1. Мотивировки

U(N) gauge group

①

$AdS_5 \times S^5 \Longleftrightarrow N=4,\ D4\ SYM$

$\frac{SO(2,4)}{SO(1,4)} \otimes \frac{SO(6)}{SO(5)}$ conf $\otimes$ $SO(6) \sim SU(4)$

$(A_m,\ \varphi^{[ik]} = \frac{1}{2}\varepsilon^{ikem}\bar{\varphi}_{[em]},\ \psi^i_{\alpha\dot\alpha})$

Решение IIB D10 $\ni$ $\frac{6}{}$ D3-брана $\frac{4}{}$

супергравитации с макс. симм.

слева реализуется

$SO(2,4) \otimes SO(6) \subset SU(2,2|4)$

(4+6) ← 3 браны, 6 d=10

16 + 16 SUSY (32)

N=4, 4D суперконф.

6 скал. + 16 вейл. ферм.

$\varphi^{[ik]},\ \psi^i_\alpha$ ← 16

(6) + 8 + $A^m(x)$

$$T_a T_b \textcircled{1} - T_a \textcircled{1} T_b - T_b T_a \textcircled{1} + T_b \textcircled{1} T_a$$

$$+ \textcircled{1} T_a T_b - \textcircled{1} T_b T_a = \text{I}$$

$$\text{II} = \textcircled{2} T_d T_g - \textcircled{2} T_g T_d - T_d \textcircled{2} T_g +$$

$$+ T_g T_d \textcircled{2} + T_g \textcircled{2} T_d - T_g T_d \textcircled{2} = \text{II}$$

$$\text{III} = T_c \textcircled{3} T_g - T_c T_g \textcircled{3} - \textcircled{3} T_c T_g +$$

$$+ \textcircled{3} T_g T_c + T_g T_c \textcircled{3} - T_g \textcircled{3} T_c$$

$$\text{IV} = T_c T_d \textcircled{4} - T_c \textcircled{4} T_d - T_d T_c \textcircled{4} +$$

$$+ T_d \textcircled{4} T_c + \textcircled{4} T_c T_d - \textcircled{4} T_d T_c$$

М-теория (~~сильн~~ предел сильной связи в IIA суперструны) имеет класс./кван.

$AdS_4 \times S^7$

$$\frac{SO(2,3)}{SO(1,3)} \times \frac{SO(8)}{SO(7)} \Rightarrow OSp(8|4)$$

16 + 16 SUSY (Пуанк. + конф.)

φ^I 16 ферм.

8 скал.

on-shell (8 + 8)

кол. пол?

если есть, то не удалить, имея степ. ~~свободы~~ на массив. …

$$\mathcal{L}_{CS} = \mathrm{tr}\, k\, \varepsilon^{\mu\nu\rho}\left[A_\mu \partial_\nu A_\rho + \frac{2}{3} i A_\mu A_\nu A_\rho\right]$$

$$\delta A_\mu: \quad 2\varepsilon^{\mu\nu\rho}\partial_\nu A_\rho + 2i\, \varepsilon^{\mu\nu\rho} A_\nu A_\rho = 0$$

$$\varepsilon^{\mu\nu\rho}\left\{\hat{F}_{\nu\rho} + i[A_\nu, A_\rho]\right\} = 0$$

$$F_{\nu\rho} = \partial_\nu A_\rho - \partial_\rho A_\nu + i[A_\nu, A_\rho] = 0$$

На масс. обол. инв. отн-но <u>любого</u> числа SUSY, $\delta A_\mu = 0$

$$\delta_\rho A_\mu = -a^\rho \partial_\rho A_\mu = -a^\rho\left[\partial_\mu A_\rho - i[A_\rho, A_\mu]\right]$$

$$= -\partial_\mu(a^\rho A_\rho) + i[A_\mu, (a^\rho A_\rho)] = -\nabla_\mu (a^\rho A_\rho)$$

т.е. калибр. преобр., а на масс. обол. в кал… теориях замыкание супералгебры всегда с точн. до кал. преобразов.

$$\mathcal{L}_{CS} = \mathrm{tr}\left[\varepsilon^{\mu\nu\rho}\left(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho\right) - \bar{\chi}\chi\right]$$

$$\bar{\chi} = \chi \gamma^0$$

$$\delta A_\mu = i\,\bar{\varepsilon}\,\gamma_\mu \chi = i\,\varepsilon_\alpha (\gamma^0\gamma_\mu)_{\alpha\beta}\chi_\beta$$

$$\delta \chi_\alpha = \frac{1}{2}(\gamma^{\mu\nu})_{\alpha\beta} F_{\mu\nu}$$

$$(\gamma^{\mu\nu})_{\alpha\beta} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)_{\alpha\beta} = -i\,\varepsilon^{\mu\nu\rho}(\gamma_\rho)_{\alpha\beta}$$

$$\varepsilon^{012} = 1$$

$$(\gamma_0)_{\alpha\beta} = -(\gamma_0)_{\beta\alpha}, \quad (\gamma_0\gamma_\mu)_{\alpha\beta} = (\gamma_0\gamma_\mu)_{\beta\alpha}$$

