

CPOD, August 23-29, 2010, JINR, Dubna

# Non-equilibrium phase transitions in relativistic heavy-ion collisions

Igor Mishustin

*Frankfurt Institute for Advanced Studies,  
J.W. Goethe Universität, Frankfurt am Main  
Kurchatov Institute, Russian Research Center,  
Moscow*



# Importance of fast dynamics

I.N. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. AA681 (2001) 56

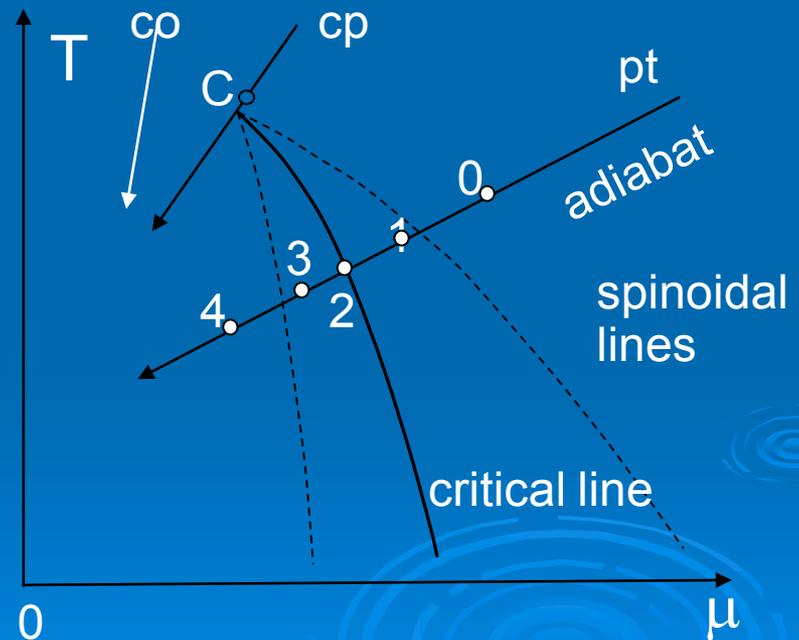
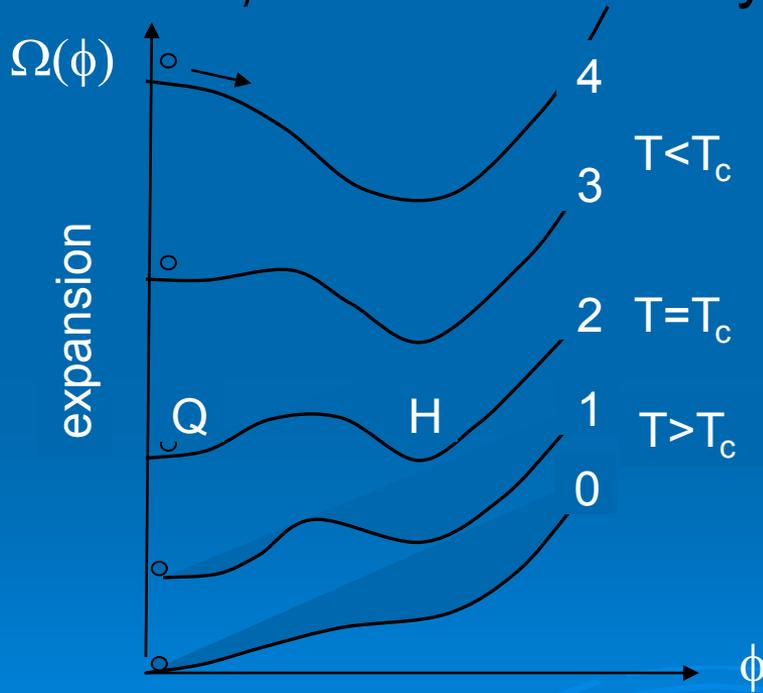
Effective thermodynamic potential leading to a 1<sup>st</sup> order transition

$$\Omega(\phi; T, \mu) = \Omega_0(T, \mu) + \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4 + \frac{c}{6}\phi^6$$

