AdS/QCD at finite density and temperature

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Outline

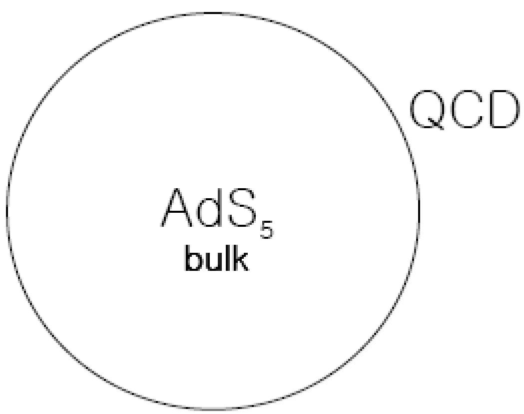
1. AdS/QCD in free space
2. Thermal AdS/QCD
3. Dense AdS/QCD
4. Summary

String Theory in higher dim. $\Leftrightarrow$ Gauge Theory in 4 dim.

Weakly coupled $\Leftrightarrow$ Strongly coupled

Maldacena 98

AdS/QCD:



$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

$$\text{4D generating functional} : Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i \int \phi_0(x)\mathcal{O}\},$$

$$\text{5D (classical) effective action} : \Gamma_5[\phi(x,z) = \phi_0(x)] ; \phi_0(x) = \phi(x, z = 0).$$

$$\text{AdS/CFT correspondence} : Z_4 = \Gamma_5.$$

AdS/CFT Dictionary

- 4D CFT (QCD) ←→ 5D AdS
- 4D generating functional ←→ 5D (classical) effective action
- Operator ←→ 5D bulk field
- [Operator] ←→ 5D mass
- Current conservation ←→ gauge symmetry
- Large Q ←→ small z
- Confinement ←→ Compactified z
- Resonances ←→ Kaluza-Klein states

AdS/QCD in free space

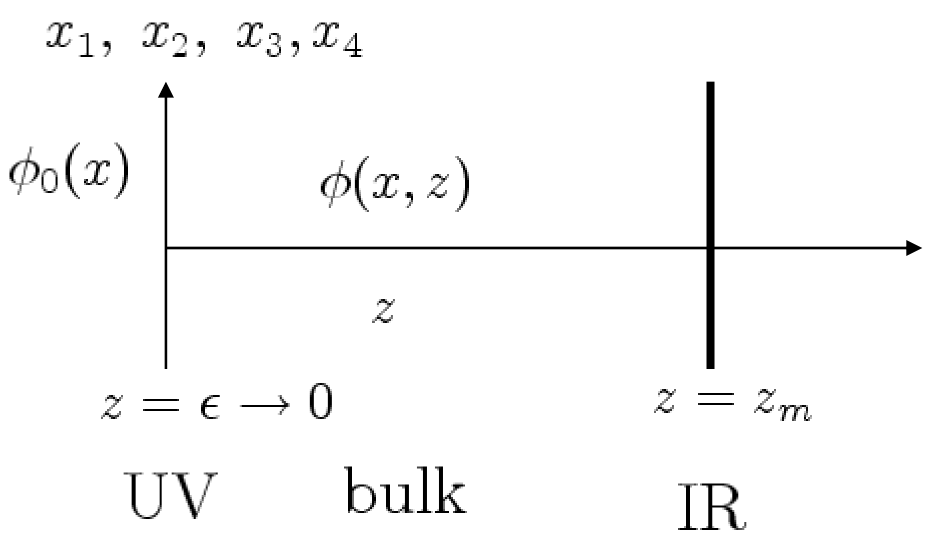
(hard wall model: dual to low energy QCD with N_f)

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)

$$S_I = \int d^4x dz \sqrt{g} \mathcal{L}_5 ,$$

$$\mathcal{L}_5 = \mathrm{Tr}\left[-\frac{1}{4g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) + |D_M\Phi|^2 - M_\Phi^2|\Phi|^2 \right] ,$$



☆ 5D field contents

Operator → 5D bulk field

$$\bar{q}_R q_L \longrightarrow \Phi(x,z)$$
$$\bar{q}_L \gamma^\mu q_L \longrightarrow L_M(x,z)$$
$$\bar{q}_R \gamma^\mu q_R \longrightarrow R_M(x,z)$$

[Operator] → 5D mass

$$(\Delta - p)(\Delta + p - 4) = m_5^2 \qquad m_\phi^2 = -3$$

Where is the chiral condensate?

