An Introduction to the Thermal Model.
I. Phenomenology
II. Theory

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Dense Matter, 29 June - 11 July 2015
Dubna, JINR

Outline

I. Phenomenology

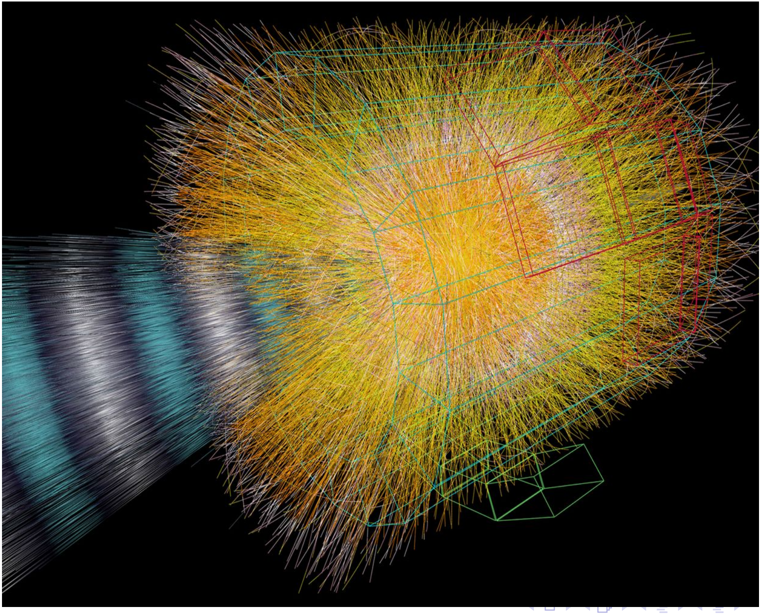
Heavy Ion Collisons at the LHC

Thermal Model

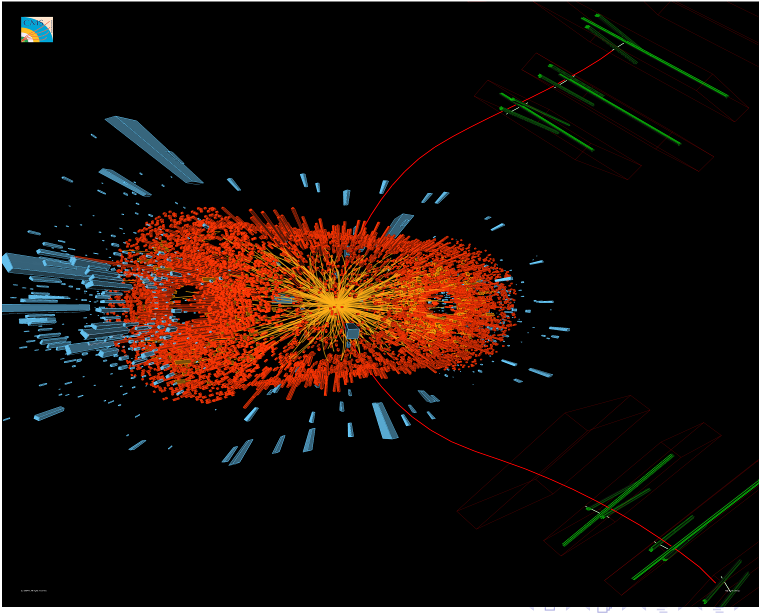
Hagedorn temperature

Heavy Ion Collisions at NICA/FAIR

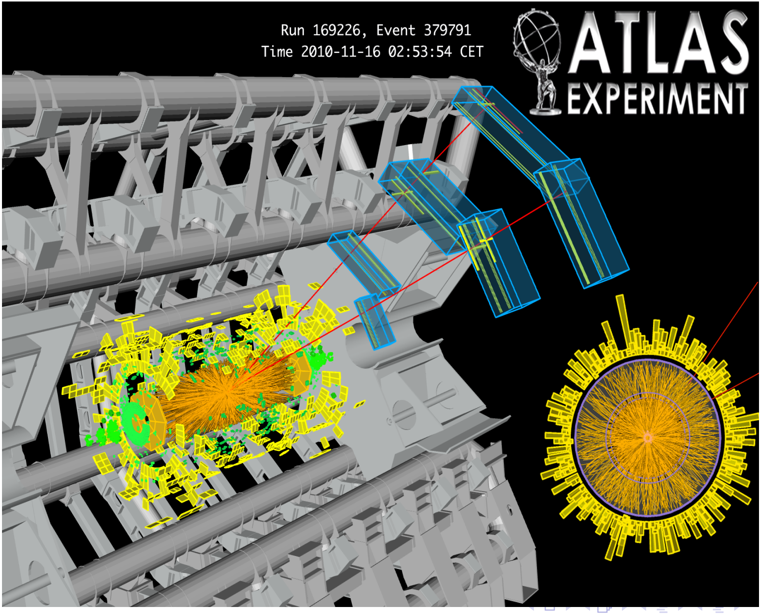
Heavy Ion Collisions in ALICE



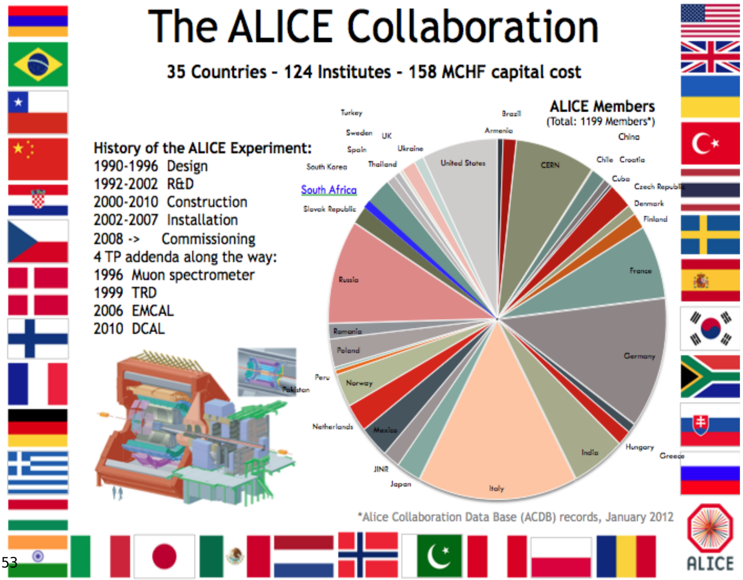
Heavy Ion Collisions in CMS



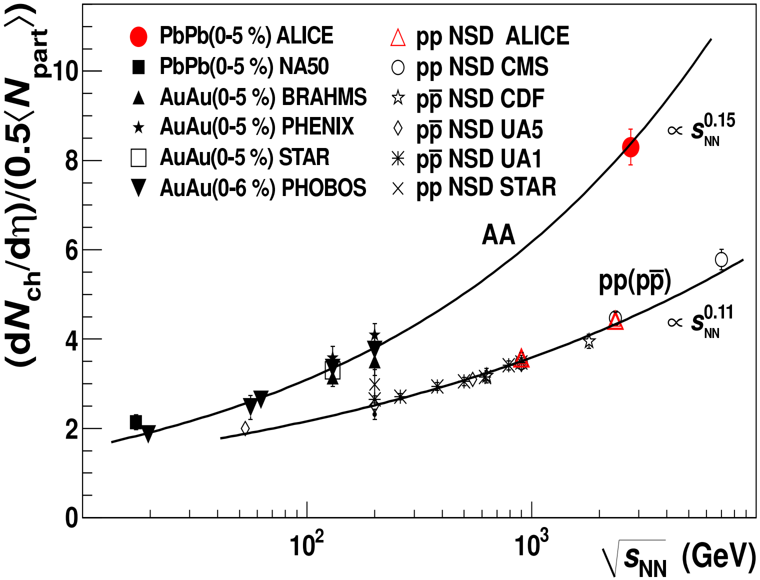
Heavy Ion Collisions in ATLAS



South Africa



Particle Multiplicity in Heavy Ion Collisions

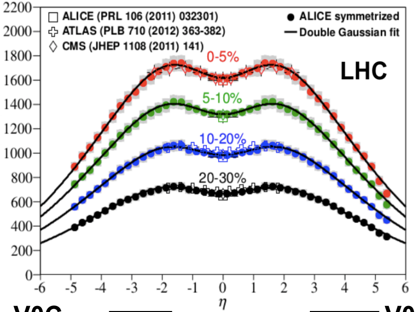


Particle Multiplicity in Heavy Ion Collisions

Acceptance for charged particles

ALICE

η coverages for $z_{vtx}=0$ (shown at last AW)



Now:
(T0 now shown)

V0C **V0A**
SPD (outer layer)
FMDC **FMDA**

After LS2:

T0C+ (ext) **T0A+** **(but not hermetic: 80%)**
V0A+ (ext)
MFT **ITS IB (middle layer)**

This is (-3.6,-2.5), i.e. the MFT+MUON acc.

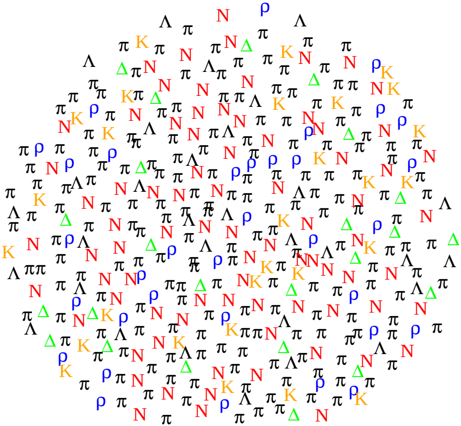
SQM DUBNA 2015

Particle Multiplicity in Heavy Ion Collisions

About 16 000 particles are produced in a heavy ion collision.

Hence: Use Concepts from Statistical Mechanics to analyze the final state
e.g. use Energy Density, Particle Density, Pressure, Temperature, Chemical Composition, ...

Hadronic Gas before Chemical Freeze-Out



J.C. and H. Satz, Z. fuer Physik C57, 135, 1993.

Thermal Equilibrium

In thermal equilibrium

$$Z = \mathrm{Tr}\; e^{-\frac{H}{T} + \frac{\mu N}{T}}$$

