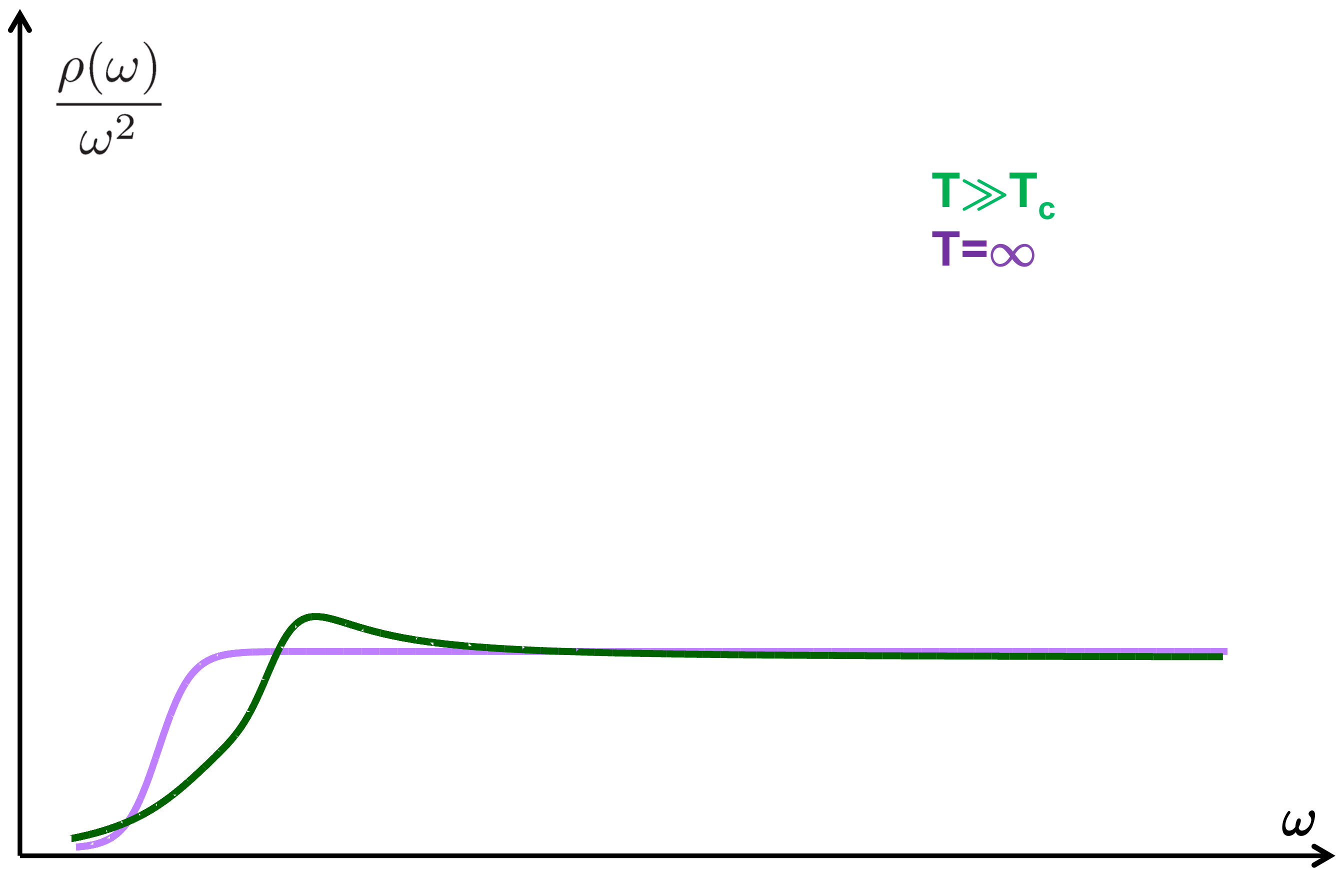

+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

+ transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\ , \qquad \eta = \frac{T}{MD}$

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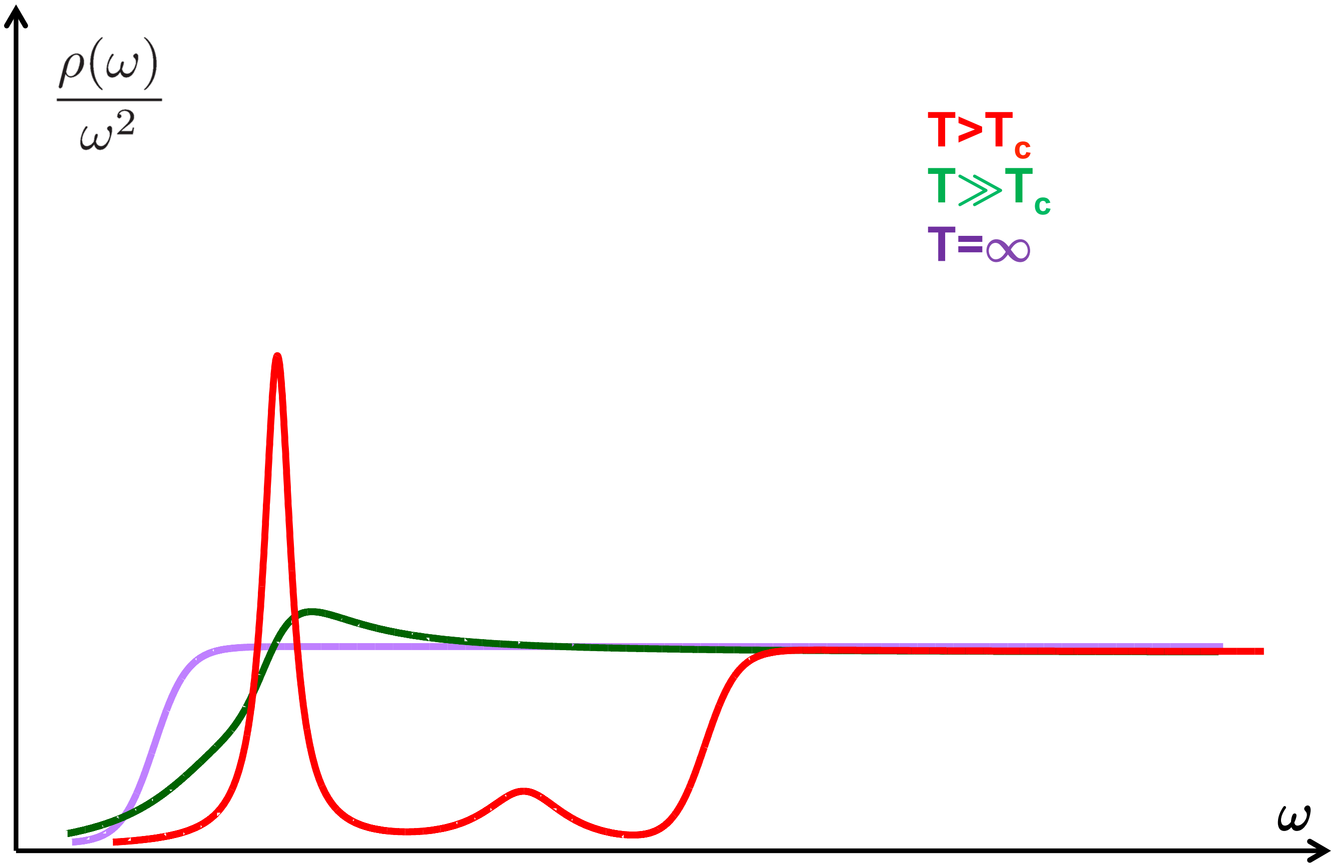

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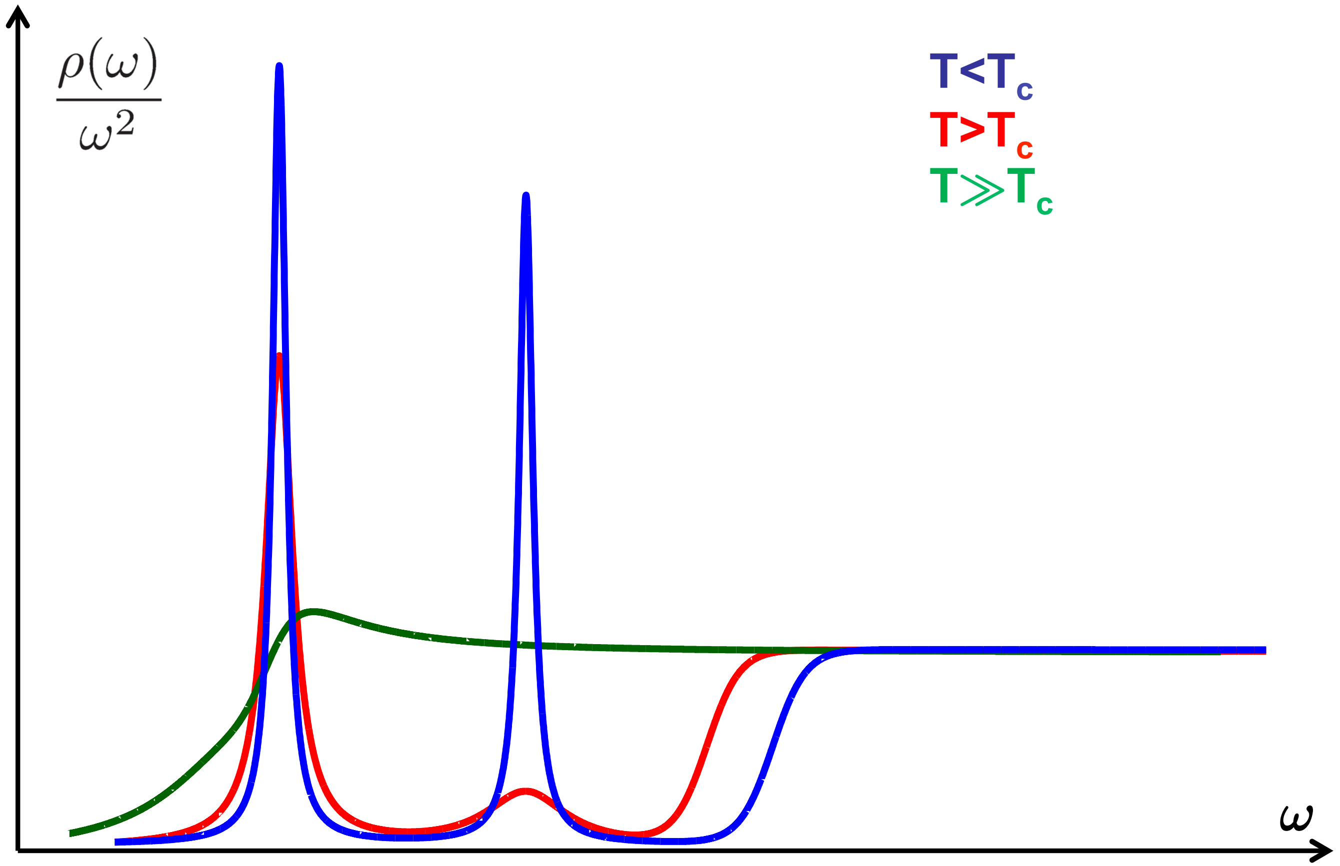

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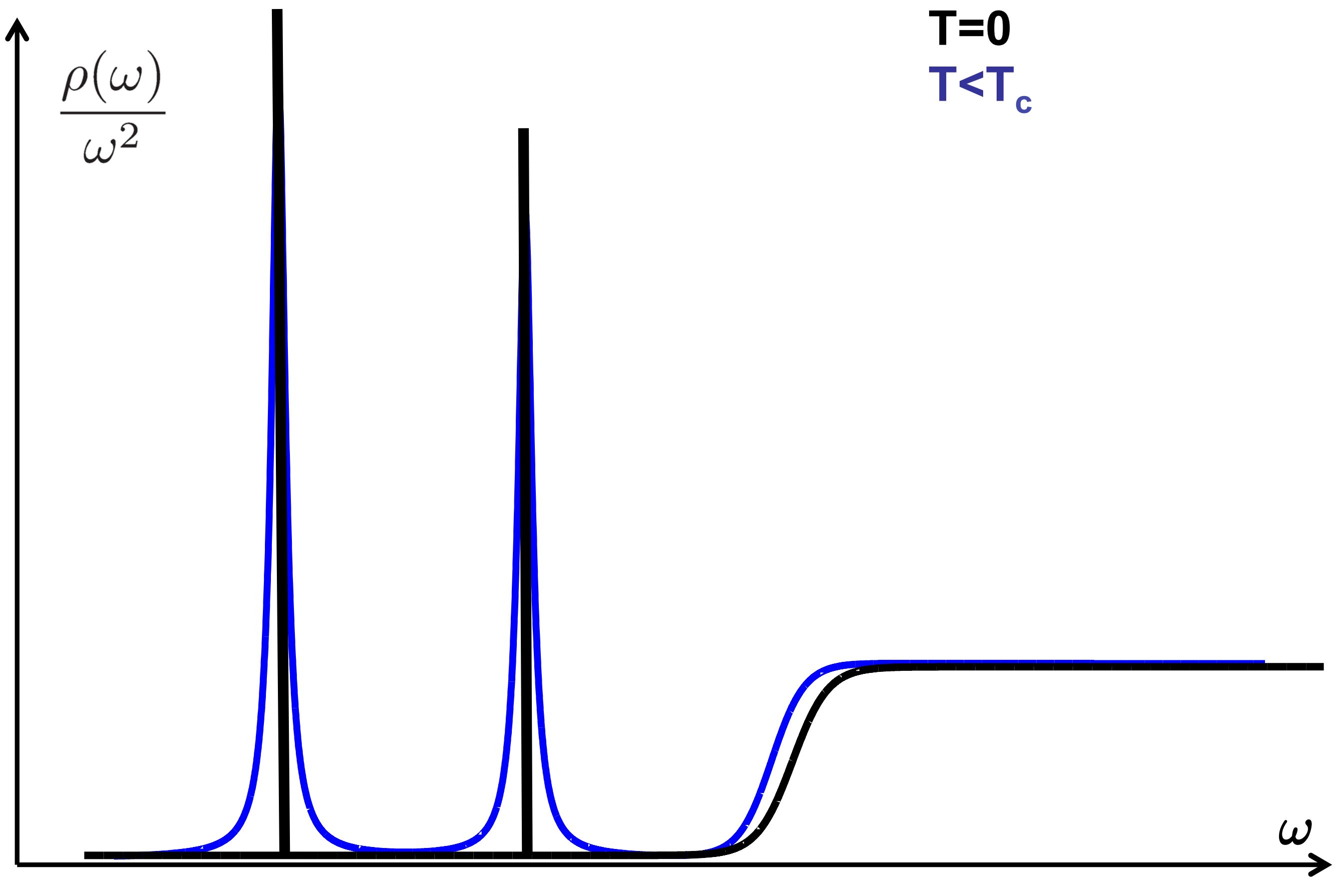

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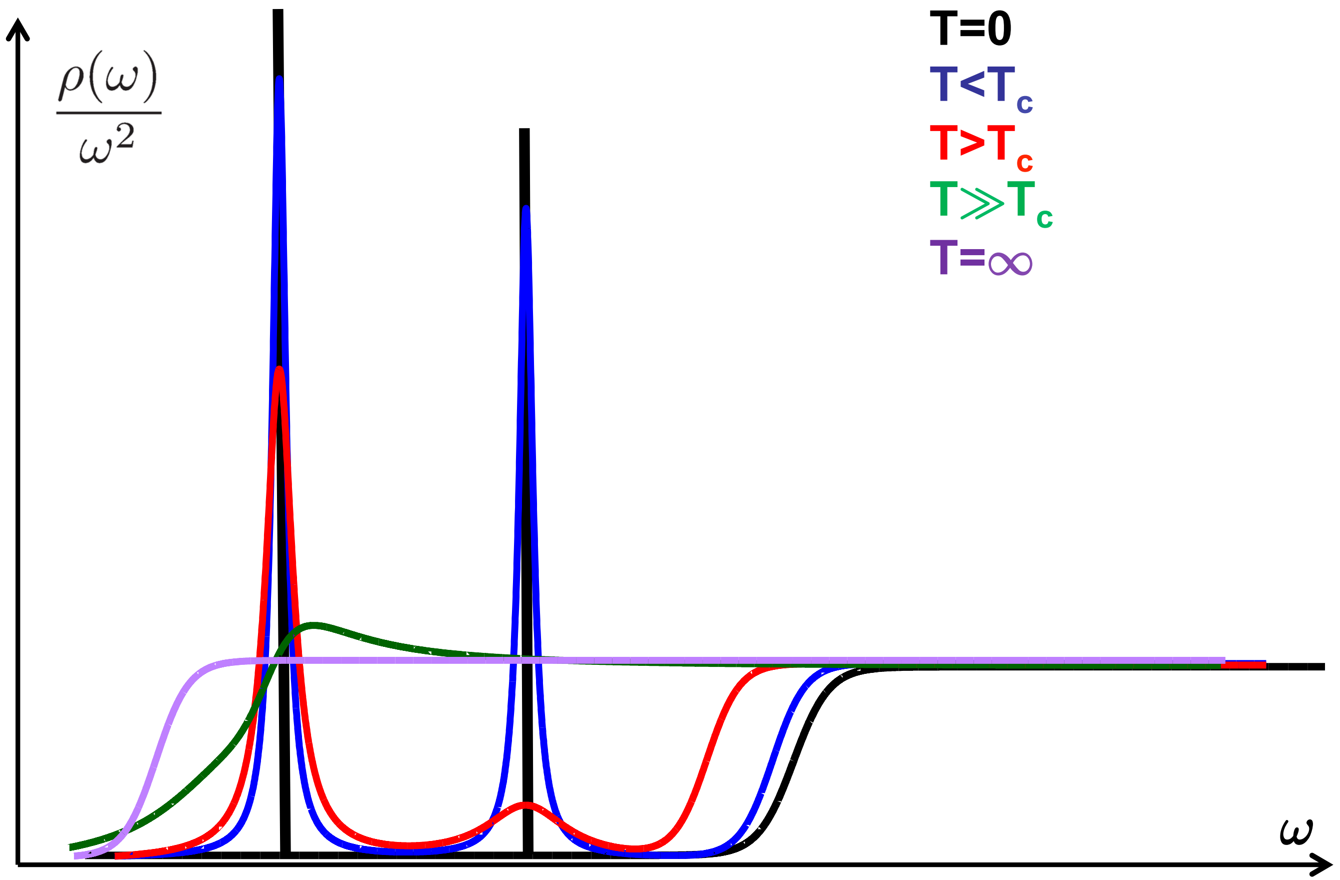

