Clusters and phase transitions in supernova equations of state — part II

Matthias Hempel, Basel University
HISS Dubna „Dense Matter“, 30.6.2015

Outline

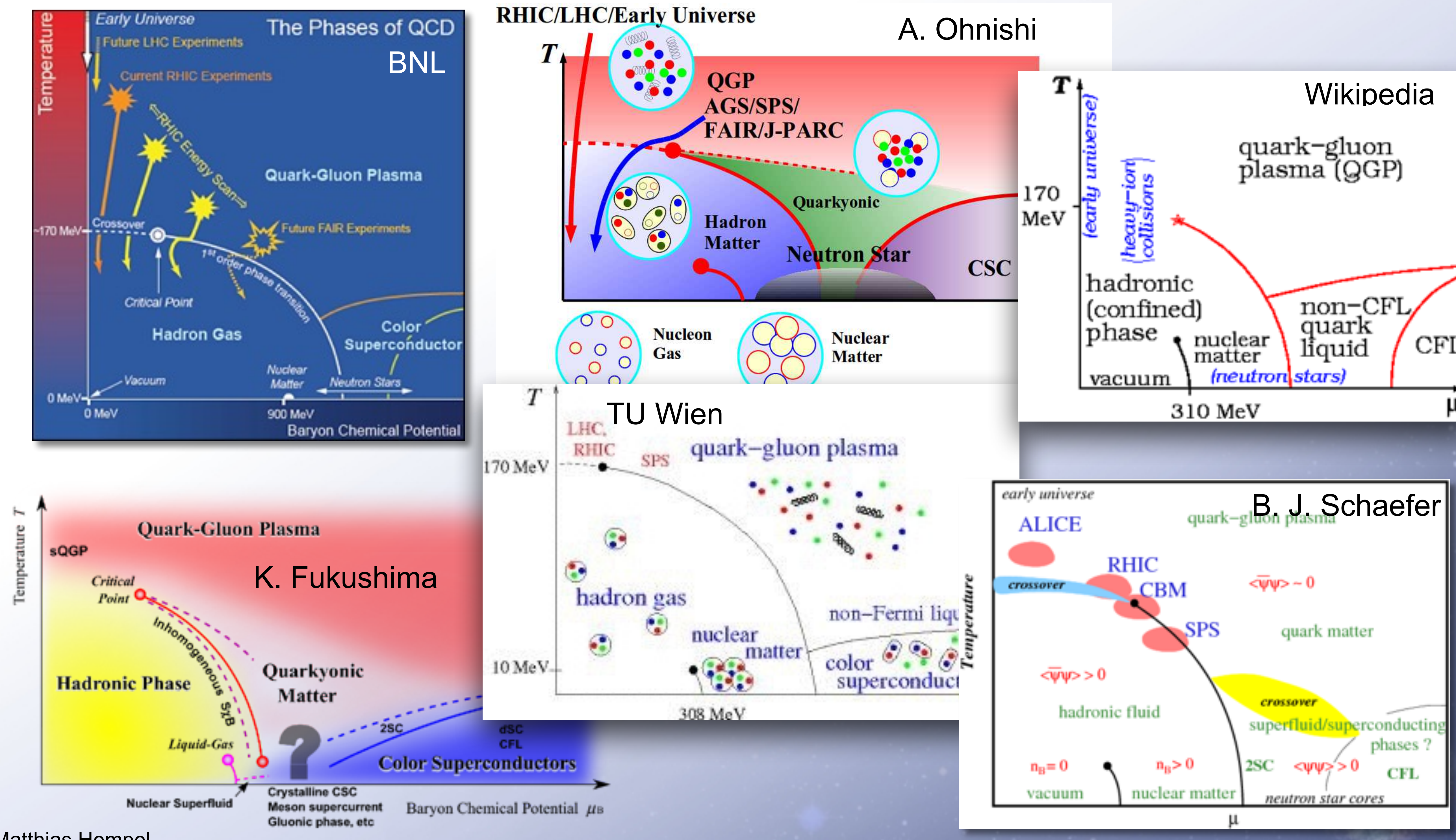
Monday	• brief astrophysical introduction • matter in supernovae, comparison with HICs • QCD phase transition in supernovae
Tuesday	• comparison liquid-gas with QCD phase transition, non-congruence • cluster in supernovae • cluster formation in nuclear matter, experimental probes

Non-congruence of the nuclear liquid-gas and QCD phase transition

[MH, V. Dexheimer, S. Schramm, I. Iosilevskiy, PRC 88 (2013)]

Introduction – QCD phase diagrams

- fundamental question: phase diagram of strongly interacting matter
- typical examples in T-μ, first order phase transitions (PT) as lines:



