Hadron production as Hawking-Unruh effect.1

Unruh effect and tunneling-Event horizon in QCD

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DUBNA

basic observation in all high energy multihadron production

thermal production pattern

Fermi, Landau, Pomeranchuk, Hagedorn

- species abundances ~ ideal resonance gas at T_H
- universal $T_H \simeq 165 \pm 15 Mev$ for all (large) $\sqrt{s}$

caveats

- strangeness suppression in elementary collisions
- strangeness suppression weakened/removed in nuclear collisions

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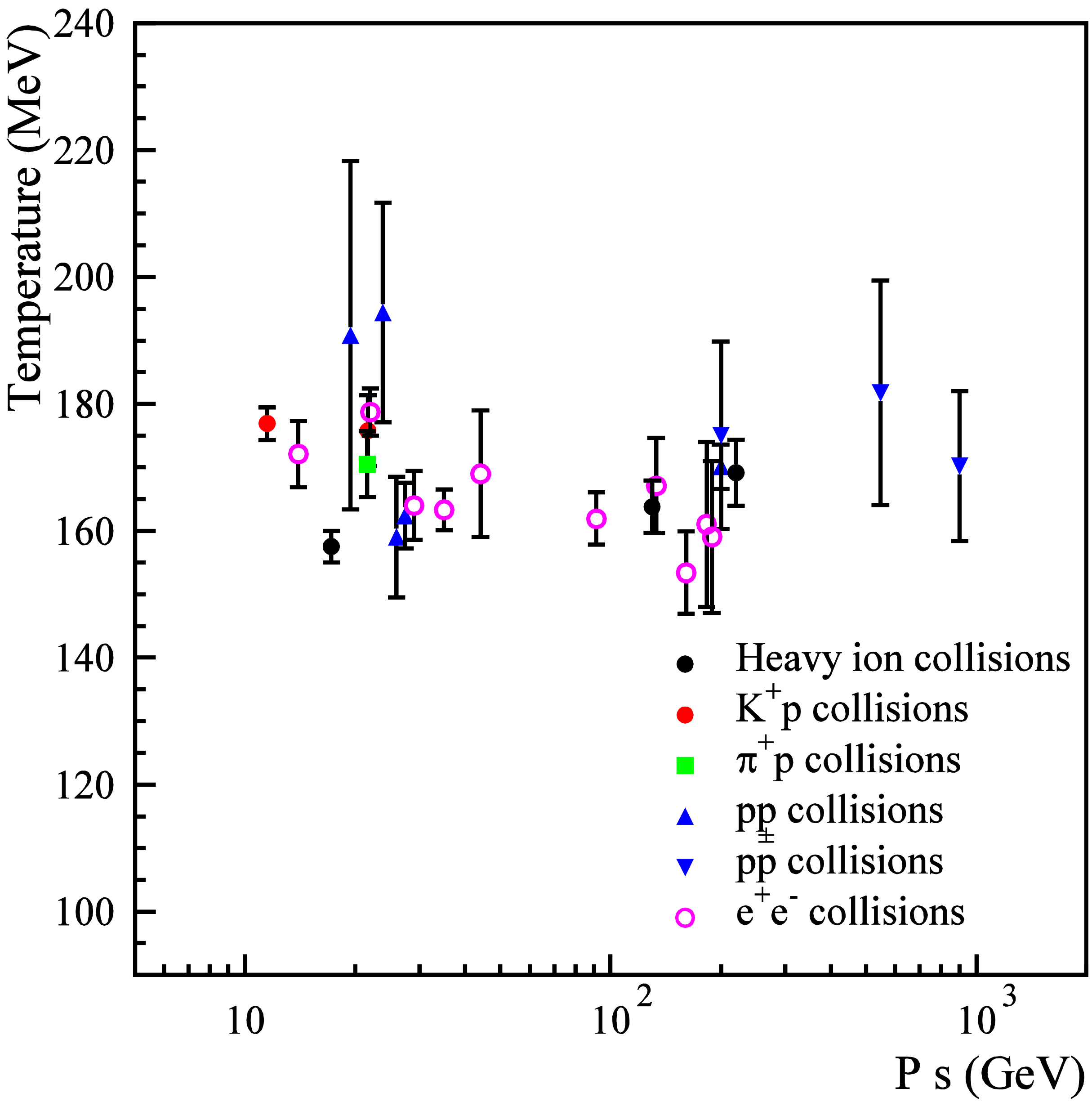
Statistical model



First Question

1) ***Why do elementary high energy collisions*** **show a statistical behavior?**

Is there another non-kinetic mechanism providing a common origin of the statistical features?

The Unruh effect

a conceptually subtle quantum field theory result

In short: the Unruh effect expresses the fact that uniformly accelerated observers in Minkowski spacetime, i.e. linearly accelerated observers with constant proper acceleration (also called Rindler observers), associate a thermal bath to the no-particle state of inertial observers (Minkowski vacuum).

$$T = \frac{\hbar a}{2\pi k c},$$

arXiv:0710.5373
The Unruh effect and its applications
Luis C. B. Crispino, Atsushi Higuchi, George E. A. Matsa

Prologo 1

$\Sigma'(X,Y,Z,T)$ Proper instantaneous r.f. $\Sigma(x,y,z,t)$ Inertial frame - $v(t)$

$$\begin{cases} x = \gamma \quad [X - v\,T] \\ t = \gamma \quad [T - v\,X] \end{cases} \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\begin{cases} X = \gamma \quad [x + v\,t] \\ T = \gamma \quad [t + v\,x] \end{cases}$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \qquad dt = \gamma\, d\tau$$

$$ds^2 = -d\tau^2 \qquad ds^2 = -dt^2 + d\mathbf{x}^2 = -dt^2\left(1 - \frac{d\mathbf{x}^2}{dt^2}\right) = -\frac{1}{\gamma^2}\,dt^2$$

$$u^\mu \equiv \frac{dx^\mu}{d\tau} = \left(\frac{d\mathbf{x}}{d\tau}, \frac{dt}{d\tau}\right) = \left(\gamma\,\frac{d\mathbf{x}}{dt}, \gamma\frac{dt}{dt}\right) = (\gamma\,\mathbf{u}, \gamma)$$

$$a^{\mu} \equiv \frac{du^{\mu}}{d\tau} = \gamma \frac{du^{\mu}}{dt} = \gamma \left(\frac{d(\gamma\,\mathbf{u})}{dt}, \frac{d\gamma}{dt} \right)$$

$$0 = \frac{d(u^{\mu}u_{\mu})}{d\tau} = u^{\mu}\frac{du_{\mu}}{d\tau} + u_{\mu}\frac{du^{\mu}}{d\tau} = 2u^{\mu}a_{\mu}$$

In the instantaneous proper frame $v = 0\,, \quad \gamma = 1 \quad \& \quad \frac{d\gamma}{dT} = \gamma^3 \left(\mathbf{v} \cdot \frac{d\mathbf{v}}{dT} \right) = 0$

$$A^{\mu} = \left(\frac{d\mathbf{u}}{dT}, 0 \right) = \left(\frac{d^2\mathbf{x}}{dT^2}, 0 \right)$$

$$A^{\mu}A_{\mu} = \alpha^2 > 0$$

proper acceleration

$$\frac{d\gamma}{dt} = \gamma^3 \left(\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} \right) = \gamma^3 v \frac{dv}{dt}$$