$$\langle N \rangle = \frac{\mathrm{Tr}\; N e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\; e^{-\frac{H}{T} + \frac{\mu N}{T}}}$$

$$\langle E \rangle = \frac{\mathrm{Tr}\; E e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\; e^{\frac{-H}{T} + \frac{\mu N}{T}}}$$

Thermal Equilibrium

Particle Number

$$
\begin{aligned}
\langle N \rangle &= \frac{\mathrm{Tr}\, N e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}} \\
&= \frac{T}{Z}\frac{\partial}{\partial \mu}\mathrm{Tr}\, e^{\frac{-H}{T} + \frac{\mu N}{T}} \\
&= T\frac{1}{Z}\frac{\partial Z}{\partial \mu} \\
&= T\frac{\partial}{\partial \mu}\ln Z
\end{aligned}
$$

Thermal Equilibrium

Average Energy

$$\langle E \rangle = \frac{\mathrm{Tr}\, H\, e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{\frac{-H}{T} + \frac{\mu N}{T}}} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} + \mu \langle N \rangle = T^2 \frac{\partial}{\partial T} \ln Z + \mu \langle N \rangle$$

Thermal Equilibrium

$$
\begin{aligned}
N_i &= g_i \, V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T}\right) e^{\frac{\mu_i}{T}} \\
&= g_i \, V \frac{4\pi}{(2\pi)^3} \int p^2 \, dp \exp\left(-\frac{\sqrt{p^2 + m_i^2}}{T}\right) e^{\frac{\mu_i}{T}} \\
&= g_i \, V \frac{4\pi}{(2\pi)^3} T^3 \int x^2 \, dx \exp\left(-\sqrt{x^2 + m_i^2 / T^2}\right) e^{\frac{\mu_i}{T}} \\
&= g_i \, V \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
\end{aligned}
$$

Full Hydrodynamic Flow

Bjorken scaling + Transverse expansion

After integration over m_T

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0}$$

where N_i^0 is the particle yield
as calculated in a fireball **AT REST!**

Effects of hydrodynamic flow cancel out in ratios.