Klebanov and Witten, 1999

$$\phi(x,z) \to z^{d-\Delta}\phi_0(x) + z^{\Delta}A(x) + \ldots, z \to \epsilon\,,$$

where $\phi_0(x)$ is the source term of 4D operator $\mathcal{O}(x)$, and

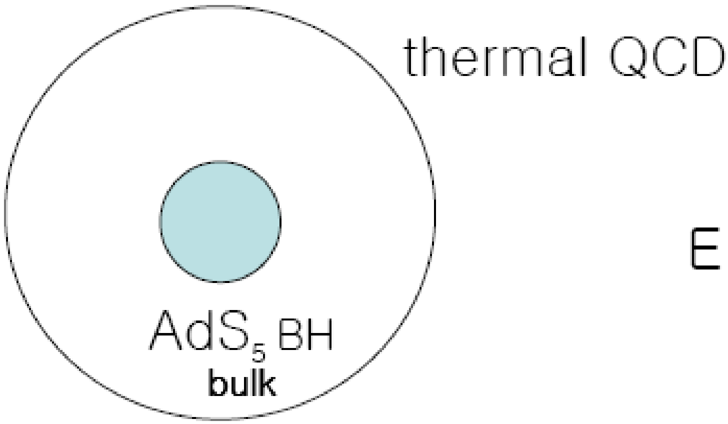
$$A(x) = \frac{1}{2\Delta - d}\langle \mathcal{O}(x)\rangle\,.$$

For example, $\mathcal{O} = \bar{q}q$, $\phi(x,z) = v(z)$:

$$v(z) = c_1 z + c_2 z^3$$
$$c_1 \sim m_q, \quad c_2 \sim \langle \bar{q}q\rangle\,.$$

Note, however, that we cannot calculate the value of the chiral condensate in bottom-up.

Hot AdS/QCD



E. Witten (1998)

$$ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \qquad f(z) = 1 - \frac{z^4}{z_h^4}$$

$$T = \frac{1}{\pi z_h}$$

That simple?

Hawking-Page transition in a cut-off AdS_5

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998),
C. P. Herzog, Phys. Rev. Lett.98, 091601 (2007)

$$I = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left(R + \frac{12}{L^2}\right) .$$

1. thermal AdS:

$$ds^2 = L^2 \left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2} \right)$$

β' : the periodicity in the timedirection, (undetermined)

2. AdS black hole: $f(z) = 1 - \frac{z^4}{z_h^4}$ $T = \frac{1}{\pi z_h}$

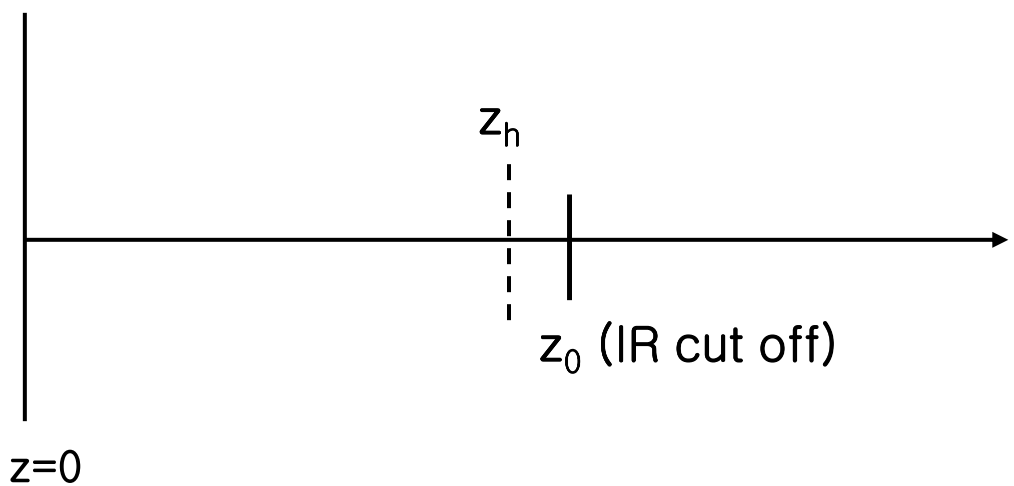
$$ds^2 = \frac{L^2}{z^2} \left(f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right) \qquad 0 \leq t < \pi z_h$$

