Optical conductivity and String Theory

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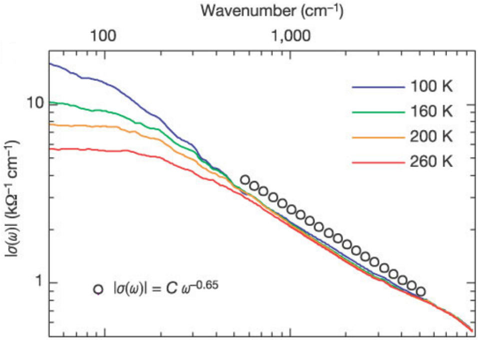
Optical conductivity



$$|\sigma(\omega)| \sim \omega^{2/3}$$

$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$ van der Marel, D et al. cond-mat/0309172

Optical conductivity

- For $\omega < T$ system exhibits Drude-like behaviour

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

- For $T < \omega < \Omega$ there is a *scaling* behaviour

$$|\sigma(\omega)| \sim \omega^{2/3}$$

- Power law correlations suggest presence of scale invariance and the underlying quantum criticality

van der Marel, D et al. cond-mat/0309172

Motivation and general concepts

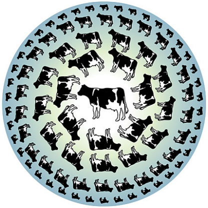
- Emergence in condensed matter physics $\text{UV} \to \text{IR}$
- Effective theories are often strongly coupled conformal field theories (sCFT) e.g. quantum critical systems
  S. Sachdev, Nature Physics, Vol. 4, Issue 3, pp. 173-185 (2008)
- AdS/CFT correspondence is a unique approach to sCFT
- However, most of the applications omitted the key ingredient of condensed matter: **the lattice**

Project

Develop a usable and realistic model of a holographic lattice and address the issue of optical conductivity

- **Holographic principle**
  Quantum gravity in $d+1$ dimensions must have a number of d.o.f. which scales like that of QFT in d dimensions

  't Hooft and Susskind '93



- String Theory realization *AdS/CFT correspondence*
  Solutions of Einstein equations in $d+1$ dimensions $\longleftrightarrow$ States in strongly coupled QFT in d dimensions

  Maldacena '97

- Charged black holes with horizonzs $\longleftrightarrow$ transitionally invariant systems at finite T and charge density

- Deformation of the theory by an operator $\mathcal{O}(x)$ of conformal dimension $\Delta = 2$, with the source term $\phi_0(x)$

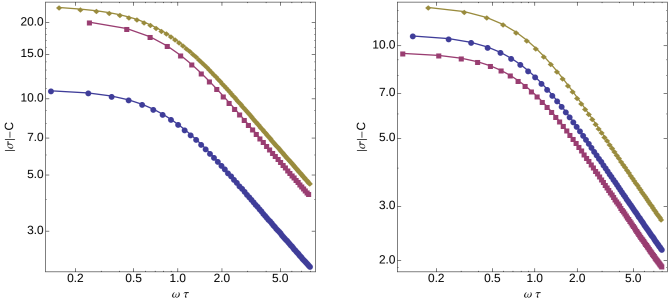
$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}_3} + \int d^3x \phi_0(x) \mathcal{O}(x)$$

  and the expectation value $\phi_1(x) = \langle \mathcal{O}(x) \rangle$

- Source $\phi_0(x) = \cos(kx)$ was used to mimic a lattice
  G. T. Horowitz, J. E. Santos, D. Tong, JHEP **1207**, 168 (2012)

- Solve dual gravity system and determine the conductivity

Conductivity: early results



- log-log plots for various parameters
- $|\sigma(\omega)| \sim A\omega^{2/3} + C$ in the frequency range $0.2 < \omega\tau < 0.8$

G. T. Horowitz, J. E. Santos, D. Tong, JHEP **1207**, 168 (2012)

Conductivity: crosscheck results

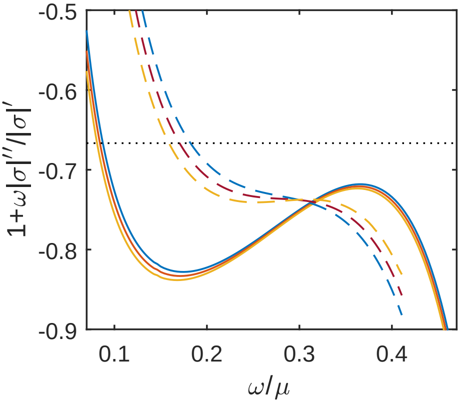
- Introduce a measure of the scaling

$$\alpha = 1 + \omega \frac{|\sigma|''}{|\sigma|'}$$

- A value $\alpha = -2/3$ would correspond to mid-infrared power-law of cuprates
- Lattices consisting of a few Fouririer modes in different arrangments have been studied

B. W. Langley, G. Vanacore, P. W. Phillips, arXiv:1506.06769 [cond-mat.str-el]

Conductivity: bad news



B. W. Langley, G. Vanacore, P. W. Phillips, arXiv:1506.06769 [cond-mat.str-el]

Dirac δ-like source

- Consider local line-like source

$$\phi_1(x) = \eta\delta(x) \tag{1}$$

- Source $\phi_1(x) = \cos(kx)$ was used to mimic a lattice
  G. T. Horowitz, J. E. Santos and D. Tong, JHEP **1207**, 168 (2012)
- Try to obtain a lattice $\sum_n \delta(x - na) \rightarrow$ holographic version of the Kronig-Penney model
- Physics of single defects $\rightarrow$ both line-like and point-like

Goal

Develop techniques to numerically solve Einstein's equations with boundary conditions (1)

- $\theta(x)$ and $m = 0 \rightarrow$ Janus solutions (analytical, 1D ODE)
  D. Bak, M. Gutperle and S. Hirano, JHEP **0305**, 072 (2003)

- $\theta(x)$ and $m = 0$ at $T > 0 \rightarrow$ Janus black holes in $d = 2 + 1$ (numerical PDE, analytical)
  D. Bak, M. Gutperle and R. A. Janik, JHEP **1110**, 056 (2011)

- $\delta(x)$ and $m^2 = -2$ with SUSY $\rightarrow$ analytical and scale invariant
  E. D'Hoker, J. Estes, M. Gutperle, D. Krym, JHEP **0906**, 018 (2009)

  $\rightarrow$ Nontrivial profiles of $p-$form gauge fields
  $\rightarrow$ Difficult to generalize to $T > 0$ and $\mu \neq 0$
  $\rightarrow$ Not very applicable to solid state physics ...

- The Dirac delta source in the AdS spacetime uniquely determines the supersymmetric potential for the scalar field

$$V_s(\phi) = -6\cosh\left(\frac{\phi}{\sqrt{3}}\right)$$

R. A. Janik, J.J, P. Witkowski, arXiv:1503.08459 [hep-th]

- The potential $V_s(\phi)$ arises in reducing $D = 11$ SUGRA on $AdS_4 \times S^7$

M. Cvetic et.al. Nucl. Phys. B **558**, 96 (1999)

- For more details see my poster ...