Relative motion in the x-axis

$$\frac{d(\gamma v)}{dt} = \gamma \frac{dv}{dt} + v \frac{d\gamma}{dt} = \gamma \frac{1}{\gamma^3 \, v} \frac{d\gamma}{dt} + v \frac{d\gamma}{dt} =$$

$$= \left(\frac{1}{\gamma^2 \, v} + v \right) \frac{d\gamma}{dt} = \frac{1}{v} \frac{d\gamma}{dt}$$

$$\alpha^2 = a^\mu a_\mu = \gamma^2 \left[\left(\frac{d(\gamma \mathbf{u})}{dt} \right)^2 - \left(\frac{d\gamma}{dt} \right)^2 \right]$$

$$\alpha^2 = \gamma^2 \left(1 - v^2\right) \left(\frac{d(\gamma u)}{dt} \right)^2 = \left(\frac{d(\gamma u)}{dt} \right)^2$$

$$\frac{d(\gamma u)}{dt} = \alpha \qquad t_0 = 0$$

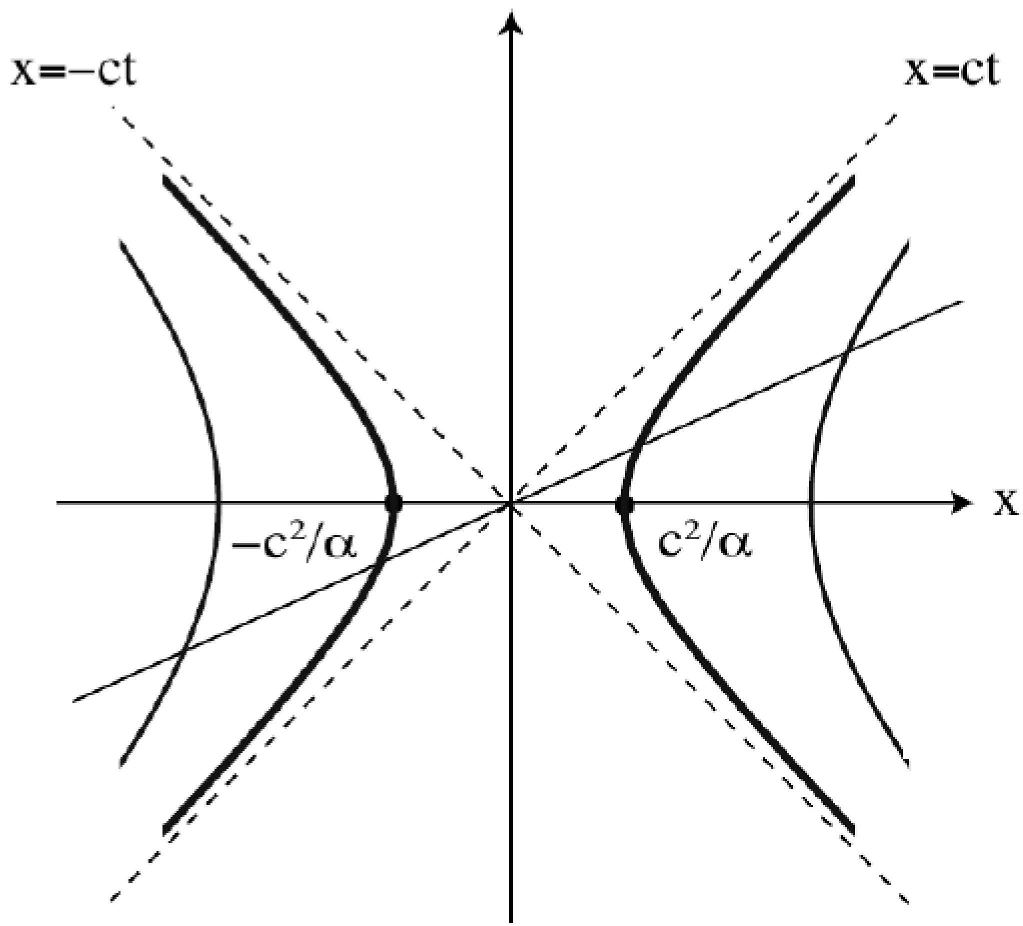
$$u(t_0) = 0.$$

$$d(\gamma u) = \alpha dt \qquad \Longrightarrow \qquad \gamma u = \alpha t$$

$$u(t) = \frac{dx}{dt} = \frac{\alpha t}{\sqrt{1 + \alpha^2 t^2}}$$

$$x(t_0) = \alpha^{-1}$$

$$x(t) = \frac{1}{\alpha}\sqrt{1 + \alpha^2 t^2} \qquad \Longrightarrow \qquad \boxed{x^2 - t^2 = \frac{1}{\alpha^2}}$$



x=−ct
x=ct
x
−c²/α
c²/α

Rindler coordinates

$$x^2 - c^2\,t^2 = \left(\frac{c^2}{\alpha}\right)^2$$

$$\begin{cases} c\,t = \dfrac{c^2}{a}\,e^{\frac{a\,\xi}{c^2}}\,\sinh\left(\dfrac{a\,\lambda}{c}\right) \\ \\ x = \dfrac{c^2}{a}\,e^{\frac{a\,\xi}{c^2}}\,\cosh\left(\dfrac{a\,\lambda}{c}\right) \end{cases} \qquad \alpha = a\,e^{-\frac{a\,\xi}{c^2}}$$

$$J = c\,e^{\frac{2\,a\,\xi}{c^2}} \in \,]0, \infty[$$

$$\begin{cases} \lambda = \dfrac{c}{a}\,\tanh^{-1}\left(\dfrac{c\,t}{x}\right) \\ \xi = \dfrac{c^2}{2a}\,\ln\left[\left(\dfrac{a}{c^2}\right)^2 \left(x^2 - c^2\,t^2\right)\right] \end{cases}$$

$$x = ct \quad \Longrightarrow \quad \lambda = \frac{c}{a}\,\tanh^{-1}(1) = +\infty$$

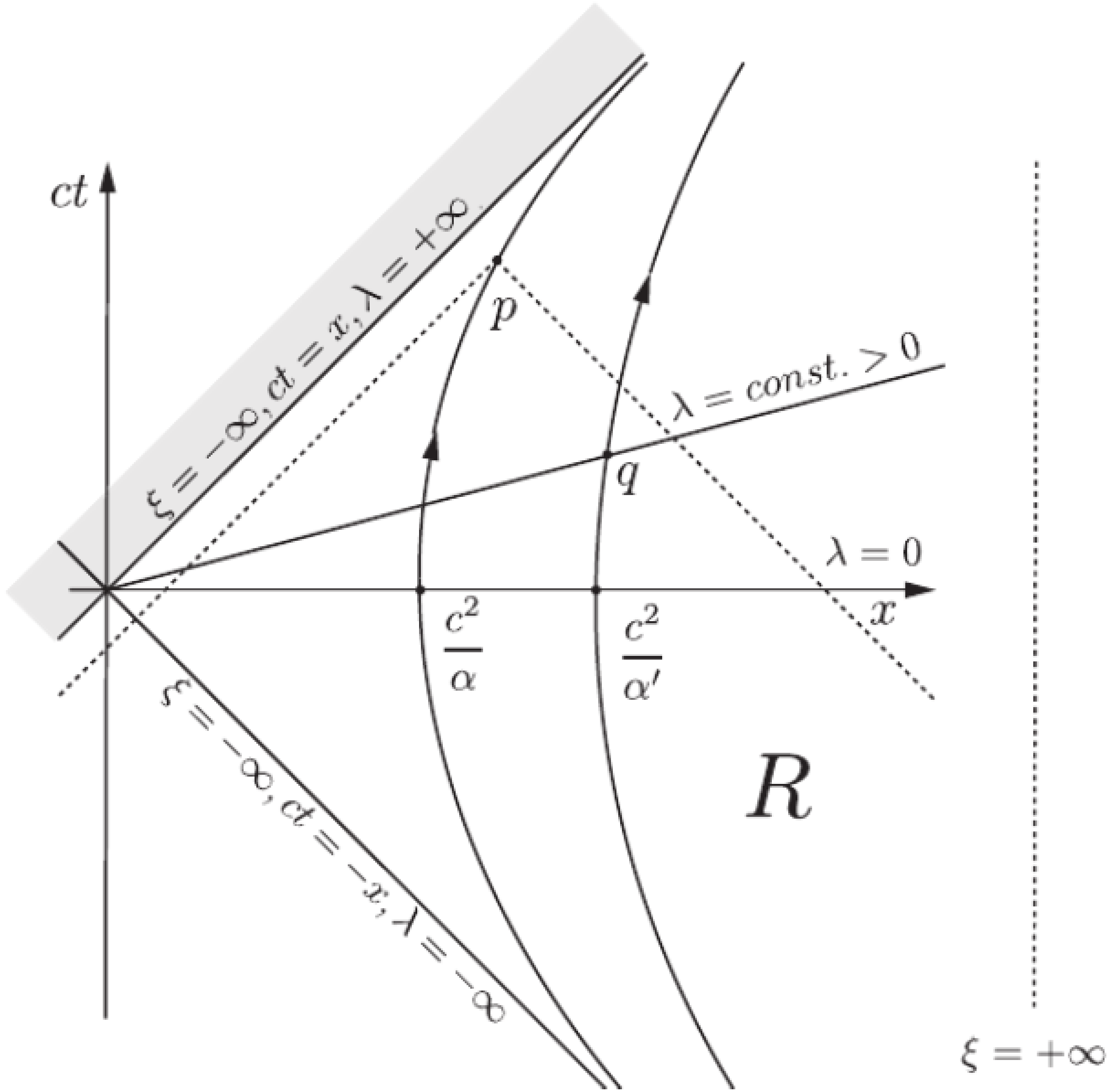
$$x = -ct \quad \Longrightarrow \quad \lambda = \frac{c}{a}\,\tanh^{-1}(-1) = -\infty$$