Transition between two backgrounds ⟷ (De)confinement transition

$$\beta' = \pi z_h \sqrt{f(\epsilon)}$$

$$\begin{aligned}\Delta V &= \lim_{\epsilon \to 0} \left(V_2(\epsilon) - V_1(\epsilon) \right) \\ &= \begin{cases} \frac{L^3 \pi z_h}{\kappa^2} \frac{1}{2 z_h^4} & z_0 < z_h \\ \frac{L^3 \pi z_h}{\kappa^2} \left(\frac{1}{z_0^4} - \frac{1}{2 z_h^4} \right) & z_0 > z_h \end{cases}\end{aligned}$$

$$T_c = 2^{1/4} / (\pi z_0)$$

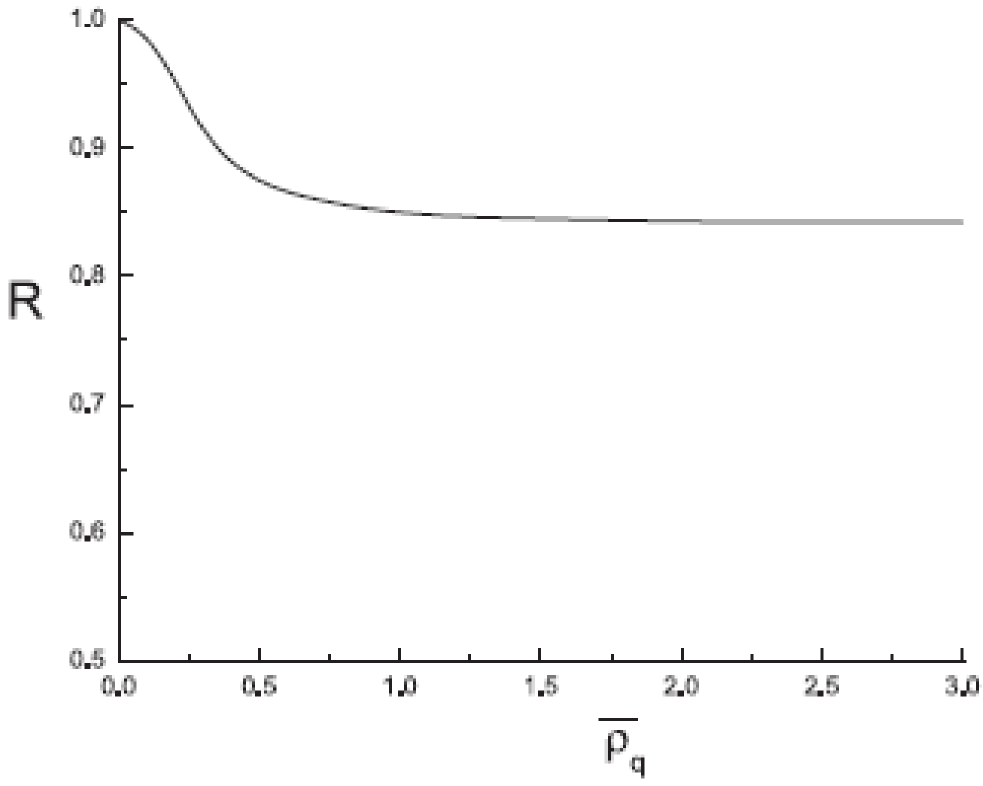


Consequences of the Hawking-Page transition

- Low temperature: no stable AdS black hole. thermal AdS background (Euclideanized zero temp. background)
- High temperature: AdS black hole
- No temperature dependence at low temperature without N_c corrections, which is consistent with large N_c finite temp. QCD.
- Therefore, in AdS/QCD, both bottom-up and top-down, you may not be able to do much with temperature dependence of observables in confined phase.
- The (De)confinement transition is first order.