$a, b, c$  are functions of  $T$  and  $\mu$

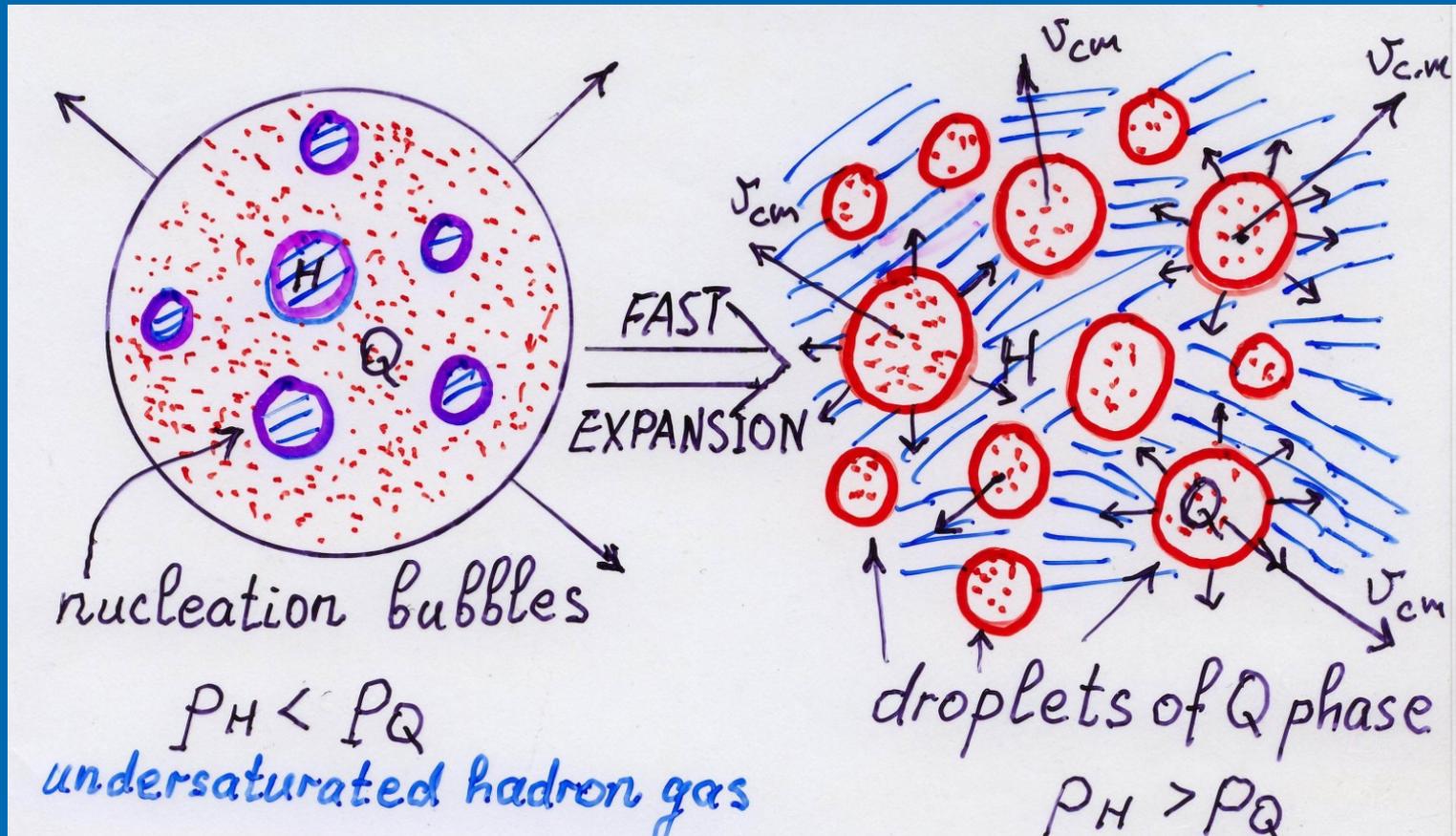
$$\frac{\partial \Omega}{\partial \phi} = 0 \Rightarrow P = -\Omega(\phi_{eq})$$

Equilibrium  $\phi$  is determined by



In rapidly expanding system 1-st order transition is delayed until the barrier between two phases disappears