$$\xi = -\infty$$

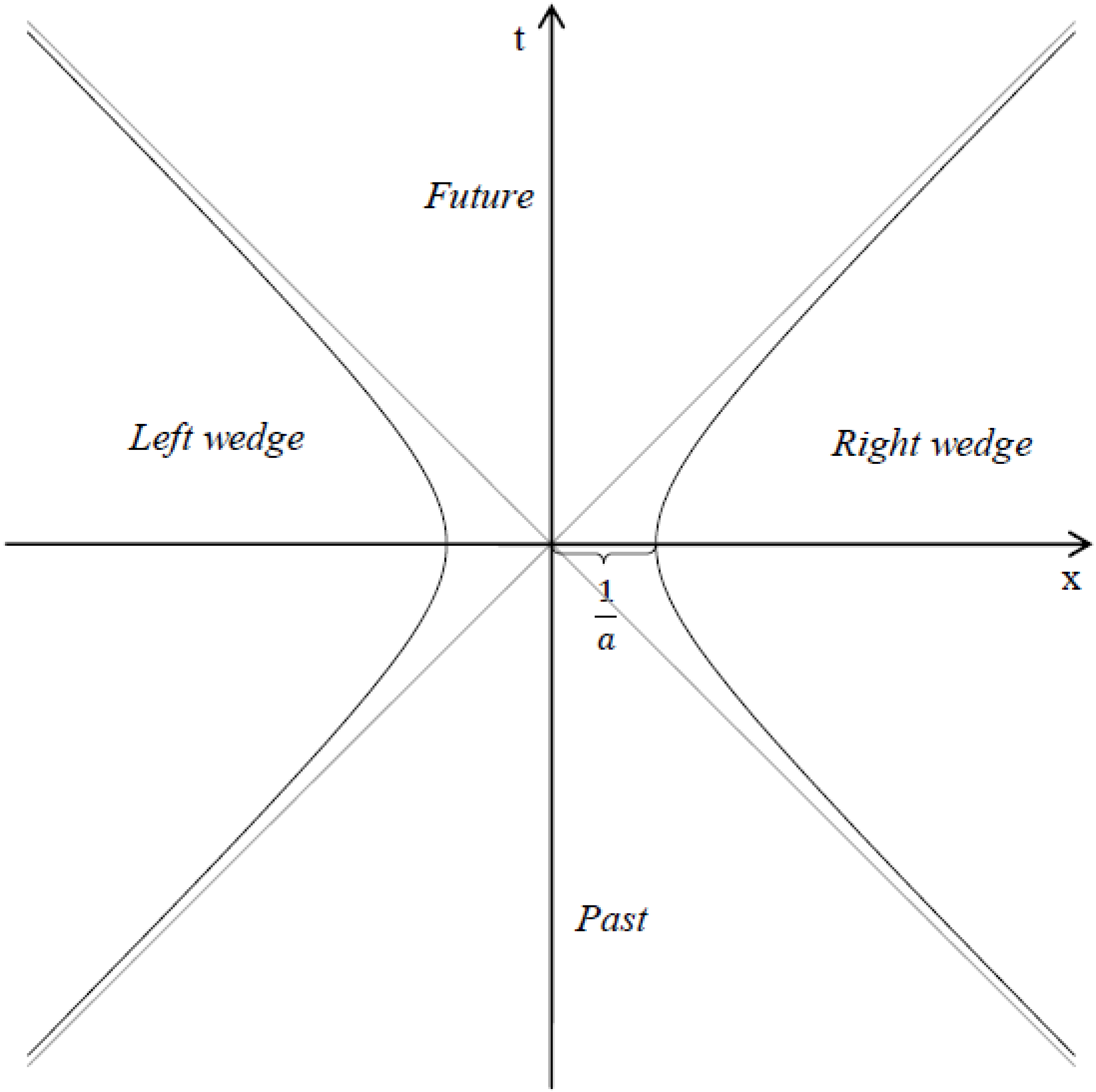
L.C. $\lambda = \pm\infty$ & $\xi = -\infty$

$$\lambda = cost. \qquad \frac{x}{ct} = \coth\left(\frac{a\,\lambda}{c}\right) = cost$$

$$\xi = cost \qquad x^2 - c^2\,t^2 = \left(\frac{c^2}{a}\right)^2 e^{\frac{2\,a\,\xi}{c^2}} = cost$$


ct
ξ = −∞, ct = x, λ = +∞
p
λ = const. > 0
q
λ = 0
x
c²/α
c²/α′
ξ = −∞, ct = −x, λ = −∞
R
ξ = +∞

$$|t| < -x$$



$$|t| < x$$

FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.

2D Rindler metric

$$\begin{cases} c\,t = \dfrac{c^2}{a}\, e^{\frac{a\,\xi}{c^2}} \sinh\left(\dfrac{a\,\lambda}{c}\right) \\[2ex] x = \dfrac{c^2}{a}\, e^{\frac{a\,\xi}{c^2}} \cosh\left(\dfrac{a\,\lambda}{c}\right) \end{cases}$$

$$ds^2 = dx^2 - c^2\, dt^2$$

$$dt = (\partial_\lambda t)\ d\lambda + (\partial_\xi t)\ d\xi$$
$$dx = (\partial_\lambda x)\ d\lambda + (\partial_\xi x)\ d\xi$$

$$ds^2 = e^{\frac{2\,a\,\xi}{c^2}} \left(d\xi^2 - c^2\, d\lambda^2\right)$$

conformal to Minkowski metric with $\Omega = e^{\frac{a\,\xi}{c^2}}$

$$g_{\alpha\beta} = \begin{pmatrix} g_{\xi\xi} & g_{\xi\lambda} \\ g_{\lambda\xi} & g_{\lambda\lambda} \end{pmatrix} = \begin{pmatrix} e^{\frac{2\,a\,\xi}{c^2}} & 0 \\ 0 & -e^{\frac{2\,a\,\xi}{c^2}} \end{pmatrix}$$

Eddington Finkelstein

$$\begin{cases} v &= c\,\lambda + \xi \\ u &= c\,\lambda - \xi \end{cases} \Longrightarrow \begin{cases} \lambda &= \dfrac{v+u}{2\,c} \\ \xi &= \dfrac{v-u}{2} \end{cases}$$

$$ds^2 = -e^{\frac{a\,(v-u)}{c^2}}\,dv\,du$$

Kruskal

$$\begin{cases} U &= -\dfrac{c^2}{a}\,e^{-\frac{a}{c^2}\,u} \\ \\ V &= \dfrac{c^2}{a}\,e^{\frac{a}{c^2}\,v} \end{cases} \qquad ds^2 = -dU\,dV$$