BNL

A. Ohnishi

Wikipedia

TU Wien

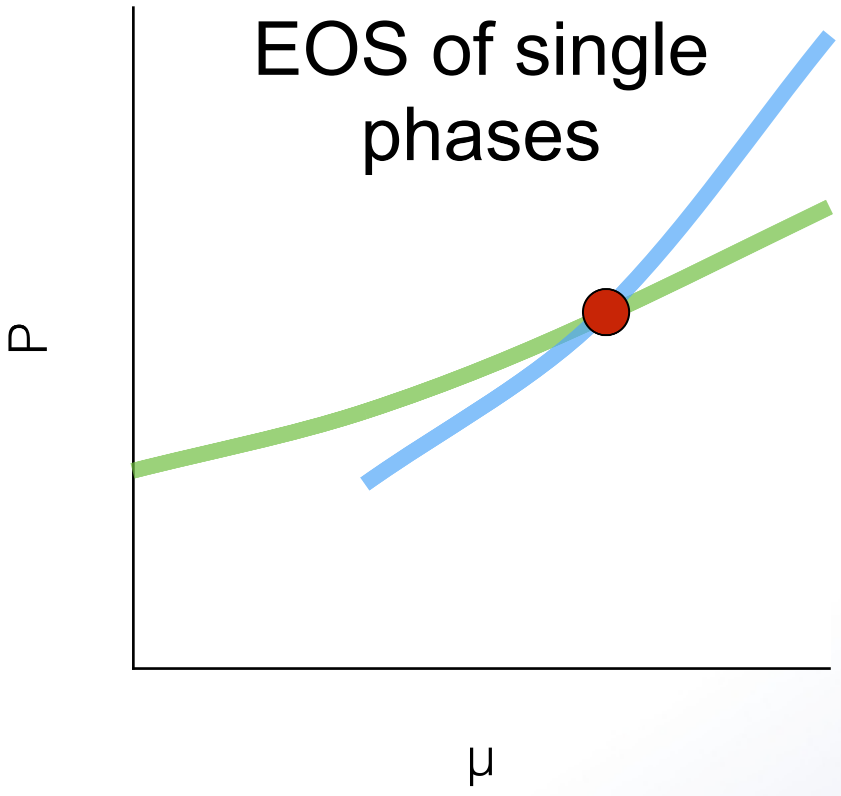
B. J. Schaefer

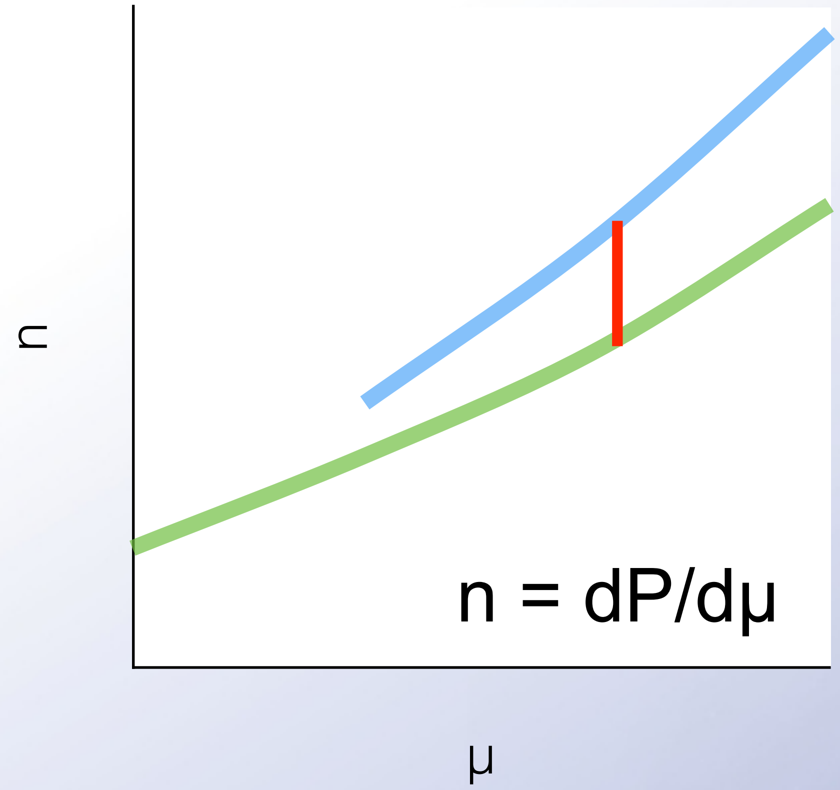
K. Fukushima

Matthias Hempel
Dubna, 30.6.2015

Basics of phase transitions

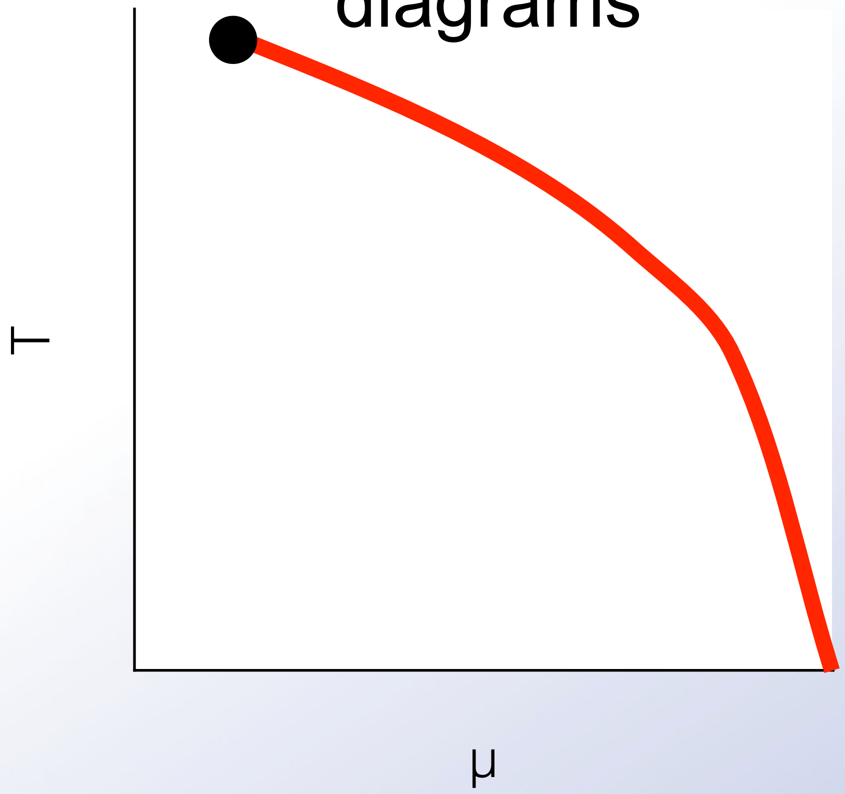
T = const.,
EOS of single phases

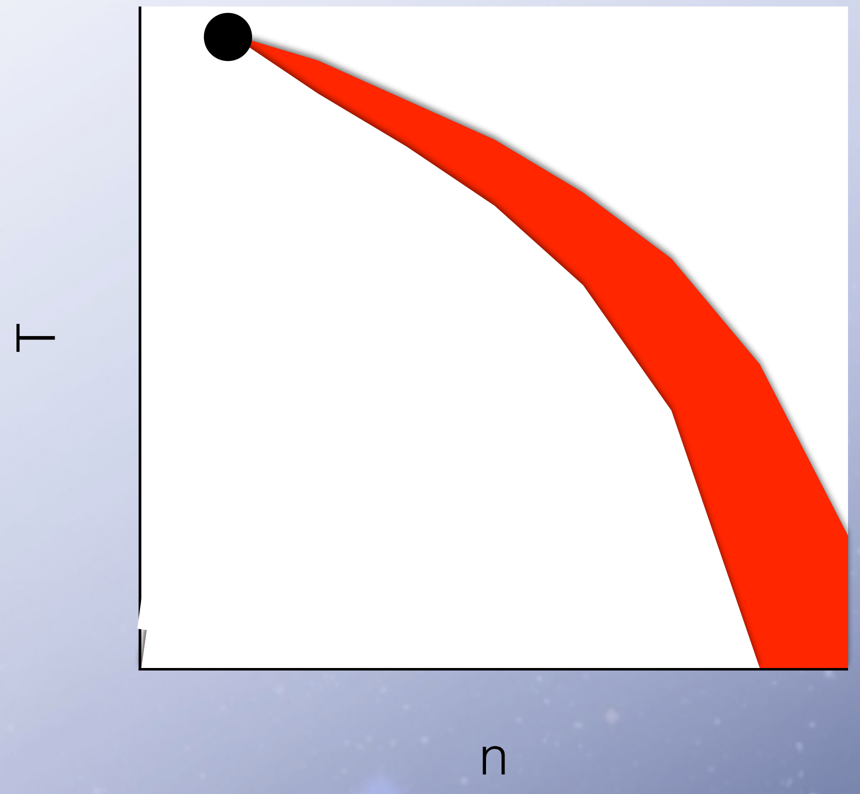




$n = dP/d\mu$

phase
diagrams



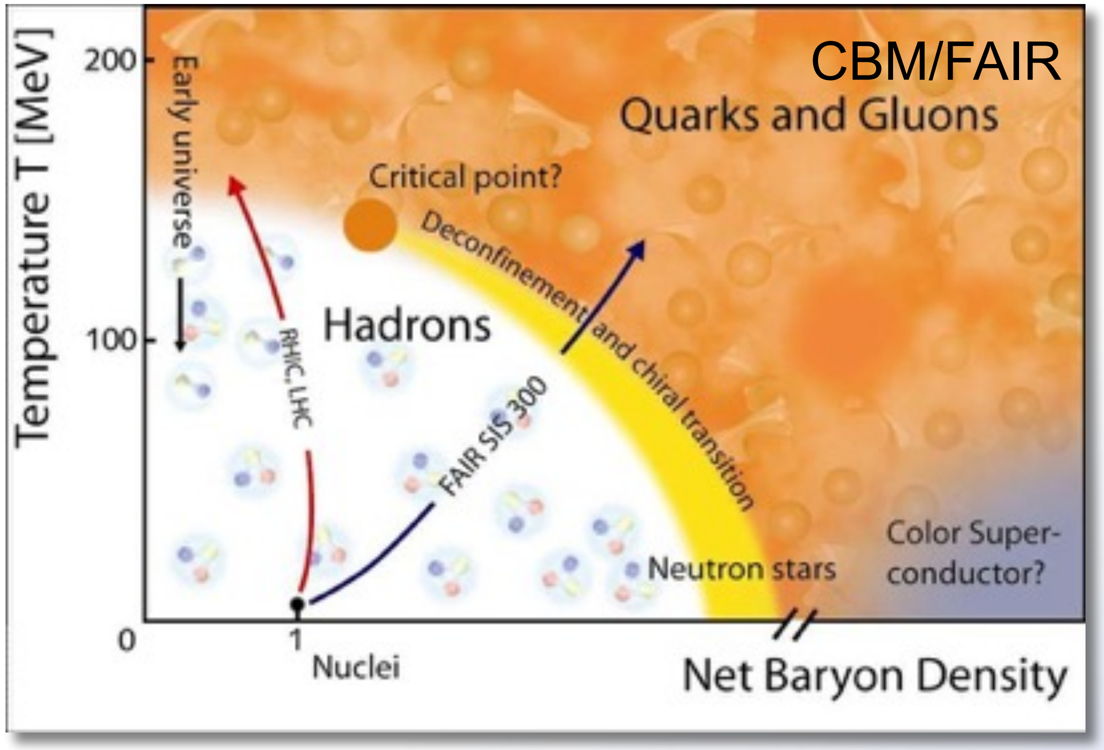


$P^{I} = P^{II}$
$\mu^{I} = \mu^{II}$
$T^{I} = T^{II}$

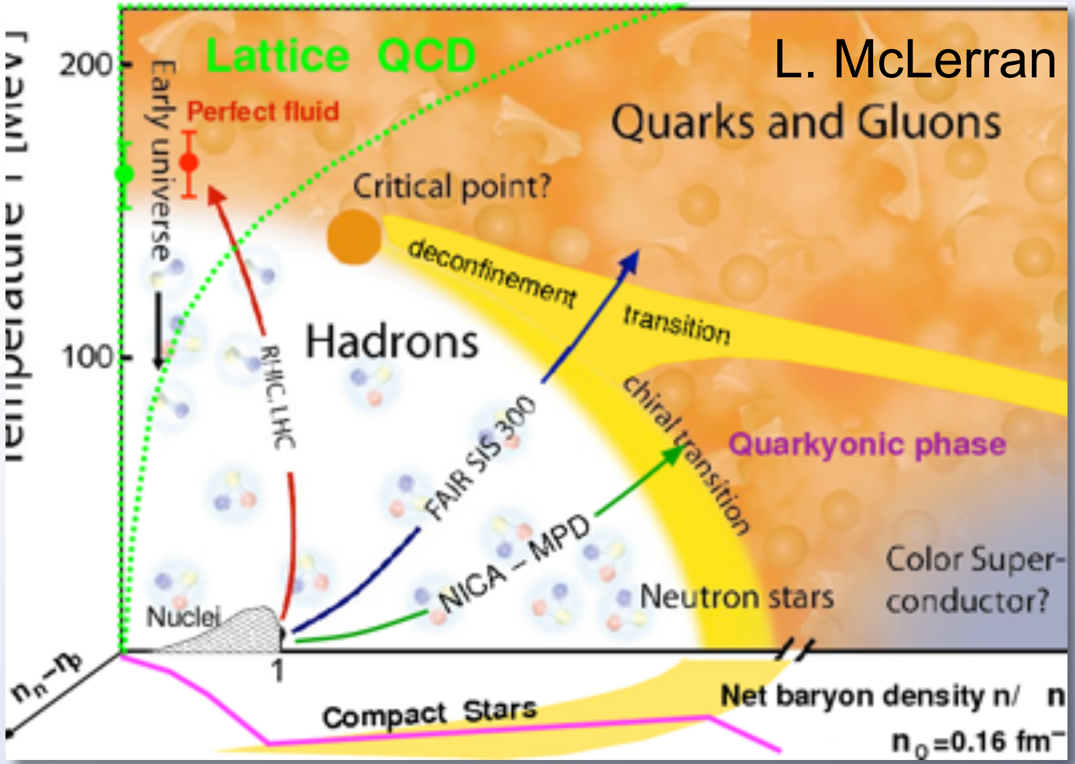
- different densities of two phases at phase transition point
- extended range of two-phase coexistence in density
- similarly for all extensive variables: entropy, energy

Introduction – QCD phase diagrams

- two-dimensional area in T-ρ:



- additional axis: asymmetry, strangeness, lepton fraction, magnetic fields,…:



- the phase diagrams of the previous slide correspond to “congruent“ first order PTs
- for asymmetric systems the PTs are typically “non-congruent“ and manifest themselves in principally different forms

Non-congruence

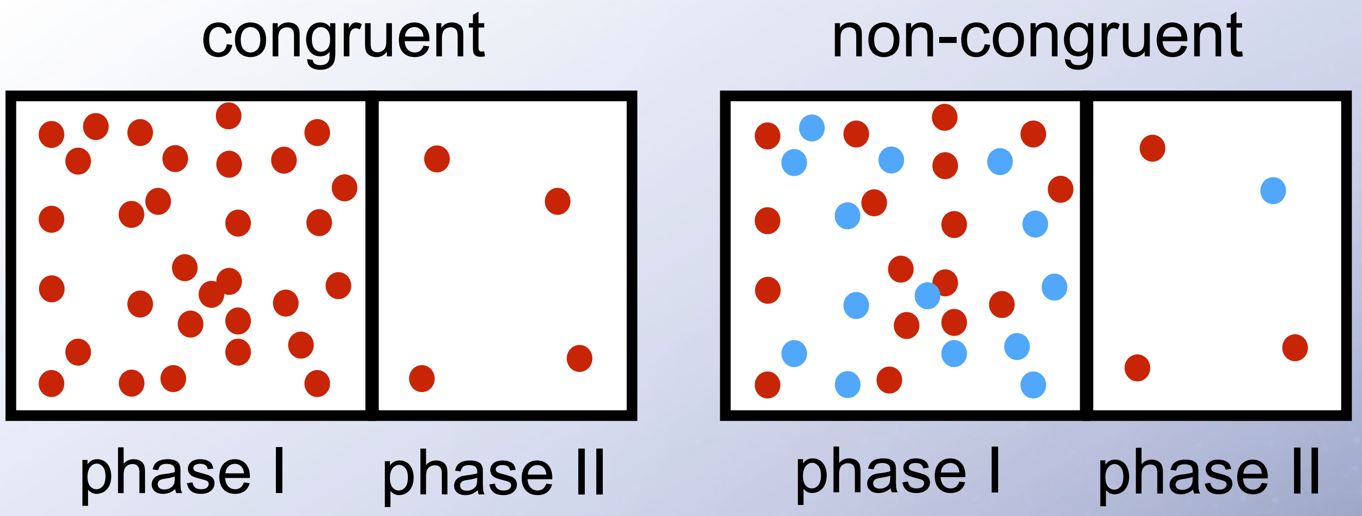
[MH, V. Dexheimer, S. Schramm, I. Iosilevskiy, PRC 88 (2013)]

- first order PTs with more than one globally conserved charge: non-congruent PTs
- only one globally conserved charge: congruent
- fundamental differences, because the concentrations of the charges can be exchanged between the two phases, “distillation“

- term “non-congruent“ is commonly used for PTs in terrestrial applications with chemically reacting plasmas *[Iosilevskiy, Acta Phys. Polon. B (2010)]*
- term introduced in astro and nuclear physics by I. Iosilevskiy

- known in heavy-ion collisions (HICs), nuclear physics and astrophysics under the terms “Maxwell“ and “Gibbs“

[C. Greiner, Koch, and Stöcker, PRL (1987)]
[Glendenning, PRD (1992)]



Setup

- liquid-gas phase transition of nuclear matter: non-linear relativistic mean-field model FSUgold *[Todd-Rutel and Piekarewicz, PRL (2005)]*

- QCD phase transition: Chiral SU(3) model *[Dexheimer and Schramm, PRC81 (2010)]*

- in all calculations: neglect of all Coulomb interactions, “Coulomb-less“ approximation (cf. works by Gulminelli, Raduta, Typel, …)

- solve for thermal, mechanical, and chemical equilibrium

Chiral SU(3) model

[Dexheimer and Schramm, PRC81 (2010)]

- meson-exchange based effective relativistic mean-field model
- coupling constants are fitted to vacuum masses of baryons and mesons, nuclear matter properties, hyperon optical potentials, and lattice data
- quarks (u,d,s) and hadrons (baryon octet) are included as a chemical mixture of quasi-particle degrees of freedom

- populated degrees of freedom regulated by “Polyakov“ potential U

$$\begin{aligned} U = {} & \left(a_0 T^4 + a_1 \mu_B^4 + a_2 T^2 \mu_B^2\right)\Phi^2 \\ & + a_3 T_0^4 \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4), \end{aligned}$$

- via contribution of Φ to masses of particles:

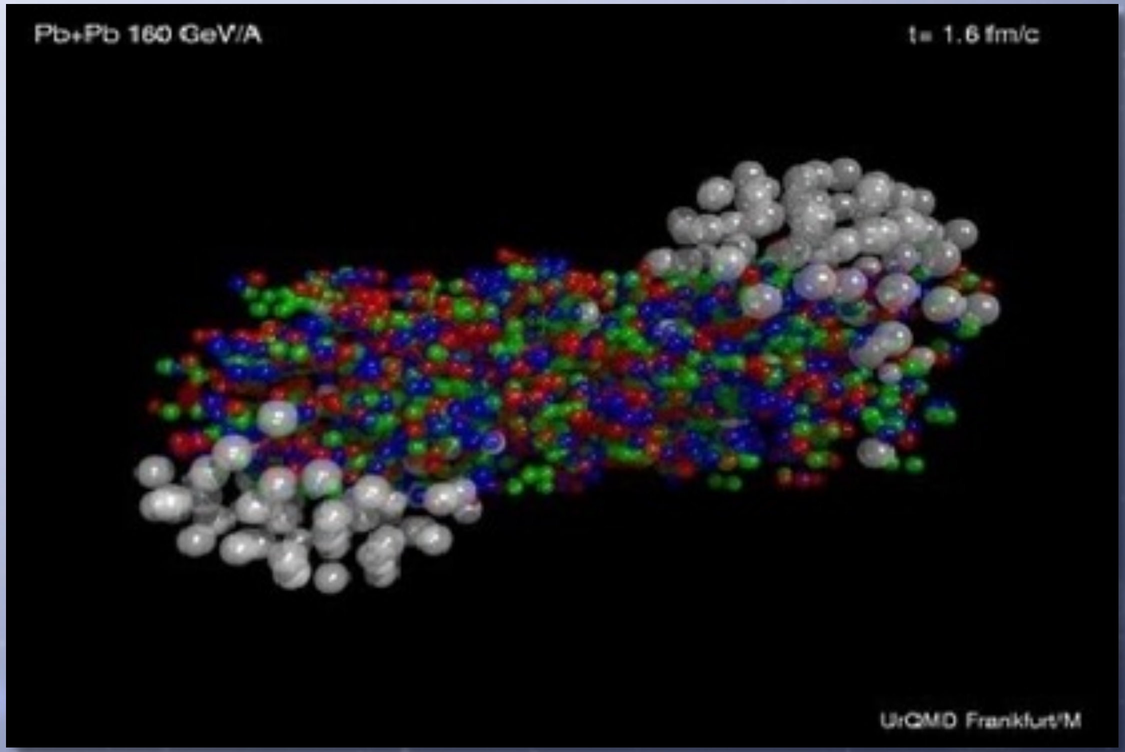
$$\begin{aligned} M_B^* &= g_{B\sigma}\sigma + g_{B\delta}\tau_3\delta + g_{B\zeta}\zeta + M_{0_B} + g_{B\Phi}\Phi^2, \\ M_q^* &= g_{q\sigma}\sigma + g_{q\delta}\tau_3\delta + g_{q\zeta}\zeta + M_{0_q} + g_{q\Phi}(1 - \Phi). \end{aligned}$$

- zero Temperature: baryons at low densities, quarks at high densities, first order PT in-between

Conditions in heavy-ion collisions

- at least two conserved charges: (net) baryon number B and (net) isospin or equivalently the charge to baryon ratio Y_Q=Z/A
- → one can expect that PTs in HICs are generally non-congurent
- ^{208}Pb + ^{208}Pb: Y_Q=0.39
- ^{197}Au + ^{197}Au: Y_Q=0.40

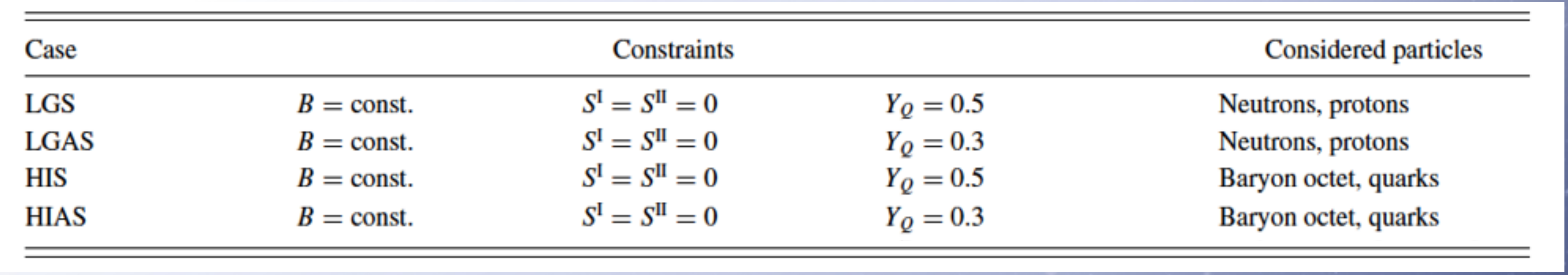
- additional conserved charge: (net) strangeness S=0



Scenarios

- liquid-gas symmetric (LGS)
- liquid-gas asymmetric (LGAS)
- heavy-ion symmetric (HIS)
- heavy-ion asymmetric (HIAS)

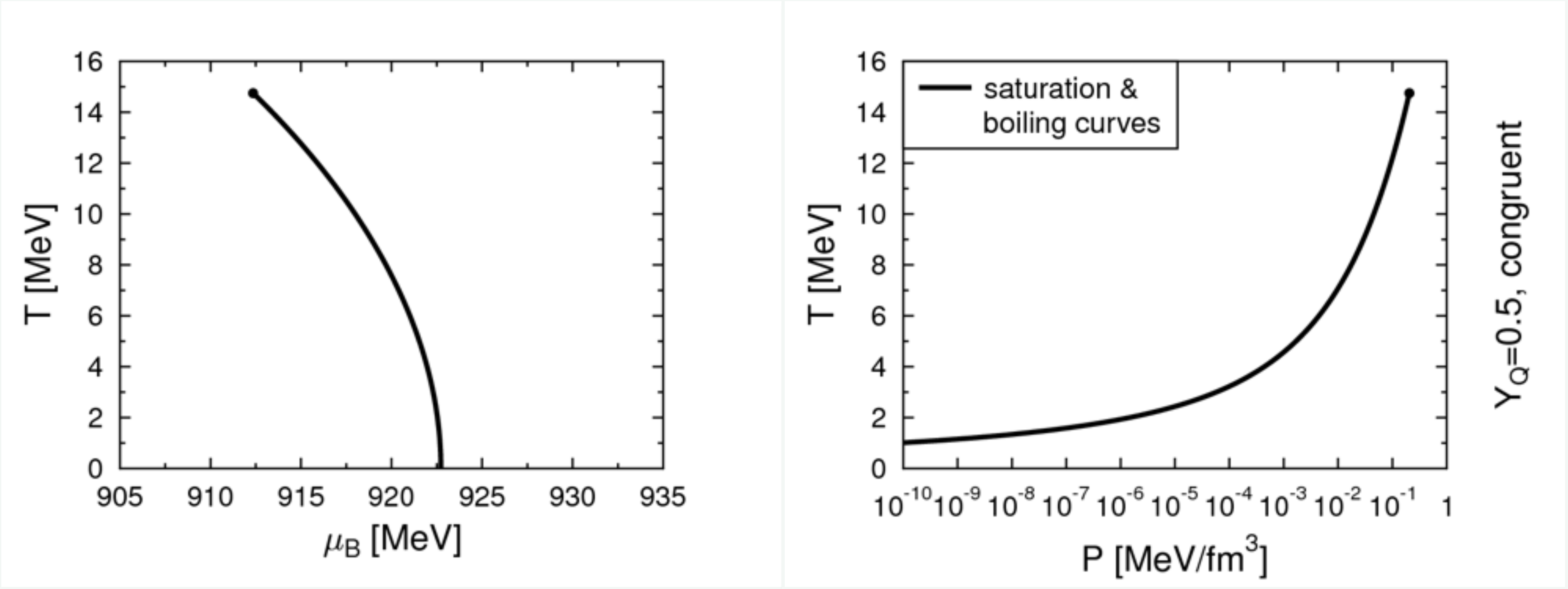
- symmetric: Y_Q=0.5, asymmetric: Y_Q=0.3
- zero strangeness locally



Case	Constraints			Considered particles
LGS	$B = \text{const.}$	$S^{\mathrm{I}} = S^{\mathrm{II}} = 0$	$Y_Q = 0.5$	Neutrons, protons
LGAS	$B = \text{const.}$	$S^{\mathrm{I}} = S^{\mathrm{II}} = 0$	$Y_Q = 0.3$	Neutrons, protons
HIS	$B = \text{const.}$	$S^{\mathrm{I}} = S^{\mathrm{II}} = 0$	$Y_Q = 0.5$	Baryon octet, quarks
HIAS	$B = \text{const.}$	$S^{\mathrm{I}} = S^{\mathrm{II}} = 0$	$Y_Q = 0.3$	Baryon octet, quarks

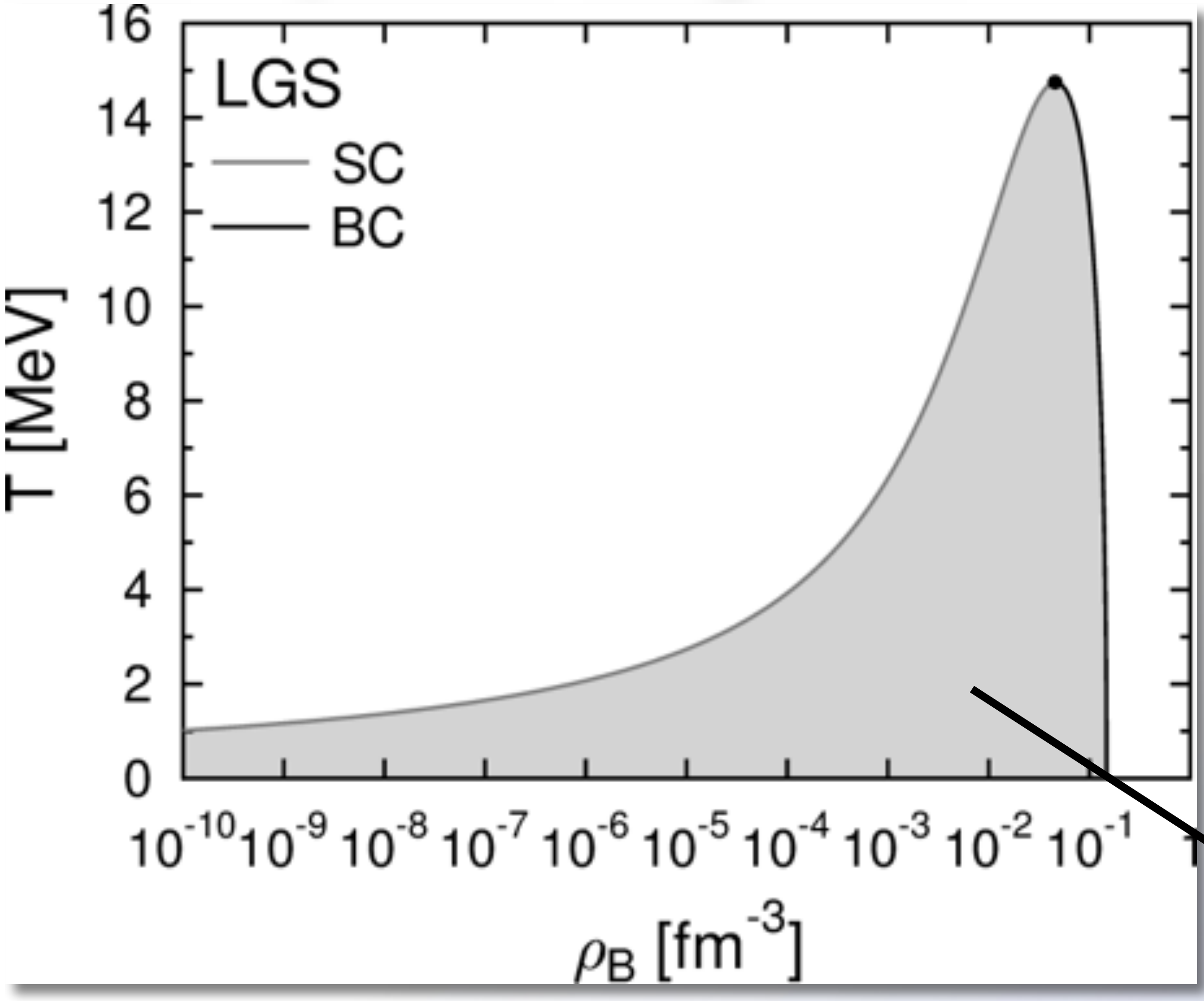
nuclear liquid-gas PT

LGS – phase diagrams

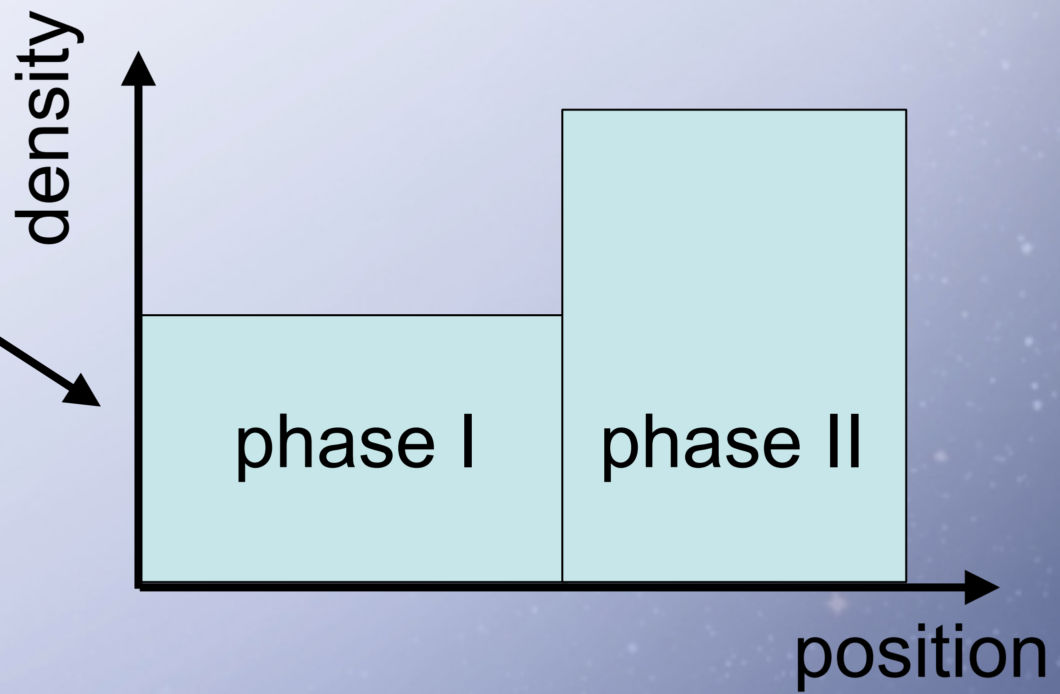


- first order phase transition line terminates at critical end point (CEP)