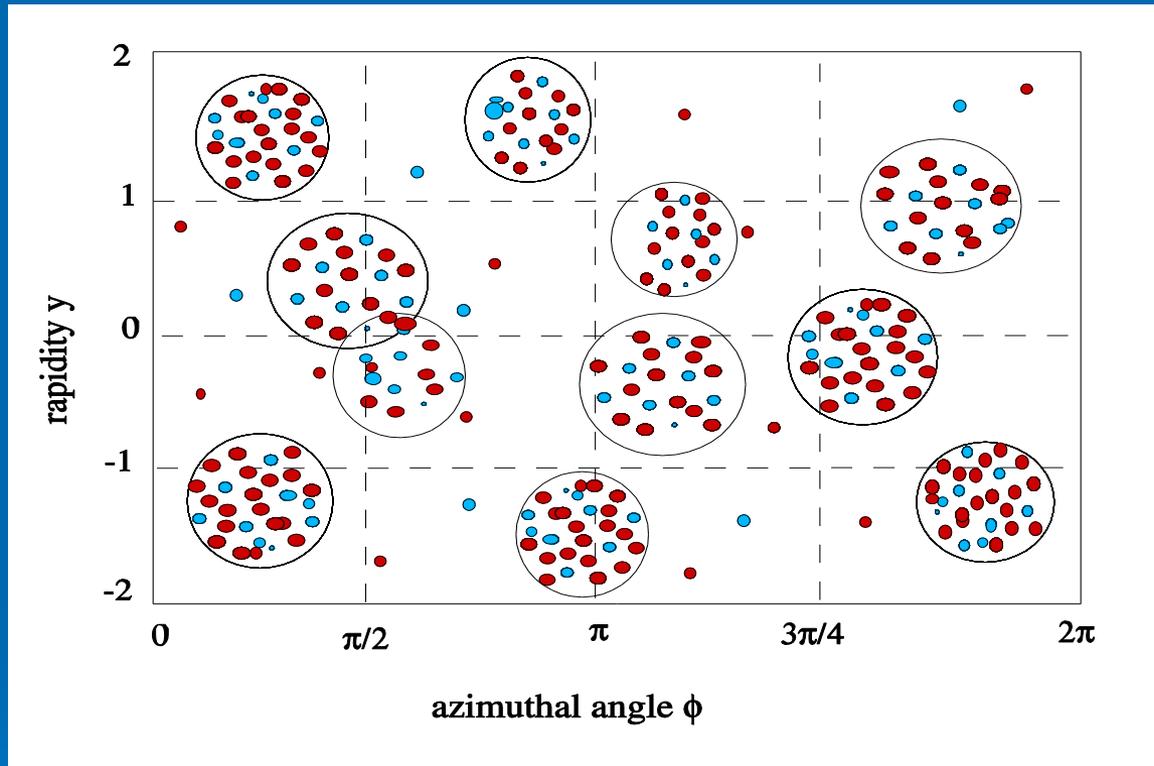
# Dynamical fragmentation 1



In the course of fast expansion the system enters spinodal instability when Q phase becomes unstable and splits into quark droplets/hadron resonances  
Csernai&Mishustin 1995, Mishustin 1999, Rafelski et al. 2000, Randrup et al.2003

Extreme possibility - direct transition from quarks to hadrons without mixed phase

# Experimental signal of droplets in the rapidity-azimuthal angle plane



Look for event-by-event fluctuations of hadron distributions in momentum space associated with emission from quark droplets. Such measurements should be done in the broad energy range!