Thermal Equilibrium

$$
\begin{aligned}
n_i &= g_i \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} \\
\epsilon_i &= g_i \frac{1}{2\pi^2} T m_i^3 \left[K_1\left(\frac{m_i}{T}\right) + 3\frac{T}{m} K_2\left(\frac{m_i}{T}\right)\right] e^{\frac{\mu_i}{T}} \\
s_i &= g_i \frac{1}{2\pi^2} m_i^3 \left[K_1\left(\frac{m_i}{T}\right) + \frac{4T}{m} K_2\left(\frac{m_i}{T}\right) - \frac{\mu_i}{m} K_2\left(\frac{m_i}{T}\right)\right] e^{\frac{\mu_i}{T}} \\
P_i &= g_i \frac{1}{2\pi^2} T^2 m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
\end{aligned}
$$

Chemical Equilibrium

In equilibrium

$$E_1 + E_2 + \cdots = E_3 + E_4 + E_5 + \cdots \tag{1}$$

for the chemical potentials

$$\mu_1 + \mu_2 + \cdots = \mu_3 + \mu_4 + \mu_5 + \cdots \tag{2}$$

As an example

$$\pi^0 + p \leftrightarrow \pi_0 + p + \pi^0 \tag{3}$$

leads to

$$\mu_{\pi^0} + \mu_p = \mu_{\pi^0} + \mu_p + \mu_{\pi^0} \tag{4}$$

which leads to

$$\mu_{\pi^0} = 0 \tag{5}$$

Chemical Equilibrium

In equilibrium

$$B + B \leftrightarrow B + B + B + \bar{B} \tag{6}$$

$$dE = -pdV + TdS + \mu_B dN_B + \mu_{\bar{B}} dN_{\bar{B}}$$

Due to baryon number conservation one has

$$N_B - N_{\bar{B}} = \text{constant}$$

and

$$dN_B = dN_{\bar{B}}$$

The energy is a minimum for

$$dE = (\mu_B + \mu_{\bar{B}})dN_B = 0 \tag{7}$$

$$\mu_B = -\mu_{\bar{B}} \tag{8}$$

Chemical Equilibrium

In equilibrium

$$N_B = g\ V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T} + \frac{\mu_B}{T}\right)$$
$$N_{\bar{B}} = g\ V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T} - \frac{\mu_B}{T}\right)$$

$$\begin{aligned} N_B &= N_{\bar{B}} \rightarrow \mu_B = 0 \\ N_B &\geq N_{\bar{B}} \rightarrow \mu_B \geq 0 \\ N_B &\leq N_{\bar{B}} \rightarrow \mu_B \leq 0 \end{aligned}$$

	Chemical Equilibrium	No Chem. Equil.
π	$\exp\left[-\frac{E_\pi}{T}\right]$	$\exp\left[-\frac{E_\pi}{T}+\frac{\mu_\pi}{T}\right]$
N	$\exp\left[-\frac{E_N}{T}+\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_N}{T}\right]$
$\overline{N}$	$\exp\left[-\frac{E_N}{T}-\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_{\overline{N}}}{T}\right]$
Λ	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_B}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_\Lambda}{T}\right]$
$\overline{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T}-\frac{\mu_B}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_{\overline{\Lambda}}}{T}\right]$
K	$\exp\left[-\frac{E_K}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_K}{T}\right]$
$\overline{K}$	$\exp\left[-\frac{E_K}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_{\overline{K}}}{T}\right]$

The number of particles of type *i* is determined by:

$$\boxed{N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)}$$

For bosons:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) - 1}$$

For fermions:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) + 1}$$

Chemical Equilibrium

Only conserved quantum numbers matter for chemical equilibrium: In equilibrium

$$\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S + C_i\mu_C + .. \qquad (9)$$

g_i	m_i	stat	S_i	B_i	Q_i	Particle i
1	0.140	-1	0	0	1.	π^+
1	0.135	-1	0	0	0.	π^0
1	0.140	-1	0	0	-1.	π^-
1	0.547	-1	0	0	0.	η
3	0.770	-1	0	0	1.	ρ^+
3	0.770	-1	0	0	0.	ρ^0
3	0.770	-1	0	0	-1.	ρ^-
3	0.782	-1	0	0	0.	ω
1	0.958	-1	0	0	0.	η'
1	0.980	-1	0	0	0.	f_0
1	0.982	-1	0	0	1.	a_0^+
1	0.982	-1	0	0	0.	a_0^0
1	0.982	-1	0	0	-1.	a_0^-
3	1.019	-1	0	0	0.	ϕ
3	1.170	-1	0	0	0.	
3	1.230	-1	0	0	1.	
3	1.230	-1	0	0	0.	
3	1.230	-1	0	0	-1.	
3	1.229	-1	0	0	1.	
3	1.229	-1	0	0	0.	
3	1.229	-1	0	0	-1.	
5	1.275	-1	0	0	0.	
3	1.282	-1	0	0	0.	
1	1.297	-1	0	0	0.	
1	1.300	-1	0	0	1.	
1	1.300	-1	0	0	0.	

The Role of Resonances

Example: ρ's

$$\rho \rightarrow \pi^+\pi^-$$

Final, observed, number of π^+ is given by

$$N_{\pi^+} = N_{\pi^+}(\text{thermal}) + N_{\pi^+}(\text{resonance decays})$$

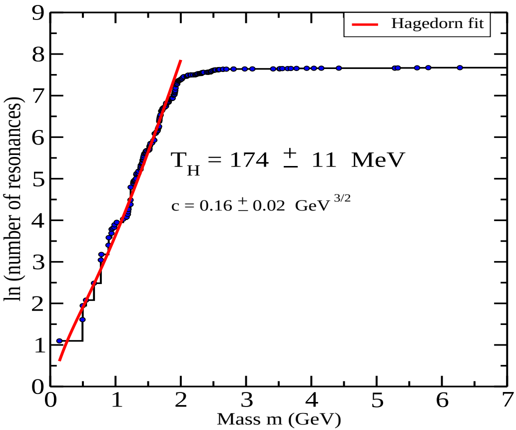
depending on the temperature, over 80% of observed pions are due to resonance decays

The Role of Resonances

Resonances are very important in the thermal model.

Without resonances the thermal model doesn't work.

THE HAGEDORN TEMPERATURE.



Keep on adding the number of hadronic resonances.