$$\gamma_0 = \sigma_2, \quad \gamma^1 = i\sigma_3, \quad \gamma^2 = i\sigma_1,$$

$$\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1)$$

$$\theta^\alpha \theta_\alpha = \theta^1\theta_1 + \theta^2\theta_2$$
$$= -\theta_2\theta_1 + \theta_1\theta_2$$
$$= 2\theta_1\theta_2$$
$$= (-i)$$

$$\sigma_1\sigma_3 = -i\,\sigma_2$$

$$(\sigma_1)_{12}(\sigma_3)_{22} = -i\,(\sigma_2)_{12}$$
$$-1 = -i\,(\sigma_2)_{12}$$

$$\theta\gamma_0\theta =$$
$$2\theta_1(\gamma_0)_{12}\theta_2 = -2i\,\theta_1\theta_2$$

$$\theta^\alpha\theta_\alpha = i\,\theta\gamma_0\theta$$

$$\frac{1}{2}(\bar\theta\theta)\,\delta C = i(\bar\theta\theta)\left(+\frac{1}{2}\right)(\gamma^0\gamma^\mu)_{\rho\gamma}(\gamma^0)_{\beta\lambda}(\gamma^0)_{\rho\beta}\,\varepsilon_\gamma\partial_\mu\psi_\lambda$$
$$= \frac{i}{2}(\bar\theta\theta)\,\bar\varepsilon\gamma^\mu\partial_\mu\psi$$

(5)

$$\delta S = \theta_\delta\, \theta^{\mu\nu}(\ell_e)\,\theta^r = r\, \delta_\theta\, (\ell_o \ell_w)^{\mu\nu}\, \varepsilon_r\, \delta_\theta\, (\ell_o)^{r\mu}\, e^{\mu\nu}\, h^r$$

$$\delta h^\mu = -r\,(\ell_w)^{\nu\mu}\,\varepsilon^\nu\, e^\mu\, b + \rho\,\varepsilon^\mu$$

$$\delta \rho = -i\, \underline{\varepsilon}\, \ell_w\, e^\mu\, h$$

$$r = ?$$

$$\theta_\delta\,(\ell_e)^{\nu\mu}\, h^\mu\, \delta = r\,\theta_\delta\,(\ell_w)^{\nu\mu}\,\varepsilon^\nu\, e^\mu\, b$$

$$- \varepsilon^r \left[(\ell_e)^{\nu\mu}\, h^\mu + \varepsilon\,(\ell_o)^{r\ell}\,\theta^\ell\,\rho \right]$$

$$= +r\,(\underline{\theta}\,\ell_w\,\varepsilon)\left[e^\mu\, b + \theta\,\ell_o\, e^\mu\, h\right] -$$

$$\theta_\delta\,\theta_\delta = \frac{1}{2}\,\varepsilon^{\delta\delta}\,\theta_r\,\theta^r$$

$$= \frac{(\omega)^2}{2}\,(\ell_o)^{\delta\delta}\,\underline{\theta}\,\theta$$

$$= -\frac{1}{2}\,(\ell_o)^{\delta\delta}\,(\theta\theta)$$

$$\delta \psi = +\underline{\varepsilon}\, h$$

$$\delta \Phi = -\delta x^m\, e_m{}^\mu\, \Phi - \delta\theta^r\,(e^\theta)\,\Phi$$

$$=$$

$$\delta\theta^r = \varepsilon^r, \quad \delta x^m = -i\,\underline{\varepsilon}\,\ell_w\,\theta = +r\,\underline{\theta}\,\ell_w\,\varepsilon$$

$$\Phi = d + \theta\,\ell_o\, h + \frac{1}{2}\,\theta\,\ell_o\,\theta\,\rho$$

$$\mathcal{L}_{inv} = \frac{1}{2}\partial^\mu \varphi^a \partial_\mu \varphi_a + \frac{i}{2}\bar{\psi}^a \gamma^\mu \partial_\mu \psi^a +$$

$$+ \frac{1}{2} C^a C^a + t_{abcd}\, \varphi^a \varphi^b \left(\frac{1}{3}\varphi^c C^d - \frac{1}{2}\bar{\psi}^c \psi^d\right)!$$

$C^a + \frac{1}{3}$ ✓ в ед. массы

$\dim \varphi = 1/2$, $\dim \psi = 1$, $\dim C = 3/2$, $\dim \theta = -1/2$

$\dim \mathcal{L}_{inv} = 3$, $\dim A_\mu = 1$

$$\sim \bar{D}\Phi^a D\Phi^a + t_{(abcd)}\Phi^a\Phi^b\Phi^c\Phi^d$$

$$D_\alpha = \frac{\partial}{\partial\bar{\theta}_\alpha} + i(\bar{\theta}\gamma^\mu)_\alpha \partial_\mu$$