# Critical slowing down in the 2<sup>nd</sup> order phase transition

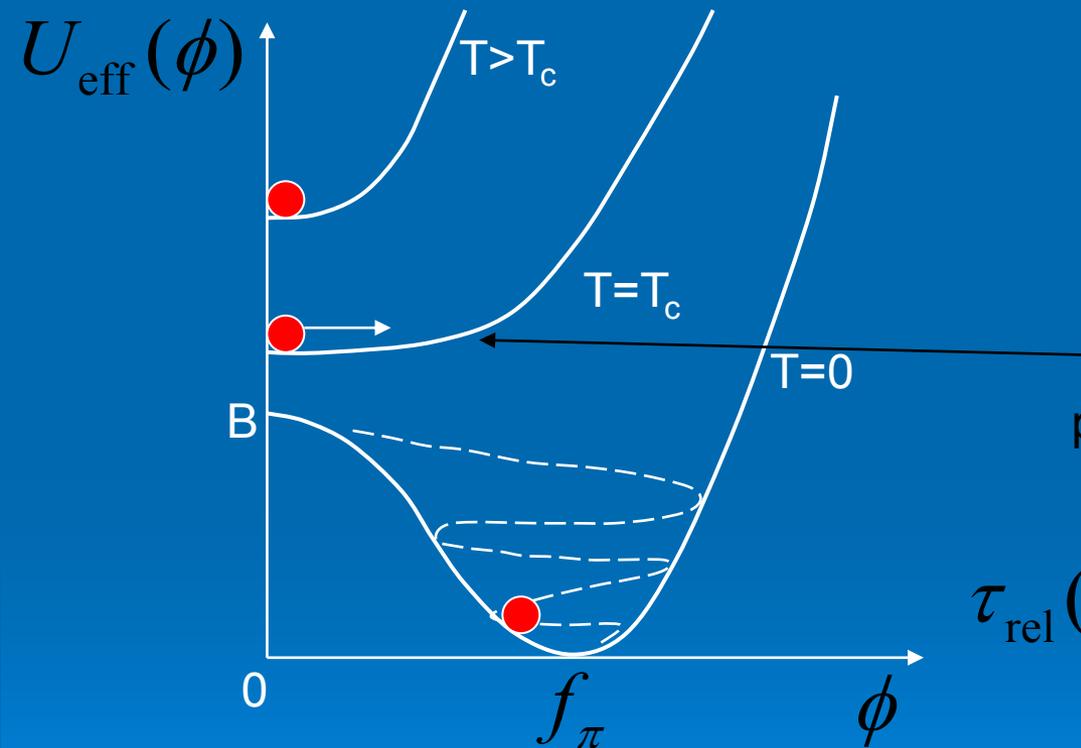
Fluctuations of the order parameter evolve according to the equation

$$\frac{d\delta\phi}{dt} = -\gamma \frac{\partial\Omega}{\partial\phi} \approx -\frac{\delta\phi}{\tau_{\text{rel}}}$$

In the vicinity of the critical point the relaxation time for the order parameter diverges-no restoring force

$$\tau_{\text{rel}}(T) \sim \frac{1}{|T - T_c|^\nu} \rightarrow \infty, \quad \nu \approx 2$$

Critical fluctuations do not develop due to the “critical slowing down” (Berdnikov, Rajagopal)



Transition goes through the spinodal decomposition (Csernai&Mishustin 1995)

# Dynamical fluctuations of the order parameter

Work in collaboration with M. Nahrgang, M. Bleicher, C. Greiner, S. Leupold,...

Linear sigma model (LσM) with constituent quarks:

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5\boldsymbol{\tau}\boldsymbol{\pi})]q + \frac{1}{2}[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi}),$$

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \quad \langle\sigma\rangle_{\text{vac}} = f_\pi \rightarrow H = f_\pi m_\pi^2$$

Mean  $\sigma$  field is the order parameter for the chiral phase transition.

Thermodynamics of LσM on the mean-field level was studied in Scavenius, Mocsy, Mishustin&Rischke, Phys. Rev. C64 (2001) 045202

CO, 2<sup>nd</sup> and 1<sup>st</sup> order chiral transitions are obtained in T- $\mu$  plane.

Here we consider  $\mu=0$  system by tune the strength of the chiral phase transition by changing the coupling constant  $g$ .

# Chiral fluid dynamics (CFD)

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134

Fluid is formed by constituent quarks and antiquarks with  
interact with the chiral field via effective mass

$$m^2 = g^2 (\sigma^2 + \pi^2)$$

CFD equations are obtained from the energy momentum  
conservation

$$\partial_\nu (T_{\text{fluid}}^{\mu\nu} + T_{\text{field}}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}^{\mu\nu} = -\partial_\mu T_{\text{field}}^{\mu\nu} = -(\partial_\mu \partial^\mu + \frac{\partial U}{\partial \sigma}) \partial^\nu \sigma$$

We solve generalized e. o. m. with friction ( $\eta$ ) and noise ( $\xi$ ):

$$\partial_\mu \partial^\mu \sigma + \frac{\partial U}{\partial \sigma} + g \langle \bar{q}q \rangle + \eta \partial_t \sigma = \xi$$