J.C. and Dawit Worku, Mod. Phys. Lett. A26 (2011) 1197; arXiv: 1103.1463

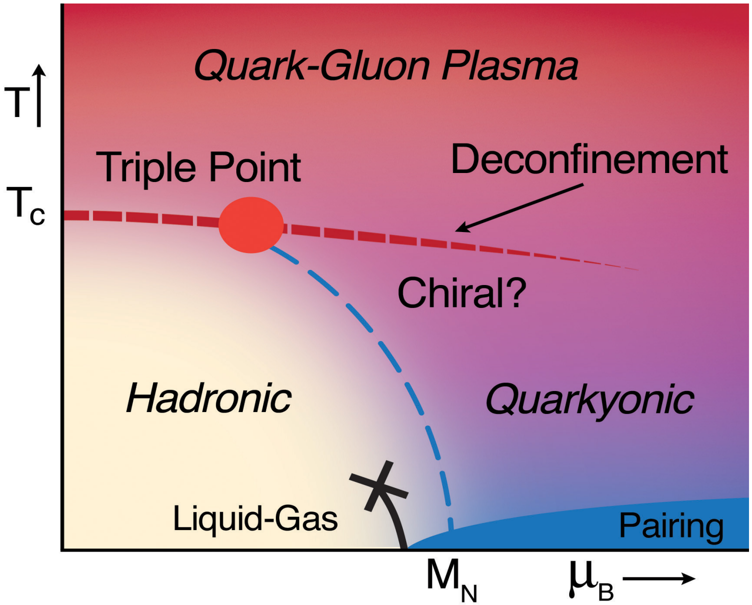
HADRONS DO NOT EXIST ABOVE THE HAGEDORN TEMPERATURE.

Thermodynamic quantities like particle density, energy density, pressure ,... all involve a summation over hadron species:

$$\sum_i \exp -\frac{E_i}{T}$$

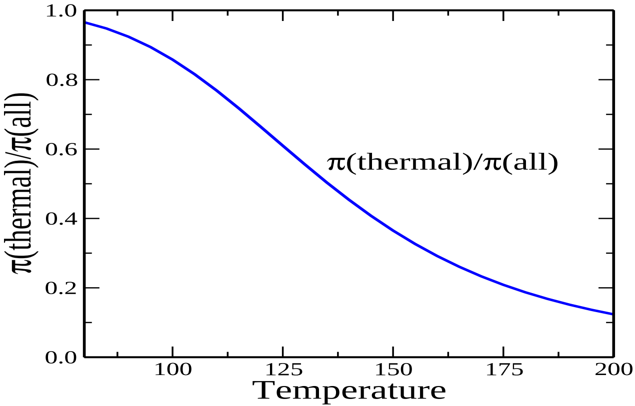
and the sum becomes (too) large due to the number of resonances.

Phase Diagram

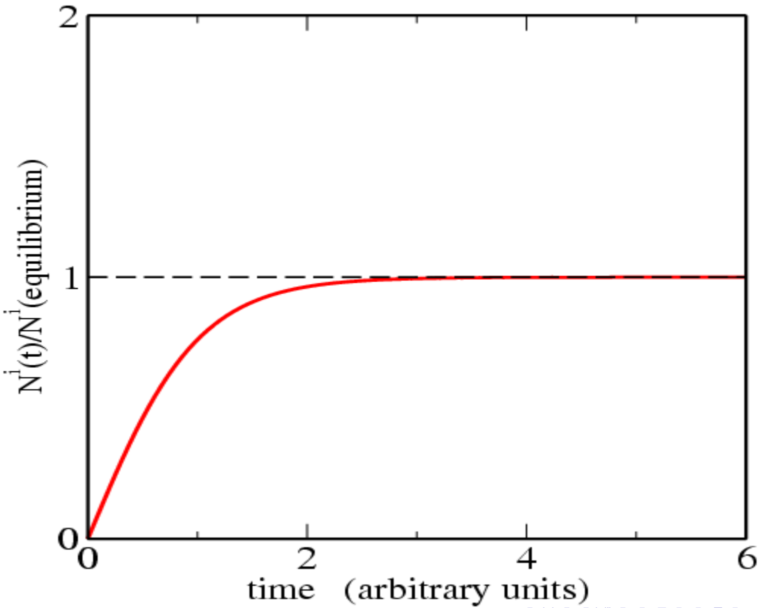


g_i	m_i	stat	S_i	B_i	Q_i	BR$\rightarrow \pi^+$	Particle i
1	0.140	-1	0	0	1.	1.000	π^+
1	0.135	-1	0	0	0.	0.000	π^0
1	0.140	-1	0	0	-1.	0.000	π^-
1	0.547	-1	0	0	0.	0.285	η
3	0.770	-1	0	0	1.	1.000	ρ^+
3	0.770	-1	0	0	0.	1.000	ρ^0
3	0.770	-1	0	0	-1.	0.000	ρ^-
3	0.782	-1	0	0	0.	0.910	ω
1	0.958	-1	0	0	0.	0.965	η'
1	0.980	-1	0	0	0.	0.521	f_0
1	0.982	-1	0	0	1.	1.285	a_0^+
1	0.982	-1	0	0	0.	0.285	a_0^0
1	0.982	-1	0	0	-1.	0.285	a_0^-
3	1.019	-1	0	0	0.	0.155	ϕ
3	1.170	-1	0	0	0.	1.000	h_1
3	1.230	-1	0	0	1.	1.500	
3	1.230	-1	0	0	0.	0.50	
3	1.230	-1	0	0	-1.	0.50	
3	1.229	-1	0	0	1.	1.91	
3	1.229	-1	0	0	0.	0.91	
3	1.229	-1	0	0	-1.	0.91	
5	1.275	-1	0	0	0.	0.69	
3	1.282	-1	0	0	0.	1.00	
1	1.297	-1	0	0	0.	1.11	
1	1.300	-1	0	0	1.	2.00	
1	1.300	-1	0	0	0.	1.50	

Importance of Resonances.



Strangeness saturation?



Strangeness saturation?

$$N_i = \boxed{\gamma_s^{|S|}} V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)$$

with

$\gamma_s < 1$ strangeness under-saturation

$\gamma_s = 1$ strangeness in chemical equilibrium

$\gamma_s > 1$ strangeness over-saturation

SPS data.

	Measurement
Pb–Pb 158A GeV	
$(\pi^+ + \pi^-)/2.$	600±30
K^+	95 ±10
K^-	50 ±5
K^0_S	60 ±12
p	140±12
$\bar{p}$	10 ±1.7
ϕ	7.6±1.1
Ξ^-	4.42±0.31
$\overline{\Xi^-}$	0.74±0.04
$\bar{\Lambda}/\Lambda$	0.2±0.04

SPS data.

SPS: Chemical Freeze-Out Parameters:

$$
\begin{aligned}
T &= 156.0 \pm 2.4\mathrm{MeV} \\
\mu_B &= 239 \pm 12\mathrm{MeV} \\
\gamma_s &= 0.862 \pm 0.036
\end{aligned}
$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

AGS data.