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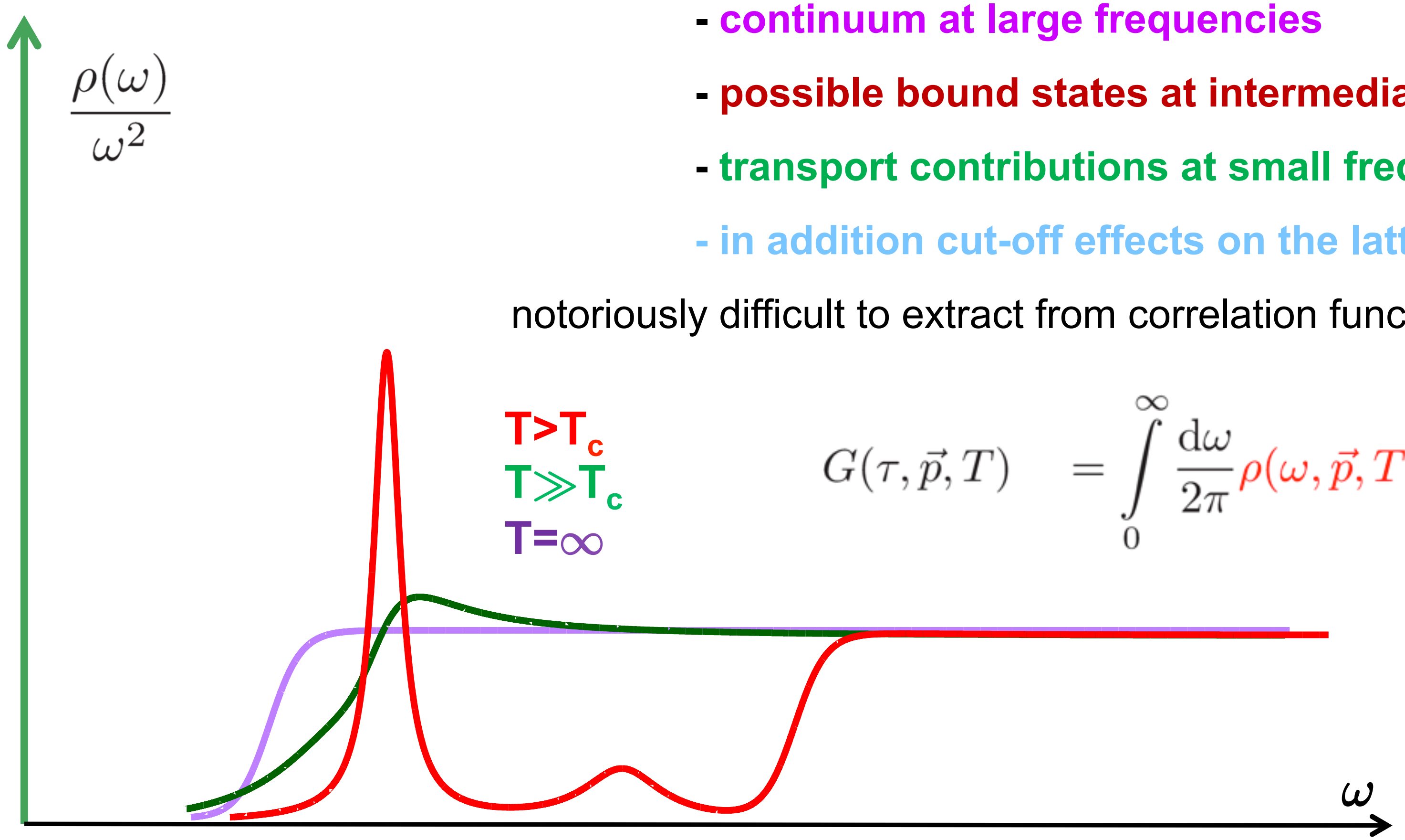
Vector spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions



$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

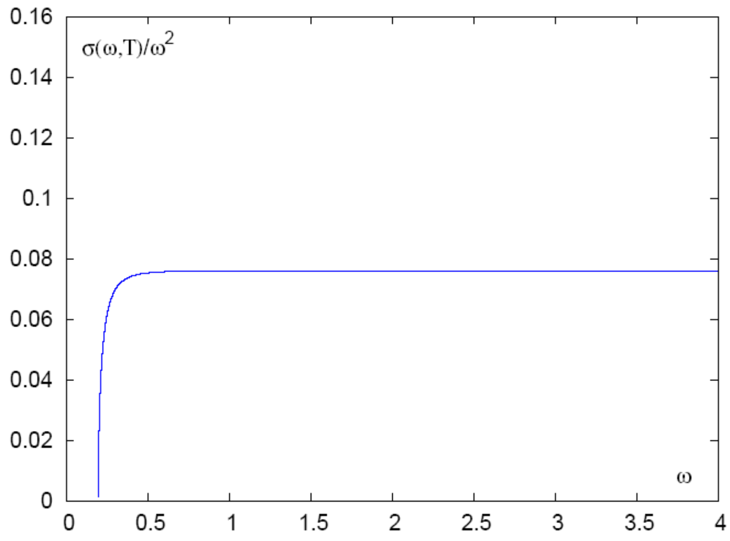
+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\,\omega\delta(\omega)$

+ (narrow) transport peak at small ω: $\rho(\omega \ll T) = 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

Free spectral functions – lattice vs. continuum

Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

$$\sigma_H = \frac{N_c}{8\pi^2}\Theta(\omega^2 - 4m^2)\ \omega^2 \tanh(\frac{\omega}{4T})$$
$$\times\sqrt{1-(\frac{2m}{\omega})^2}\left[a_H + (\frac{2m}{\omega})^2\ b_H\right]$$
$$+\ \frac{N_c}{3}\ \frac{T^2}{2}\ f_H\ \omega\delta(\omega)$$



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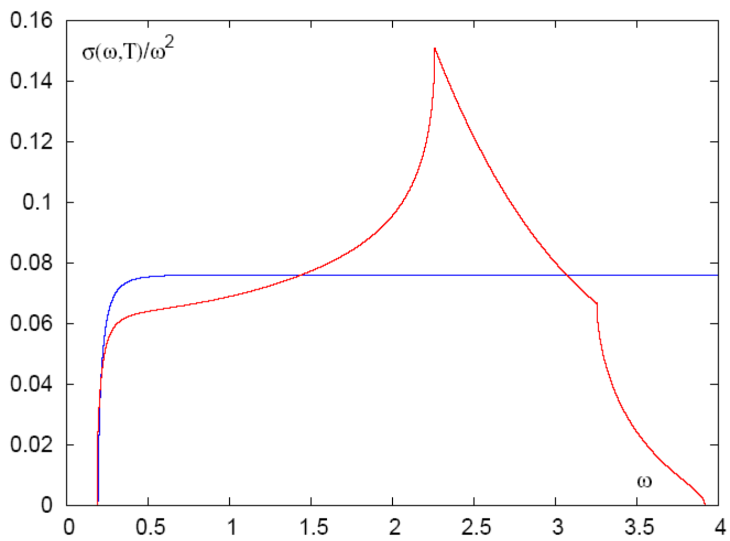
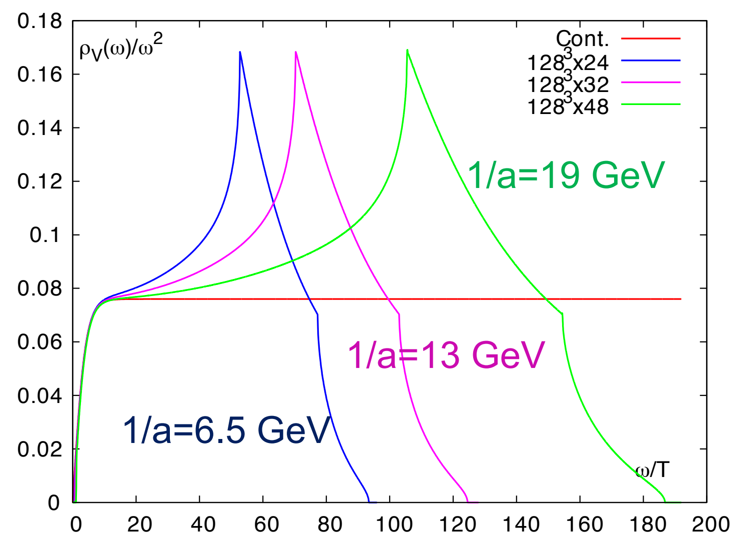
$$\sigma_H = \frac{N_c}{8\pi^2}\Theta(\omega^2-4m^2)\ \omega^2\tanh(\frac{\omega}{4T}) \times\sqrt{1-(\frac{2m}{\omega})^2}\left[a_H+(\frac{2m}{\omega})^2\ b_H\right] + \ \frac{N_c}{3}\ \frac{T^2}{2}\ f_H\ \omega\delta(\omega)$$

Lattice cut-off effects (here for Wilson fermions):

$$\omega_{max} = 2\log(7+ma)$$





we will perform the continuum extrapolation to get rid of the cut-off effects

Free spectral functions – lattice vs. continuum

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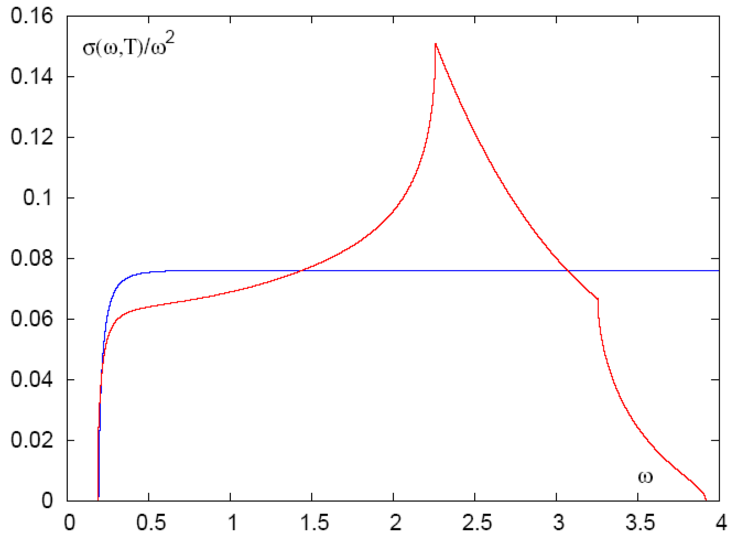
with interactions:

$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2}$$

zero mode contribution at $\omega \simeq 0$ [Umeda 07]

[Petreczky+Teaney 06
Aarts et al. 05]

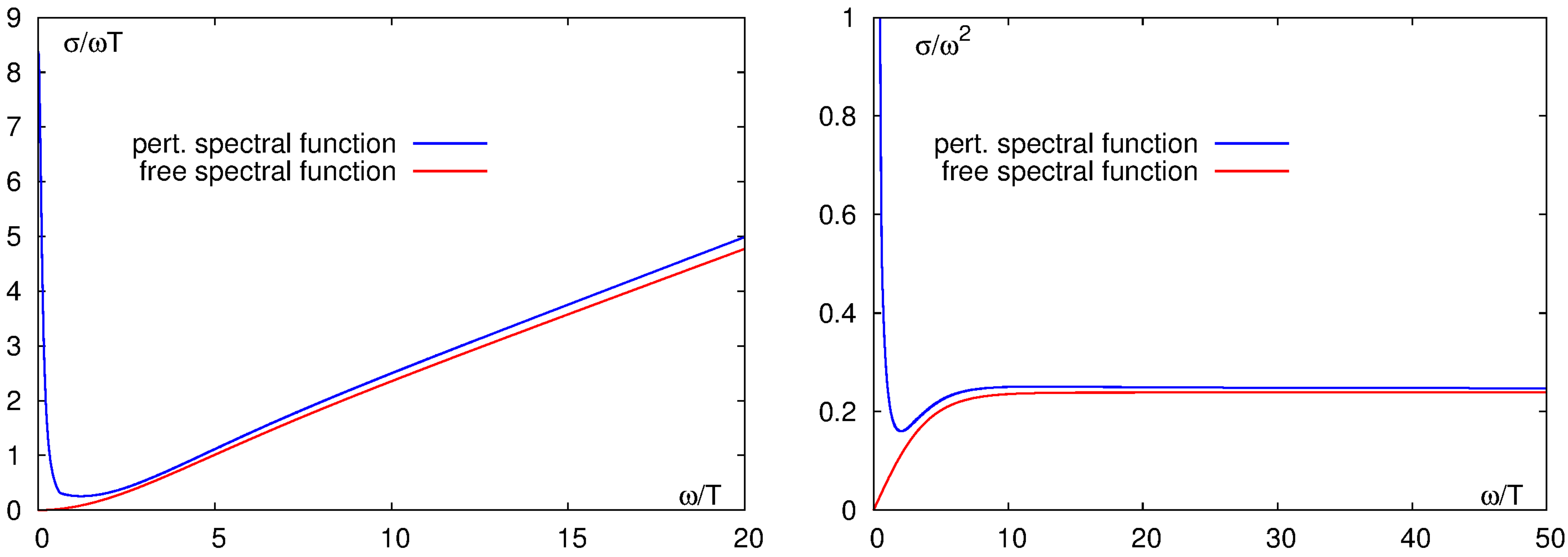


Vector spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) \quad = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

Example:

compare two spectral functions – with and without transport peak



What are the contributions of the different scales?