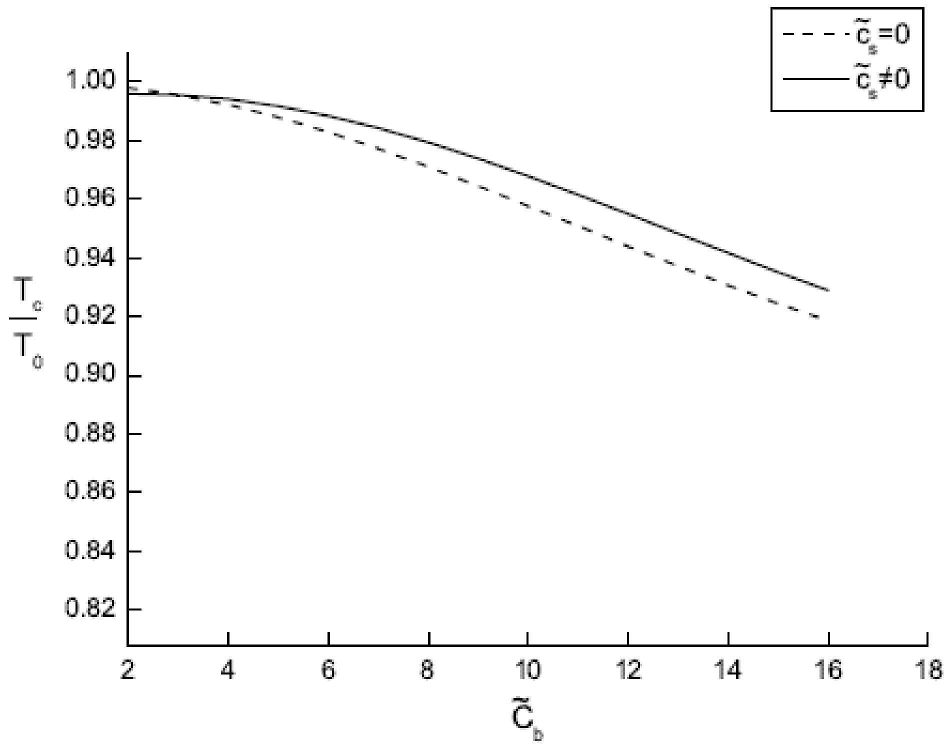
Example: critical temperature with finite density

Y. Kim, et al, PRD 2007.



Example: Critical temperature with strangeness

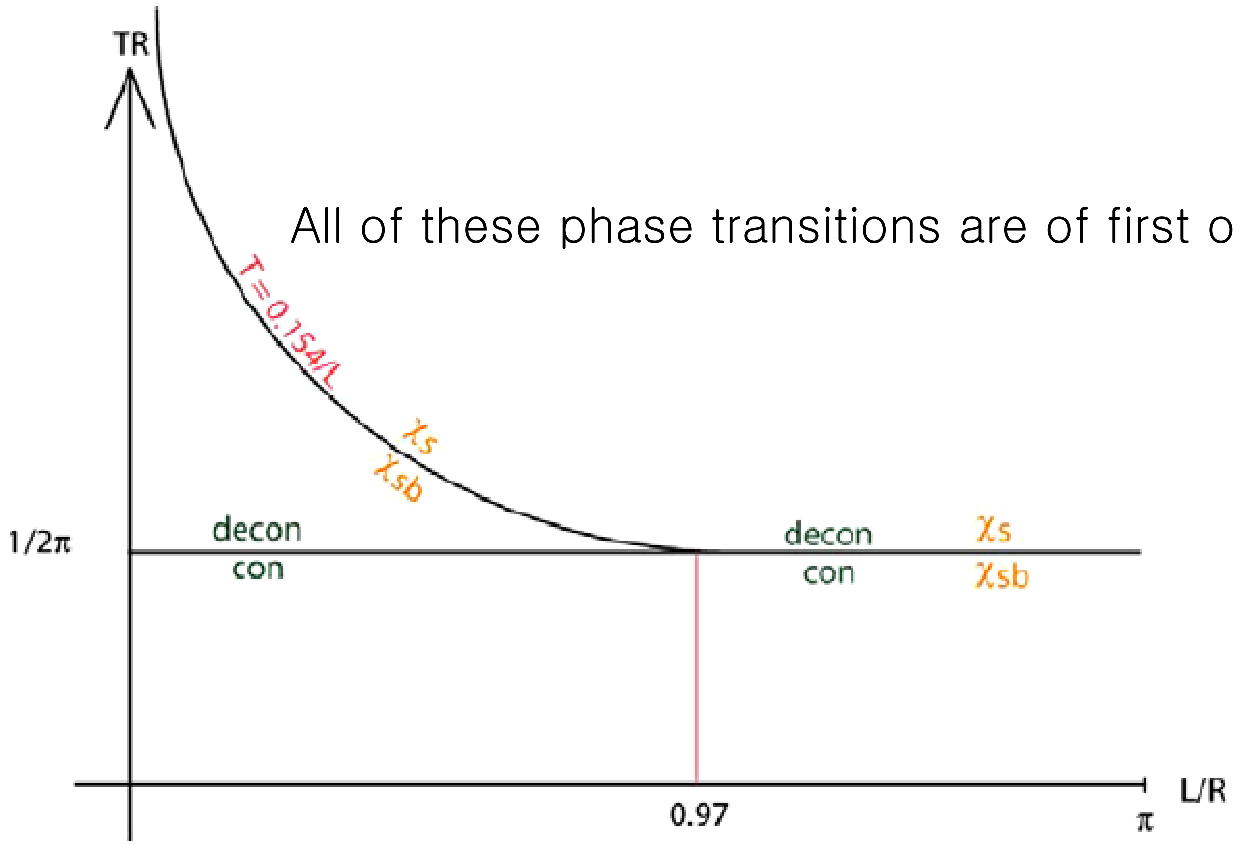
K.-I. Kim, Y. Kim, S. H. Lee, e-Print: arXiv:0709.1772 [hep-ph]