LGS – phase diagrams



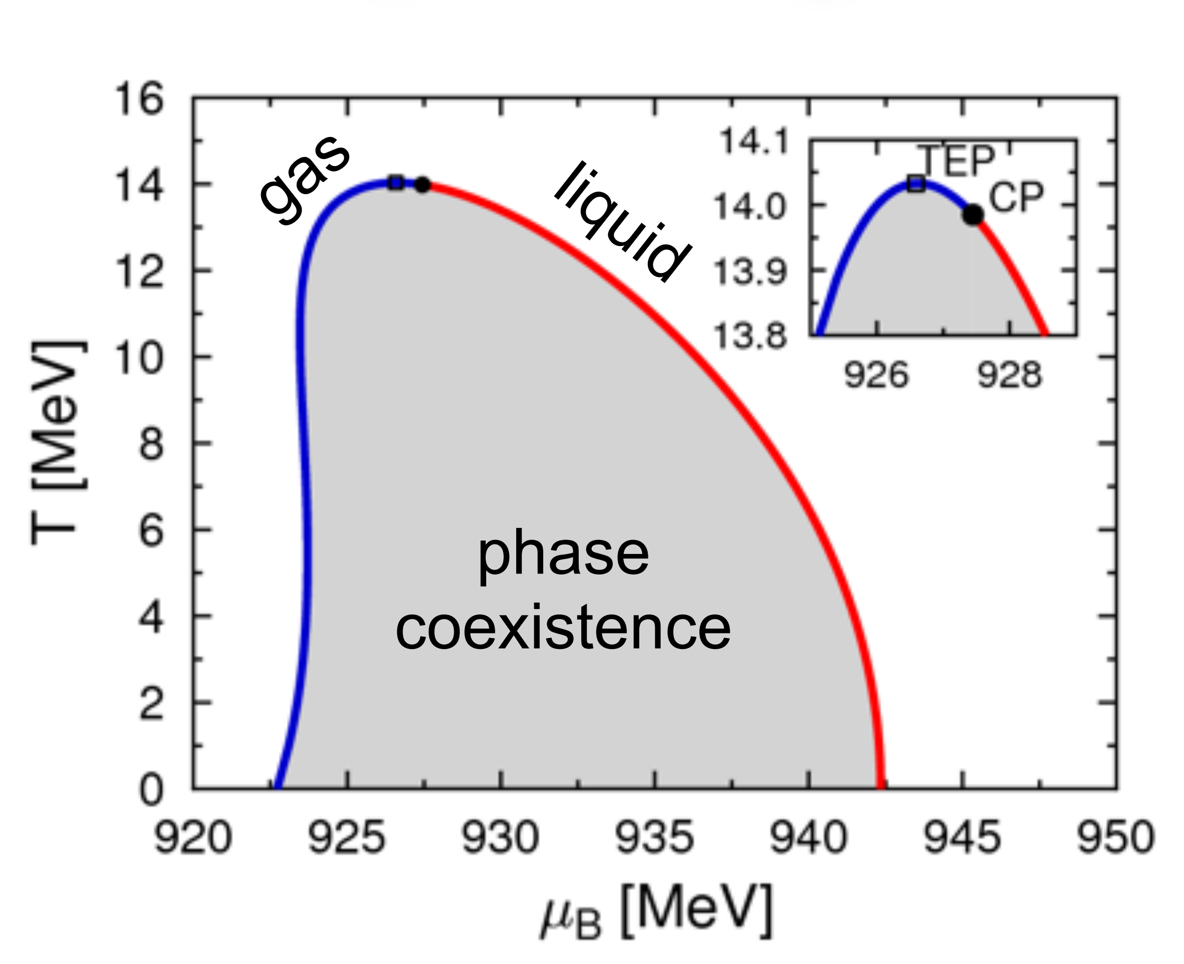
- discontinuity of the density
- extended two-phase coexistence region in terms of density (still congruent)



SC: saturation curve

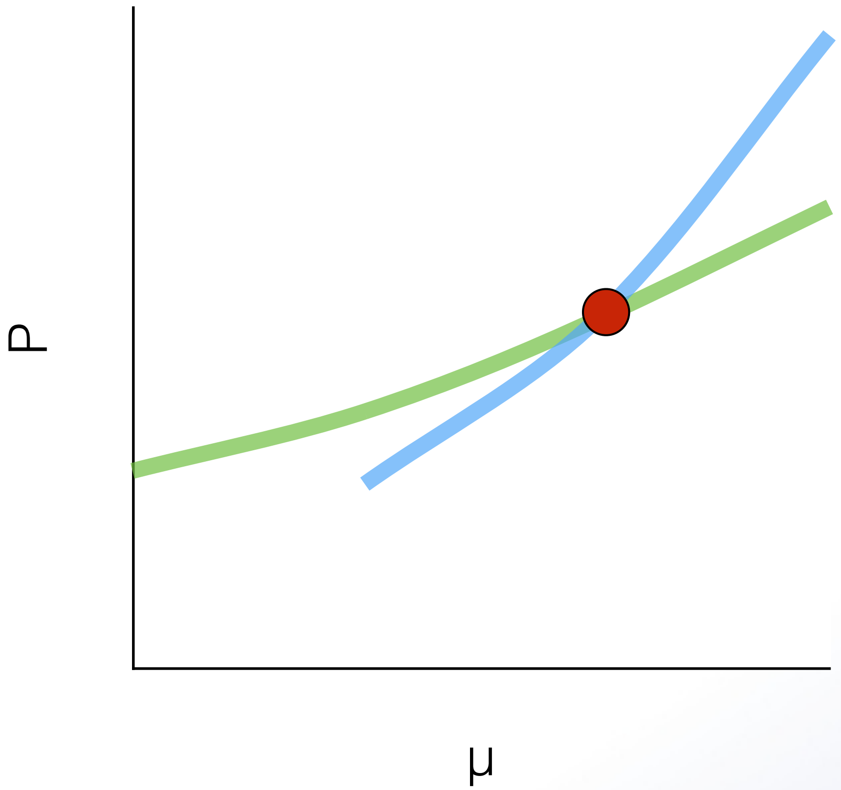
BC: boiling curve

LGAS – phase diagrams



- phase transition lines turn to two-dimensional phase coexistence regions
- saturation and boiling curves do not coincide