Vector spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

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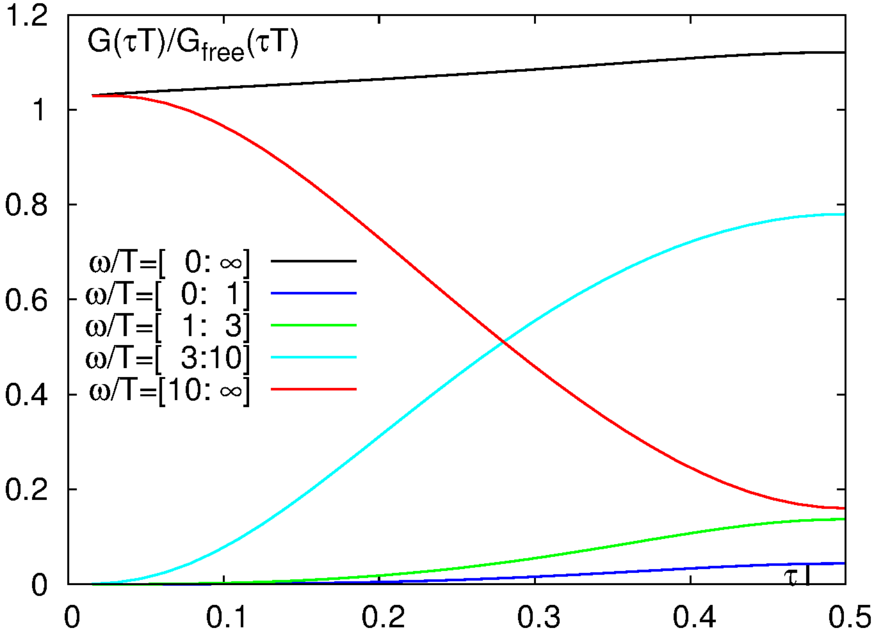
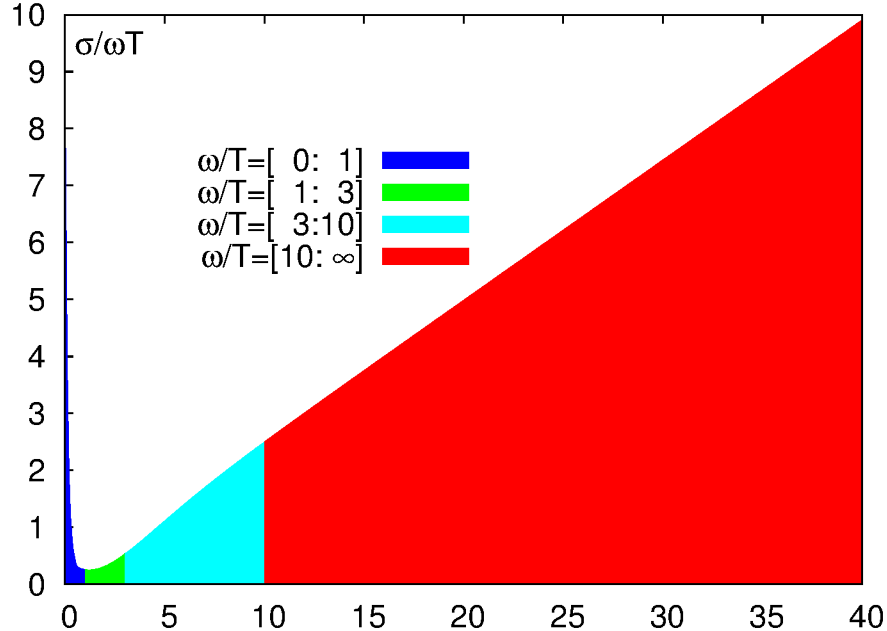




What are the contributions of the different scales?

- corresponding correlations functions are very similar, up to 10% at large τT
- only very small contribution from the transport peak
- very accurate data required

Spectral functions at high temperature

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\rho_{00}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega)$$
$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)$$

δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{00}(\omega)$+$\rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{00} is protected, the δ-funtion in ρ_{ii} gets smeared:

$$\kappa = \frac{\alpha_s}{\pi}$$

at leading order

Ansatz:

$$\rho_{00}(\omega) = 2\pi\chi_q \omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2 \tanh(\omega/4T)$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

["Thermal dilepton rate and electrical conductivity…",
H.T.-Ding, OK et al., PRD83 (2011) 034504]

Electrical Conductivity

Electrical Conductivity ⟷ slope of spectral function at ω=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

$$C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2 = \begin{array}{lll} 5/9\ e^2 & \text{for} & n_f = 2 \\ 6/9\ e^2 & \text{for} & n_f = 3 \end{array}$$

Using our Ansatz for $\rho_{ii}(\omega)$:

$$\frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW}\ C_{em}$$

Vector correlation function on large & fine lattices

[H.T.-Ding, OK et al., PRD83 (2011) 034504]

Quenched SU(3) gauge configurations at T/T_c=1.5 (separated by 500 updates)

Lattice size $N_\sigma^3 N_\tau$ with N_σ = 32 – 128
N_τ = 16, 24, 32, 48

Temperature: $T = \frac{1}{aN_\tau}$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to the chiral limit, $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

Volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	$1/a$[GeV]	a[fm]	#conf
16	32	6.872	1.4124	0.13495	0.829	6.43	0.031	60
16	48	6.872	1.4124	0.13495	0.829	6.43	0.031	62
16	64	6.872	1.4124	0.13495	0.829	6.43	0.031	77
16	128	6.872	1.4124	0.13495	0.829	6.43	0.031	129
24	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	19.30	0.010	431

cut-off dependence & continuum extrapolation

close to continuum

New results at 1.1 and 1.2 T_c

[H.T.Ding, A.Francis, OK, F.Meyer, M.Müller et al., arXiv:1301.7436,1312:5609,1412.5869]

PRACE-Project:
Thermal Dilepton Rates and
Electrical Conductivity in the QGP
(JUGENE Bluegene/P in Jülich)

	$1.1\ T_c$	$1.2\ T_c$					
N_σ	N_τ	N_τ	β	κ	$1/a$[GeV]	a[fm]	#Confs
96	32	28	7.192	0.13440	9.65	0.020	250
144	48	42	7.544	0.13383	13.21	0.015	300
192	64	56	7.793	0.13345	19.30	0.010	240

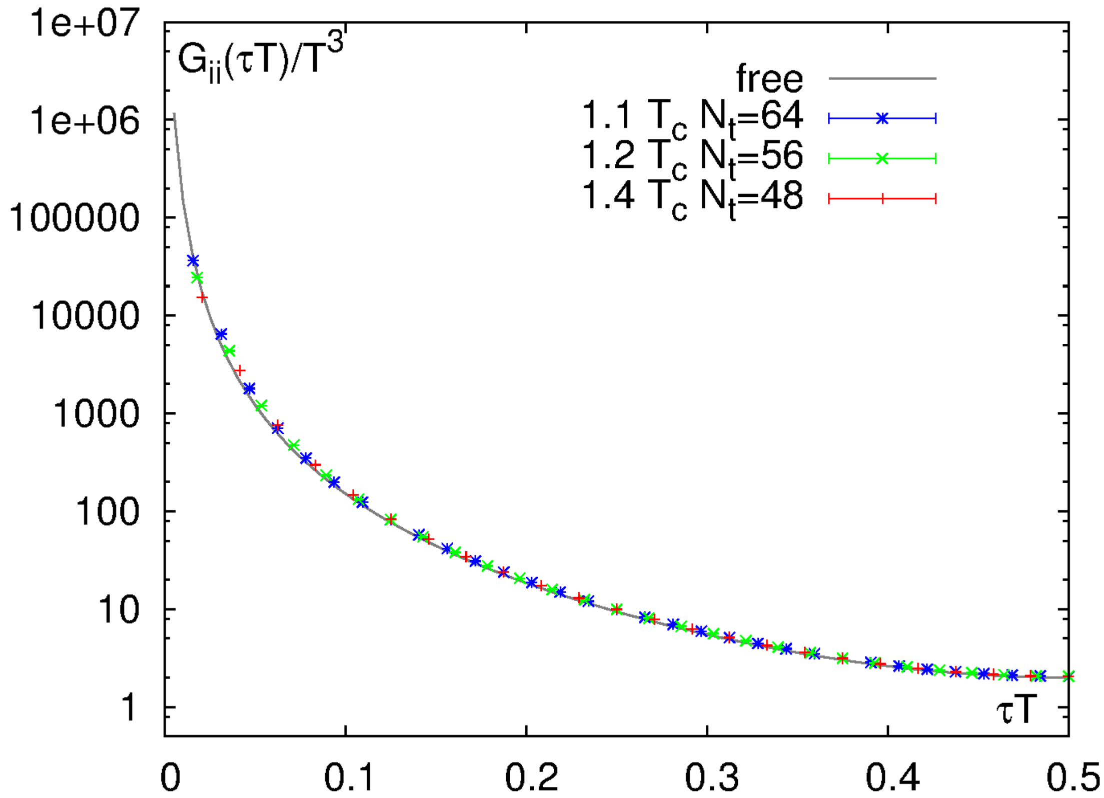
study of T-dependence of dilepton rates and electrical conductivity

fixed aspect ratio $N_\sigma/N_\tau = 3$ and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

Vector correlation function



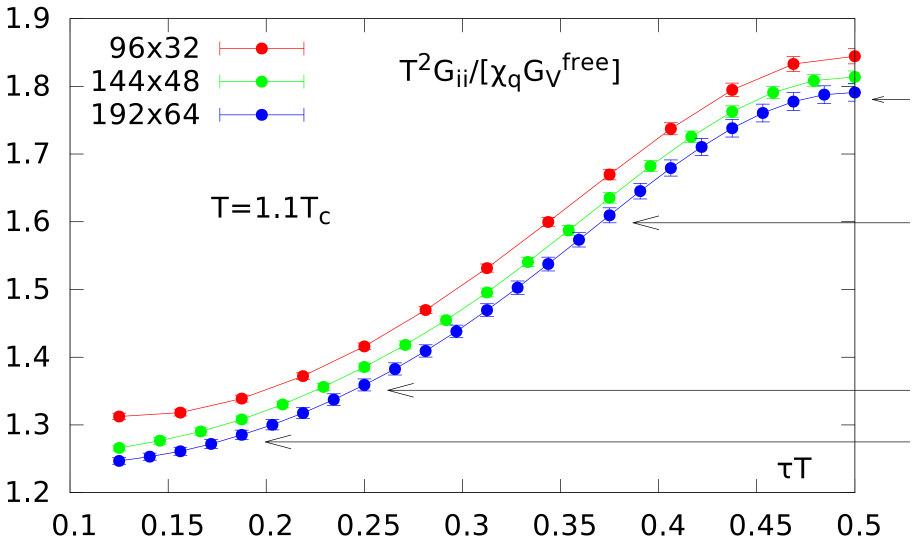
compared to free (non-interacting) correlator:

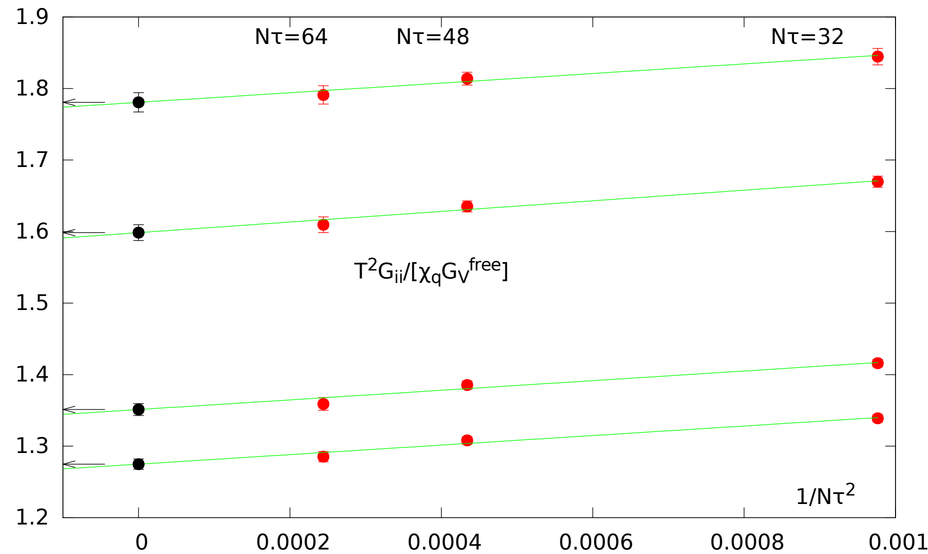
$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T) \frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

hard to distinguish differences due to different orders of magnitude in the correlator

→ in the following we will use $G_V^{free}(\tau)$ as a normalization

Continuum extrapolation



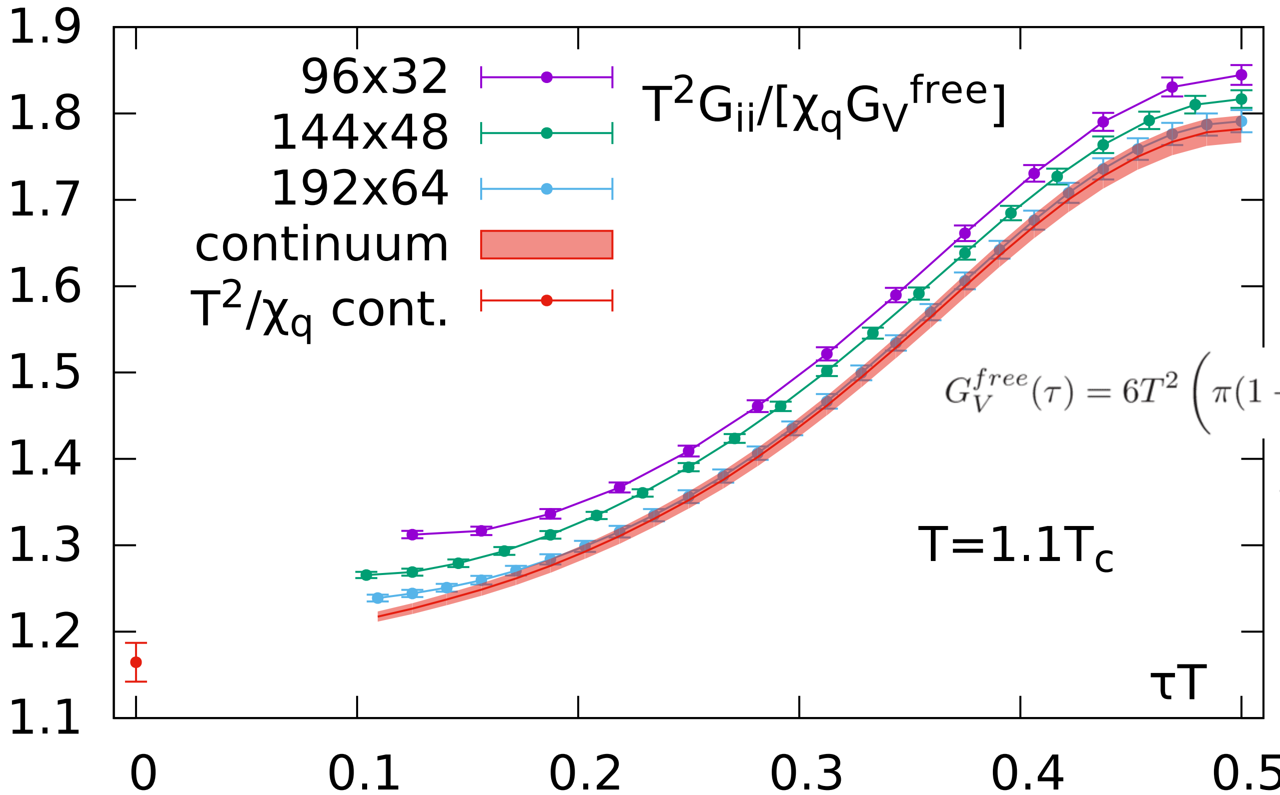


we interpolate the correlator for each lattice spacing

and perform the continuum limit a→0 at each distance τT

cut-off effects are visible at all distances on finite lattices

Continuum extrapolation



normalized by free correlator

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

and quark number susceptibility

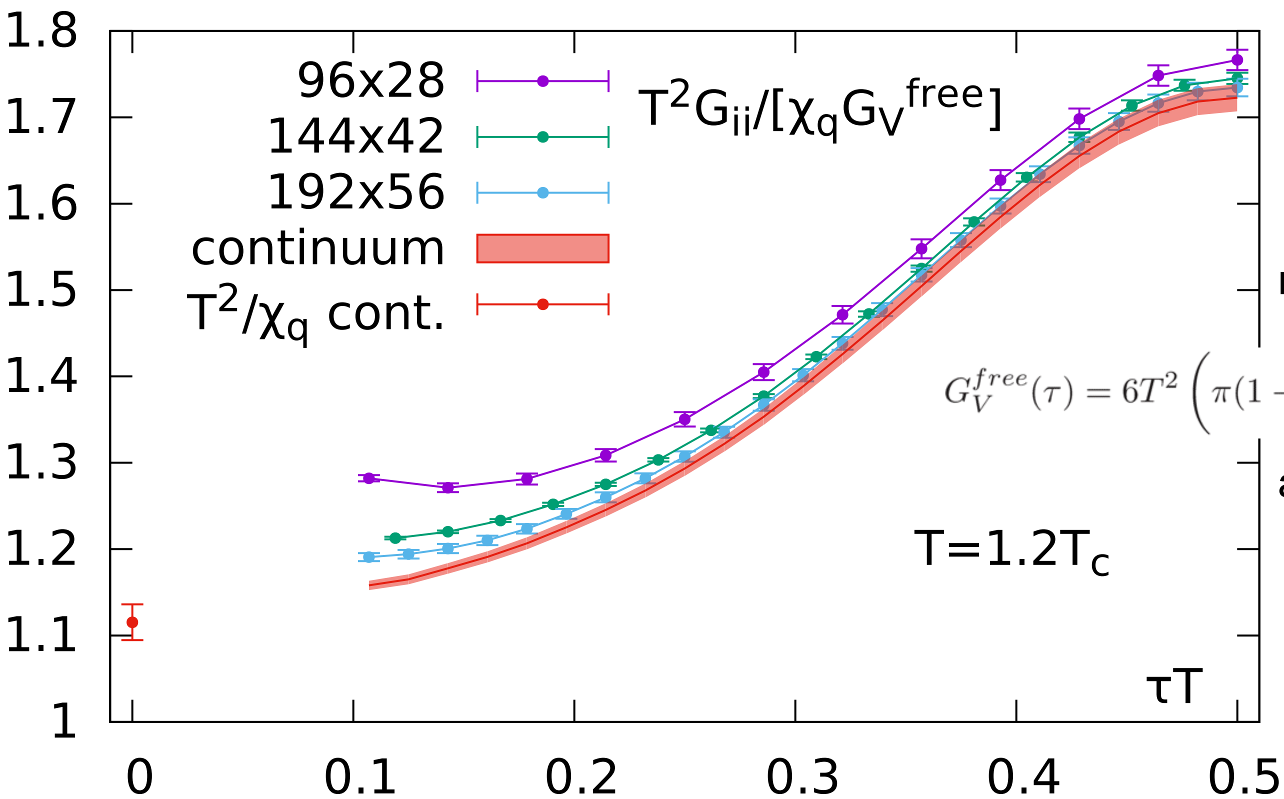
cut-off effects are visible at all distances on finite lattices but