The critical temperature at finite strange density. The solid line is for $\tilde{c}_s \neq 0$ while the dashed line is for $\tilde{c}_s = 0$, where $\tilde{c}_s = c_s z_m^3$ and $\tilde{c}_b = c_b z_m^3$.

Example: Sakai-Sugimoto model at finite temperature

O. Aharony et al. / Annals of Physics 322 (2007) 1420–1443



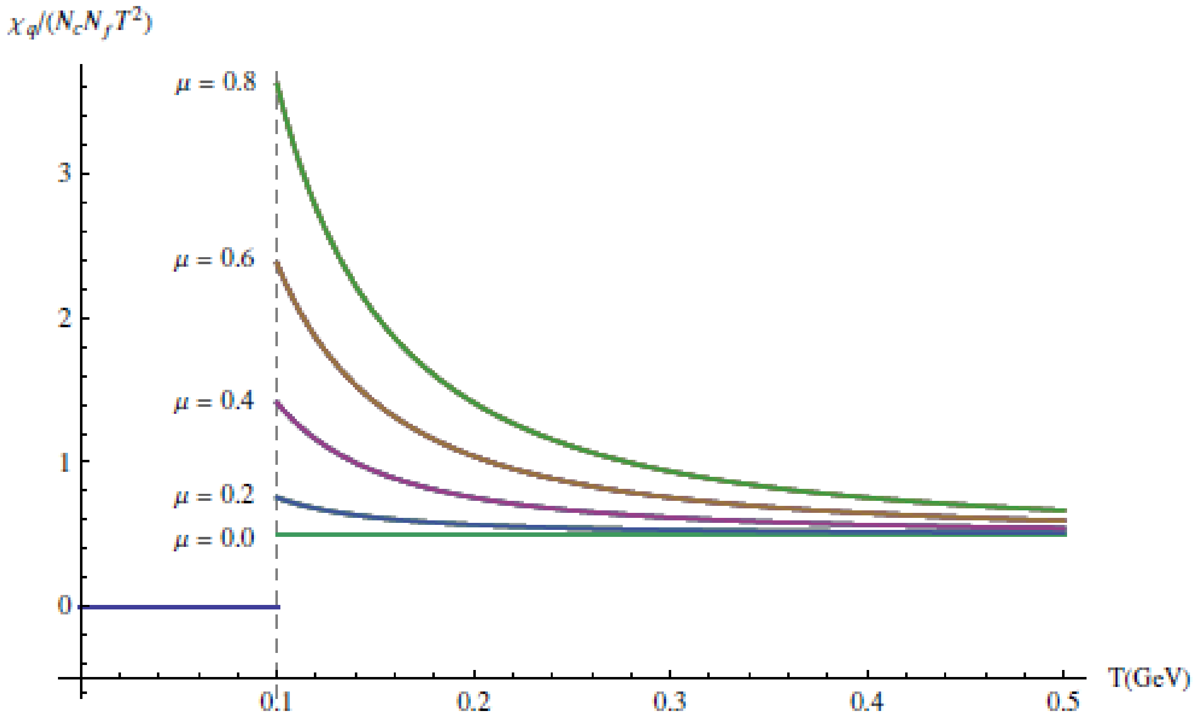
All of these phase transitions are of first order

R : compactification scale (on a circle of radius)