Horizon

$$(U,V)_+ = \left(0, e^{\frac{a}{c^2}v} < \infty\right)$$
$$(U,V)_- = \left(e^{-\frac{a}{c^2}u} < \infty, 0\right)$$

Regular at the horizon

- Uniform acceleration → hyperbolic motion in Minkowski s-t
- Rindler metric and coordinates for the accelerated observes

$$ds^2 = e^{\frac{2\,a\,\xi}{c^2}}\left(d\xi^2 - c^2\,d\lambda^2\right)$$

- Rindler metric does not cover the entire Minkowski (t-x) plane but…

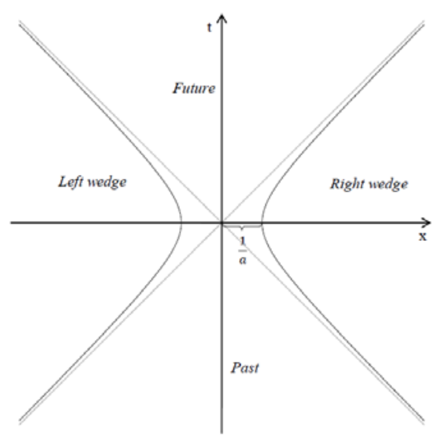


FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.

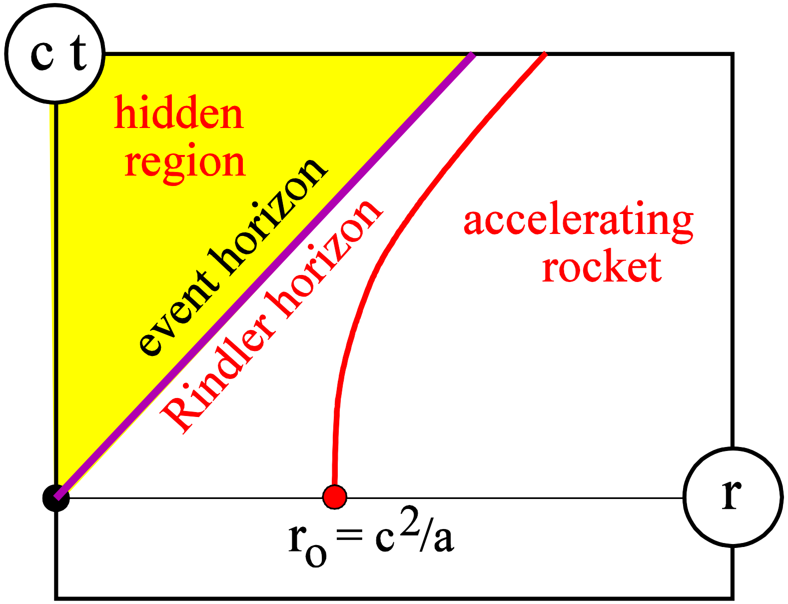
$$(U,V)_{R.W} = \left(-\frac{c^2}{a}\,e^{-\frac{a}{c^2}\,u},\ \frac{c^2}{a}\,e^{\frac{a}{c^2}\,v}\right)$$ is regular on the L-C

Extension of the Rindler metric in the whole space -time

$$(U,V)_{L.W} = \left(\frac{c^2}{a}\,e^{\frac{a}{c^2}\,u},\ -\frac{c^2}{a}\,e^{-\frac{a}{c^2}\,v}\right)$$

$$(U,V)_{F} = \left(\frac{c^2}{a}\,e^{-\frac{a}{c^2}\,u},\ \frac{c^2}{a}\,e^{\frac{a}{c^2}\,v}\right)$$

$$(U,V)_{P} = \left(-\frac{c^2}{a}\,e^{-\frac{a}{c^2}\,u},\ -\frac{c^2}{a}\,e^{\frac{a}{c^2}\,v}\right)$$



Event horizon is a space-time membrane

Prologo 2

$$\left(\nabla_\mu \nabla^\mu - m^2\right)\,\phi = 0$$

$$\left[\widehat{\phi}\left(\mathbf{x},t\right),\widehat{\phi}\left(\mathbf{x}',t\right)\right] = \left[\widehat{\pi}\left(\mathbf{x},t\right),\widehat{\pi}\left(\mathbf{x}',t\right)\right] = 0$$

$$\left[\widehat{\phi}\left(\mathbf{x},t\right),\widehat{\pi}\left(\mathbf{x}',t\right)\right] = i\,\delta^3\left(\mathbf{x},\mathbf{x}'\right)$$

$$\widehat{\pi}\left(\mathbf{x},t\right) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\widehat{\phi}}} = \dot{\widehat{\phi}}\left(\mathbf{x},t\right)$$

$$\widehat{\phi}\left(x\right) = \sum_i \left[\widehat{a}_i\, f_i\left(x\right) + \widehat{a}_i^\dagger\, f_i^*\left(x\right)\right]$$

annihilation operator

positive frequency normal modes

negative frequency normal modes

creation operator

$\{f_i\}$

complete set of wave functions

$$\widehat{a}_i = (f_i, \phi)_{KG}$$
$$\widehat{a}_i^\dagger = (\phi, f_i)_{KG}$$

definition

$$(f_A, f_B)_{KG} = i \int d^3\mathbf{x}\, (f_A^*\, \partial_t\, f_B - f_B\, \partial_t\, f_A^*)$$
$$= i \int d^3\mathbf{x}\, (f_A^*\, \pi_B - f_B\, \pi_A^*)$$

$$[\widehat{a}_i, \widehat{a}_j] = -\left(f_i, f_j^*\right) = 0$$
$$\left[\widehat{a}_i^\dagger, \widehat{a}_j^\dagger\right] = -\left(f_i^*, f_j\right) = 0$$
$$\left[\widehat{a}_i, \widehat{a}_j^\dagger\right] = \left(f_i, f_j\right) = \delta_{ij}$$

$$\widehat{a}_i\ |0\rangle \ = \ 0 \qquad \forall i$$

Bogoliubov transformations

$$\left\{ f_i^{(A)} \right\} \qquad \left\{ f_I^{(B)} \right\}$$

two complete sets of positive energy eigenfunctions

$$\Downarrow$$

$$f_I^{(B)} = \sum_i \left[\alpha_{Ii}\, f_i^{(A)} + \beta_{Ii}\, f_i^{(A)*} \right]$$

$$f_I^{(B)*} = \sum_i \left[\alpha_{Ii}^*\, f_i^{(A)*} + \beta_{Ii}^*\, f_i^{(A)} \right]$$

$$f_i^{(A)} = \sum_I \left[\alpha_{Ii}^*\, f_I^{(B)} - \beta_{Ii}\, f_I^{(B)*} \right]$$

$$f_i^{(A)*} = \sum_I \left[\alpha_{Ii}\, f_I^{(B)*} - \beta_{Ii}^*\, f_I^{(B)} \right]$$

