2.1 a) $Y_Q = \frac{n_p + n_d}{n_n + n_p + 2n_d} \overset{!}{=} 0.5$

try $n_n = n_p \Rightarrow Y_Q = 0.5$ ✓

$n_n = n_p \Rightarrow \mu_n = \mu_p$, $\underline{\mu_Q = 0}$

in the following therefore always use

$\mu_n = \mu_p = \mu \quad \mu_d = 2\mu$

b) $X_d = \frac{2n_d}{2n_n + 2n_d} = \frac{1}{\frac{n_n}{n_d} + 1}$

$g_n = g_p = 2 \quad g_d = 3 \qquad M_n = M \quad M_d = 2M - B_d$

use $M \simeq 939$ MeV $\qquad B_d = 2b_d \simeq 2.2$ MeV

($b_d$: binding energy per baryon) $\sim 1.1$ MeV

$$n_n = g_n \left(\frac{MT}{2\pi}\right)^{3/2} \exp\left(\frac{\mu - M}{T}\right)$$

$$n_d = g_d \left(\frac{2MT}{2\pi}\right)^{3/2} \exp\left(\frac{2\mu - 2M + B_d}{T}\right)$$

$$\Rightarrow \frac{n_n}{n_d} = \frac{2}{3}\left(\frac{1}{2}\right)^{3/2} \exp\left(-\frac{\mu - M + B_d}{T}\right)$$

- $T = $ const, $\mu \to \infty \Rightarrow \frac{n_n}{n_d} \to 0$, $X_d \to 1$
- $\mu = $ const, $T \to \infty \Rightarrow \frac{n_n}{n_d} \to \frac{2}{3}\left(\frac{1}{2}\right)^{3/2}$

but: $n_B \to \infty$ in this case, ~~too~~. It's more meaningful to consider $n_B = $ const instead of $\mu = $ const.
To achieve this, let's try

$\mu - M \propto -\frac{3}{2} T \ln(aT)$

in this case:

$n_n = g_n \left(\frac{M}{2\pi a}\right)^{3/2} = \text{const}$, $n_d \to 0$, and thus also $n_B = \text{const}$

we get $\underline{\underline{X_d \to 0}}$

- $\mu = \text{const}$, $T \to 0$
  - $\mu - M < -B_d \Rightarrow \frac{n_n}{n_d} \to \infty$, $X_d \to 0$
  - $\mu - M > -B_d \Rightarrow \frac{n_n}{n_d} \to 0$, $X_d \to 1$

Overall, these limits illustrate the behavior of the toy model: deuterons appear at "high" densities, and disappear at high temperatures. That's reasonable.

However, it is completely unrealistic that high density matter (e.g. above $n_B^0$) would consist of only deuterons. Furthermore, for $T \to 0$ we have to consider the right quantum statistics and cannot use Maxwell-Boltzmann. It doesn't make sense to apply this toy model for these conditions.

c) $X_d \overset{!}{=} \frac{1}{2}$, $T = 5\,\text{MeV}$

$\Rightarrow \frac{n_n}{n_d} = 1$

$\Rightarrow \frac{3}{2} 2^{3/2} = \exp\left(-\frac{\mu - M + B_d}{T}\right)$

$\mu - M = -B_d - T \ln(3\sqrt{2}) = -9.4\,\text{MeV}$

$n_n = \frac{1}{4} n_B$

$n_B = 4 n_n = 8 \left(\frac{939 \cdot 5}{2\pi}\right)^{3/2} \text{MeV}^3 \exp\left(-\frac{9.4}{5}\right) = 2.5 \cdot 10^4\,\text{MeV}^3$

$$\mathrm{MeV}^3 = \frac{\mathrm{MeV}^3}{(\hbar c)^3} = \frac{1}{197.3^3}\ \mathrm{fm}^{-3}$$

$$\Rightarrow n_B = 0.0032\ \mathrm{fm}^{-3} \simeq 0.02\, n_B^0$$

that's good!

d) from Stachel et al: $N_p \sim 30$, $N_d \sim 0.01$

at LHC: $\mu \sim 0$

$$\Rightarrow 300 = \frac{2}{3}\left(\frac{1}{2}\right)^{3/2} \exp \frac{936.8\,\mathrm{MeV}}{T}$$

$$\Rightarrow T \simeq 131\ \mathrm{MeV}$$

not so bad

2.2) $$K_c^{id}(d) = \frac{n_d}{n_n n_p} = \frac{n_d}{n_n^2}$$

$$= \frac{3}{4} \frac{\left(\frac{2MT}{2\pi}\right)^{3/2}}{\left(\frac{MT}{2\pi}\right)^3} \exp \frac{B_d}{T}$$

$$= 6 \left(\frac{\pi}{MT}\right)^{3/2} \exp \frac{B_d}{T}$$

$$= 6 \left(\frac{0.0033}{T/\mathrm{MeV}}\right)^{3/2} \mathrm{MeV}^{-3} \exp\left(\frac{2.2\,\mathrm{MeV}}{T}\right)$$

$$= 4.6 \cdot 10^7 \left(\frac{0.0033}{T/\mathrm{MeV}}\right)^{3/2} \exp\left(\frac{2.2\,\mathrm{MeV}}{T}\right) \mathrm{fm}^3$$

define $K_c(d) = K_c^{id}(d) \exp\left(\frac{\Delta}{T}\right)$

$$\Rightarrow \Delta = T \ln \frac{K_c(d)}{K_c^{id}(d)}$$

| T (MeV) | $K_c^{id}(d)$ (fm$^3$) | $\Delta$ (MeV) |
|---|---|---|
| 5.1 | 1165 | 1.1 |
| 6.1 | 830 | -3.9 |
| 7.0 | 645 | -7.4 |
| 8.0 | 507 | -10.4 |
| 9.0 | 412 | -13.4 |
| 10.0 | 344 | -17.5 |
| 11.0 | 292 | -24.7 |

The values are in a similar order of magnitude as from the quantum statistical model. Note that Typel et al. report $\frac{\Delta}{2}$, and the densities shown are very low.