	Measurement
Au–Au 11.6*A* GeV	
Participants	363±10
K^+	23.7±2.9
K^-	3.76±0.47
π^+	133.7±9.9
Λ	20.34±2.74
p/π^+	1.234±0.126
$\bar{p}$	$>$0.0185±0.0018

AGS data.

AGS: Chemical Freeze-Out Parameters:

$$
\begin{aligned}
T &= 130.6 \pm 5.5\mathrm{MeV} \\
\mu_B &= 594 \pm 26\mathrm{MeV} \\
\gamma_s &= 0.883 \pm 0.124
\end{aligned}
$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

SIS data.

	Measurement
Au–Au 1.7*A* GeV	
π^+/p	0.052±0.013
K$^+/\pi^+$	0.003±0.00075
π^-/π^+	2.05±0.51
η/π^0	0.018±0.007

SIS data.

SIS: Chemical Freeze-Out Parameters:

$$
\begin{aligned}
T &= 49.7 \pm 1.1 \mathrm{MeV} \\
\mu_B &= 818 \pm 15 \mathrm{MeV} \\
\gamma_s &= 1 \ (\mathrm{fixed})
\end{aligned}
$$

J. C., H. Oeschler and K. Redlich)
Physical Review C59, (1999) 1663.

RHIC data.

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu, Phys. Rev. C71, 0409071 (2005)

Ratio	Experiment	Central	Mid-Central	Peripheral
$\pi^-_{(2)}/\pi^+_{(2)}$	BRAHMS	0.990±0.100		
	PHENIX	0.960±0.177	0.920±0.170	0.933±0.172
	PHOBOS	1.000±0.022		
	STAR	1.000±0.073	1.000±0.073	1.000 ± 0.073
$K^+_{(2)}/K^-_{(2)}$	PHENIX	1.152±0.240	1.292±0.268	1.322±0.284
	PHOBOS	1.099±0.111		
	STAR	1.109±0.022	1.105±0.036	1.120±0.040
$\bar{p}_{(1)}/p_{(1)}$	PHENIX	0.680±0.149	0.671±0.142	0.717±0.157
$\bar{p}_{(2)}/p_{(2)}$	BRAHMS	0.650±0.092		
	PHOBOS	0.600±0.072		
	STAR	0.714±0.050	0.724±0.050	0.764±0.053
$\bar{\Lambda}_{(1)}/\Lambda_{(1)}$	PHENIX	0.750±0.180	0.798±0.197	0.795±0.197
$\bar{\Lambda}_{(2)}/\Lambda_{(2)}$	STAR	0.719±0.090	0.739±0.092	0.744±0.100
$\bar{\Xi}^+_{(2)}/\Xi^-_{(2)}$	STAR	0.840±0.053	0.822±0.114	0.815±0.096
$\bar{\Omega}^+/\Omega^-$	STAR	1.062±0.410		
$K^-_{(2)}/\pi^-_{(2)}$	PHENIX	0.151±0.030	0.134±0.027	0.116±0.023
	STAR	0.151±0.022	0.147±0.022	0.130±0.019
$K^0_S/\pi^-_{(2)}$	STAR	0.134±0.022	0.131±0.022	0.108±0.018
$\bar{p}_{(1)}/\pi^-_{(2)}$	PHENIX	0.049±0.010	0.047±0.010	0.045±0.009
$\bar{p}_{(2)}/\pi^-_{(2)}$	STAR	0.069±0.019	0.067±0.019	0.067±0.019
$\Lambda_{(1)}/\pi^-_{(2)}$	STAR	0.043±0.008	0.043±0.008	0.039±0.007
$\Lambda_{(2)}/\pi^-_{(2)}$	PHENIX	0.072±0.017	0.068±0.016	0.074±0.017
$< K^{*0} > /\pi^-_{(2)}$	STAR	0.039±0.011		
$\phi/\pi^-_{(2)}$	STAR	0.022±0.003	0.021±0.004	0.022±0.004
$\Xi^-_{(2)}/\pi^-_{(2)}$	STAR	0.0093±0.0012	0.0072±0.0011	0.0060±0.0008

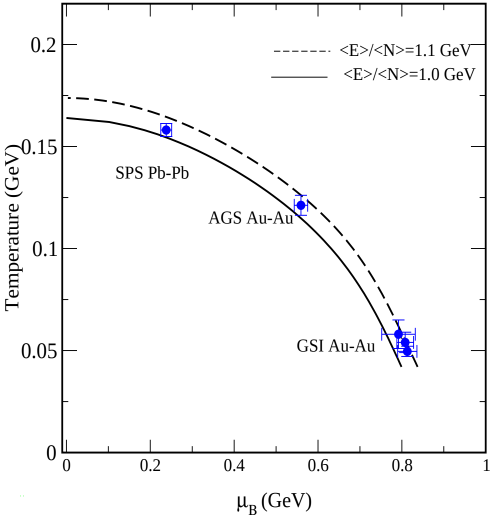
RHIC data.

RHIC: Chemical Freeze-Out Parameters:

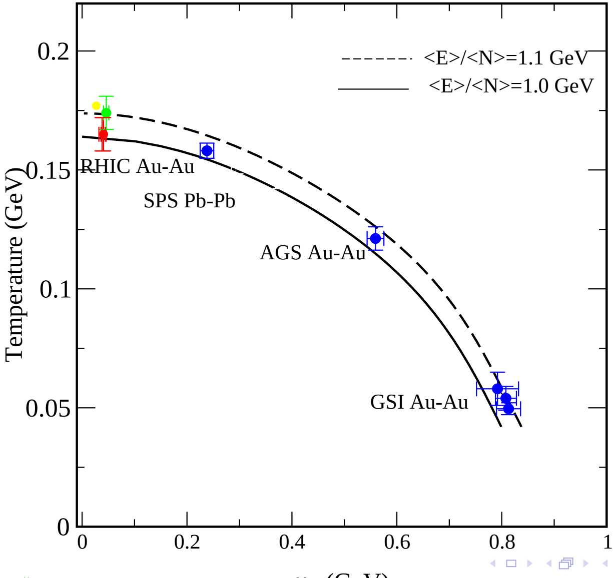
$$
\begin{aligned}
T &= 169 \pm 4.2 \mathrm{MeV} \\
\mu_B &= 39.6 \pm 6 \mathrm{MeV} \\
\gamma_s &= 0.9 \pm 0.1
\end{aligned}
$$