The coefficients of the previous expansions can be written as

$$(f_A, f_B)_{KG} = i \int d^3\mathbf{x}\, (f_A^* \,\partial_t\, f_B - f_B \,\partial_t\, f_A^*)$$

$$\alpha_{Ii} = \left(f_i^{(A)}, f_I^{(B)}\right)_{KG} = \left(f_I^{(B)}, f_i^{(A)}\right)_{KG}^*$$

$$\beta_{Ii} = -\left(f_i^{(A)*}, f_I^{(B)}\right)_{KG} = \left(f_I^{(B)*}, f_i^{(A)}\right)_{KG}$$

$$\widehat{\phi}\,(x) = \sum_i \left[\widehat{a}_i^{A}\, f_i^{A}(x) + \widehat{a}_i^{\dagger A} f_i^{*A}(x)\right]$$

$$\widehat{\phi}\,(x) = \sum_i \left[\widehat{a}_i^{B}\, f_i^{B}(x) + \widehat{a}_i^{\dagger B} f_i^{*B}(x)\right]$$

$$f_I^{(B)} = \sum_i \left[\alpha_{Ii}\, f_i^{(A)} + \beta_{Ii}\, f_i^{(A)*}\right]$$

$$f_I^{(B)*} = \sum_i \left[\alpha_{Ii}^*\, f_i^{(A)*} + \beta_{Ii}^*\, f_i^{(A)}\right]$$

$$\widehat{a}_i^{(A)} = \sum_I \left[\alpha_{Ii} \, \widehat{a}_I^{(B)} + \beta_{Ii}^* \, \widehat{a}_I^{(B)\dagger} \right]$$

$$\widehat{a}_I^{(B)} = \sum_i \left[\alpha_{Ii}^* \, \widehat{a}_i^{(A)} - \beta_{Ii}^* \, \widehat{a}_i^{(A)\dagger} \right]$$

Bogoliubov transformations

$$\widehat{a}_i^{(A)} \left|0_{(A)}\right\rangle = 0$$

$$\widehat{a}_I^{(B)} \left|0_{(A)}\right\rangle$$

$$= \sum_i \left[\alpha_{Ii}^* \, \widehat{a}_i^{(A)} - \beta_{Ii}^* \, \widehat{a}_i^{(A)\dagger}\right] \left|0_{(A)}\right\rangle =$$

$$- \sum_i \beta_{Ii}^* \, \widehat{a}_i^{(A)\dagger} \left|0_{(A)}\right\rangle \neq 0$$

The ground state associated with the complete basis $\left\{f_i^{(A)}\right\}$ can be different from the ground state associated with the complete basis $\left\{f_I^{(B)}\right\}$

$$\widehat{N}_I^{(B)} \equiv \widehat{a}_I^{(B)\dagger}\,\widehat{a}_I^{(B)}$$

Number operator in $\left\{ f_I^{(B)} \right\}$

$$\widehat{N}_I^{(B)}\left|0_{(B)}\right\rangle \equiv \widehat{a}_I^{(B)\dagger}\,\widehat{a}_I^{(B)}\left|0_{(B)}\right\rangle = 0$$

$$\left\langle 0_{(A)} \left| \widehat{N}_I^{(B)} \right| 0_{(A)} \right\rangle = \sum_i \left|\beta_{Ii}\right|^2$$

$$\widehat{a}_i^{(A)} = \sum_I \left[\alpha_{Ii}\,\widehat{a}_I^{(B)} + \beta_{Ii}^*\,\widehat{a}_I^{(B)\dagger}\right]$$

$$\widehat{a}_I^{(B)} = \sum_i \left[\alpha_{Ii}^*\,\widehat{a}_i^{(A)} - \beta_{Ii}^*\,\widehat{a}_i^{(A)\dagger}\right]$$

Therefore if one the coefficient β_{Ii} is different from zero

The two sets «see» a different particle content

Unruh effect

ingredients

$$\widehat{\phi}(x,t)$$

$$ds^2 = dx^2 - dt^2$$ Minkowski

$$ds^2 = e^{2\,a\,\xi}\left(d\xi^2 - d\lambda^2\right)$$ Rindler

Minkowski

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right) \widehat{\phi} = 0$$

$$\widehat{\phi}(x,t) = \int_0^\infty \frac{dk}{\sqrt{4\pi k}} \left(\widehat{b}_{-k}\, e^{-i\,k\,(t-x)} + \widehat{b}_{+k}\, e^{-i\,k\,(x+t)} + \right.$$
$$\left. + \widehat{b}^{\dagger}_{-k}\, e^{i\,k\,(t-x)} + \widehat{b}^{\dagger}_{k}\, e^{i\,k\,(x+t)}\right)$$

Creation and annihilation operators for + and - frequencies

$\widehat{b}_q$ e $\widehat{b}^{\dagger}_q$

Minkowski

outgoing

ingoing

$$U = t - x$$

$$V = t + x$$

$$\widehat{\phi}\,(x,t) = \widehat{\phi}_-\,(U) + \widehat{\phi}_+\,(V)$$

$$\widehat{\phi}_+\,(V) = \int_0^\infty dk\,\left[\widehat{b}_{+k}\,f_k\,(V) + \widehat{b}^\dagger_{+k}\,f^*_k\,(V)\right]$$

$$f_k\,(V) \equiv \frac{1}{\sqrt{4\,\pi\,k}}\,e^{-i\,k\,V}$$

Rindler

RW

$$u = \lambda - \xi$$

$$v = \lambda + \xi$$

$$U = -\frac{1}{a}\, e^{-a\, u}$$

$$V = \frac{1}{a}\, e^{a\, v}$$

RW $\quad V > 0$

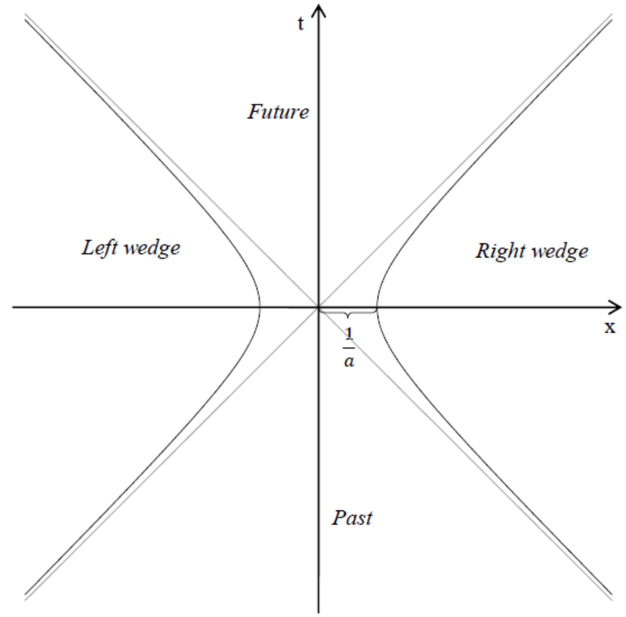


FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.

$$\begin{cases} v &= c\,\lambda + \xi \\ u &= c\,\lambda - \xi \end{cases} \Longrightarrow \begin{cases} \lambda &= \dfrac{v+u}{2\,c} \\ \xi &= \dfrac{v-u}{2} \end{cases}$$

$$\begin{cases} U &= -\dfrac{c^2}{a}\, e^{-\frac{a}{c^2}\, u} \\ V &= \dfrac{c^2}{a}\, e^{\frac{a}{c^2}\, v} \end{cases}$$

Normal modes Ingoing component

$$\widehat{\phi}_+^R(V) = \int_0^\infty d\omega \left[\widehat{a}_{+\omega}^R\, g_\omega(v) + \widehat{a}_{+\omega}^{R\dagger}\, g_\omega^*(v)\right]$$

$$g_\omega(v) = \frac{1}{\sqrt{4\,\pi\,\omega}}\, e^{-i\,\omega\,v} = \frac{1}{\sqrt{4\,\pi\,\omega}}\,(a\,V)^{-\frac{i\,\omega}{a}}$$

$$U = -\frac{1}{a}\,e^{-a\,u}$$

$$V = \frac{1}{a}\,e^{a\,v}$$

$\widehat{a}_{+\omega}^{R\dagger}$ e $\widehat{a}_{+\omega}^R$ Creation and annihilation operators in the RW