Non-congruence



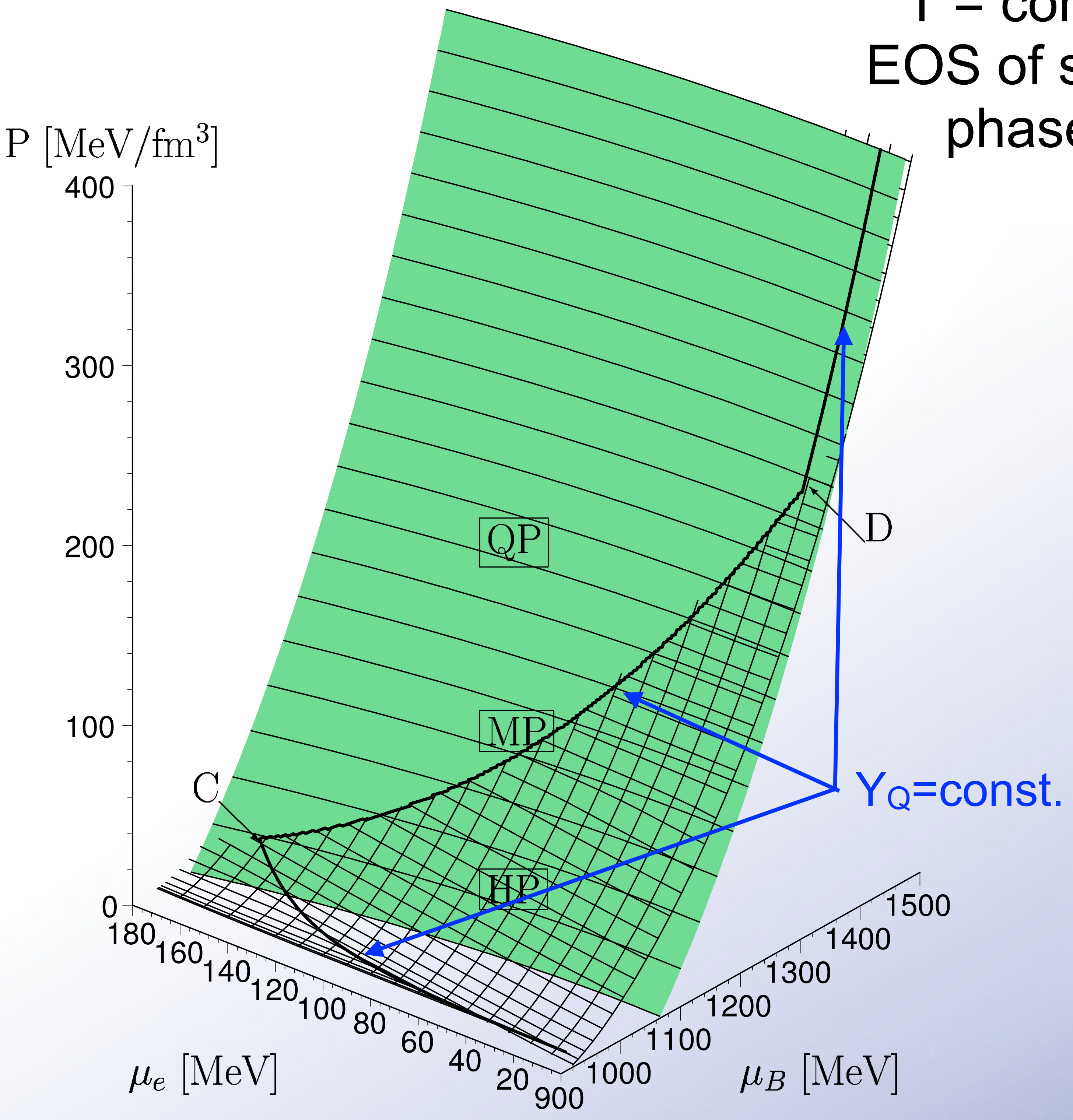
T = const.,
EOS of single
phases

$P^{I} = P^{II}$
$\mu^{I} = \mu^{II}$
$T^{I} = T^{II}$

Non-congruence

$P^I = P^{II}$
$\mu_i^I = \mu_i^{II}$
$T^I = T^{II}$

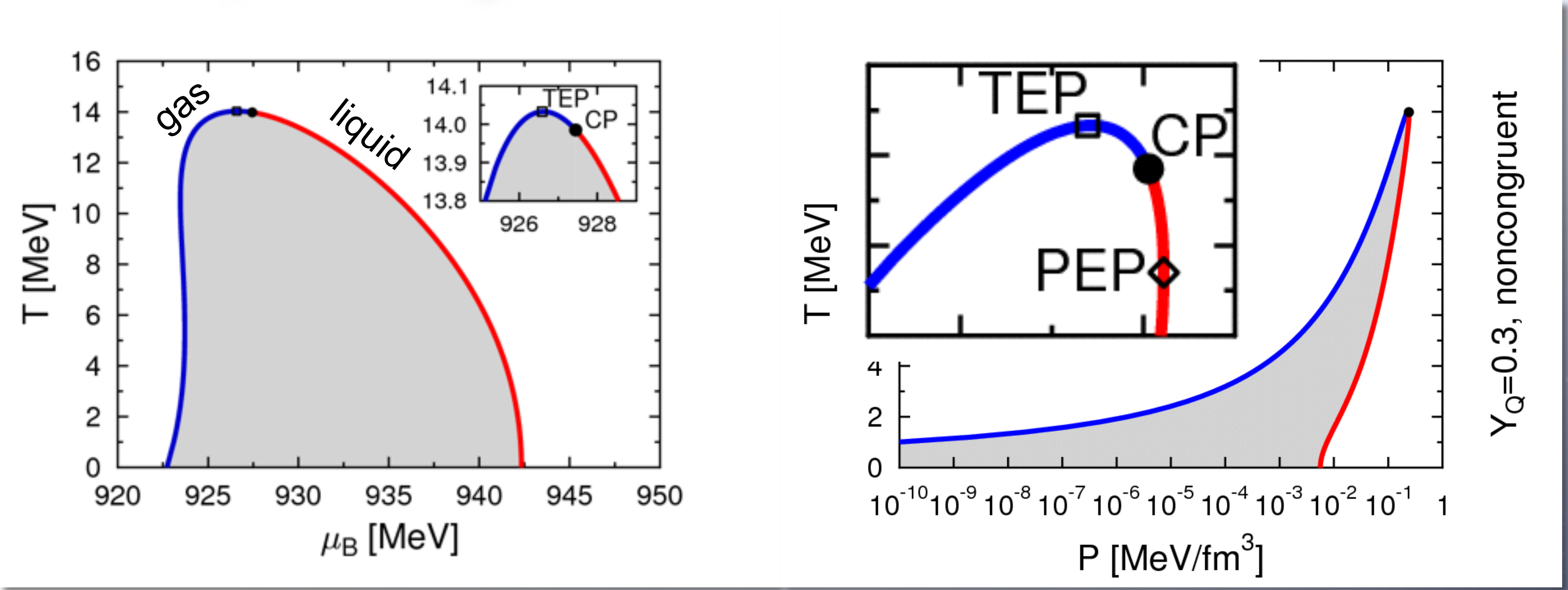
T = const.,
EOS of single
phases



- charge fraction *and* density different in two phases
- fixing of the total charge fraction → variation of the intensive variables (e.g., μ_i) along the PT
- projection onto P-T, P-μ_i planes: two-dimensional region instead of line

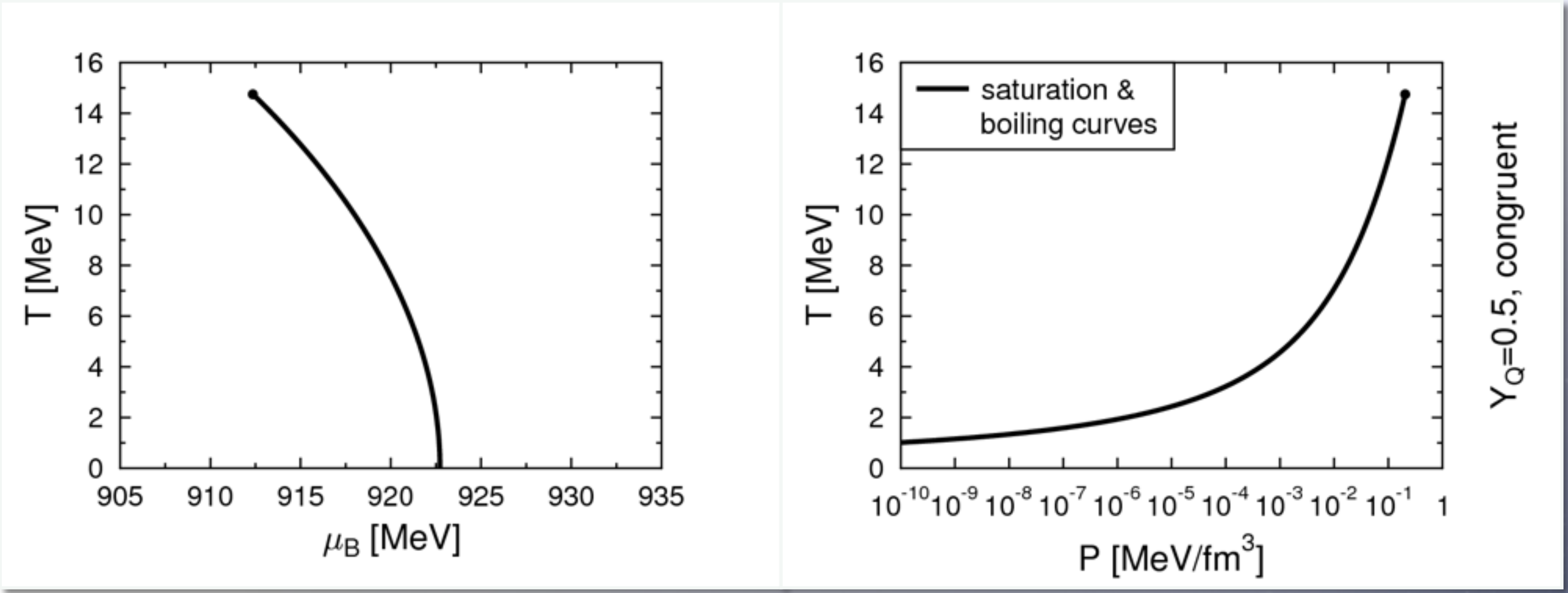
M. Hanauske, Ph.D. thesis

LGAS – phase diagrams



- phase transition lines turn to two-dimensional phase coexistence regions
- saturation and boiling curves do not coincide
- temperature endpoint (TEP), pressure endpoint (PEP) and critical point (CP) do not coincide

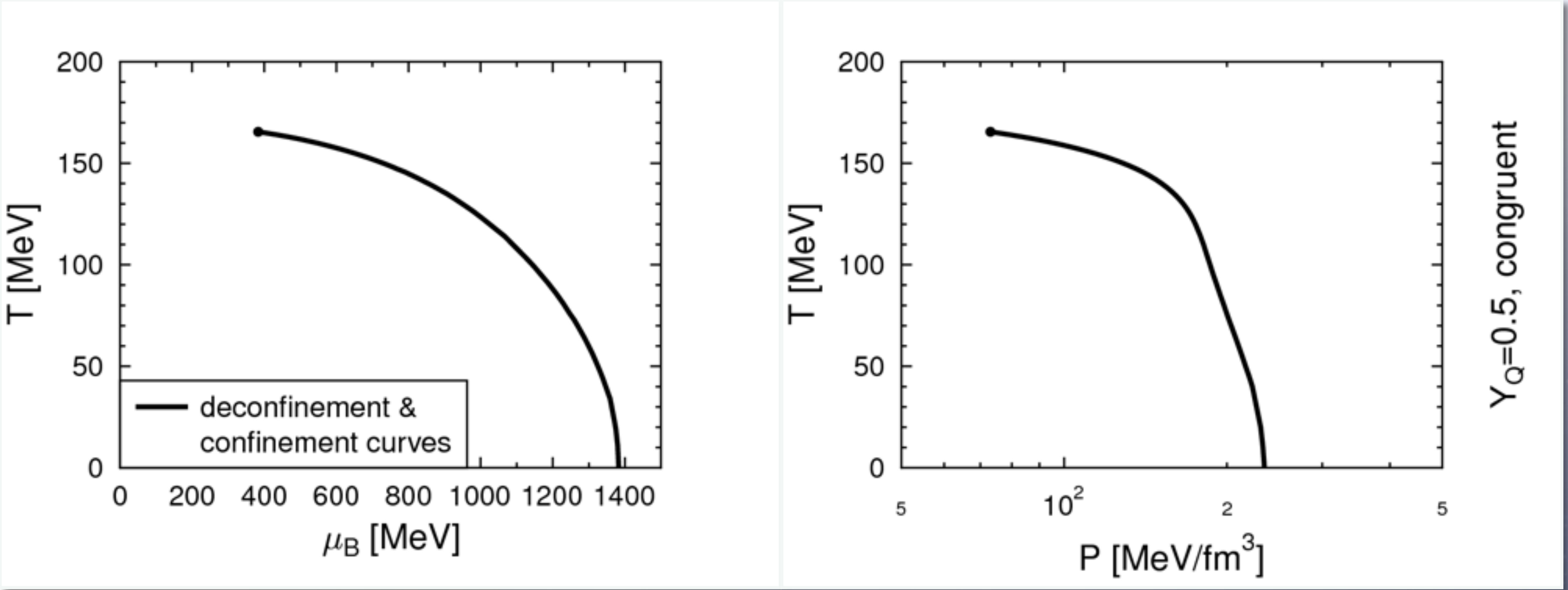
Why is LGS congruent?



- first order phase transition line terminates at critical end point (CEP)
- binary system: isospin and baryon number
- despite this the phase transition behaves congruent
- both phases stay symmetric
- symmetric nuclear matter is an “azeotrope“, due to isospin symmetry