well defined continuum limit on the correlator level

well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

Continuum extrapolation



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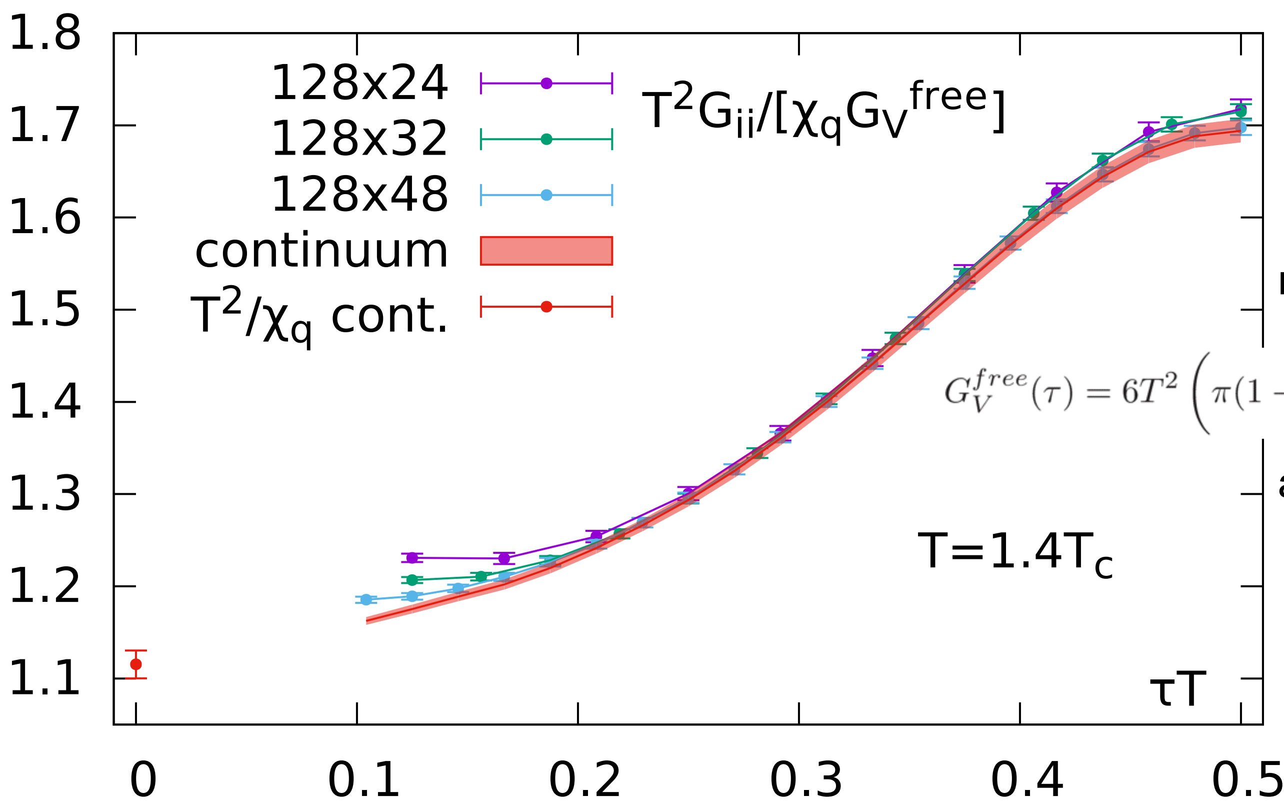

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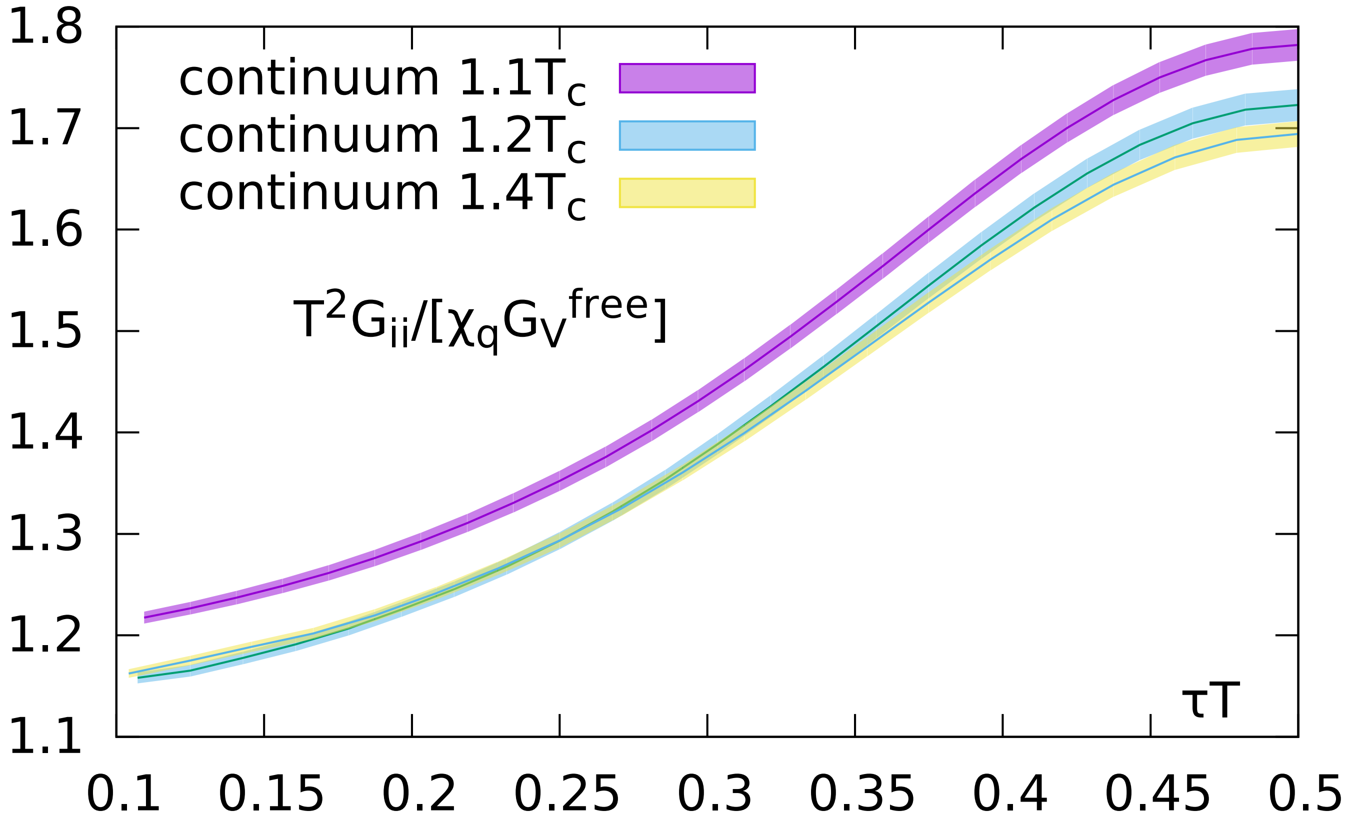
Continuum extrapolated vector correlation function

continuum extrapolated results available for three temperatures in the QGP



similar behavior in this temperature region

main difference due to different quark number susceptibility χ_q/T^2

Spectral function and electrical conductivity

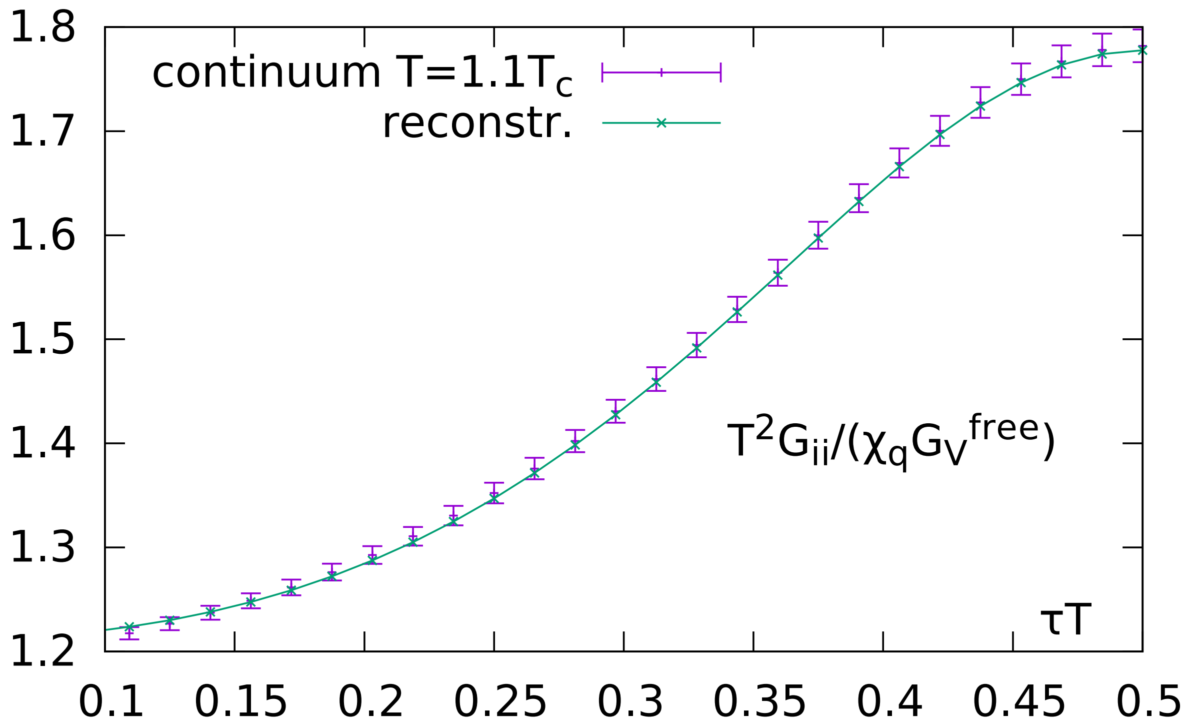
Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)$$

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$ (reconstructed correlator)

and fit to the continuum extrapolated correlators



all three temperatures are well described by this rather simple Ansatz

Spectral function and electrical conductivity

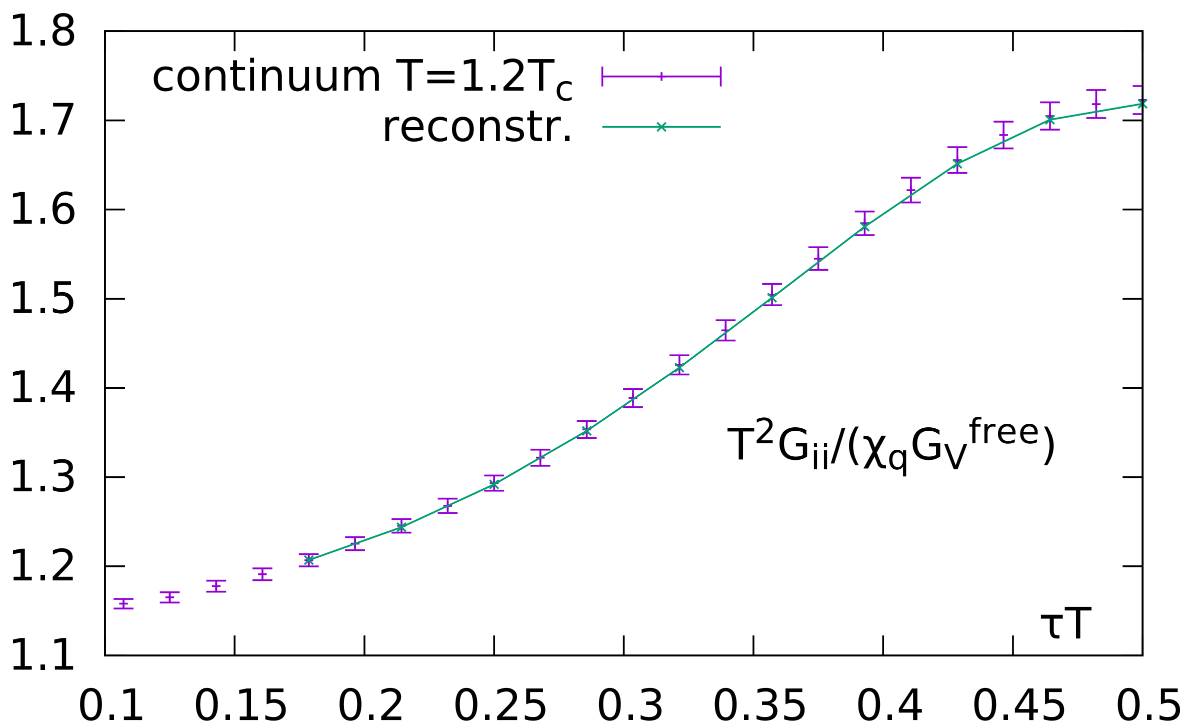
Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$ (reconstructed correlator)