Rindler

LW

Rindler $(\overline{\xi}, \overline{\lambda})$ To cover this sector

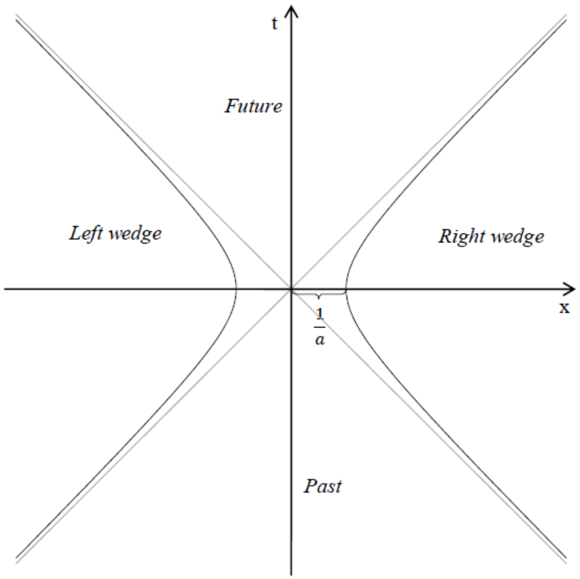


FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.

$$\overline{u} = \overline{\lambda} - \overline{\xi}$$

$$\overline{v} = \overline{\lambda} + \overline{\xi}$$

$$U = \frac{1}{a} e^{a\,\overline{u}}$$

$$V = -\frac{1}{a} e^{-a\,\overline{v}}$$

$$(U, V)_{LW} = \left(\frac{c^2}{a} e^{\frac{a}{c^2} u}, -\frac{c^2}{a} e^{-\frac{a}{c^2} v} \right)$$

$V < 0;$

$\widehat{a}^{L\dagger}_{+\omega}$ e $\widehat{a}^{L}_{+\omega}$ Creation and annihilation operators in the LW

$$V \in (-\infty, \infty)$$

$$\begin{aligned}\widehat{\phi}_{+}(V) = \theta(V)\,\widehat{\phi}^{R}_{+}(V) + \theta(-V)\,\widehat{\phi}^{L}_{+}(V) = \\ = \int_0^{\infty} d\omega \left\{ \theta(V)\left[\widehat{a}^{R}_{+\omega} g_{\omega}(v) + \widehat{a}^{R\dagger}_{+\omega} g^{*}_{\omega}(v)\right] + \right. \\ \left. + \theta(-V)\left[\widehat{a}^{L}_{+\omega} g_{\omega}(\overline{v}) + \widehat{a}^{L\dagger}_{+\omega} g^{*}_{\omega}(\overline{v})\right]\right\}\end{aligned}$$

$$g_{\omega}(v) = \frac{1}{\sqrt{4\,\pi\,\omega}}\, e^{-i\,\omega\,v}$$

set $\{f_k(V)\}$ Minkowski

$\{g_\omega(v), g_\omega(\overline{v})\}$ Rindler RW and LW

Bogoliubov Transformations

$$\theta(V)\ g_\omega(v) = \int_0^\infty dk\,[\alpha^R_{\omega k}\, f_k(V) + \beta^R_{\omega k}\, f_k^*(V)]$$

$$\theta(-V)\ g_\omega(\overline{v}) = \int_0^\infty dk\,[\alpha^L_{\omega k}\, f_k(V) + \beta^L_{\omega k}\, f_k^*(V)]$$

$$f_k(V) \equiv \frac{1}{\sqrt{4\pi k}}\, e^{-ikV}$$

Inversion

$$\beta^{L}_{\omega k} = -e^{-\frac{\pi\omega}{a}}\, \alpha^{R*}_{\omega k}$$

$$\beta^{R}_{\omega k} = -e^{-\frac{\pi\omega}{a}}\, \alpha^{L*}_{\omega k}$$

$$\theta(V)\; g_\omega(v) = \int_0^\infty dk\, \left[\alpha^{R}_{\omega k}\, f_k(V) + \beta^{R}_{\omega k}\, f^*_k(V)\right]$$

$$\theta(-V)\; g_\omega(\overline{v}) = \int_0^\infty dk\, \left[\alpha^{L}_{\omega k}\, f_k(V) + \beta^{L}_{\omega k}\, f^*_k(V)\right]$$

$$\widehat{\phi}_+(V) = \theta(V)\,\widehat{\phi}^R_+(V) + \theta(-V)\,\widehat{\phi}^L_+(V) =$$

$$= \int_0^\infty d\omega \left\{\theta(V)\left[\widehat{a}^R_{+\omega} g_\omega(v) + \widehat{a}^{R\dagger}_{+\omega} g^*_\omega(v)\right] + \right.$$

$$\left. + \,\theta(-V)\left[\widehat{a}^L_{+\omega} g_\omega(\overline{v}) + \widehat{a}^{L\dagger}_{+\omega} g^*_\omega(\overline{v})\right]\right\}$$

$$\widehat{\phi}_+(V) = \int_0^\infty d\omega \int_0^\infty \frac{dk}{\sqrt{2\pi k}} \times$$
$$\times \left[\alpha^R_{\omega k}\, e^{-i\,k\,V} \left(\widehat{a}^R_{+\omega} - e^{-\frac{\pi\omega}{a}}\, \widehat{a}^{L\dagger}_{+\omega} \right) + \right.$$
$$\left. + \, \alpha^L_{\omega k}\, e^{-i\,k\,V} \left(\widehat{a}^L_{+\omega} - e^{-\frac{\pi\omega}{a}}\, \widehat{a}^{R\dagger}_{+\omega} \right) \right] + H.c.$$

$$\widehat{\phi}_+(V) \; \left| 0_{(\mathcal{M})} \right\rangle = 0$$

$$\begin{cases} \left(\widehat{a}^R_{+\omega} - e^{-\frac{\pi\omega}{a}}\, \widehat{a}^{L\dagger}_{+\omega} \right) \left| 0_{(\mathcal{M})} \right\rangle & = 0 \\ \left(\widehat{a}^L_{+\omega} - e^{-\frac{\pi\omega}{a}}\, \widehat{a}^{R\dagger}_{+\omega} \right) \left| 0_{(\mathcal{M})} \right\rangle & = 0 \end{cases}$$

$$\left(\widehat{a}^{R}_{+\omega} - e^{-\frac{\pi\omega}{a}}\ \widehat{a}^{L\dagger}_{+\omega}\right)|0_{(\mathcal{M})}\rangle \quad = 0$$

$$\left\langle 0_{(\mathcal{M})}\left|\widehat{N}^{R}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle = \left\langle 0_{(\mathcal{M})}\left|\widehat{a}^{R\dagger}_{\omega_i}\widehat{a}^{R}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle$$

$$= e^{-\frac{2\pi\omega}{a}}\left\langle 0_{(\mathcal{M})}\left|\widehat{a}^{L}_{\omega_i}\widehat{a}^{L\dagger}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle =$$

$$\left\langle 0_{(\mathcal{M})}\left|\widehat{N}^{R}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle = e^{-\frac{2\pi\omega}{a}}\left[\left\langle 0_{(\mathcal{M})}\left|\widehat{N}^{L}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle + 1\right]$$

$$\left(\widehat{a}^{L}_{+\omega} - e^{-\frac{\pi\omega}{a}}\ \widehat{a}^{R\dagger}_{+\omega}\right)|0_{(\mathcal{M})}\rangle \quad = 0$$

$$\left\langle 0_{(\mathcal{M})}\left|\widehat{N}^{L}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle = e^{-\frac{2\pi\omega}{a}}\left[\left\langle 0_{(\mathcal{M})}\left|\widehat{N}^{R}_{\omega_i}\right|0_{(\mathcal{M})}\right\rangle + 1\right]$$

$$\left\langle 0_{(\mathcal{M})} \left| \widehat{N}^{R}_{\omega_i} \right| 0_{(\mathcal{M})} \right\rangle = \left\langle 0_{(\mathcal{M})} \left| \widehat{N}^{L}_{\omega_i} \right| 0_{(\mathcal{M})} \right\rangle$$