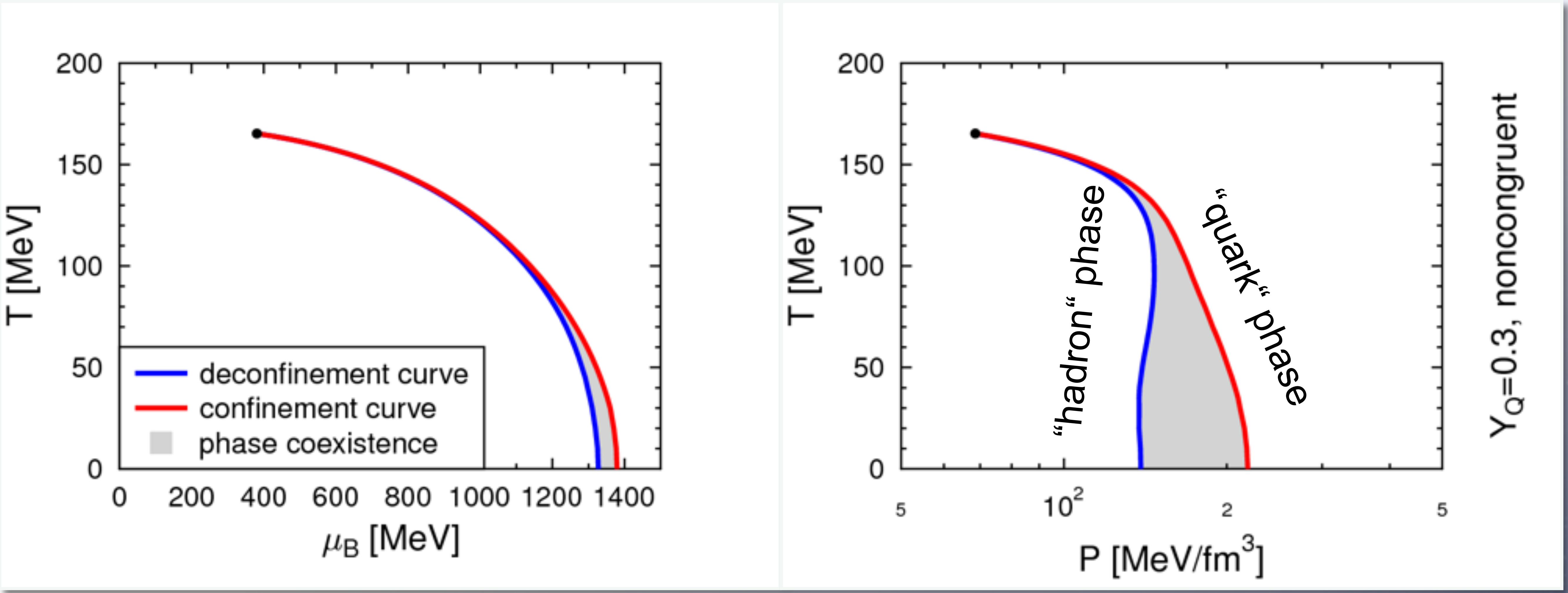
QCD PT

HIS – phase diagrams



- again “azeotrope“, due to isospin symmetry

HIAS – phase diagrams

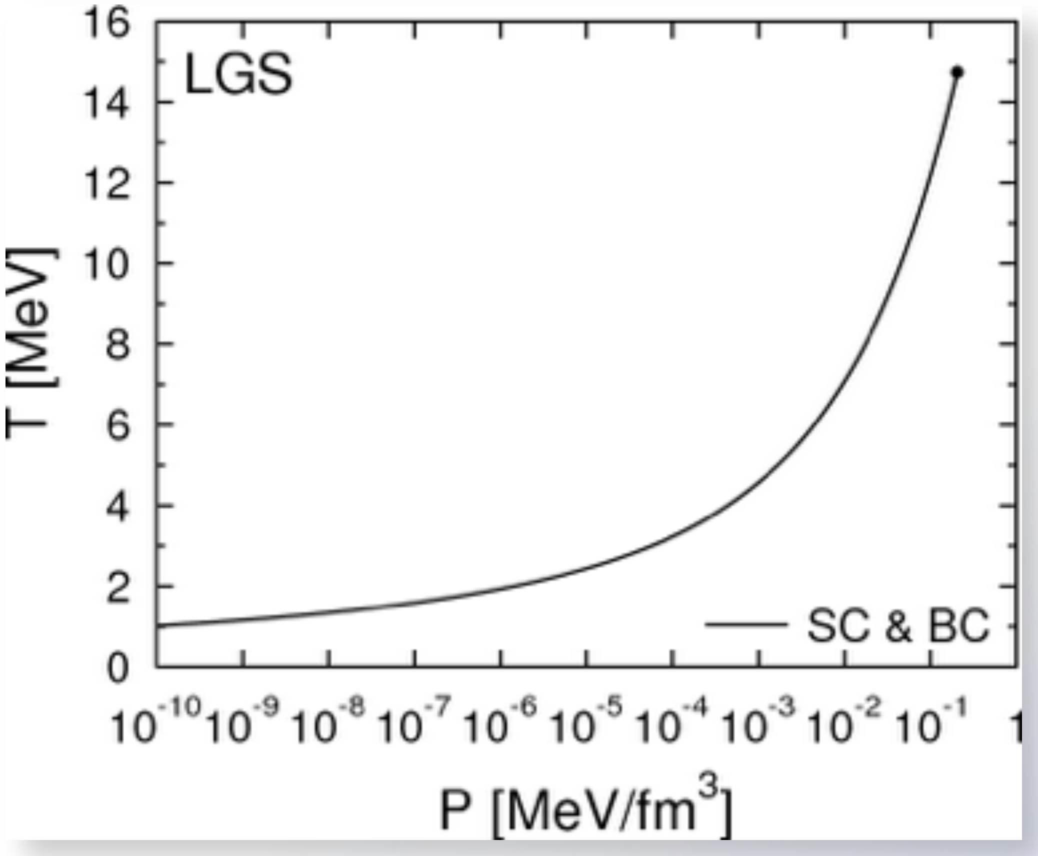


- phase transition lines turn to two-dimensional phase coexistence regions
- deconfinement and confinement curves do not coincide
- distinction between CP, TEP, and PEP not achieved, but expected
- reason: non-congruent features vanishingly small around the CP, difference compared to LG

comparison – is the QCD PT of liquid-gas type?

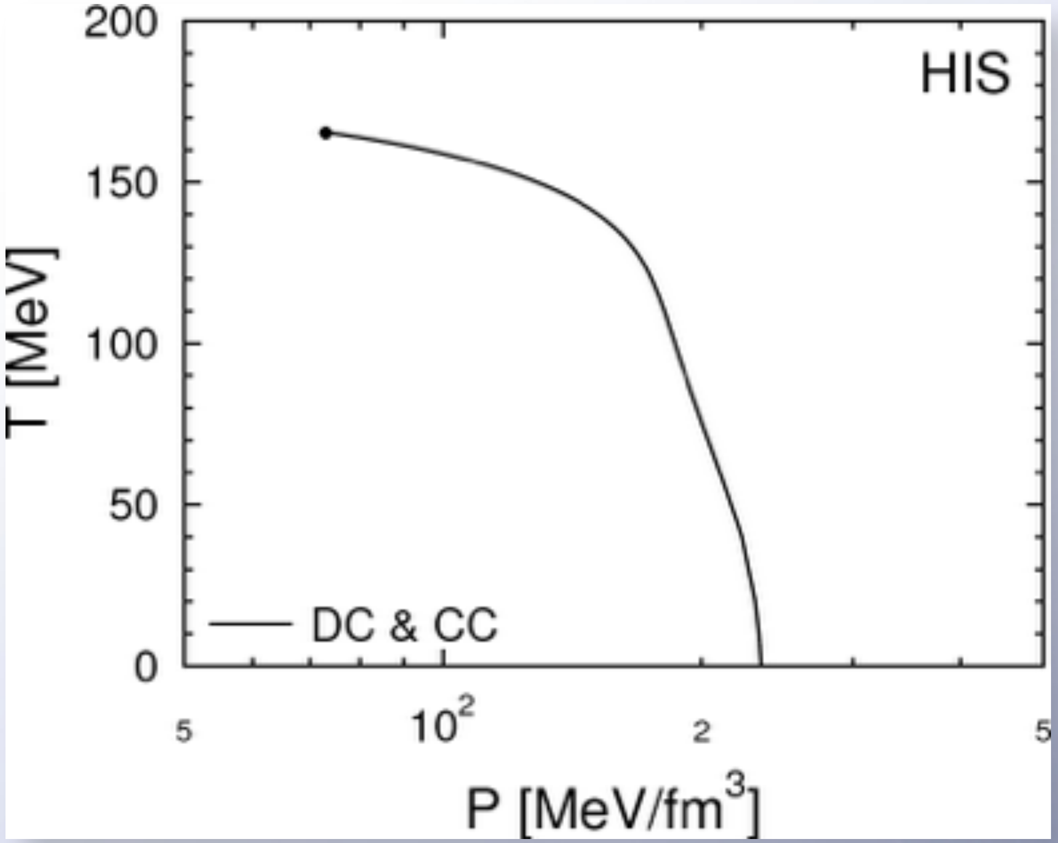
Comparison of LG and QCD PT

liquid-gas phase transition



enthalpic

chiral/deconfinement phase transition



entropic

- opposite slope as fundamental difference
- due to lower/higher entropy of the more dense phase, Clapeyron equation:

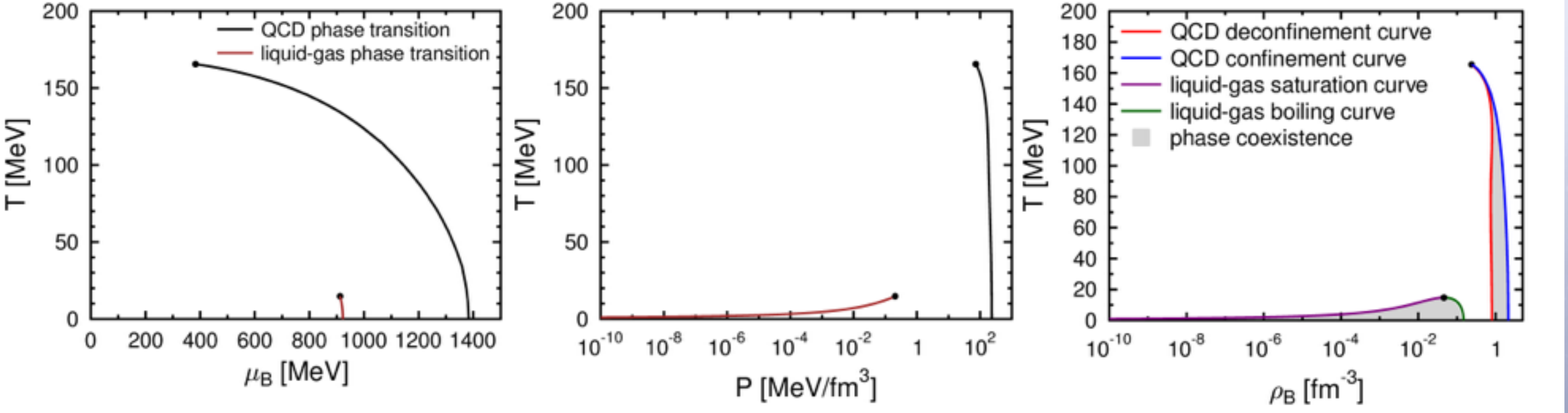
$$\frac{dP}{dT} = \frac{s^{\mathrm{I}} - s^{\mathrm{II}}}{1/\rho_B^{\mathrm{I}} - 1/\rho_B^{\mathrm{II}}}$$

[Iosilevskiy, arXiv:1403.8053]

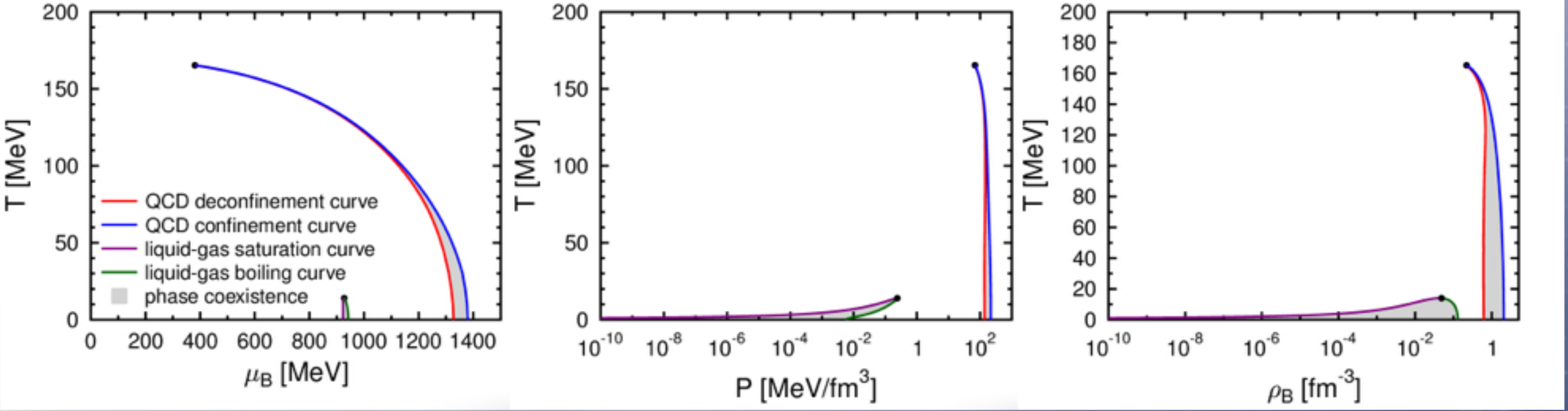
SC: saturation curve
BC: boiling curve
DC: deconfinement curve
CC: confinement curve

Phase diagrams

symmetric matter



asymmetric matter



„Realistic“ supernova EOS — cluster formation

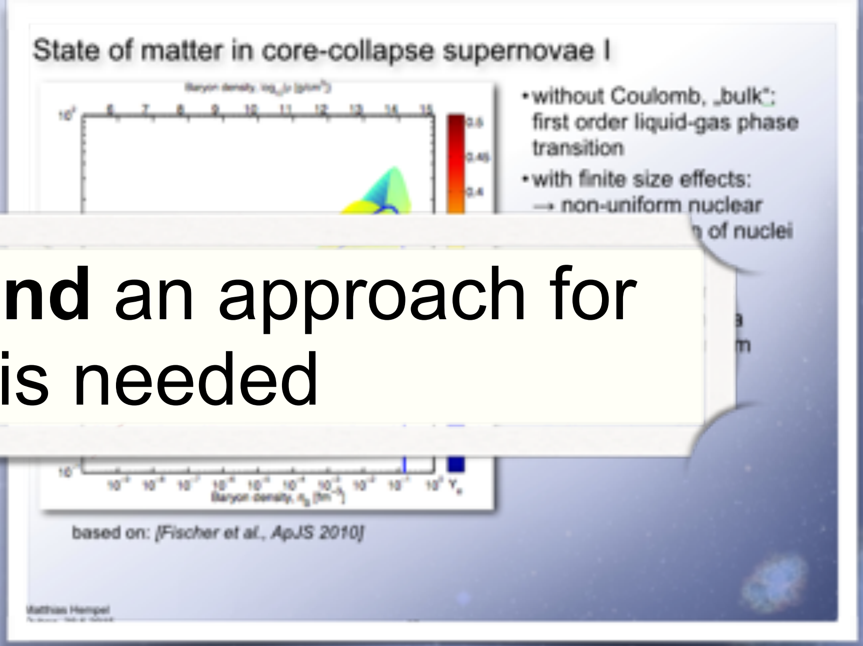
„Realistic“ supernova EOS

- until now: phase diagrams for „infinite“ thermodynamic phases, despite net electric charges, “Coulomb-less approach of bulk matter“
- Coulomb and surface energies crucial for realistic description, correlations → clusterization

