Cluster formation and phase transitions in supernova matter

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01.07.2015

1 Discussion/questions

Which parts of the lecture were not clear? Are there aspects of core-collapse supernovae, neutron stars, phase diagrams or the equation of state that you want to discuss?

2 Clusters in warm and dense nuclear matter

Consider a gas of neutrons, protons, and deuterons. The number density of nucleus $i = n, p, d$ is given as

$$n_i = g_i \left(\frac{M_i T}{2\pi}\right)^{3/2} \exp\frac{\mu_i - M_i}{T} \tag{1}$$

with the spin-degeneracy factor g_i, temperature T, mass of the nucleus M_i, and its chemical potential μ_i. The baryon number density is defined as

$$n_B = \sum_i n_i A_i \tag{2}$$

with the mass number of the nucleus A_i, and the charge fraction is defined as

$$Y_Q = \sum_i n_i Z_i / n_B \tag{3}$$

where Z_i is the charge number. Later we also use the mass fraction of a nucleus

$$X_i = A_i n_i / n_B \tag{4}$$

In the following, assume that so-called nuclear statistical equilibrium (NSE) has been established,

$$\mu_i = N_i \mu_n + Z_i \mu_p \tag{5}$$

with the neutron number N_i, which means that each nuclear species is in chemical equilibrium with all other. Furthermore, always assume to have symmetric matter with $Y_Q = 0.5$.

2.1 Limits of the ideal gas

(a) What is the value of the charge chemical potential $\mu_Q = \mu_p - \mu_n$ if one ignores the difference of the rest masses of neutrons and protons?

(b) At a fixed finite temperature, what is the value of the deuteron mass fraction in the limit of infinite density? At fixed chemical potentials, what is the limit for zero and infinite temperature?

(c) For a temperature of 5 MeV, at which density one reaches $X_d = 1/2$?

(d) At the LHC, deuterons were also observed. Considering the (approximate) proton and deuteron multiplicities shown by Stachel et al. in Fig. 1 of arXiv:1311.4662 which temperature would you assign to the fireball at freeze-out?

2.2 Equilibrium constants from heavy-ion collisions

The equilibrium constant is defined as

$$K_c[i] = \frac{n_i}{n_n^{N_i} n_p^{Z_i}} \tag{6}$$

Qin et al, PRL108, 17201 (2012) measured the deuteron equilibrium constants in low-energy heavy-ion collisions for various different temperatures and densities of the source, with the values given in Table 1.

(a) Plot and compare these values as a function of temperature with the ideal gas expression, again assuming NSE.

(b) Next, interpret the deviations from the ideal gas behavior observed in the experiment as a medium-dependence of the binding energy of the

T (MeV)	n_B (fm^{-3})	$K_c[d]$ (fm^3)
5.132638083	0.002974044	1456.804393
6.053364802	0.006655722	437.0252012
7.047677368	0.01149947	225.224263
8.009230485	0.015794475	137.9274855
9.011651885	0.020772199	93.20987024
10.0336148	0.024774121	60.27314619
11.00000000	0.027869939	31.45685198

Table 1: Temperature, density and equilibrium constant of the deuteron extracted by Qin et al. 2012 from low-energy heavy-ion collisions.

deuteron, $B_d = B_d^{\text{vac.}} + \Delta_d(T, n_B)$. Extract and plot $\Delta_d(T, n_B)$ as a function of T. If you are further interested, you can compare with the results of the Quantum Statistical model presented in Fig. 1 of Typel et al. PRC81, 015803 (2010).

(c) Is this interpretation realistic? Which other aspects should be included in the model?

2.3 Excluded volume approach

Warning: difficult!

The Helmholtz free energy of our ideal toy model is given as the sum over the included particle species:

$$F^{\text{id}}(T, V, N_n, N_p, N_d) = \sum_i F_i^{\text{id}}(T, V, N_i) \tag{7}$$

With the volume V and particle numbers N_i. To suppress nuclei at high densities, implement the so-called excluded volume approximation. Consider that each nucleon has a finite size of $V_0 = 1/n_B^0 = 1/0.16\ \mathrm{fm}^3$ (both for nucleons bound in nuclei and unbound), and that deuterons can only exist in the free volume, i.e., that is not occupied by nucleons and deuterons themselves. This should represent the medium modification of the deuteron bound state, e.g., due to Pauli blocking. In our toy model, nucleons are assumed to be not affected by medium effects, i.e., they can exist in the entire volume V.

By using the standard thermodynamic relations

$$\mu_i = \left.\frac{\partial F}{\partial N_i}\right|_{T,V,N_{j\neq i}} \tag{8}$$

and

$$P_i = -\left.\frac{\partial F}{\partial V}\right|_{T,N_i} \tag{9}$$

derive an expression for the deuteron number density $n_d = N_d/V$ as a function of the neutron number density $n_n = N_n/V$.

If you are very ambitious, having this solution at hand, you could repeat the exercises 2.1 and 2.2 with your improved model.