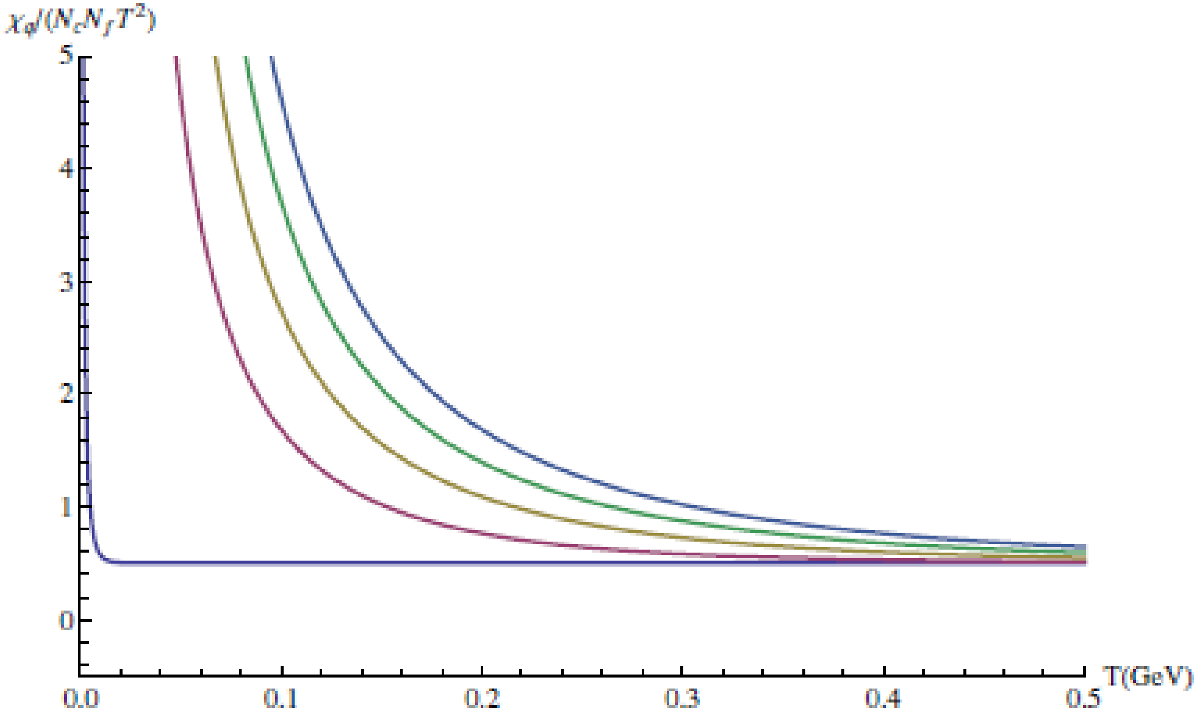
L: separation distance of D8 and anti-D8 branes

Example: quark number susceptibility

Y. Kim, Y. Matsuo, W. Sim, S. Takeuchi, T. Tsukioka, JHEP 1005:038,2010.



$\chi_q/(N_c N_f T^2)$ in the hard wall model for varying μ(GeV) with $N_c = 3$ and $N_f = 2$.



$\chi_q/(N_c N_f T^2)$ for varying $\widetilde{B} = 2\pi\alpha' B$ with $N_c = 3$ and $N_f = 2$. Here $\tilde{B} = (0, 1, 2, 3, 4)$

Existence of CEP in (T, mu) and (T, B) plane?

Dense AdS/QCD

$$\mathrm{QCD}: \ \mu_q \psi^\dagger \psi \ (= \mu_q \bar{\psi}\gamma_0 \psi) \leftrightarrow \mathrm{Gravity}: \ V_0(x,z) \to \bar{V}_0 = \mu_q + \cdots, \ \ z \to 0\,,$$