$$= \frac{1}{e^{\frac{2\pi\omega}{a}} - 1}$$

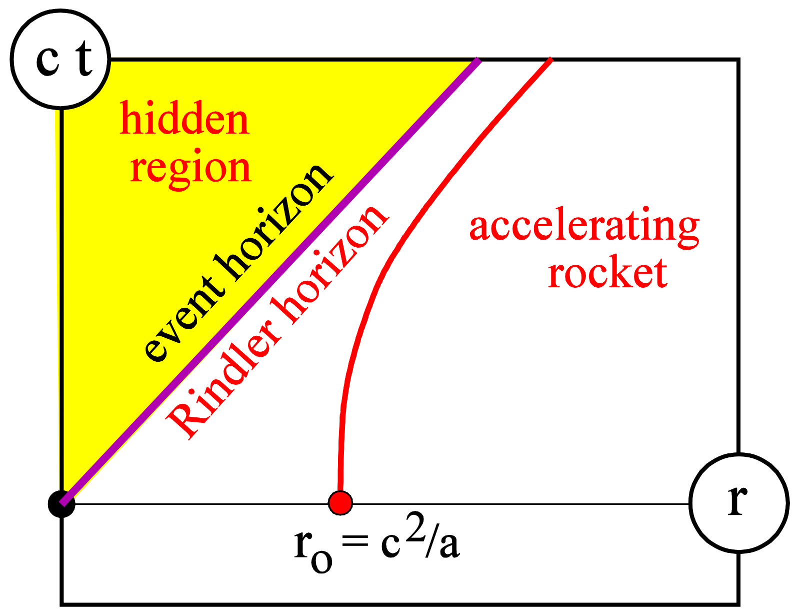
The expectation value of the number operator of Rindler particle in the RW or LW in the Minkowski vacuum is a Bose-Einstein Distribution with temperature

$$T = \frac{a}{2\pi}$$

Unruh, W. G., 1976, “Notes on black-hole evaporation,” Phys. Rev. D 14, 870–892. Unruh, W. G., 1977, in Proceedings of the 1st Marcel Grossmann Meeting on General Relativity, edited by R. Ruffini (North-Holland, Amsterdam), 527–536. Unruh, W. G., 1981, “Experimental black-hole evaporation?,” Phys. Rev. Lett. 46, 1351–1353. Unruh, W. G., 1992, “Thermal bath and decoherence of Rindler spacetimes,” Phys. Rev. D 46, 3271–3277.

Acceleration radiation in interacting field theories
Phys. Rev. D **29**, 1656 (1984)
William G. Unruh and Nathan Weiss

Unruh-Wald PRD 29(1984) 1047



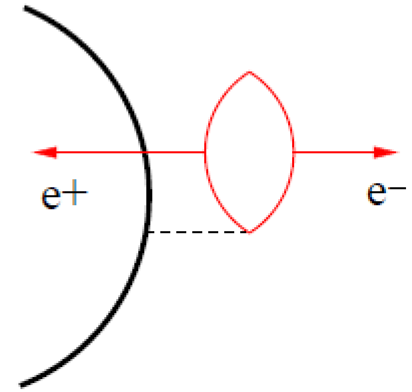
c t
hidden region
event horizon
Rindler horizon
accelerating rocket
r
$r_o = c^2/a$

Recall

In Classical Black-hole particles are confined ⇒ event horizon ⇒ no communication with outside, but...Hawking radiation [Hawking 1975]

Quantum effect ∼ uncertainty principle → vacuum fluctuation e^+e^- outside event horizon, with $\Delta E \Delta t \sim 1$ .If e^+ falls into black hole, then e^- can escape; equivalent:

e^- tunnels through event horizon



There is no information about state of system beyond event horizon; e^+ on one side, e^- on the other

$\Rightarrow$ Hawking radiation must be thermal

$$\frac{dN}{dk} \sim \exp\{-\frac{k}{T_{BH}}\}$$

with black hole temperature

$$T_{BH} = \frac{\hbar}{8\pi c GM}$$

relativistic quantum effect: disappears for $\hbar \to 0$ or $c \to \infty$

$\Rightarrow$ tunnelling through event horizon $\to$ thermal radiation

The correspondence with gravity

Unruh effect and the near horizon approximation

Rindler metric of an accelerated observer
(in spherical coordinates τ, χ, θ, ϕ)

$$ds^2 = \chi^2 a^2 d\tau^2 - d\chi^2 - \chi^2 cosh^2 a\tau(d\theta^2 + sin^2\theta d\phi^2)$$

Schwarzchild BH metric ; $\gamma = (1 - 2GM/r)$

$$ds^2 = \gamma dt^2 - \gamma^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2)$$

Coordinate transformation $\eta = \sqrt{\gamma}/k$,
where k = surface gravity and $r \rightarrow R = 2GM$

$$ds^2 = \eta^2 k^2 dt^2 - d\eta^2 - R^2(d\theta^2 + sin^2\theta d\phi^2)$$

M. K. Parikh and F. Wilczek, “Hawking radiation as tunneling,” Phys. Rev. Lett. 85 (2000) 5042

Hawking radiation as tunneling through the event horizon

Applications:

- for $F = GMm/R^2$ and Schwarzschild $R = 2M_G$ recover Hawking temperature

$$T_U = \frac{a}{2\pi} = \frac{GM}{2\pi R^2} = \frac{1}{8\pi GM}$$

Hawking temperature

- for $F = e\mathcal{E}$ recover Schwinger mechanism for production of pair (mass m) in strong field $\mathcal{E}$

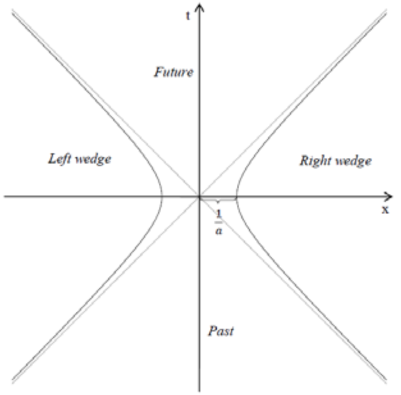
$$T_U = \frac{a}{2\pi} = \frac{e\mathcal{E}}{\pi m}$$

$$P(m, \mathcal{E}) \sim \exp\{-m/T_U\} = \exp\{-\pi m^2/e\mathcal{E}\}$$

production probability $P(m, \mathcal{E})$

R. Brout, R. Parentani, and Ph. Spindel, “Thermal properties of pairs produced by an electric field: A tunneling approach,” Nucl. Phys. B 353 (1991) 209.

Unruh radiation as tunneling through the event horizons



Minkowski spacetime

$$ds^2 = -dt^2 + dx^2$$

Rindler observer

$$t = (a^{-1} + x_R)\sinh(at_R)$$
$$x = (a^{-1} + x_R)\cosh(at_R)$$

Rindler metric $$ds^2 = -(1 + ax_R)^2 dt_R^2 + dx_R^2 \ .$$

$$det(g_{ab}) \equiv g = -(1 + ax_R)^2, \text{ vanishes.} \qquad x_R = -\frac{1}{a},$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2,$$

alternative form of the Rindler metric

$$(1 + a\,x_R) = \sqrt{|1 + 2\,a\,x_{R'}|}\ .$$

$$ds^2 = -(1 + 2\,a\,x_{R'})dt_{R'}^2 + (1 + 2\,a\,x_{R'})^{-1}dx_{R'}^2$$

the substitution $a \to GM/x_{R'}^2$ → the usual Schwarzschild metric.