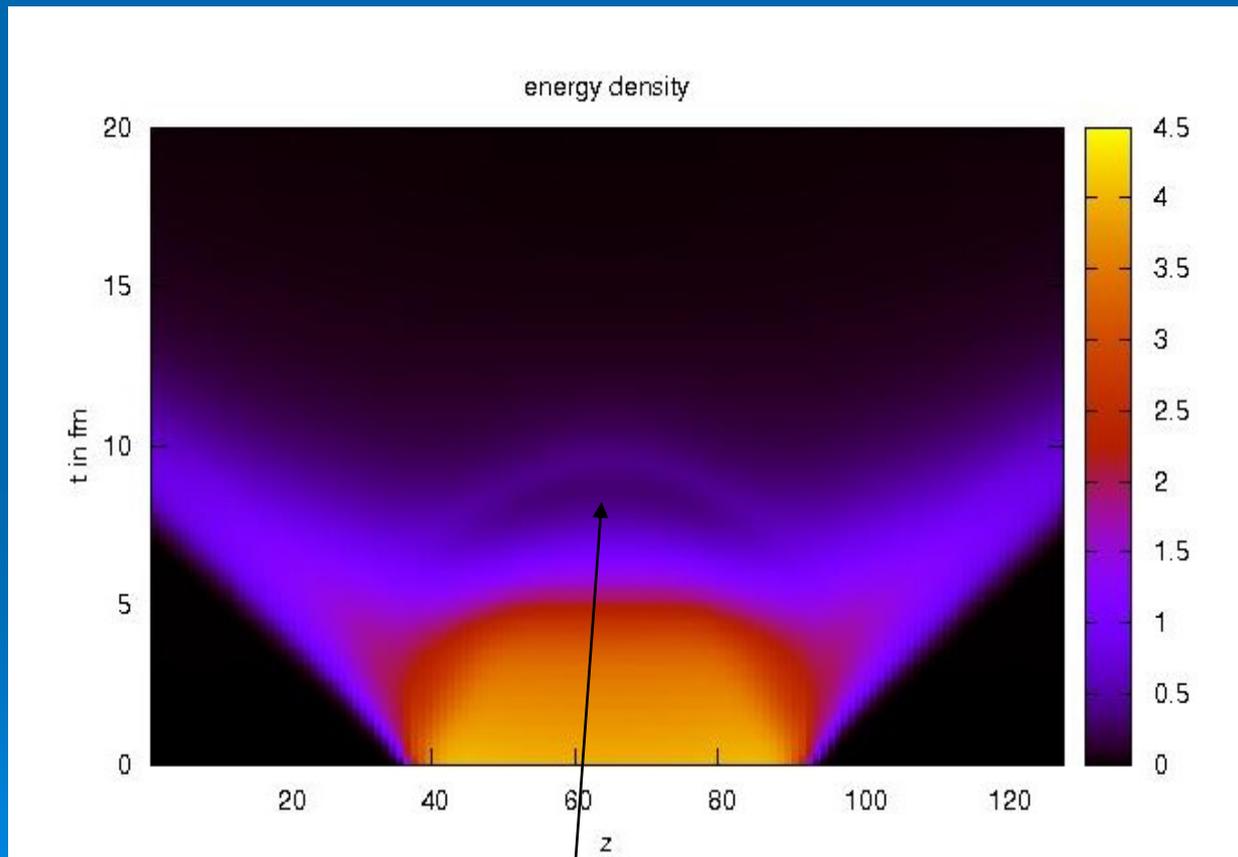
(relaxation equation)

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t) \xi(t') \rangle = \frac{2T}{V} \eta \delta(t - t')$$