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu
Phys. Rev. C71, 0409071 (2005)

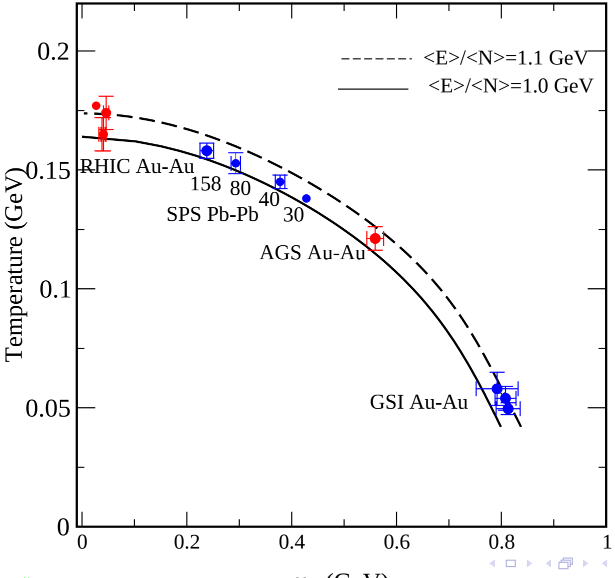
E/N in 1999


<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
μ_B (GeV)
0
0.05
0.1
0.15
0.2
0
0.2
0.4
0.6
0.8
1

E/N in 2000

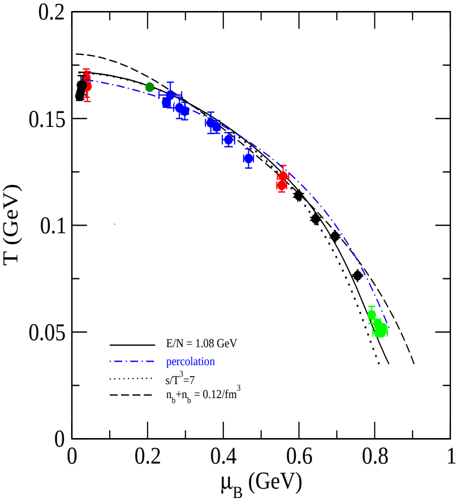

<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
RHIC Au-Au
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
0.2
0.15
0.1
0.05
0
0.2
0.4
0.6
0.8
1

E/N in 2005

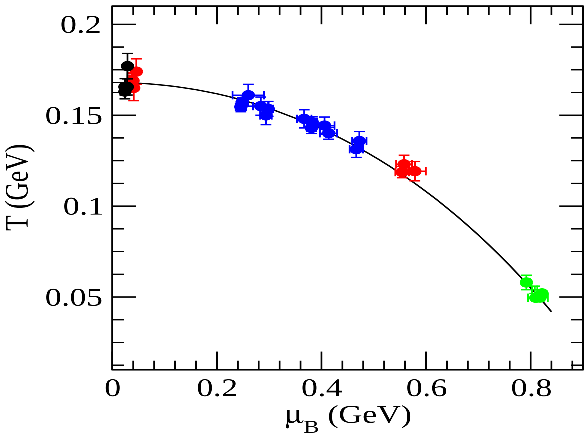


<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
RHIC Au-Au
158
80
40
30
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
0
0.05
0.1
0.15
0.2
0.2
0.4
0.6
0.8
1

Chemical Freeze-Out: Status in 2005



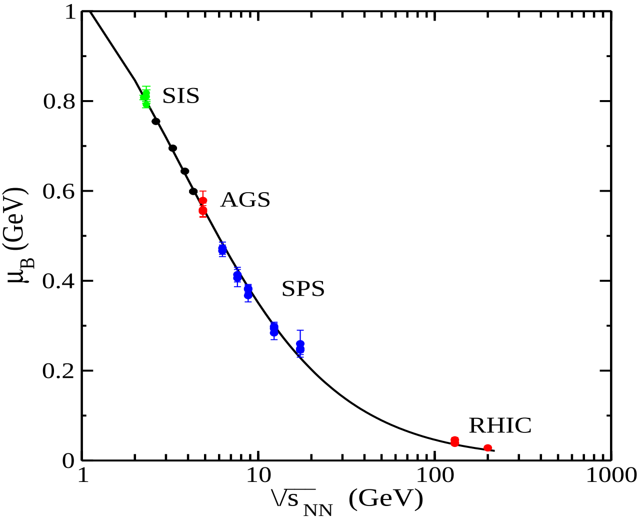
Chemical Freeze-Out: Status in 2005



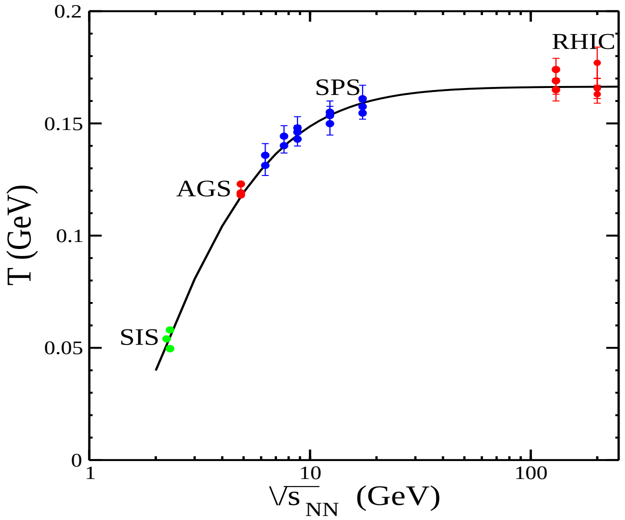
$$T = 0.166 - 0.139\,\mu_B^2 - 0.053\,\mu_B^4$$

J.C., H. Oeschler, K. Redlich, S. Wheaton, PR C73, 034905 (2006)

μ_B as a function of $\sqrt{s_{NN}}$

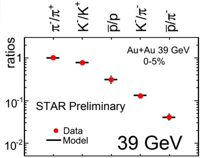


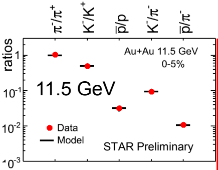
T as a function of $\sqrt{s_{NN}}$

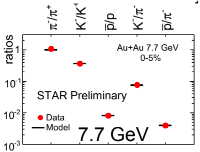


Freeze-out Conditions

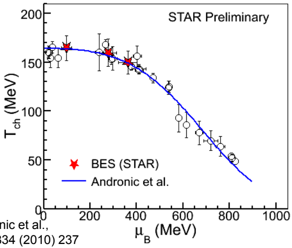
QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.