and fit to the continuum extrapolated correlators

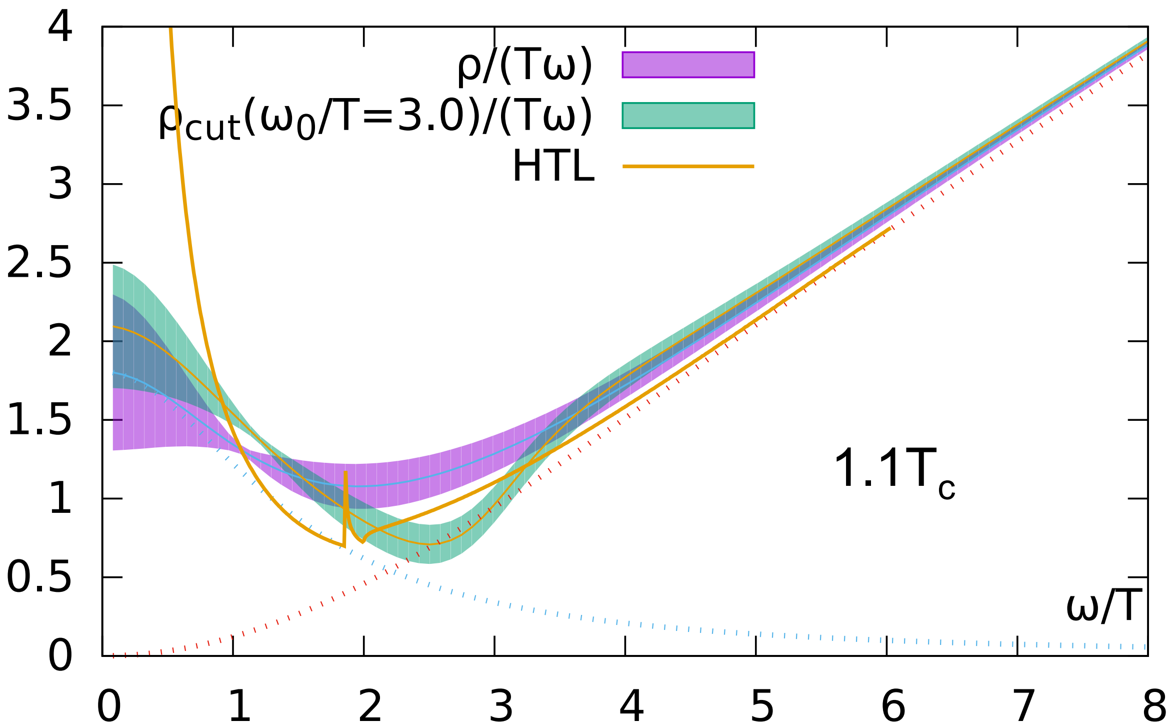


all three temperatures are well described by this rather simple Ansatz

Spectral function and electrical conductivity

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

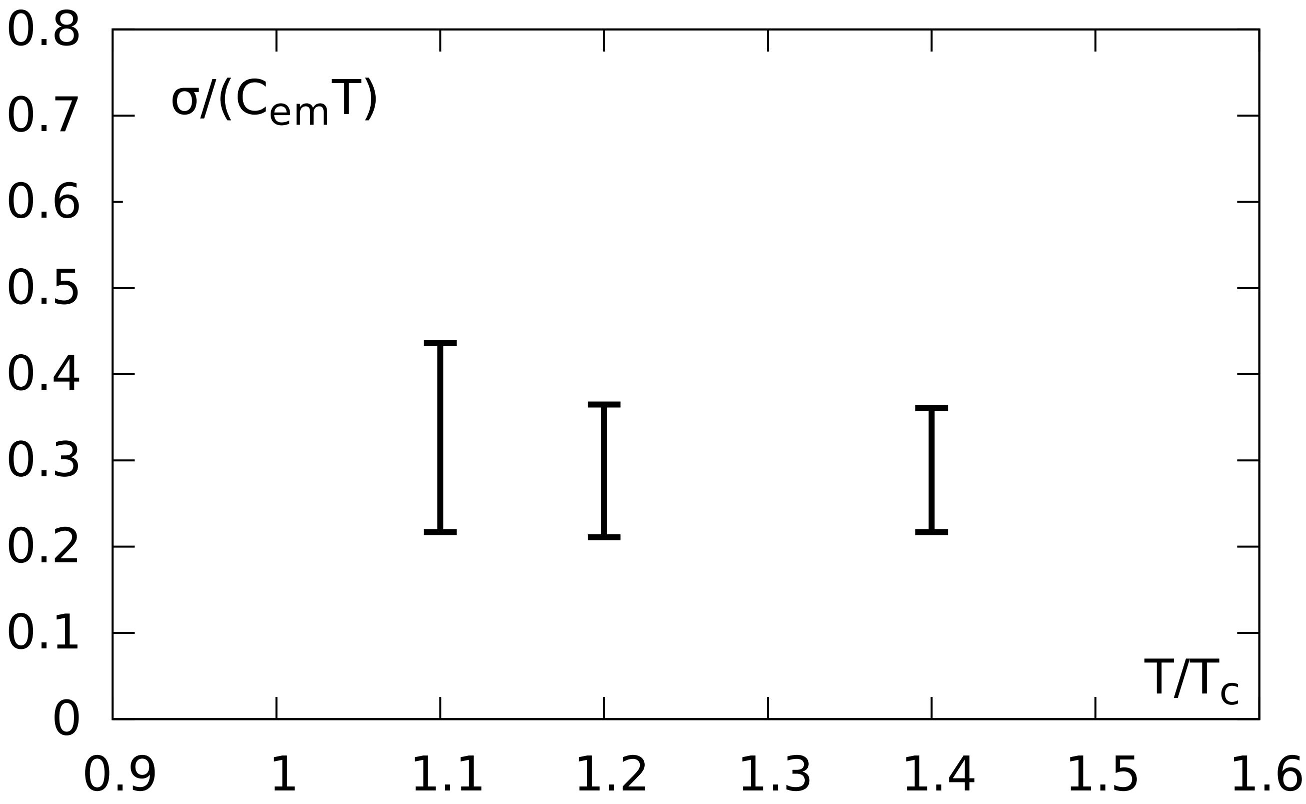
systematic uncertainties (within this Ansatz) estimated by varying the truncation

in all fits the covariance matrix is estimated by a bootstrap analysis

Electrical conductivity

T-dependence of the **electrical conductivity**:

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$



similar studies using dynamical clover Wilson (w/o continuum limit):

A.Amato et al., arXiv:1307.6763

B.B.Brandt et al., JHEP 1303 (2013) 100

previous studies using staggered fermions (need to distinguish ρ_{even} and ρ_{odd}):

S.Gupta, PLB 597 (2004) 57

G.Aarts et al., PRL 99 (2007) 022002

Dileptonrate directly related to vector spectral function:

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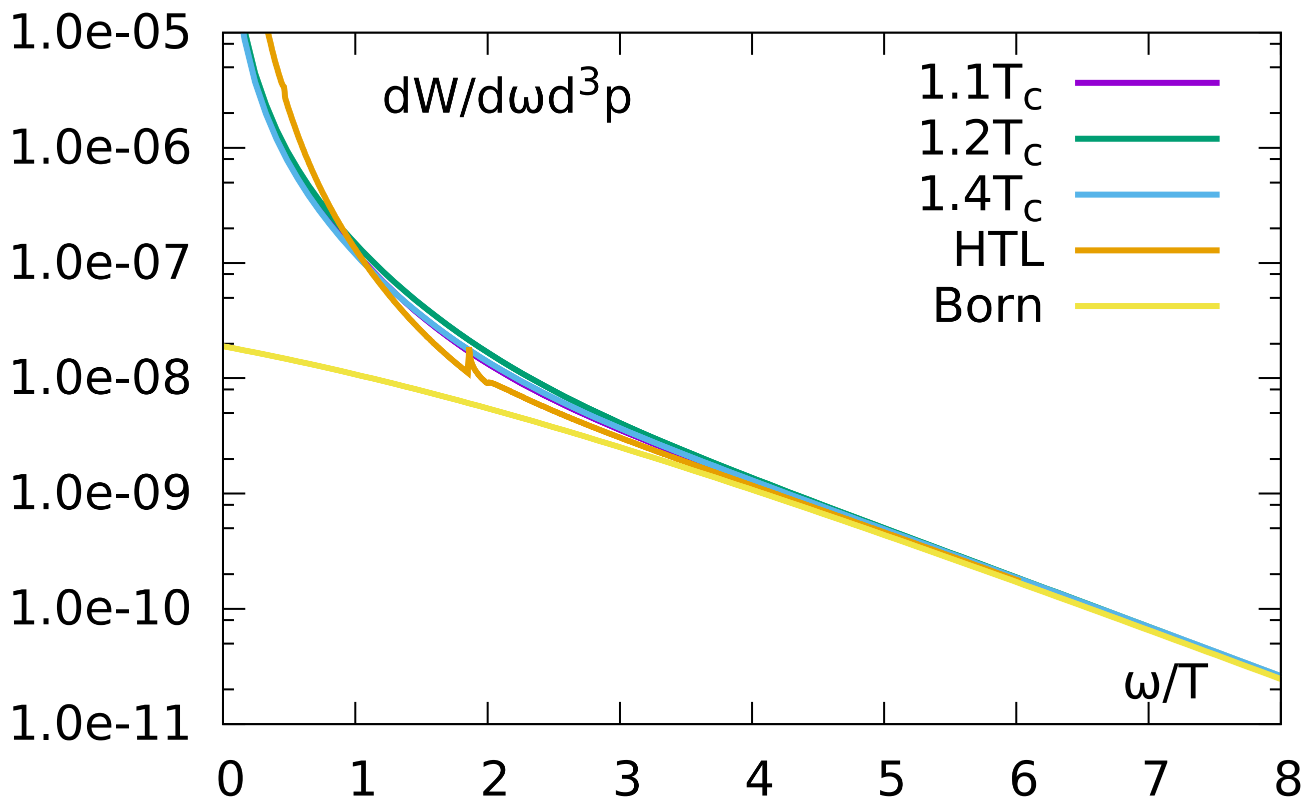

Hard thermal loop (HTL)
[E.Braaten, R.D.Pisarski, NP B337 (1990) 569]

Comparison to perturbation theory

So far we have used a simple Ansatz for the spectral function:

$$
\begin{aligned}
\rho_{00}(\omega) &= 2\pi\chi_q\omega\delta(\omega) \\
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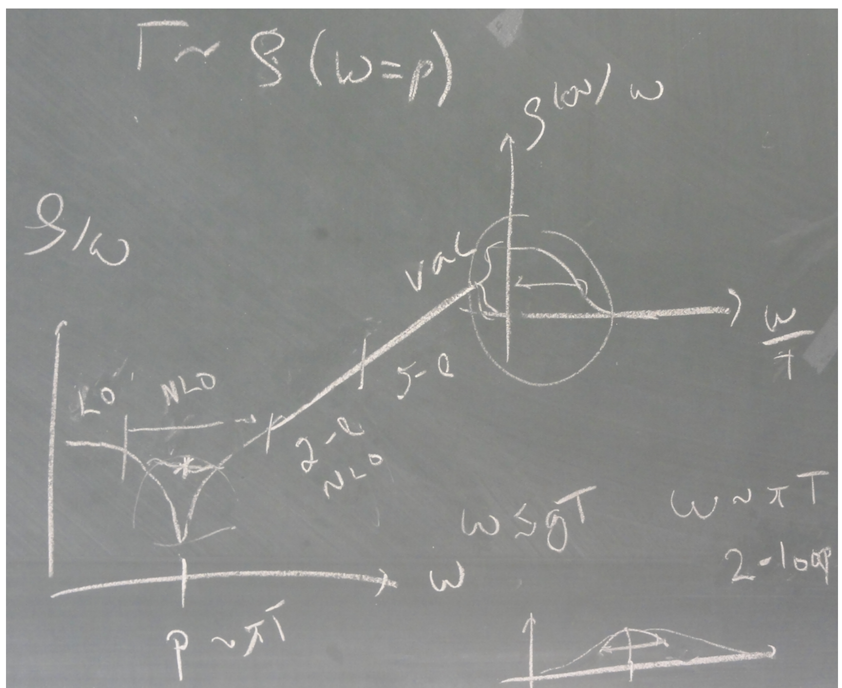
\end{aligned}
$$

Can we do better including more information from perturbation theory?

Problem: different scales contribute to the spectral function

different perturbative techniques required depending on the energy regimes



perturbation theory – vacuum spectral function

At very high energies, due to asymptotic freedom

→ perturbation should be working

→ thermal effects should be suppressed

→ "vacuum physics"

5-loop vacuum spectral function:

$$\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)$$

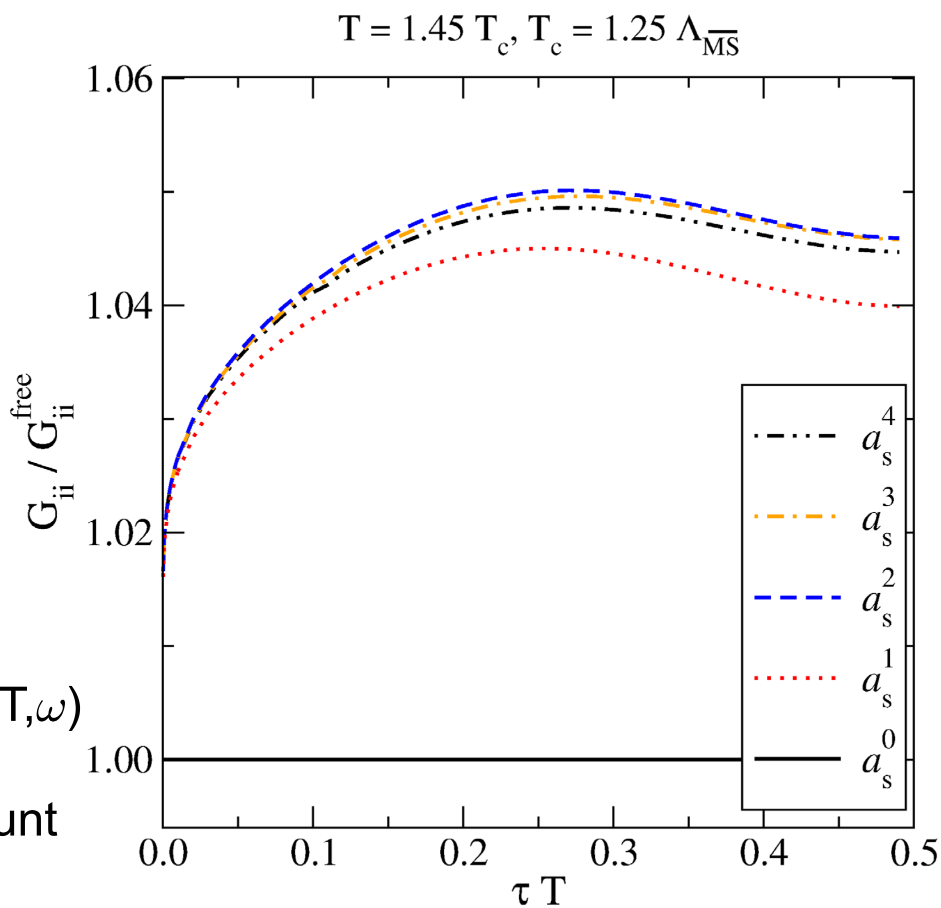
$$\begin{aligned} R(\omega^2) &= r_{0,0} + r_{1,0}\, a_s + \left(r_{2,0} + r_{2,1}\,\ell\right) a_s^2 \\ &+ \left(r_{3,0} + r_{3,1}\,\ell + r_{3,2}\,\ell^2\right) a_s^3 \\ &+ \left(r_{4,0} + r_{4,1}\,\ell + r_{4,2}\,\ell^2 + r_{4,3}\,\ell^3\right) a_s^4 + \mathcal{O}(a_s^5) \end{aligned}$$

using 3-loop α_s and l=log(μ^2/ω^2)

using a renormalization scale μ=(1..5)max(πT,ω)

taking leading order thermal effect into account

$$\rho_{ii}^{(T)}(\omega) \equiv \frac{3\omega^2}{4\pi}[1 - 2n_{\rm F}(\frac{\omega}{2})]R(\omega^2) + \pi\chi_{\rm q}^{\rm free}\omega\delta(\omega)$$



[Y.Burnier and M.Laine, arXiv 1201.1994]

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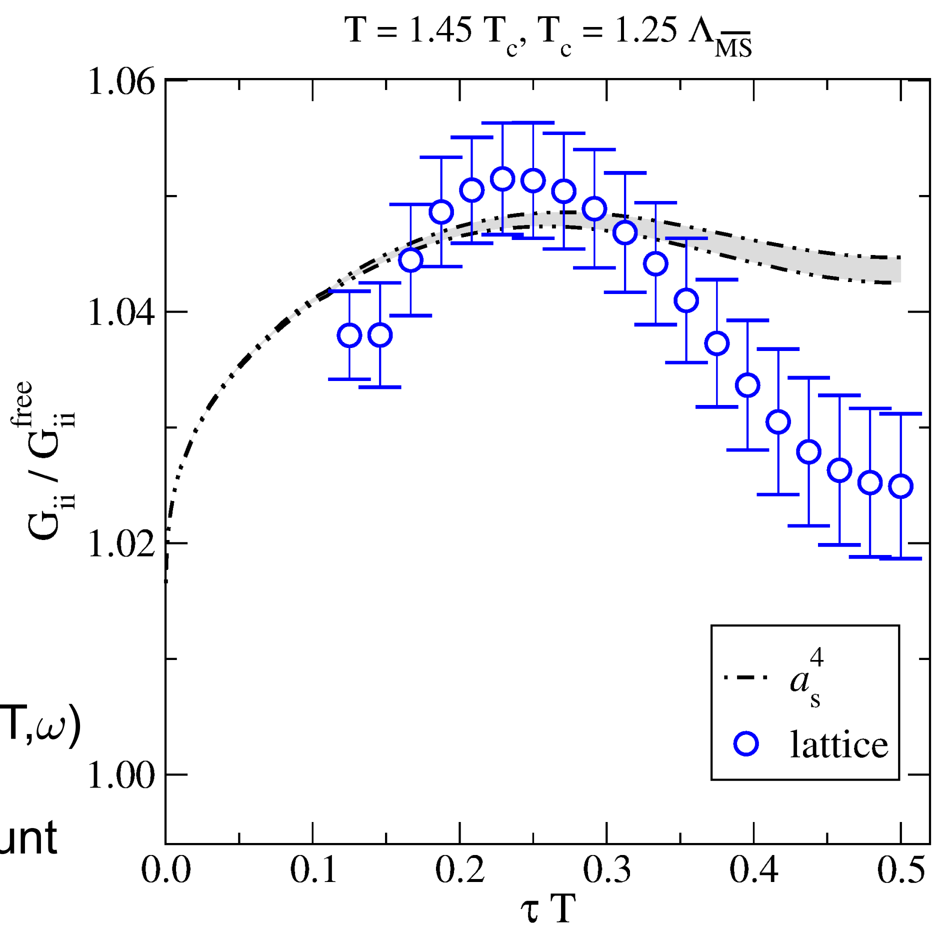