$$V_0(x,z) = \bar{V}_0 + V_0'(x,z)\,.$$

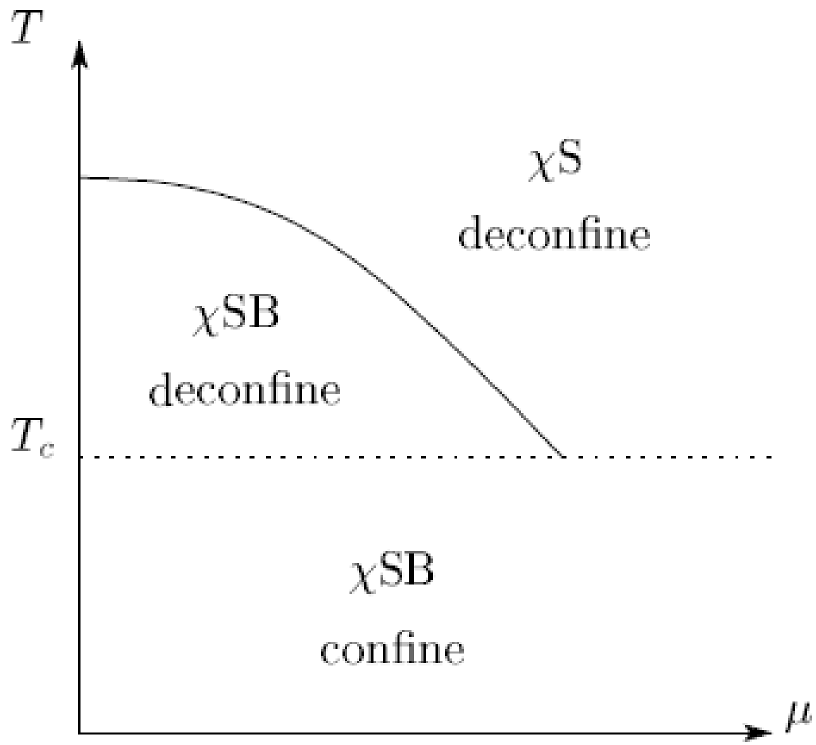
$$S_X = \int d^5x \left[\sqrt{g}|DX|^2 + 3|X^2|\right]\,,\ D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu}.$$

$$\left[\partial_z^2 - \frac{3}{z}\partial_z + \frac{3}{z^2}\right]X_0 = 0, \quad X_0 = c_1 z + c_2 z^3\,.$$

$$\delta S_X \sim |X|^2(F_L^2 + F_R^2) \longrightarrow (\partial_z \bar{V}_0)^2 X_0 \ \text{in the EoM for}\ X_0$$

Example: Sakai-Sugimoto model with finite chemical potential

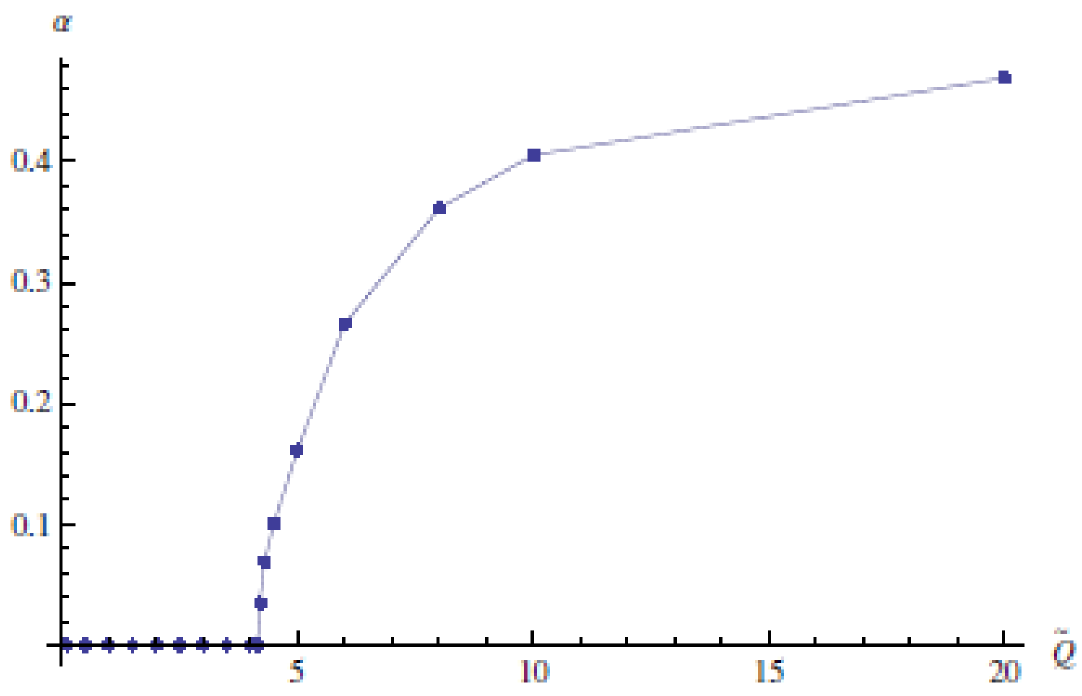
N. Horigome and Y. Tanii, JHEP 0701:072,2007.