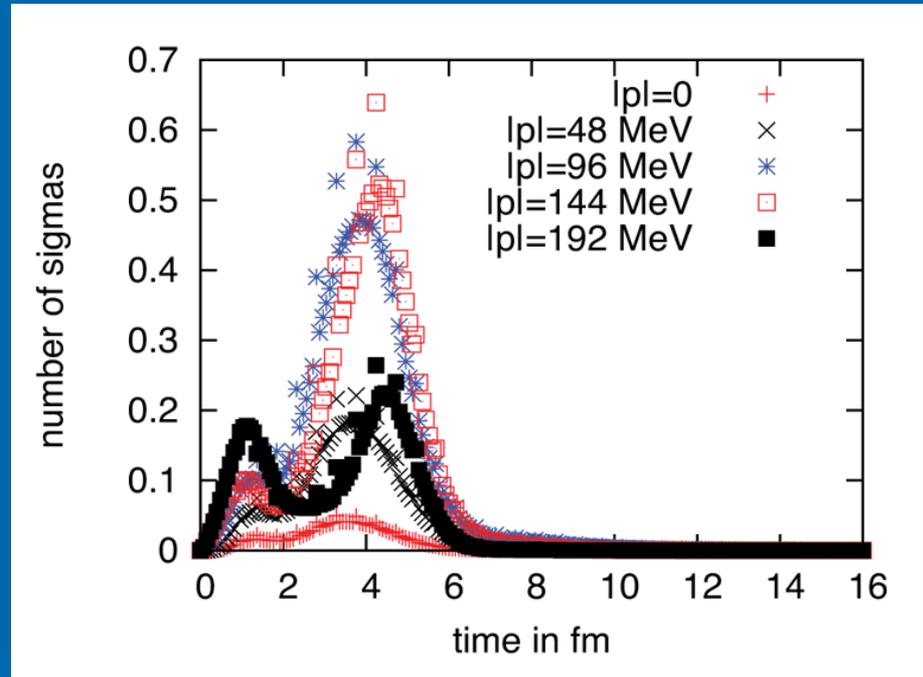
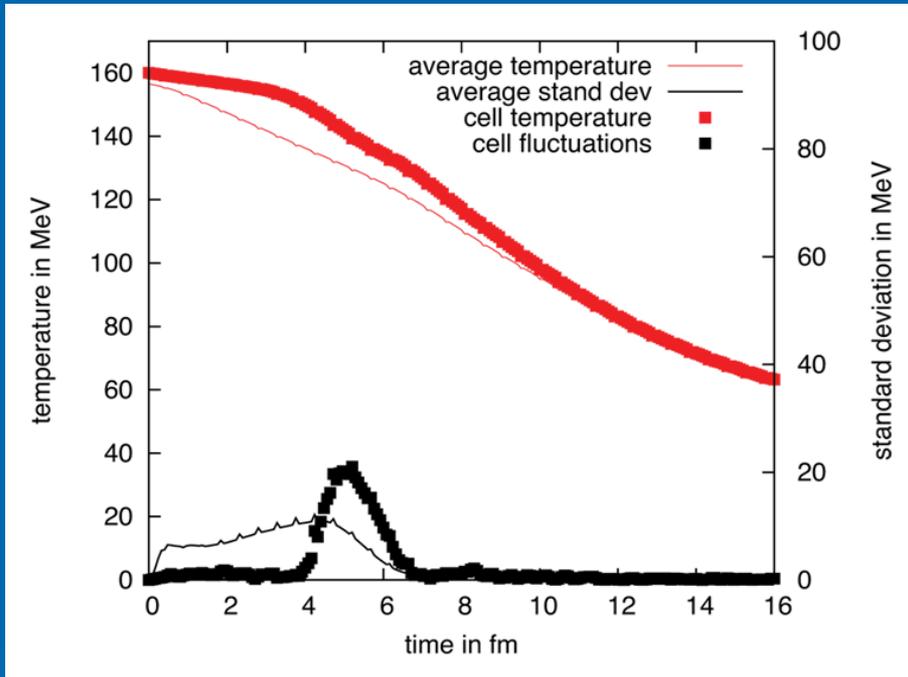
(fluctuation-dissipation  
theorem)