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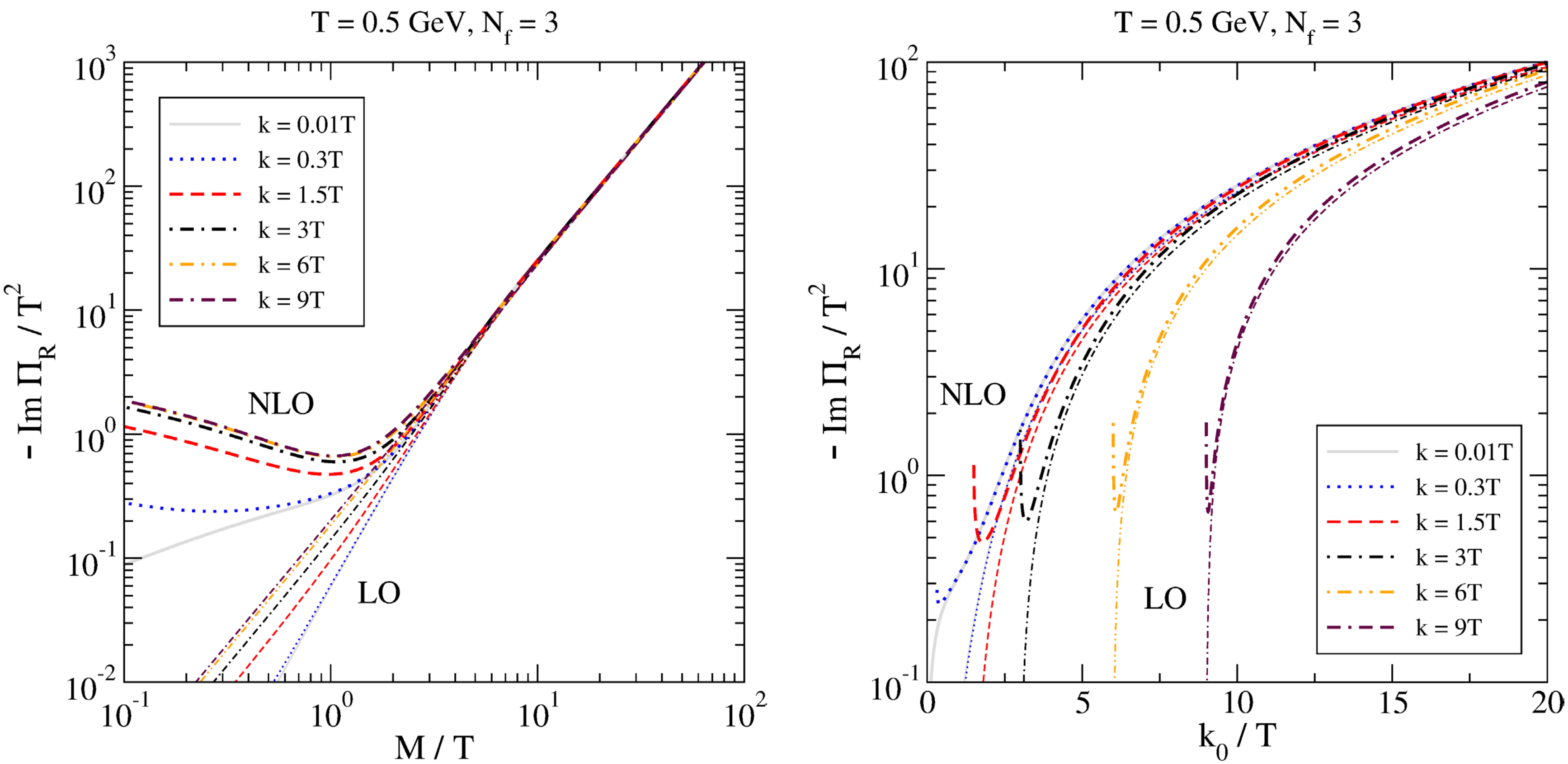
perturbation theory – thermal corrections

Thermal corrections in the intermediate frequency regime required [Altherr+Aurenche 1989]

[Moore+Robert 2006]

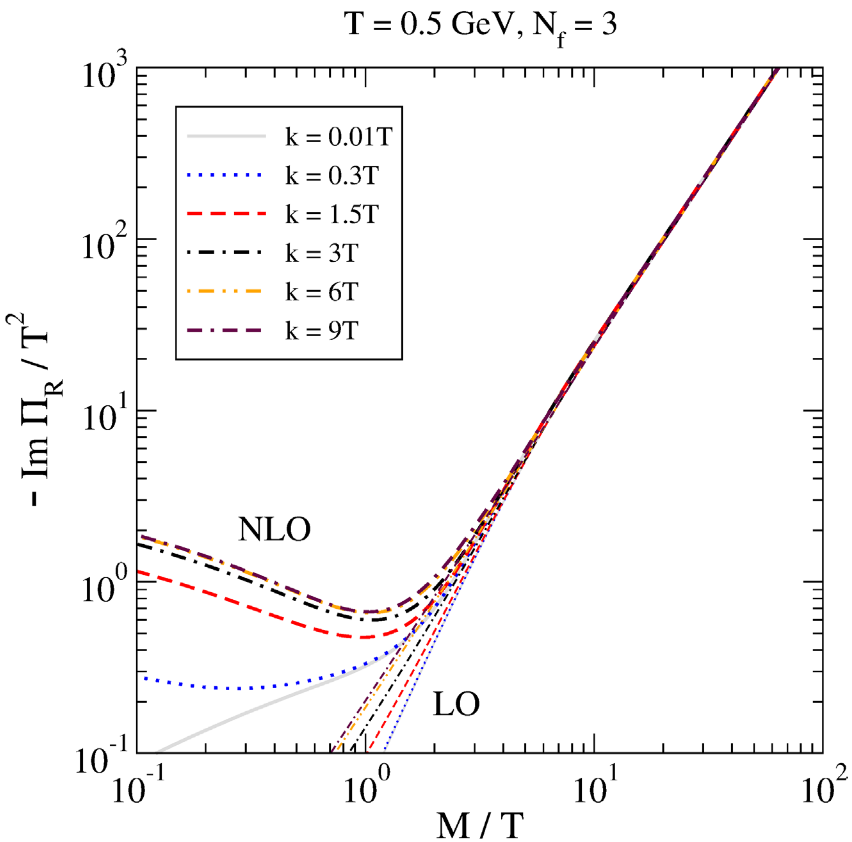


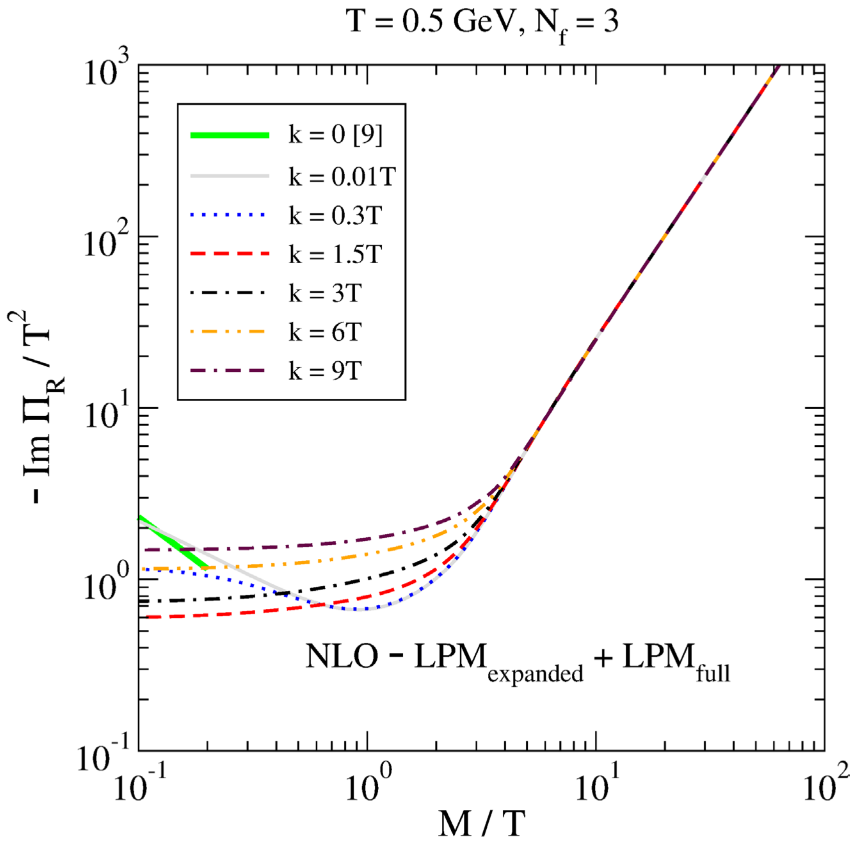
[M.Laine, JHEP 11 (2013) 120, arXiv:1310.0164]

perturbation theory – thermal corrections

Thermal corrections in the intermediate frequency regime required [Altherr+Aurenche 1989]

and proper treatment of the small frequency regime [Ghiglieri+Moore 2014] [Moore+Robert 2006]



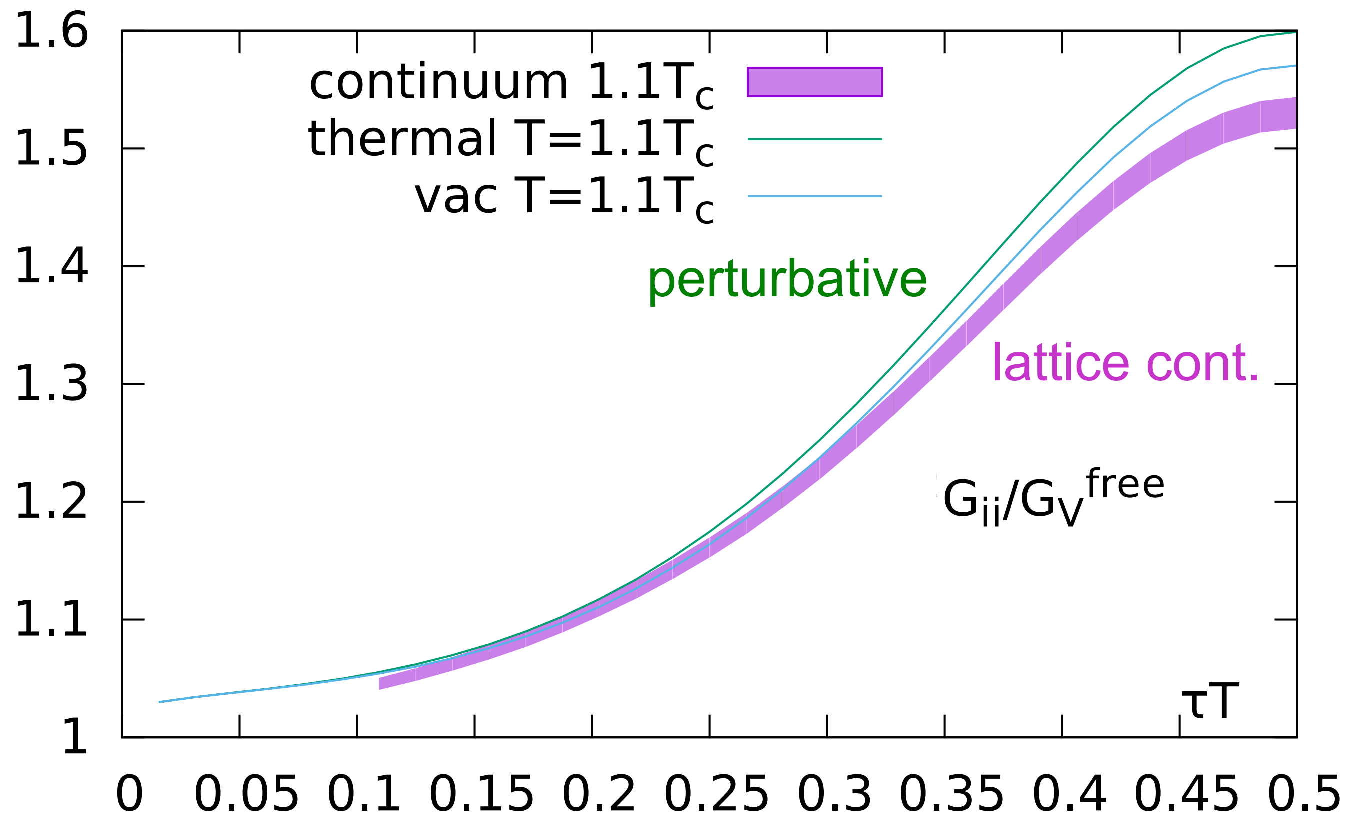


interpolation between different regimes [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

progress in perturbation theory in the past years → compare to lattice QCD results

Comparison of lattice and perturbative results

[J. Ghiglieri, OK, M.Laine, F.Meyer, preliminary]



interpolation of different perturbative regimes [J.Ghiglieri and M.Laine arXiv:1502.0579]

small and intermediate distance well described by perturbative results

lattice results at large distances below the perturbative

→ electrical conductivity smaller on lattice

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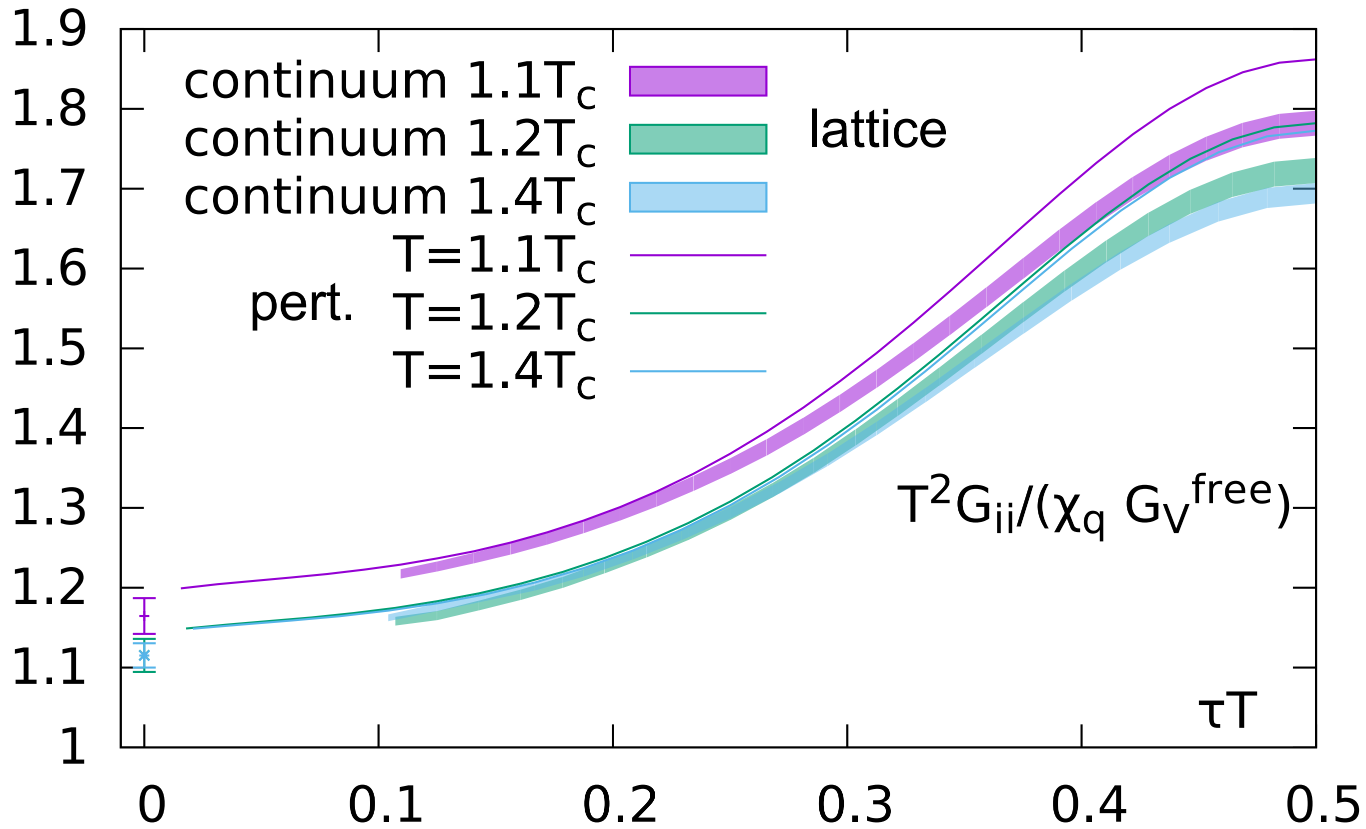