- nuclear matter in the liquid phase → nucleons bound in nuclei
- nuclear matter in the gas phase → unbound nucleons

- the formation of clusters replaces/quenches the first-order phase transition of bulk matter
- connected to „pasta phases“

yesterday:



State of matter in core-collapse supernovae I

- without Coulomb, „bulk“: first order liquid-gas phase transition
- with finite size effects: → non-uniform nuclear
of nuclei

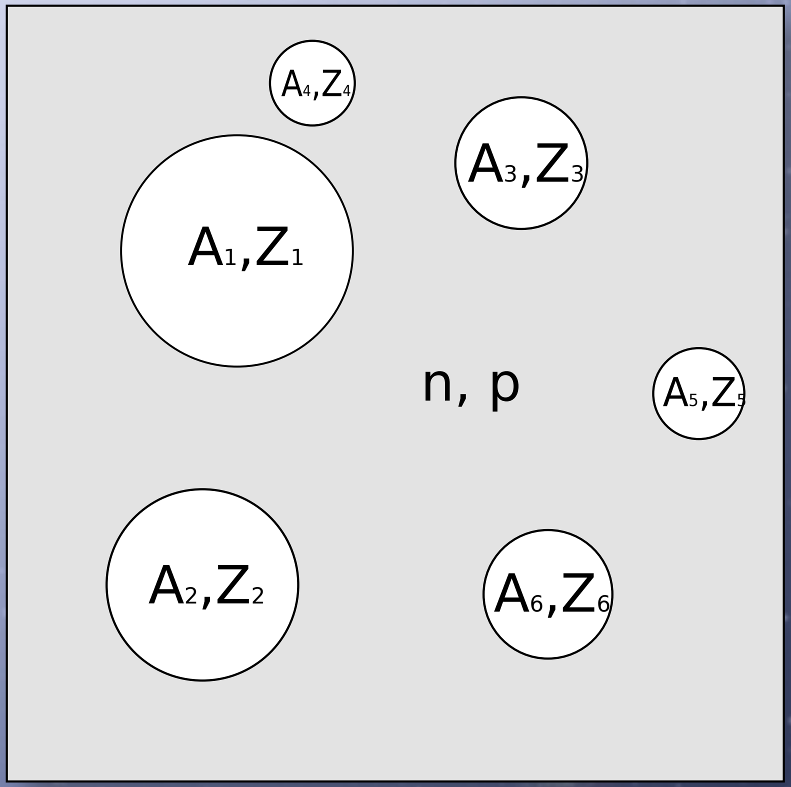
based on: *[Fischer et al., ApJS 2010]*

a model for the nuclear interactions **and** an approach for formation of nuclei/clusters is needed

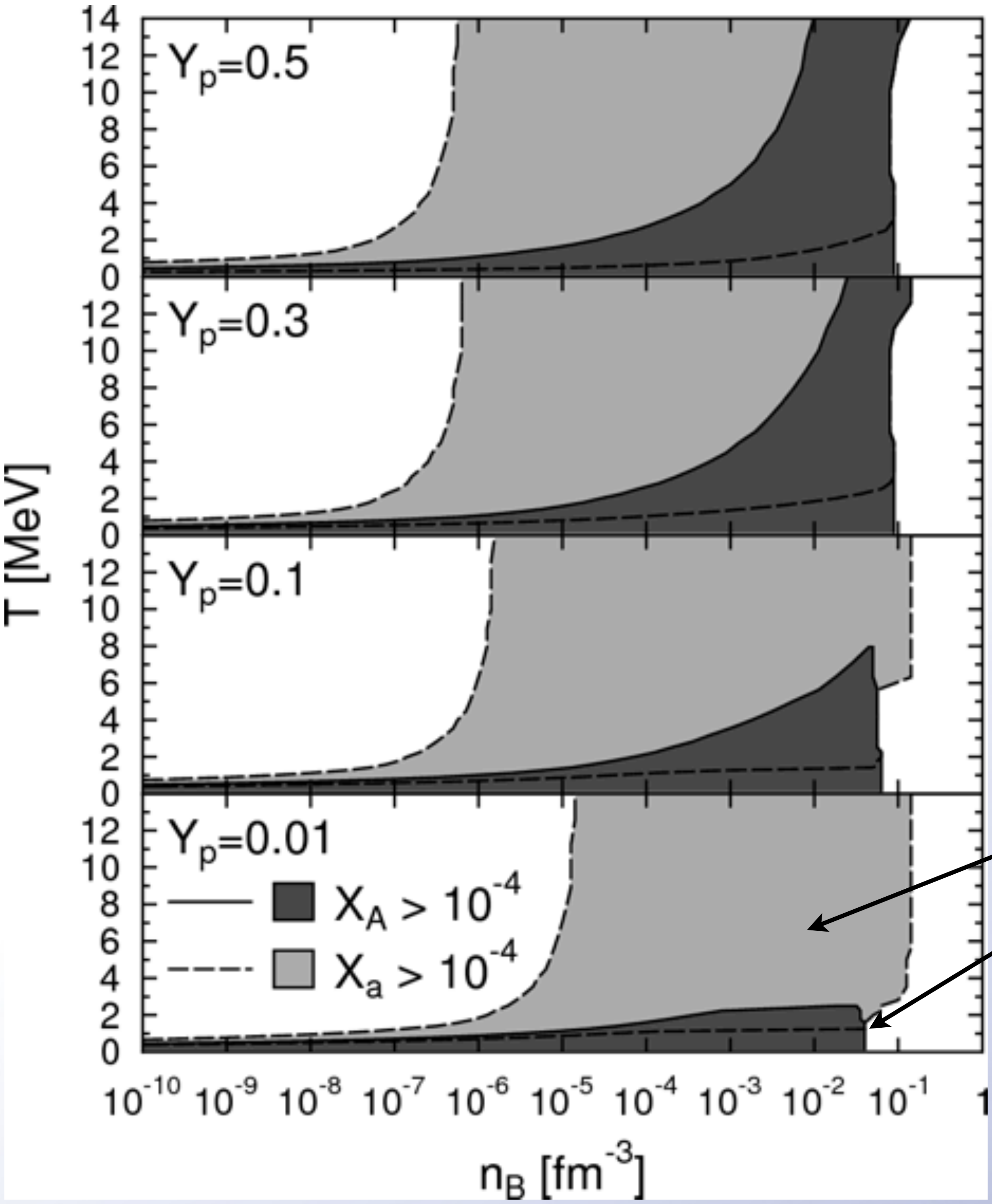
EOS model: excluded volume NSE with interactions

- chemical mixture of nuclei and interacting nucleons in nuclear statistical equilibrium (NSE)
- nucleon interactions: relativistic mean-field (RMF)
- phenomenological medium modifications of nuclei/nuclear clusters

MH, J. Schaffner-Bielich; NPA 837 (2010) (HS)



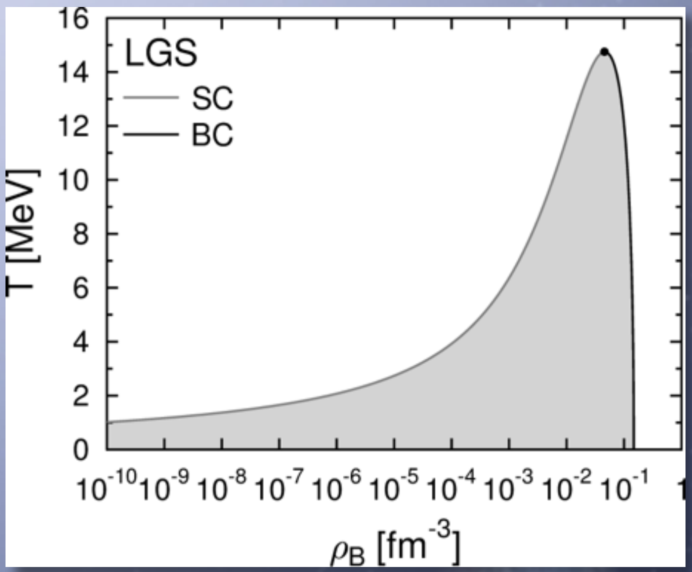
Nuclei in supernova matter



- analogy to liquid-gas phase diagram
- dissolution of nuclei at low densities and large T
- fraction of nuclei increases with Y_p
- transition to uniform nuclear matter at ~½ n_0
- almost no first order phase transition any more (not shown here)

light nuclei ($Z < 5$)

heavy nuclei ($Z \geq 6$)



Constraining supernova EOS with equilibrium constants from heavy-ion collisions

Phys. Rev. C 91, 045805 (2015)

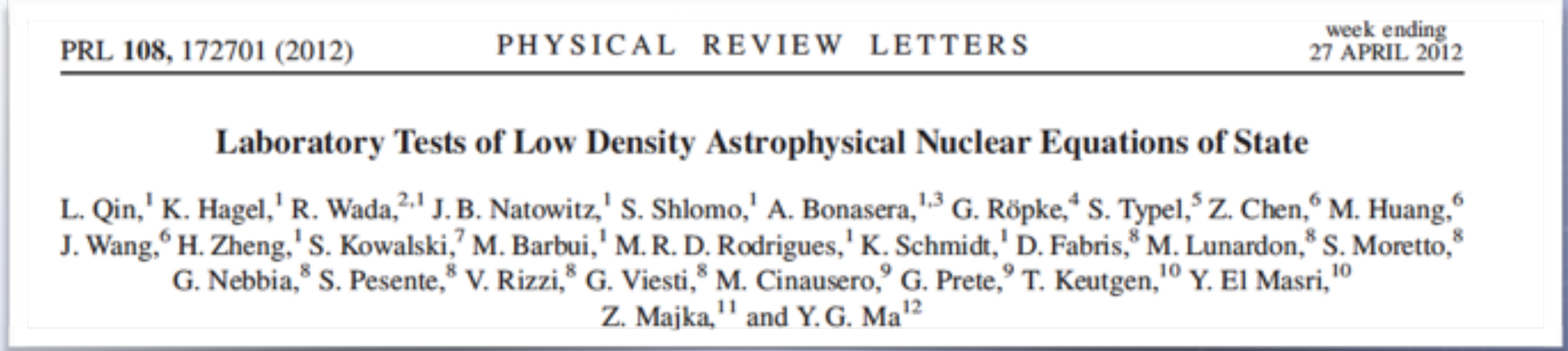
in collaboration with:
Gerd Röpke, Stefan Typel, Joe Natowitz, Kris Hagel,
Kohsuke Sumiyoshi, Shun Furusawa

Light cluster in supernova matter

- can be more abundant than protons (cf. Sumiyoshi & Röpke 2008)
- modify cooling of proto-neutron star
- relevant for nucleosynthesis in neutrino-driven winds (Arcones et al. 2008)
- supernova dynamics (?)
- see: O'Connor et al. 2007, Sumiyoshi and Röpke 2008, Arcones et al. 2008, Hempel et al. 2012, Furusawa et a. 2013, …

- EOS: various different theoretical approaches

cluster formation can be probed in the lab, „femtonova“



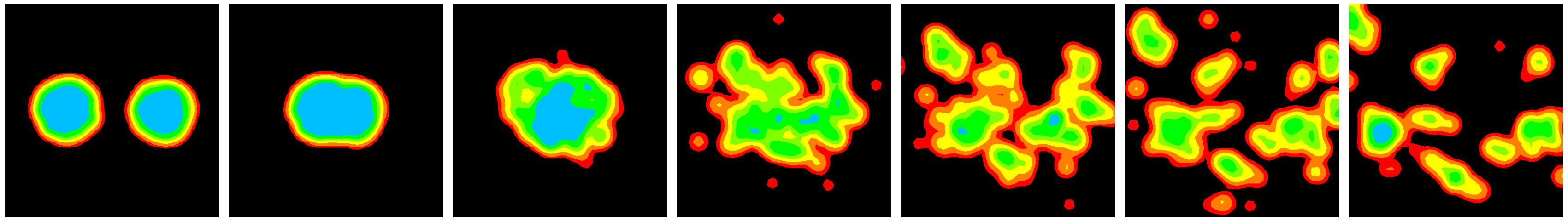
PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Original work of Qin et al.