However, what is also important are attractive mean-field interactions of the nucleons. If these are included in the model the values of $-\Delta$ were reduced, ie. the ~~[illegible]~~ binding energy shift of the deuteron was less.

2.3) $F = F_n + F_p + F_d$ , $V_0 = \frac{1}{n_0^0} = \frac{1}{0.16}\,\mathrm{fm}^3$

$$F_n = F_n^{id}(T, V, N_n) \,,\quad F_p^{id}(T, V, N_p)$$

$$F_d = F_d^{id}(T, \bar{V}, N_d) \qquad \bar{V} = V - V_0(N_n + N_p + 2N_d)$$

$$\mu_i = \frac{\partial F}{\partial N_i}\Big|_{N_{j\neq i}, V}$$

$\bar{V}$ is the free volume

$$\mu_d = \mu_d^{id}(T, \bar{V}, N_d) + \frac{\partial \bar{V}}{\partial N_d}\frac{\partial F_d^{id}}{\partial \bar{V}}\Big|_{T, N_d}$$

$$= \mu_d^{id}(T, \bar{V}, N_d) - 2V_0 \frac{\partial F_d^{id}}{\partial \bar{V}}$$

note: $\frac{\partial F_i^{id}}{\partial V}\Big|_{T,N_i} = -p_i^{id} = -\frac{N_i}{V}T$

$$\Rightarrow \mu_d = \mu_d^{id}(T, \bar{V}, N_d) + 2V_0 \frac{N_d}{\bar{V}} T$$

$$\mu_n = \mu_n^{id}(T, V, N_n) + \frac{\partial \bar{V}}{\partial N_n}\frac{\partial F_d^{id}}{\partial \bar{V}}\Big|_{T, N_d}$$

$$= \mu_n^{id}(T, V, N_n) + V_0 \frac{N_d}{\bar{V}} T \quad (\mu_p \text{ analogue})$$

$$\mu_i^{id} = M_i + T\ln\left\{\frac{n_i}{g_i}\left(\frac{2\pi}{M_i T}\right)^{3/2}\right\}$$

$$\Rightarrow \mu_i^{id}(T, V, N_i) = \mu_i^{id}(T, n_i) \quad \text{with} \quad n_i = \frac{N_i}{V}$$

$$\Rightarrow \mu_d = \mu_d^{id}(T, \bar{n}_d) + 2V_0 \bar{n}_d T \,, \qquad \bar{n}_d = \frac{N_d}{\bar{V}}$$

$$\boxed{\begin{aligned} \mu_n &= \mu_n^{id}(T, n_n) + V_0 \bar{n}_d T \\ \Rightarrow \mu_d &= \mu_d^{id}(T, n_d) + T\ln\frac{\bar{n}_d}{n_d} + 2V_0\bar{n}_d T \end{aligned}}$$

(cannot be solved for $n_n$ or $n_d$)

better than $\frac{\bar{n}_d}{n_d}$ :

$$\frac{n_d}{\bar{n}_d} = \frac{\bar{V}}{V} = 1 - V_0 \, \frac{N_n + N_p + 2N_d}{V} = 1 - \frac{n_B}{n_B^0}$$

next use NSE:

$$\mu_d = 2\mu_n$$

$$\Rightarrow \cancel{2}\, \mu_d^{id} - T \ln\left(1 - \frac{n_B}{n_B^0}\right) = 2\mu_n^{id}$$

$$\Rightarrow \quad n_d = g_d \left(\frac{M_d T}{2\pi}\right)^{3/2} \exp\left\{ \frac{2\mu_n^{id} - M_d + T \ln\left(1 - \frac{n_B}{n_B^0}\right)}{T} \right\}$$

$$n_d = \left(1 - \frac{n_B}{n_B^0}\right) g_d \left(\frac{M_d T}{2\pi}\right)^{3/2} \exp\left\{ \frac{2\mu_n^{id} - M_d}{T} \right\}$$

$$n_d = \left(1 - \frac{n_B}{n_B^0}\right) n_d^{id}\left(2\mu_n^{id}, T\right)$$

for $n_B \to n_B^0$ we get $n_d \to 0$, i.e. the deuteron is suppressed!

let's use $\quad n_B = 2n_n + 2n_d$

$$\Rightarrow n_d\left(1 + 2\,\frac{1}{n_B^0}\, n_d^{id}\right) = \left(1 - \frac{2n_n}{n_B^0}\right) n_d^{id}$$

$$\Rightarrow \boxed{n_d = \frac{1 - \frac{2n_n}{n_B^0}}{1 + \frac{2n_d^{id}}{n_B^0}}\, n_d^{id}} \quad \text{whereas} \quad n_d^{id} = n_d^{id}(2\mu_n^{id}, T)$$

$$\text{and} \quad \mu_n^{id} = \mu_n^{id}(n_n, T)$$

thus we have obtained an explicit equation for $n_d$ as a function of $n_n$! At least something...

With this we could solve