$\int d^3x\, d^2\theta$ $\dim \mu = -2$

$$\partial^\mu \varphi^a \to \nabla^\mu \varphi^a = \partial_\mu \varphi^a + i (A_\mu)^{ab} \varphi^b$$

$$\mathcal{L}^{\text{м}}_{inv} = \frac{1}{2}(\nabla_\mu \varphi)^2 + \frac{i}{2}\bar{\Psi}\gamma^\mu \nabla_\mu \Psi +$$

$$+ \frac{1}{2} c^a c^a + i\varphi^a(\bar{\chi}^{ab}\psi^b) + f_{abcd}\varphi^a\varphi^b\Big[\frac{1}{3}\varphi^c c^d -$$

$$- \frac{1}{2}\bar{\Psi}^c \psi^d\Big]$$

$$\begin{cases} \delta A_\mu = i\,\bar{\varepsilon}\gamma_\mu \chi \\ \delta \chi = \frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu}\varepsilon \\ \delta\varphi^a = \bar{\varepsilon}\psi^a, \quad \delta\psi^a = -i\gamma^\mu \varepsilon \nabla_\mu \varphi^a + c^a \varepsilon \\ \delta c^a = -i\,\bar{\varepsilon}\gamma^\mu \nabla_\mu \psi^a + i\,\bar{\varepsilon}\chi^{ab}\varphi^b. \end{cases}$$

$$\mathcal{L}_{\text{норм}} = \mathcal{L}_{CS} + \mathcal{L}_{\text{м}(inv)}.$$

Обобщение на $N=2$, $d=3$

$$(A_\mu, \chi) \Longrightarrow (\underset{②}{A_\mu}, \underset{④}{\chi}, \chi^*, \underset{①}{D}, \underset{①}{\sigma})$$

Материя: $d=4 \to d=3$ редукция кир. мул

$$\Phi = \varphi + \bar\theta\psi + (\bar\theta\theta)F \qquad ④+④$$

комп. $\quad$ комп. = ④ $\quad$ комп.

$$\mathcal{L}_{CS} = \mathrm{tr}\left[\left(\varepsilon^{\mu\nu\rho}(A_\mu\partial_\nu A_\rho + \tfrac{2i}{3}A_\mu A_\nu A_\rho\right) - \bar\chi\chi + 2D\sigma\right]$$

$$\mathcal{L}_m = (D_\mu\Phi)_A (D^\mu\bar\Phi)^A + i\bar\psi_A\gamma^\mu(D_\mu\psi)^A +$$

$$+ F_A F^{A*} + \ldots + W_F + W_F^*$$

(... — члены с вспом. полями D и σ)

W_F из $\int d^3x\, d^2\theta\, W(\Phi) + \int d^3x\, d^2\bar\theta\, \bar W(\bar\Phi)$ (4-го порядка)

$N = 8$?

$$\mathcal{L}_m = \partial_\mu \varphi^I \partial^\mu \varphi^I + i \bar{\psi}^A \gamma^\mu \partial_\mu \psi^A$$

$$\delta \varphi^I = \bar{\varepsilon}^{\dot{A}} \Gamma^I_{\dot{A}A} \psi^A$$

$$\delta \psi^A = -i \Gamma^I_{A\dot{A}} \gamma^\mu \partial_\mu \varphi^I \varepsilon^{\dot{A}}$$

$\Gamma^I_{A\dot{A}}$ — $SO(8)$ инв. тензор, связ. 3 независимых представ. 8 группы $SO(8)$ (Триальность)

Что такое аналог $N=4$ SYM для сп. $AdS_4 \otimes S^7$?

$U(N)$ gauge theory с полями материи в присоед. предст. и вместе с кал. полем — в одном мультиплете! Как? Стартовал из $N=2$ описания и добавить нужную материю

– по аналогии с $\mathcal{N}=4$ SYM в $\mathcal{N}=1$ суперполях

$4 + 12 = 16$ ферм.

$$\mathcal{L}_{\mathcal{N}=4} = \mathcal{L}^{\text{вект}}_{\mathcal{N}=1} + \int \bar{\Phi}^A_{\frac{1}{2}} e^{-2V} \Phi^A_{\frac{1}{2}} \, d^4x\, d^4\theta + \quad (L^2)$$

$\Phi^A \to$ фунд.

$$+ \lambda \int d^4x\, d^2\theta\, \varepsilon_{ABC} \operatorname{Tr}\left(\Phi^A_{\frac{1}{2}} \Phi^B_{\frac{1}{2}} \Phi^C_{\frac{1}{2}}\right) + \text{c.c. } \quad (L^3)$$

представл. $SU(3)$

при опр. λ возникает скрытая $SU(4)$ симм.: $SU(3) \times U(1) \to SU(4)$!

В CS случае:

$$\int \lambda\, \varepsilon_{ABCD} \operatorname{Tr}\left(\Phi^A_{\frac{1}{L^{1/2}}} \Phi^B_{\frac{1}{L^{1/2}}} \Phi^C_{\frac{1}{L^{1/2}}} \Phi^D_{\frac{1}{L^{1/2}}}\right) d^3x\, d^2\theta \equiv O\, L^2 \,!$$

$$SU(4) \times U(1)$$