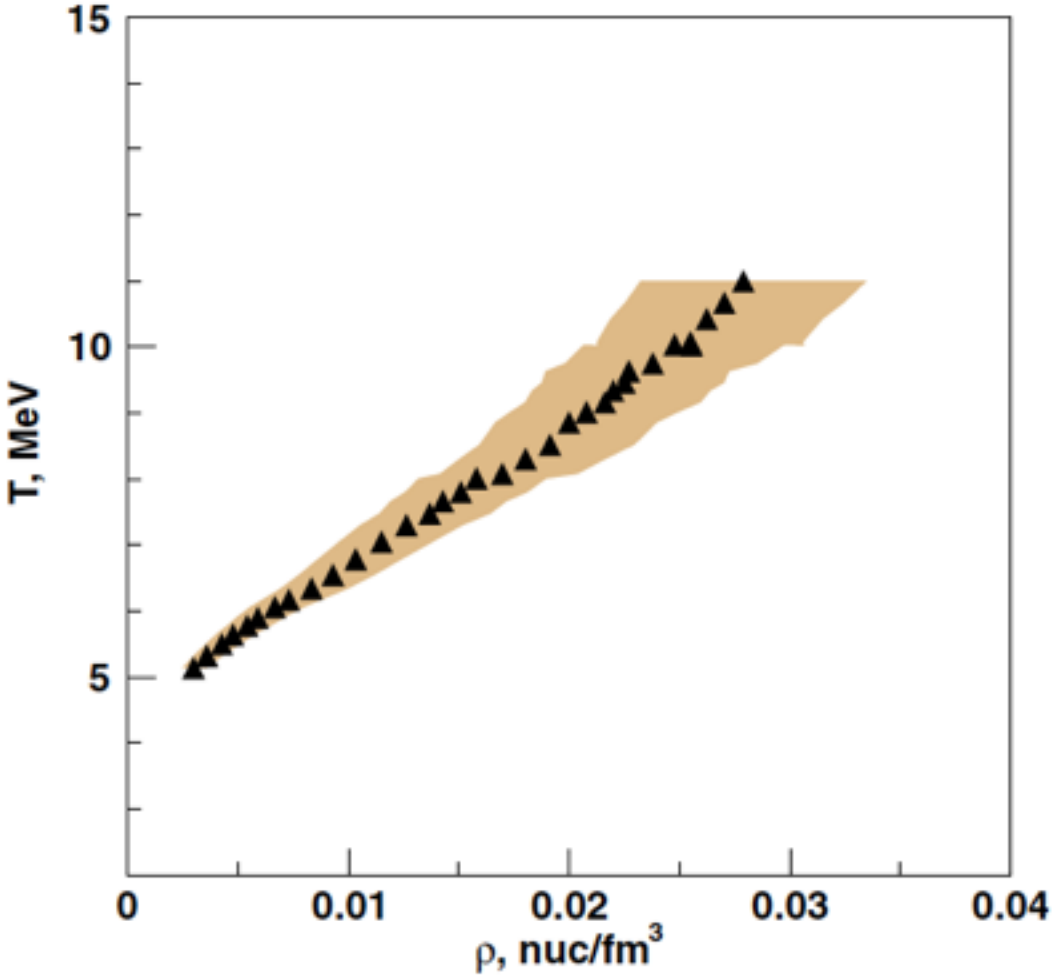
- Qin et al. PRL108 (2012): measured charged particle and neutron yields at Texas A&M with low-energy heavy ion collisions



Akira Ono An event of central collision of Xe + Sn at 50 MeV/nucleon (AMD calculation)

- density extraction: thermal coalescence model of Mekjian
- temperature: double isotope yield ratios

- conditions similar as in core-collapse supernovae



Basic aspects of equilibrium constants

- primary observable used by Qin et al.: equilibrium constant
- defined by particle yields or number densities

$$K_c[i] = \frac{n_i}{n_n^{N_i} n_p^{Z_i}}$$

- ideal gas:

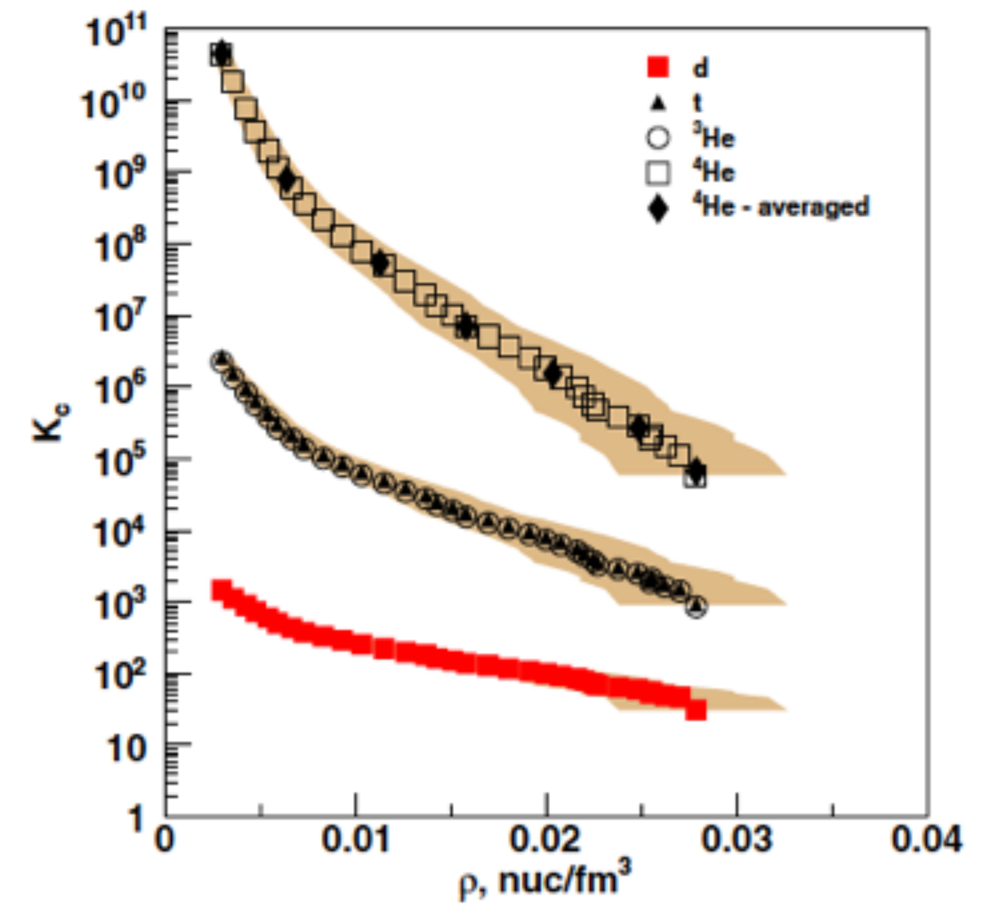
$$n_i^{\mathrm{id}}(T,\mu_i) = g_i \left(\frac{M_i T}{2\pi}\right)^{3/2} \exp\left(\frac{1}{T}(\mu_i - M_i)\right)$$

- using nuclear statistical equilibrium

$$\mu_i = N_i \mu_n + Z_i \mu_p$$

- K_c^{id} only a function of temperature, e.g.:

$$K_c^{\mathrm{id}}[\alpha] = \frac{1}{2}\left(\frac{2\pi}{mT}\right)^{9/2} \exp\left(\frac{B_\alpha}{T}\right)$$



- thus also „composition independent“ (Qin et al. 2012)
- not true for an non-ideal (i.e. interacting) system

Comparison of conditions in SN and HIC

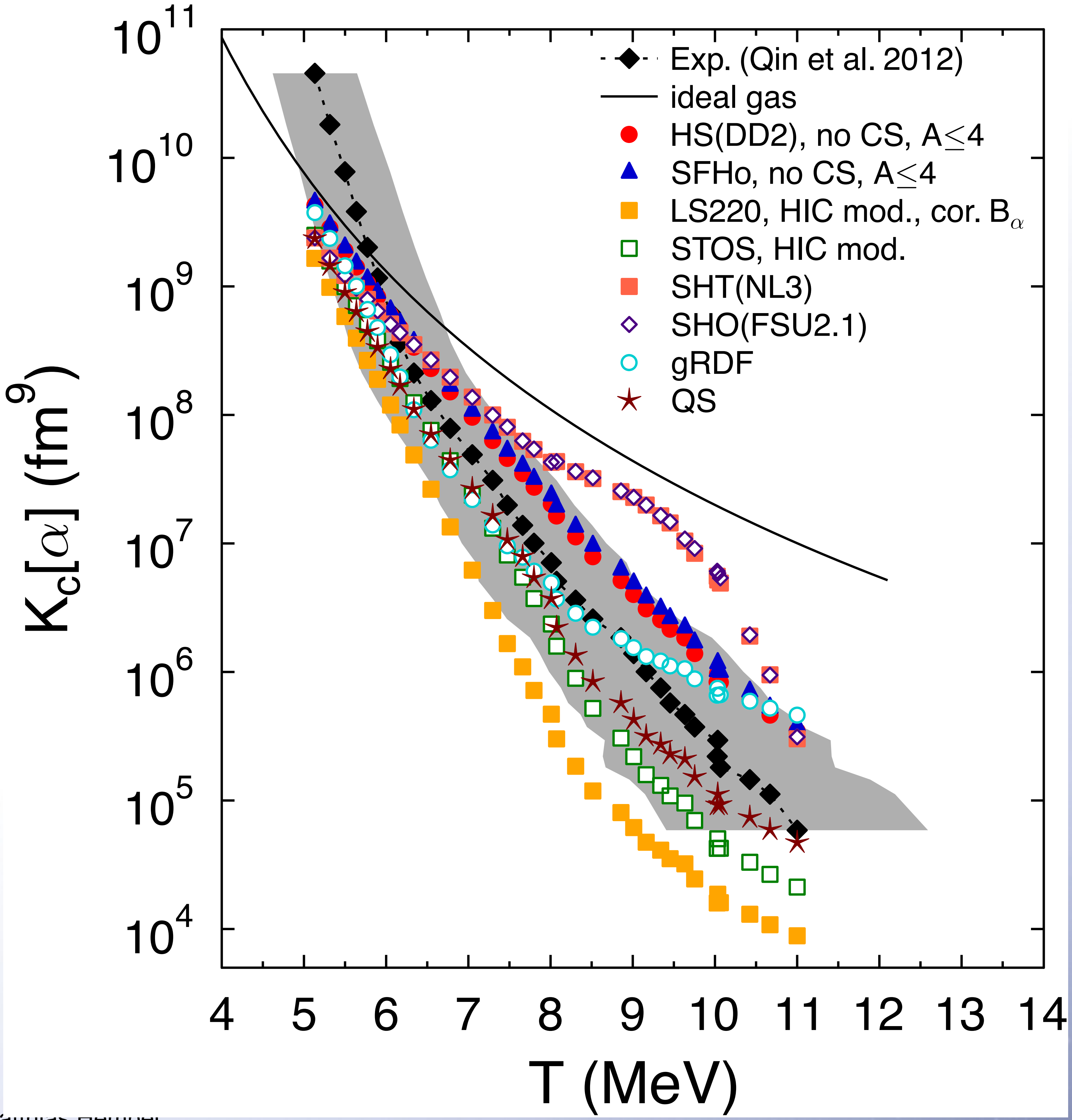
	supernovae	heavy ion collisions
dynamic timescales	ms	fm/c
equilibrium	weak eq. only partly	only strong eq.
temperatures	0 - 100 MeV	10 - 200 MeV
charge neutrality	yes	no
asymmetry	moderate	low
highest densities	$< 2\text{-}4\ \rho_0$	$< 4\text{-}5\ \rho_0$

relevant systematic differences between matter in heavy-ion collisions and supernovae

- isospin asymmetry; in the experiment: Y_Q~0.41
- Coulomb interactions/charge neutrality
- limited number of participating nucleons

Constraining cluster formation in SN EOS

[MH, Hagel, Natowitz, Röpke, Typel, PRC 91, 045805 (2015)]



- comparison with various theoretical approaches, all modified to account for differences SN ↔ HIC
- ideal gas behavior ruled out, nuclear clusters encounter medium modification (Pauli blocking)
- see also: Natowitz et al. PRL 2010, Typel et al. PRC 2010

Conclusions

- phase transitions with more than one globally conserved charge are generally *non-congruent*
- → different dimensionality of phase coexistence regions (e.g. T-P or T-μ)

- principle difference between liquid-gas and QCD PT: *enthalpic* vs. *entropic*

- in supernova matter phase transitions are „quenched“/replaced by cluster formation; partly due to charge neutrality and Coulomb interactions

- cluster formation in the supernova EOS can be constrained with low-energy heavy-ion collisions
- systematic differences of matter in heavy-ion collisions and supernovae are important, but can be taken into account