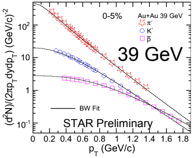


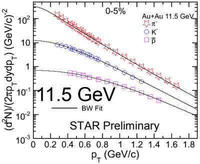
Chemical freeze-out:
Particle ratios

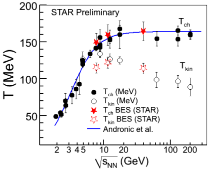


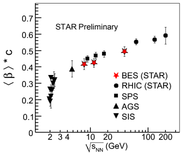
Andronic et al.,
NPA 834 (2010) 237

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.





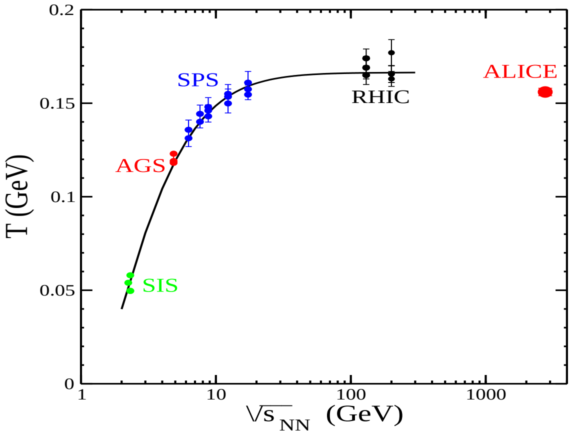




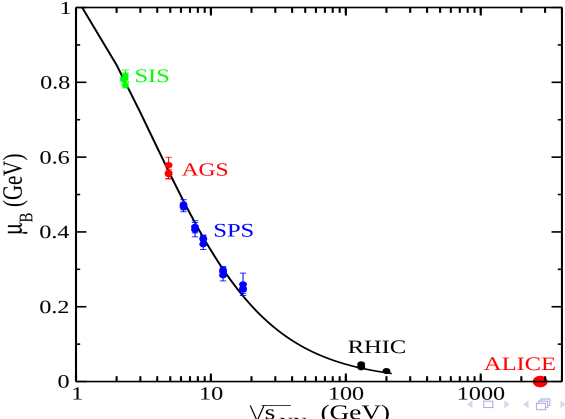
Kinetic freeze-out : Momentum distributions

L. Kumar, Energy scan, 27th May

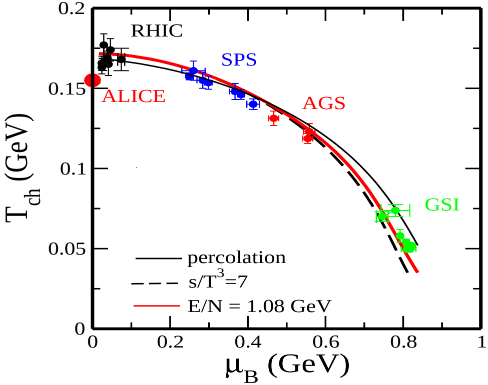
M. Floris: Nucl. Phys. A931, (2014) 103-112



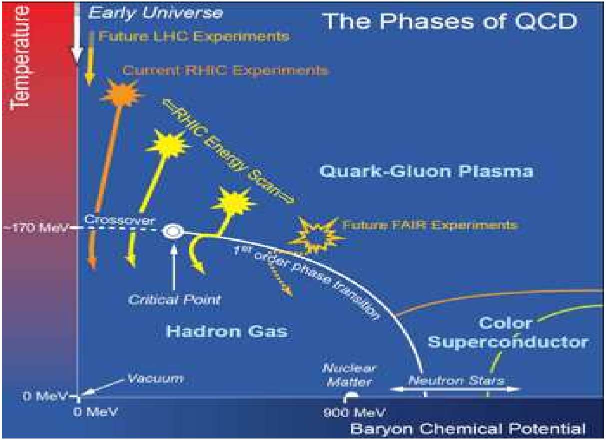
M. Floris: Nucl. Phys. A931, (2014) 103-112

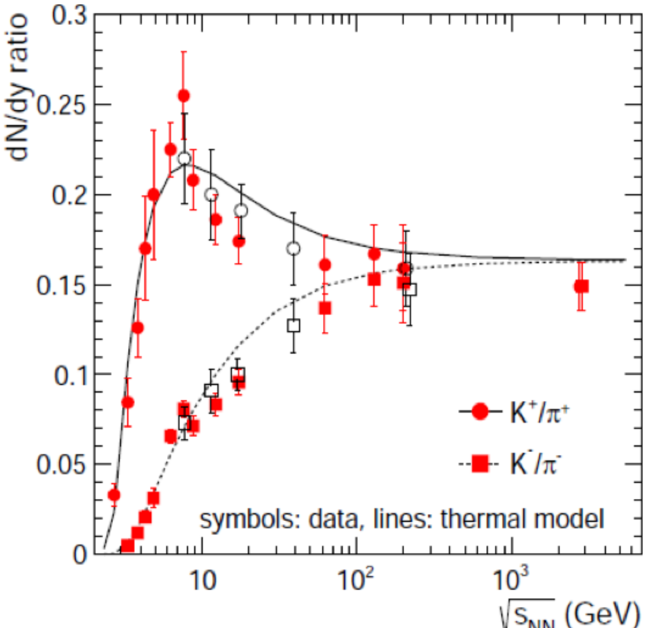


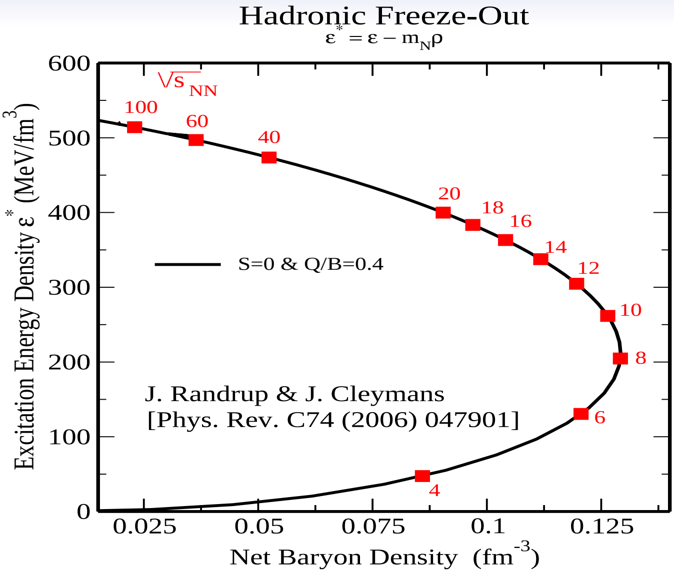
M. Floris: Nucl. Phys. A931, (2014) 103-112