# Dynamical evolution of chiral fluid in Bjorken-like expansion

$$\eta = 2.2 / fm, \quad g=5.5 \text{ (1st order transition)}$$



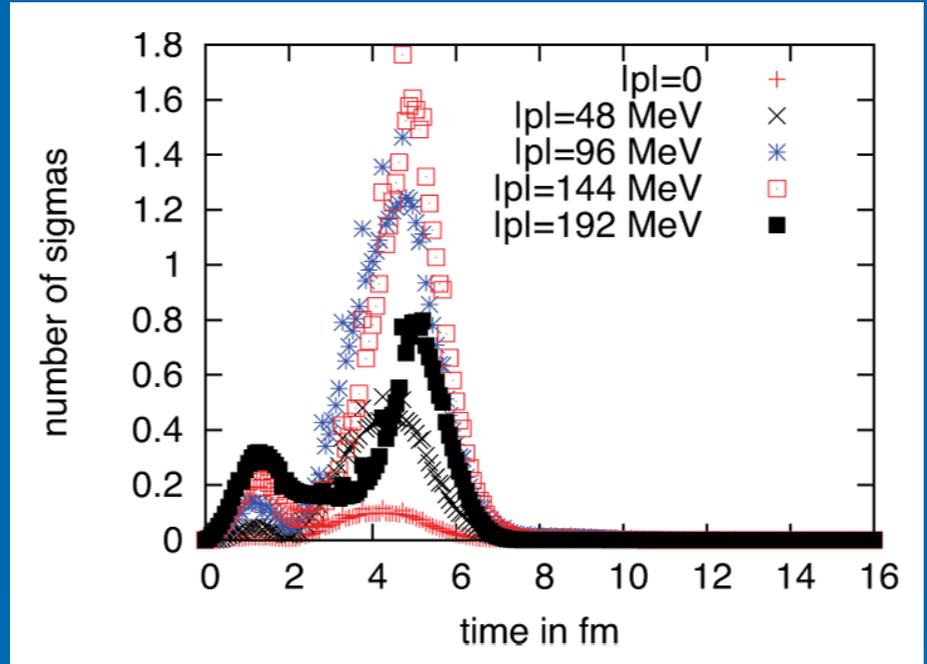
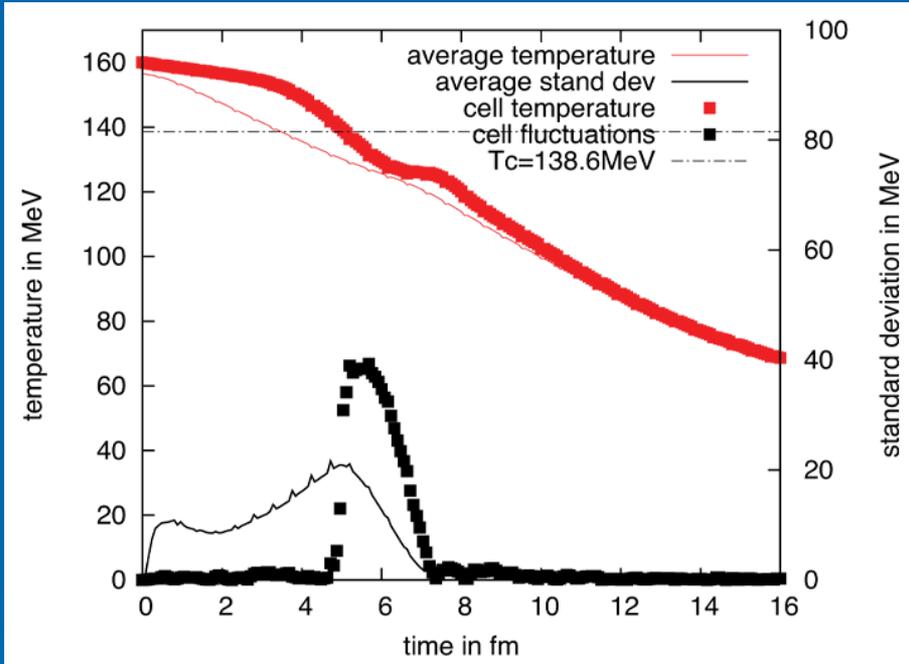
# Crossover transition