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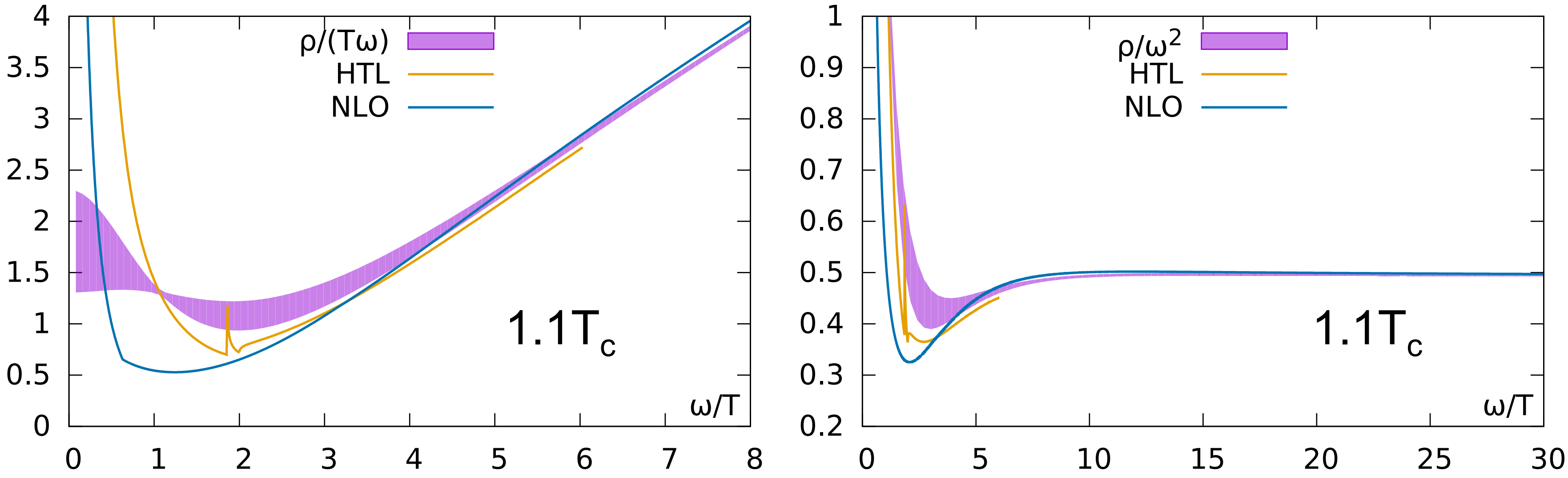

differences between our Ansatz and the perturbative results

at intermediate frequencies

→ important for thermal dilepton rates

at small frequencies extrapolate linearly in ω to 0

→ important for the electrical conductivity

Next step: improve the Ansatz by incorporating more perturbative information

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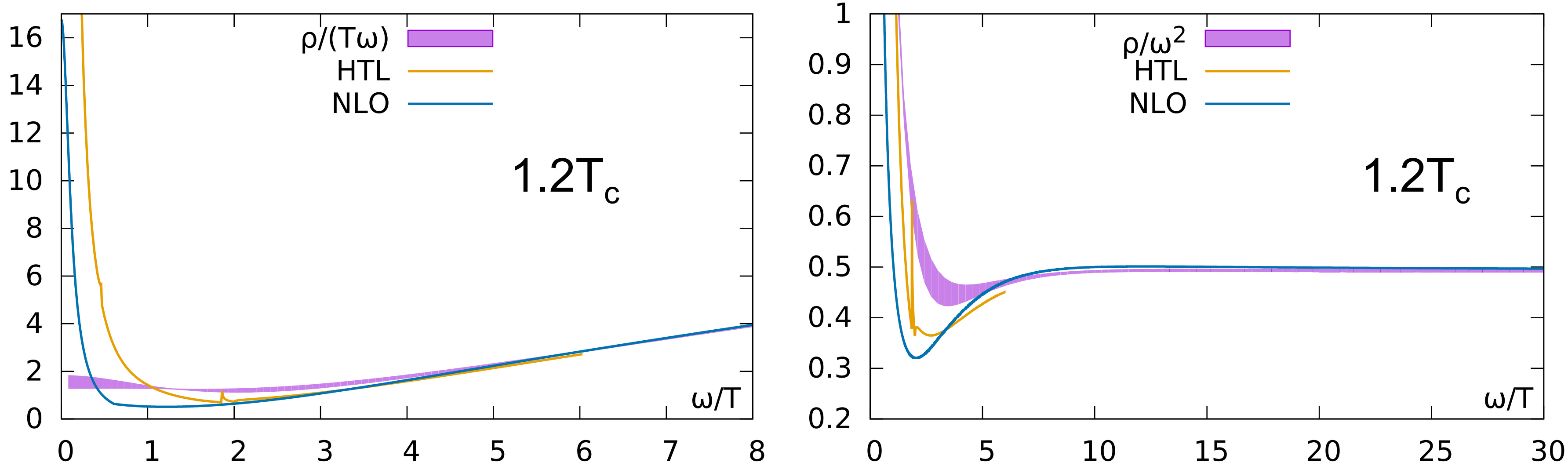

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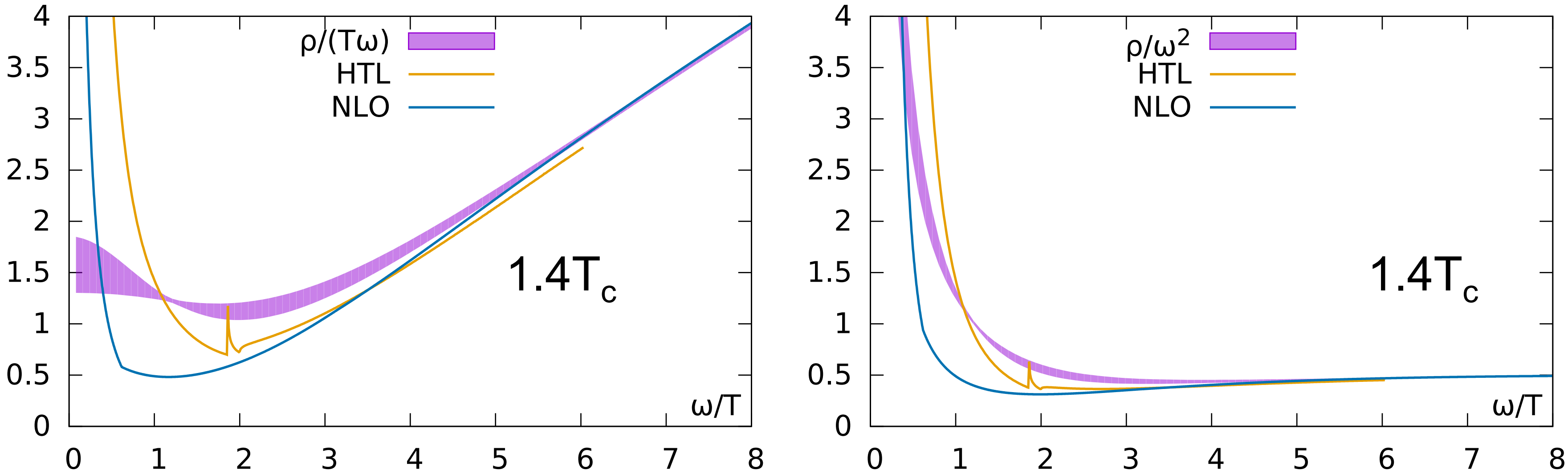

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Dilepton rates

[J. Ghiglieri, OK, M.Laine, F.Meyer, preliminary]

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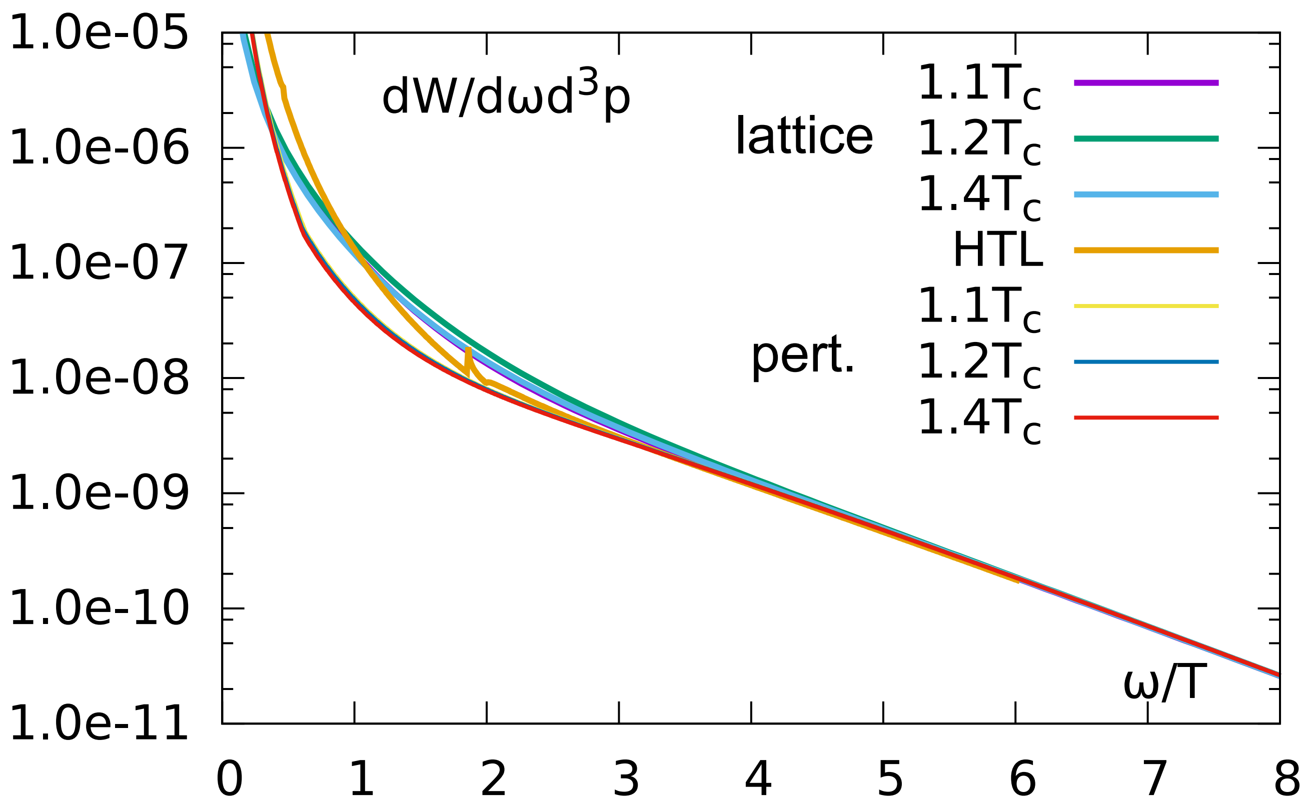
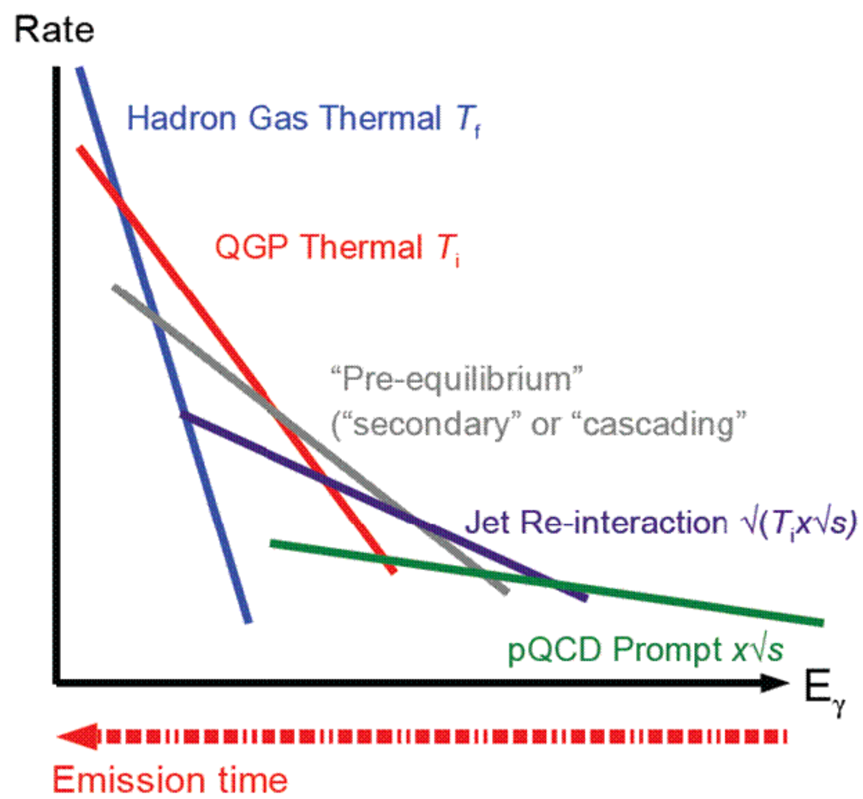


Hard thermal loop (HTL)

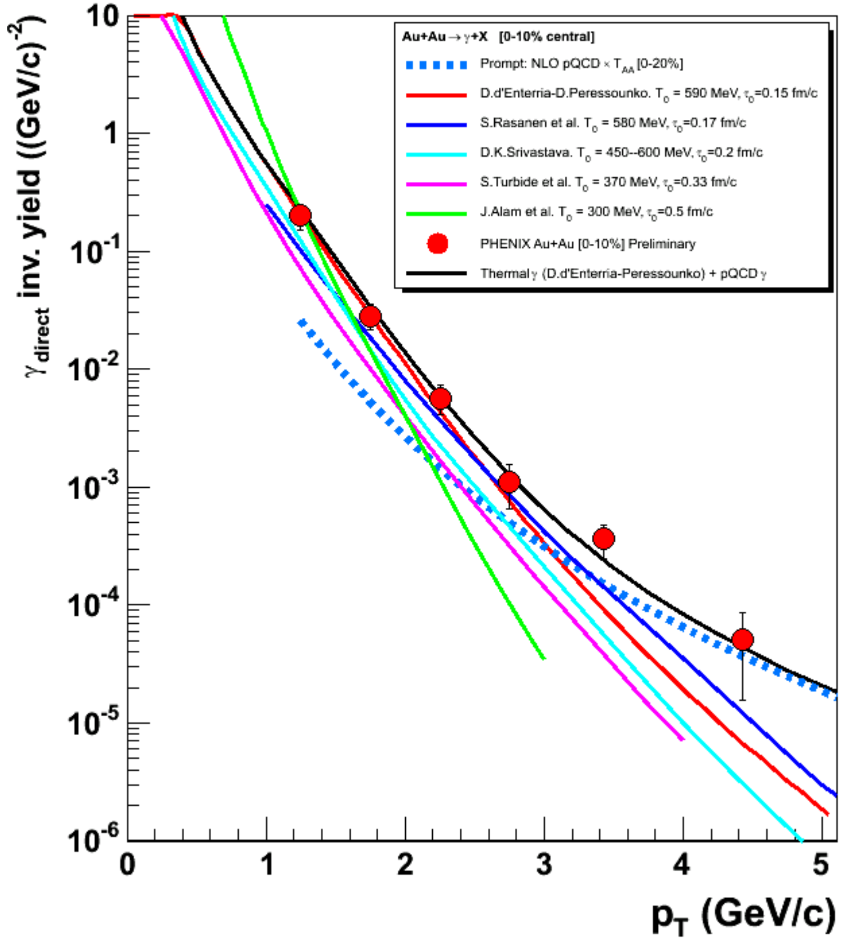
[E.Braaten, R.D.Pisarski, NP B337 (1990) 569]

Hard Probes in Heavy Ion Collisions - Photons

Direct and fragmentation photon relative contribution



[Fleuret 2009]



Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V(\omega = |\vec{p}|, T)$$

relevant energy region accessible on the lattice

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

but here the spectral function is needed at one specific frequency ω=p

→ additional information required to constrain the spectral function

→ use perturbative input whenever possible

→ need a good Ansatz to fit the correlation function

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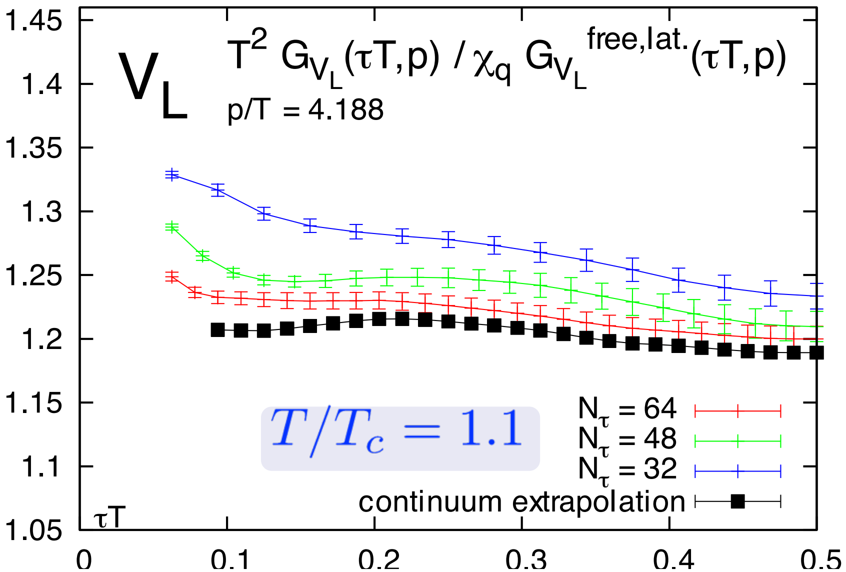
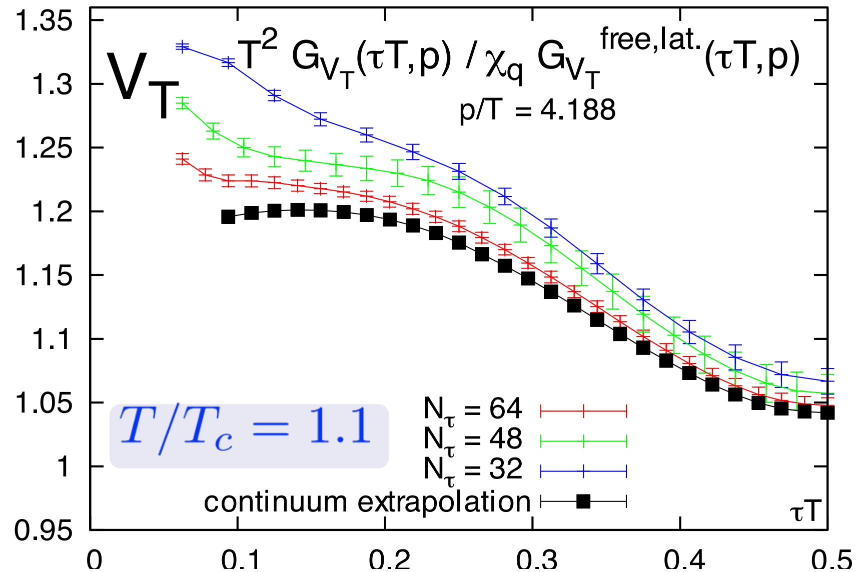
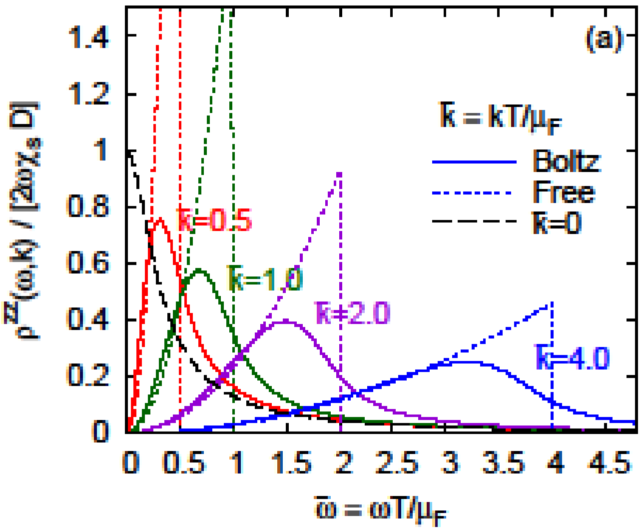
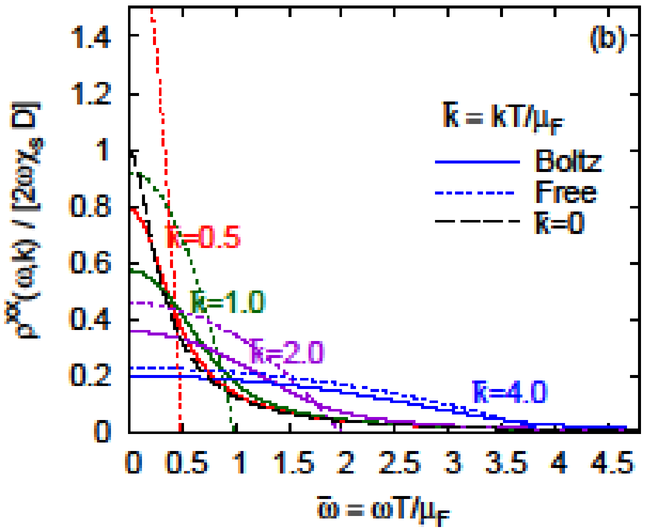
Non-zero momentum





indications for non-trivial behavior of spectral functions at small frequencies:



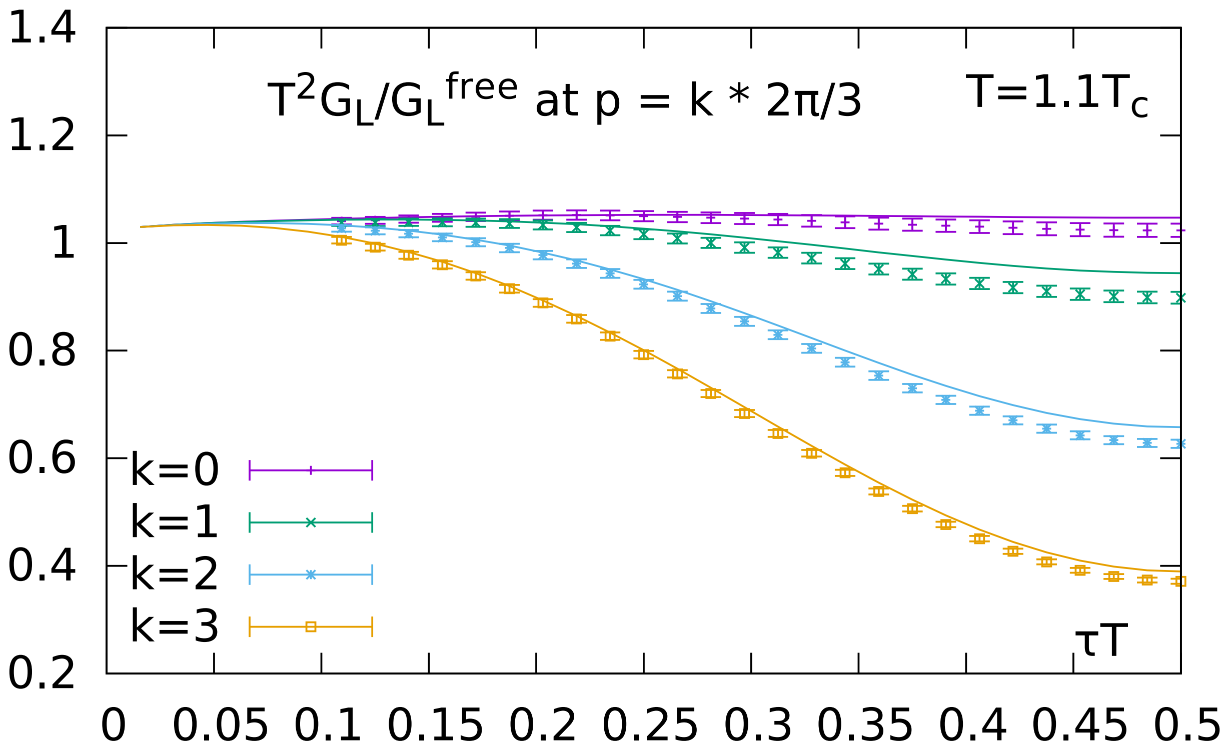


[Hong+Teaney, PRC82 (2010)044908]

Non-zero momentum

[J. Ghiglieri, OK, M.Laine, F.Meyer, preliminary]

comparison to the “vacuum” perturbative result



already very comparable for different momenta in the longitudinal channel

Non-zero momentum

[J. Ghiglieri, OK, M.Laine, F.Meyer, preliminary]

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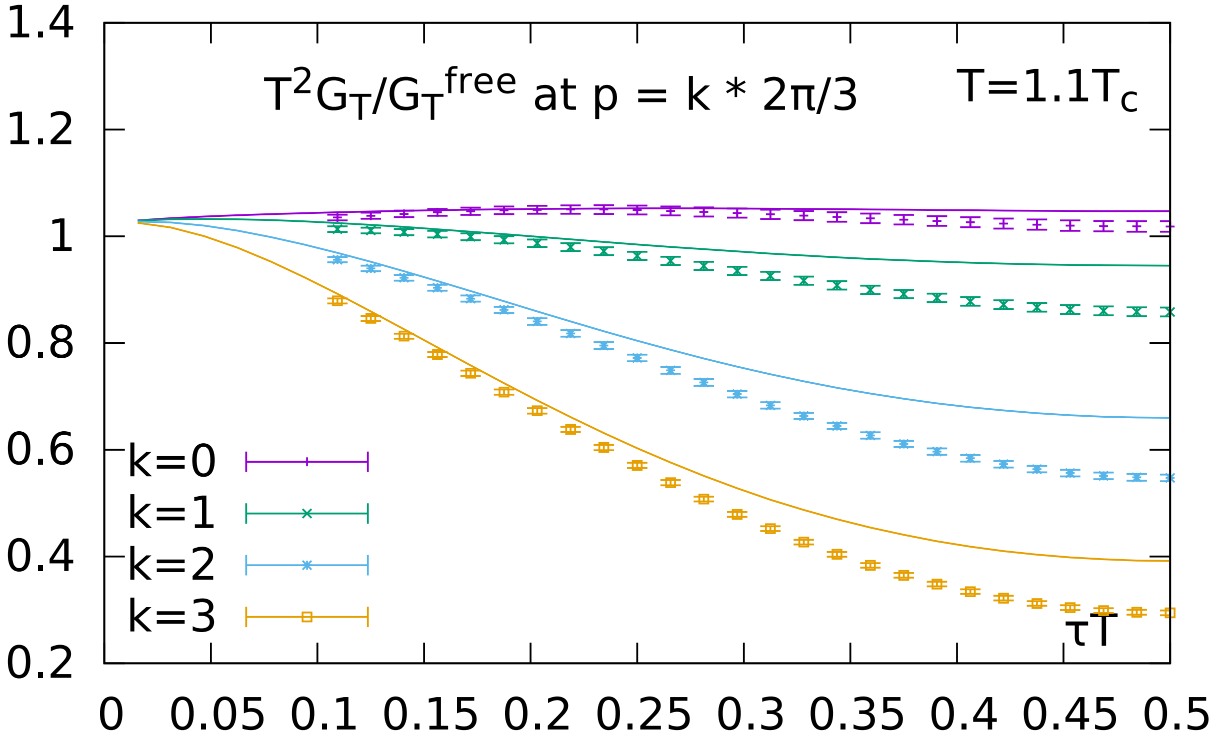


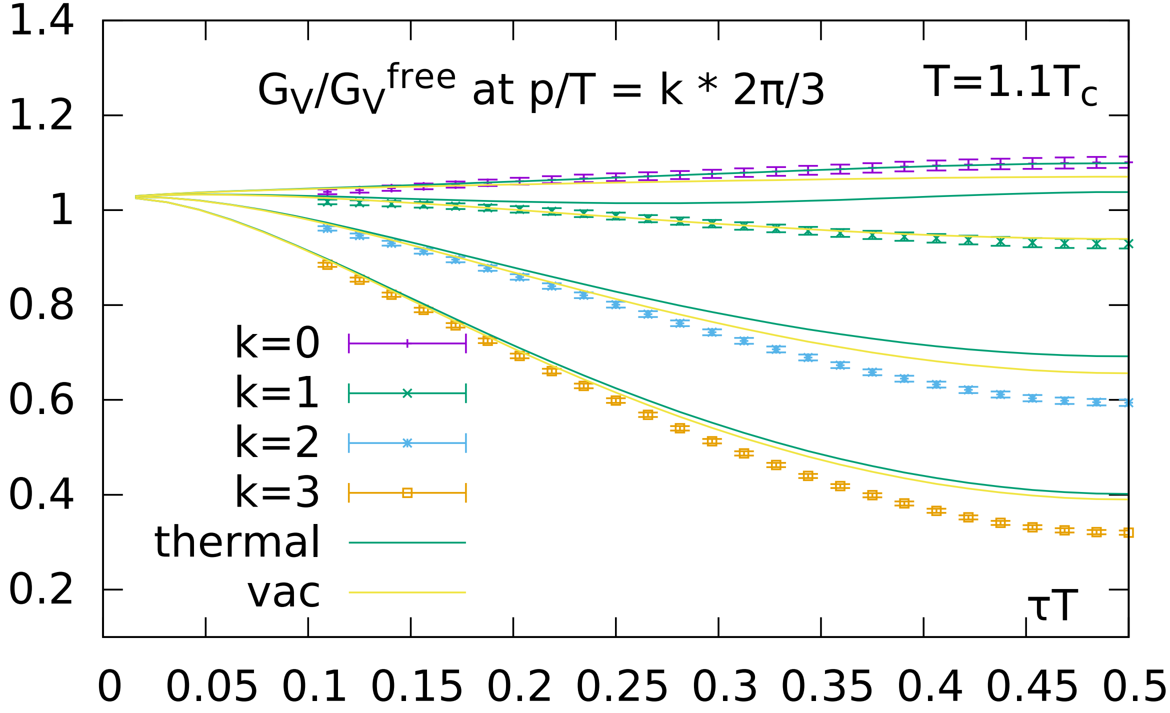
already very comparable for different momenta in the longitudinal channel

but differences become larger in the transverse channel

Non-zero momentum

[J. Ghiglieri, OK, M.Laine, F.Meyer, preliminary]

interpolation of hard and soft contributions [Laine, Ghiglieri, Moore et al.] :



thermal perturbative results so far only for the full vector channel

→ still need to understand different contributions

→ incorporate perturbative results in a fit Ansatz in the future

→ try to extract thermal photon rates from lattice correlation functions

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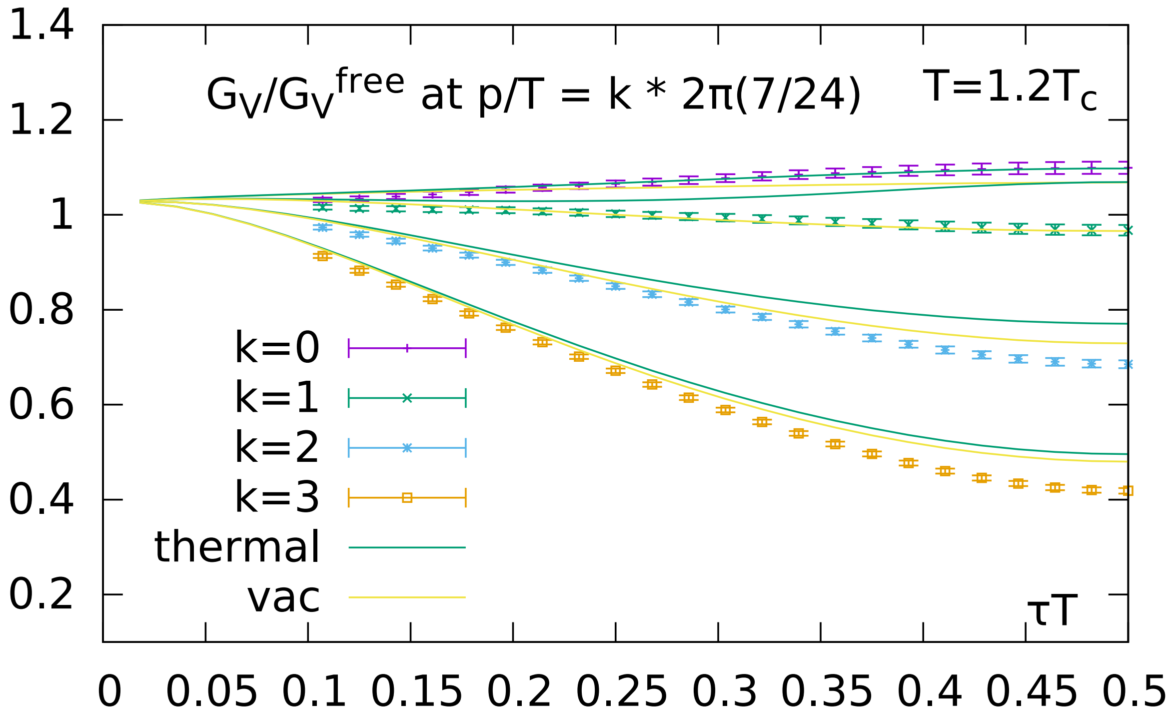

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Conclusions:

Detailed knowledge of the **vector correlation function** in the region $1.1 \leq T/T_c \leq 1.5$

→ **continuum extrapolation** of correlation function and thermal moments

continuum $G_V(\tau T)$ well reproduced by **Breit-Wigner plus continuum** Ansatz for $\sigma_V(\omega)$ in the temperature region $1.1 \leq T/T_c \leq 1.5$

→ **electrical conductivity** σ/T shows small temperature effects

→ **Dilepton rate** approaches leading order Born rate for $\omega/T \geq 4$
enhancement at small ω/T

first comparison to perturbation theory very promising

Outlook:

include more perturbative result for $\sigma_V(\omega)$ in the Ansatz

vector spectral function at **non-zero momentum** → **thermal photon rates**

especially close to T_c effects of dynamical quarks need to be included