Phase Diagram




dN/dy ratio
0.3
0.25
0.2
0.15
0.1
0.05
0
10
10²
10³
√sNN (GeV)
K⁺/π⁺
K⁻/π⁻
symbols: data, lines: thermal model


Hadronic Freeze-Out
ε* = ε − m_N ρ
√s_NN
100
60
40
20
18
16
14
12
10
8
6
4
S=0 & Q/B=0.4
J. Randrup & J. Cleymans
[Phys. Rev. C74 (2006) 047901]
Excitation Energy Density ε* (MeV/fm³)
Net Baryon Density (fm⁻³)
600
500
400
300
200
100
0
0.025
0.05
0.075
0.1
0.125

Strangeness in Heavy Ion Collisions
vs
Strangeness in pp - collisions

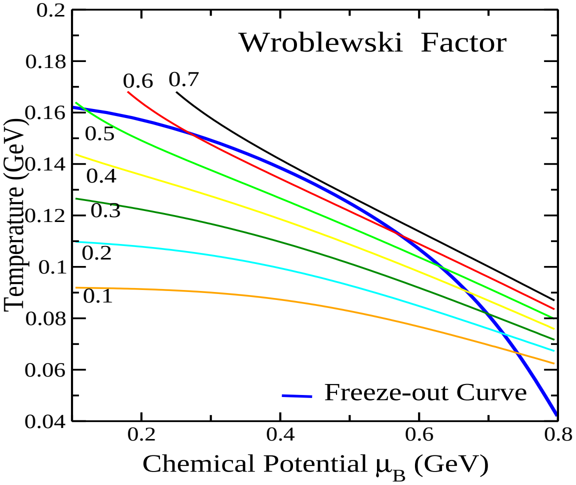
Use the Wroblewski factor

$$\lambda_s = \frac{2\left\langle s\bar{s}\right\rangle}{\left\langle u\bar{u}\right\rangle + \left\langle d\bar{d}\right\rangle}$$

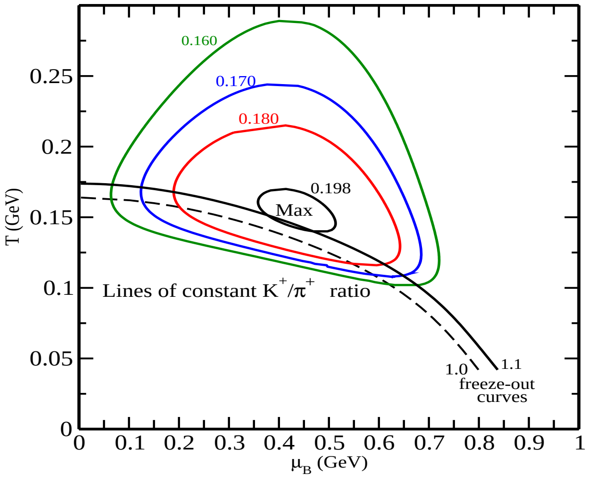
This is determined by the number of **newly** created quark – anti-quark pairs and **before** strong decays, i.e. before ρ's and Δ's decay.

Limiting values :
$\lambda_s = 1$ all quark pairs are equally abundant, SU(3) symmetry.
$\lambda_s = 0$ no strange quark pairs.

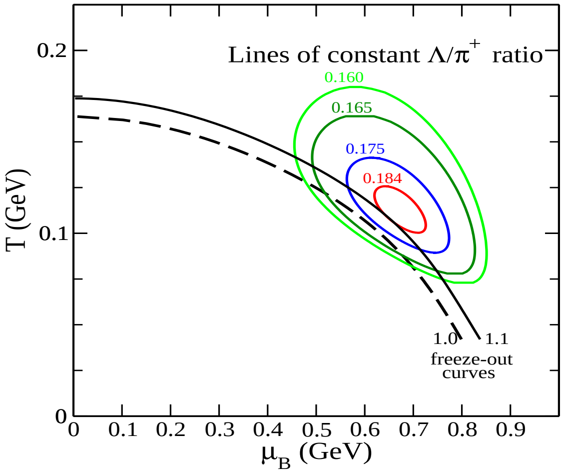
Maxima in particle ratios : Wroblewski factor

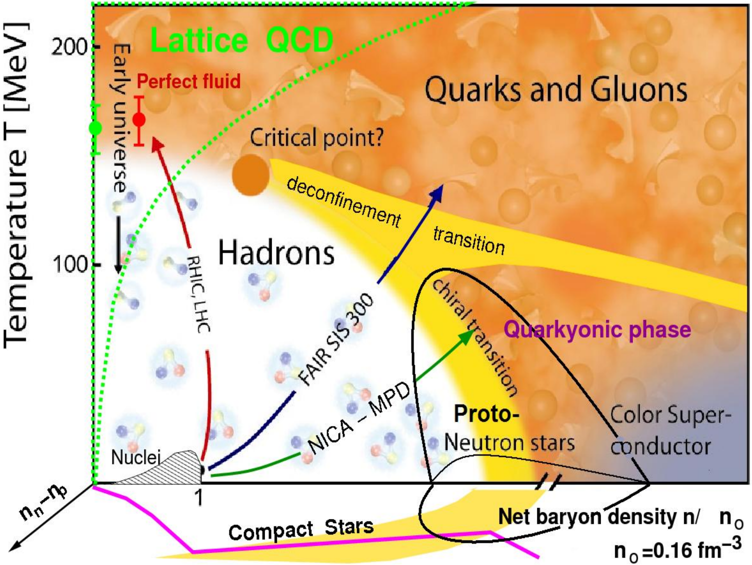


Maxima in particle ratios : K^+/π^+



Maxima in particle ratios : Λ/π^+




Temperature T [MeV]
200
100
Lattice QCD
Perfect fluid
Early universe
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
chiral transition
Quarkyonic phase
Proto-
Neutron stars
Color Super-
conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_0
$n_0 = 0.16\ fm^{-3}$

Good Luck NICA