Fluctuations are rather weak, effective number of sigmas is small

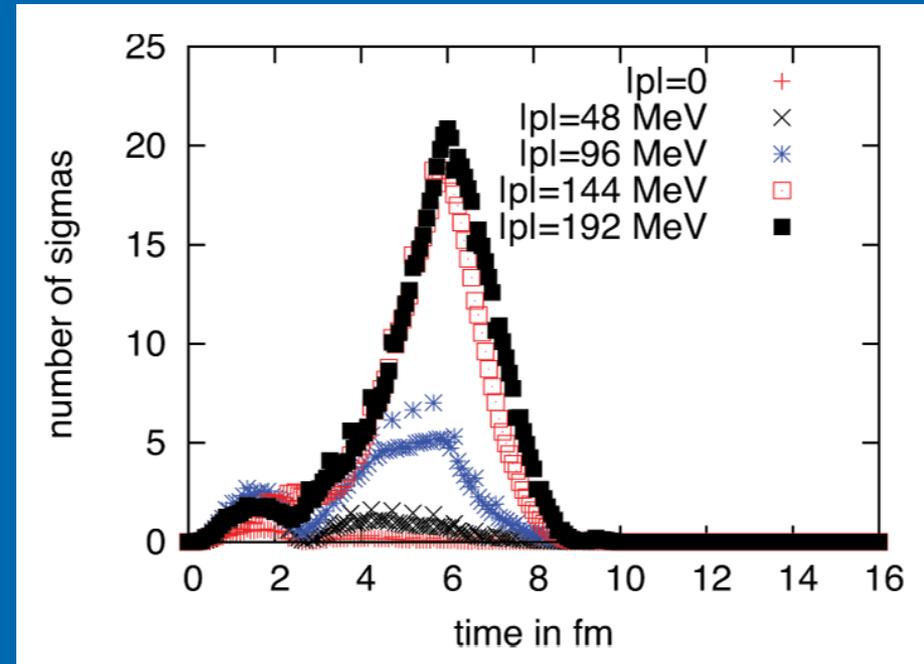
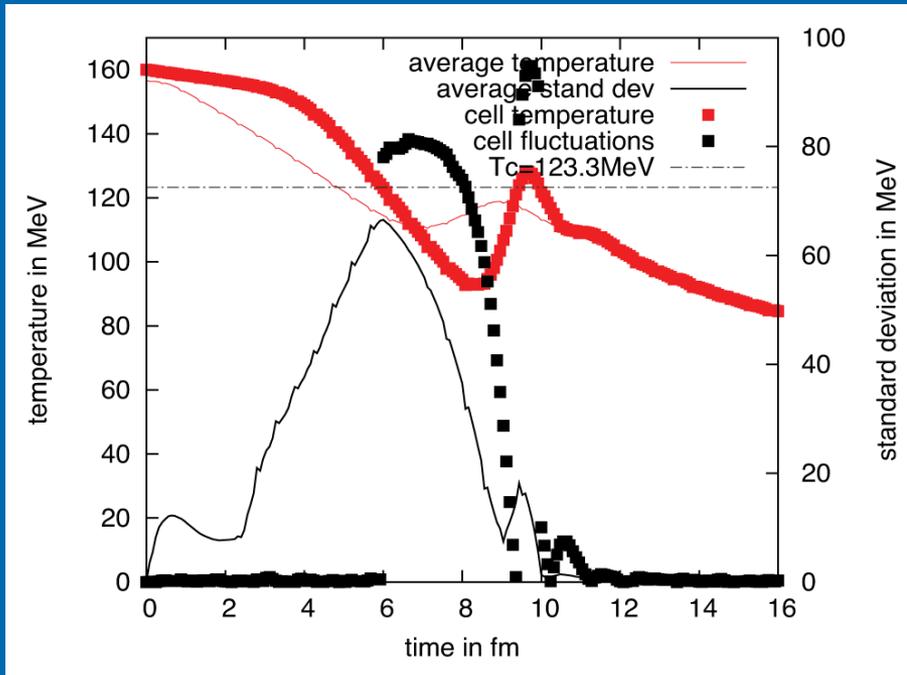
$$\frac{dN_{\sigma}}{d^3k} = \frac{1}{(2\pi)^3} \frac{1}{2\omega_k} [\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2]$$

# Second order transition with critical point (g=3.63)



Fluctuations are stronger, but no traces of divergence are seen

# Strong first order transition ( $g=5.5$ )



- ➔ Strong supercooling and reheating effects are clearly seen.
- ➔ Sharp rise of fluctuations after 6 fm/c, when the barrier in thermodynamic potential disappears. Effective number of sigmas increases by two orders of magnitude!

# Conclusions

- Phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium
- 2<sup>nd</sup> order phase transitions (with CEP) are too weak to produce significant observable effects
- Non-equilibrium effects in a 1<sup>st</sup> order transition (spinodal decomposition, strong fluctuations of order parameter) may help to identify the phase transition