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2,$$

But also

$$ds^2 = -dt^2 + dx^2$$

$$t = \frac{\sqrt{1 + 2ax_{R'}}}{a} \sinh(at_{R'})$$
$$x = \frac{\sqrt{1 + 2ax_{R'}}}{a} \cosh(at_{R'})$$

for $x_{R'} \geq -\frac{1}{2a}$

$$t = \frac{\sqrt{|1 + 2ax_{R'}|}}{a} \cosh(at_{R'})$$
$$x = \frac{\sqrt{|1 + 2ax_{R'}|}}{a} \sinh(at_{R'})$$

for $x_{R'} \leq -\frac{1}{2a}$

Unruh radiation via WKB method
D.A.Singleton http.//dx.doi.org.10.5772/53898

WKB approximation tells us how to find the transmission probability in terms of the incident wave and transmitted wave amplitudes. The transition probability is in turn given by the exponentially decaying part of the wave function over the non–classical (*tunneling*) region

$$\Gamma_{QM} \propto e^{-\mathrm{Im}\frac{1}{\hbar}\oint p_x dx} .$$

scalar field
metric $g_{\mu\nu}$

$$\phi = \phi_0 e^{\frac{i}{\hbar} S(t,\vec{x})}$$

$$g^{\mu\nu}\partial_\nu(S)\partial_\mu(S) + m^2 = 0 .$$

Now for stationary spacetimes the action S can be split into a time and space part

$$S(t,\vec{x}) = Et + S_0(\vec{x})$$

$$ds^2 = -(1 + 2\,a\,x_{R'})dt_{R'}^2 + (1 + 2\,a\,x_{R'})^{-1}dx_{R'}^2$$

$$-\frac{1}{(1 + 2\,a\,x_{R'})}(\partial_t S)^2 + (1 + 2\,a\,x_{R'})(\partial_x S)^2 + m^2 = 0\ .$$

$$S(t, \vec{x}) = Et + S_0(\vec{x})$$

$$-\frac{E}{(1 + 2\,a\,x_{R'})^2} + (\partial_x S_0(x_{R'}))^2 + \frac{m^2}{1 + 2\,a\,x_{R'}} = 0$$

$$S_0^{\pm} = \pm \int_{-\infty}^{\infty} \frac{\sqrt{E^2 - m^2(1 + 2\,a\,x_{R'})}}{1 + 2\,a\,x_{R'}}\, dx_{R'}\ .$$

the + sign corresponds to the ingoing particles (i.e., particles that move from right to left) and the − sign to the outgoing particles (i.e., particles that move left to right).]

this integral has a pole along the path of integration at $x_{R'} = -\frac{1}{2a}$.

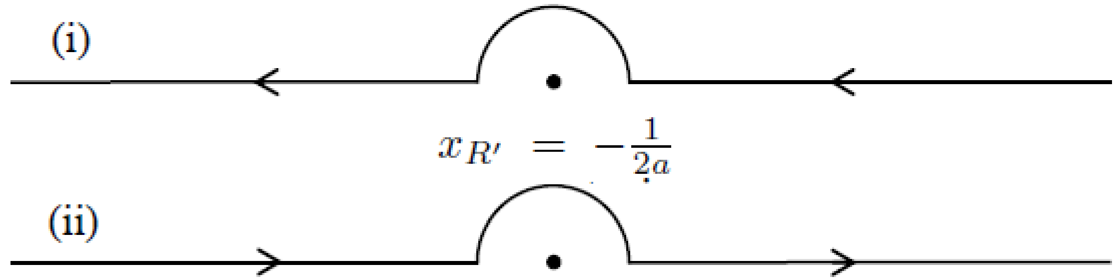


FIG. 2: Contours of integration for (i) the ingoing and (ii) the outgoing particles.

a semi–circular contour which we parameterize as $x_{R'} = -\frac{1}{2a} + \epsilon e^{i\theta}$, where $\epsilon \ll 1$ and θ goes from 0 to π for the ingoing path and π to 0 for the outgoing path.

$$S_0^{\pm} = \pm \int_{-\infty}^{\infty} \frac{\sqrt{E^2 - m^2(1 + 2\,a\,x_{R'})}}{1 + 2\,a\,x_{R'}}\; dx_{R'} \; .$$

for ingoing $(+)$ particles is

$$S_0^{+} = \int_0^{\pi} \frac{\sqrt{E^2 - m^2 \epsilon e^{i\theta}}}{2a\epsilon e^{i\theta}}\; i\epsilon e^{i\theta} d\theta = \frac{i\,\pi\,E}{2a} \; ,$$

and for outgoing (−) particles,

$$S_0^- = -\int_\pi^0 \frac{\sqrt{E^2 - m^2 \epsilon e^{i\theta}}}{2a\epsilon e^{i\theta}}\, i\epsilon e^{i\theta} d\theta = \frac{i\,\pi\,E}{2a}\ .$$

the total action $S(t,\vec{x}) = Et + S_0(\vec{x})$,

$$\Gamma \propto e^{-\frac{1}{\hbar}[\mathrm{Im}(\oint p_x dx) - E\,\mathrm{Im}(\Delta t)]}\ .$$

$$t = \frac{\sqrt{1+2ax_{R'}}}{a}\sinh(at_{R'}) \qquad t = \frac{\sqrt{|1+2ax_{R'}|}}{a}\cosh(at_{R'})$$

$$x_{R'} \geq -\frac{1}{2a}, \qquad x_{R'} \leq -\frac{1}{2a}.$$

$$t_{R'} \to t_{R'} - \frac{i\pi}{2a}$$

$$\sinh(at_{R'}) \to \sinh\left(at_{R'} - \frac{i\pi}{2}\right) = -i\cosh(at_{R'})$$

$$\sqrt{1+2ax_{R'}} \to i\sqrt{|1+2ax_{R'}|}$$

$$S(t,\vec{x}) \;=\; S_0(\vec{x}) + Et$$

$$E\Delta t \;=\; -\frac{i\pi E}{2a}.$$

When the horizon is crossed once, the total action $S(t,\vec{x})$ gets a contribution of $E\Delta t = -\frac{iE\pi}{2a}$, and for a round trip, as implied by the spatial part $\oint p_x dx$, the total contribution is $E\Delta t_{total} = -\frac{iE\pi}{a}$.

$$S_0^+ = \int_0^\pi \frac{\sqrt{E^2 - m^2\epsilon e^{i\theta}}}{2a\epsilon e^{i\theta}}\, i\epsilon e^{i\theta} d\theta = \frac{i\,\pi\,E}{2a}\;,$$

$$S_0^- = -\int_\pi^0 \frac{\sqrt{E^2 - m^2\epsilon e^{i\theta}}}{2a\epsilon e^{i\theta}}\, i\epsilon e^{i\theta} d\theta = \frac{i\,\pi\,E}{2a}\;.$$

$$\Gamma \propto e^{-\frac{1}{\hbar}[\mathrm{Im}(\oint\, p_x dx) - E\mathrm{Im}(\Delta t)]}\;.$$

$$\mathrm{Im}(\oint\, p_x dx) - E\mathrm{Im}(\Delta t) = \frac{\pi E}{a} + \frac{\pi E}{a}$$

$$T = \frac{\hbar a}{2\pi kc};$$

Uniform acceleration

↓

Event Horizon

↓

Universal thermal behavior

↓

In QCD ?

Confinement $V \to \sigma r$

Conjecture

Physical vacuum ⟺ **Event horizon for colored constituents**

⇓

Thermal hadron production ⟺ **Hawking-Unruh radiation in QCD**

P.C., D.Kharzeev and H.Satz -- D.Kharzeev and Y.Tuchin (temperature)

F.Becattini, P.C., J.Manninen and H.Satz (strangeness suppression in e+e-)

P.C. and H.Satz (strangeness enhancement in heavy ion collisions)

P.C., A. Iorio and H.Satz (entropy and freeze-out)

Questions

1) ***Why do elementary high energy collisions***
show a statistical behavior?

2) ***Why is strangeness production universally suppressed***
in elementary collisions?

3) ***Why (almost) no strangeness suppression in nuclear collisions?***

4) ***Why hadron freeze-out for s/T^3 = 7 or E/N=1.08 Gev***

5) ***Why thermalization in so short time (0.5- 1 fm/c)***