The phase diagram of the dual gauge theory.

Example: Nuclear to strange matter transition in D4/D6/D6 model

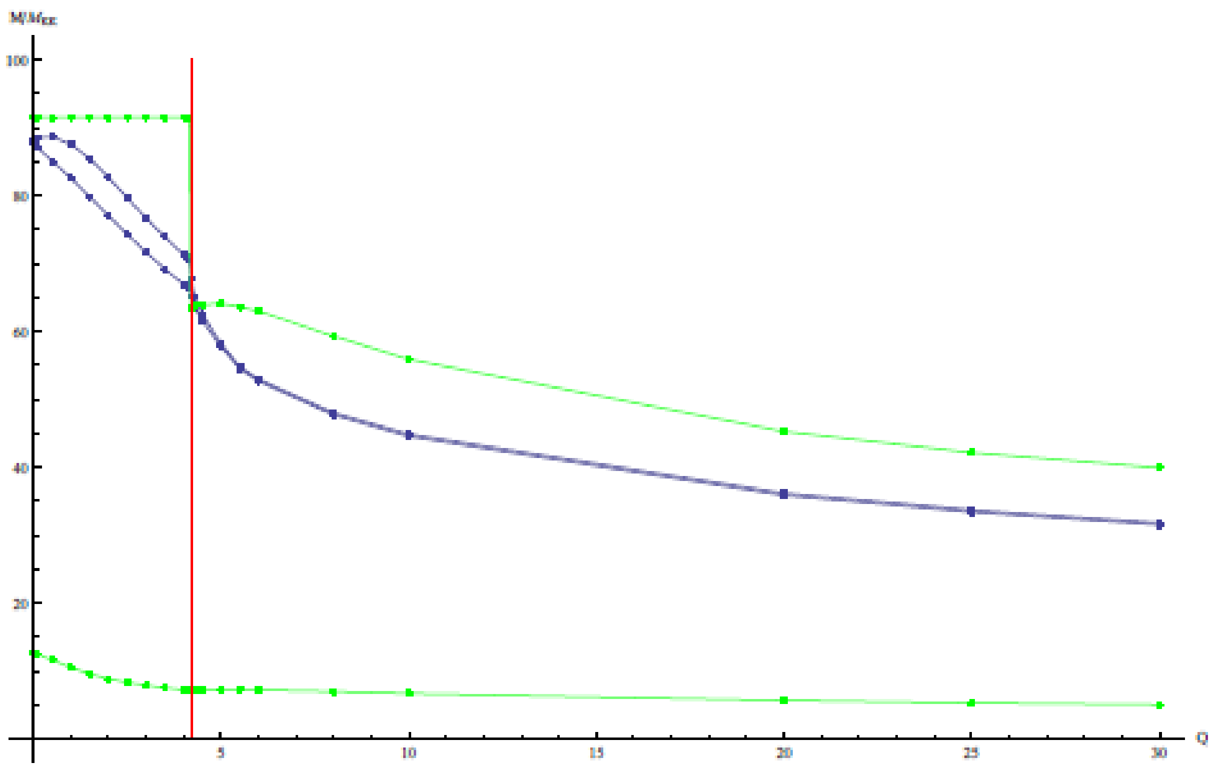
Y. Kim, Y. Seo, and S.-J. Sin, JHEP 1003:074,2010.



Density dependence of α, the fraction of the strange quarks.

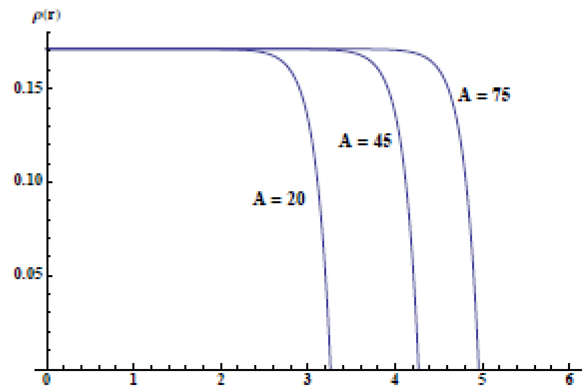
Example: Meson mass in asymmetric dense matter

Y. Kim, Y. Seo, I. J. Shin, and S.-J. Sin, to appear.

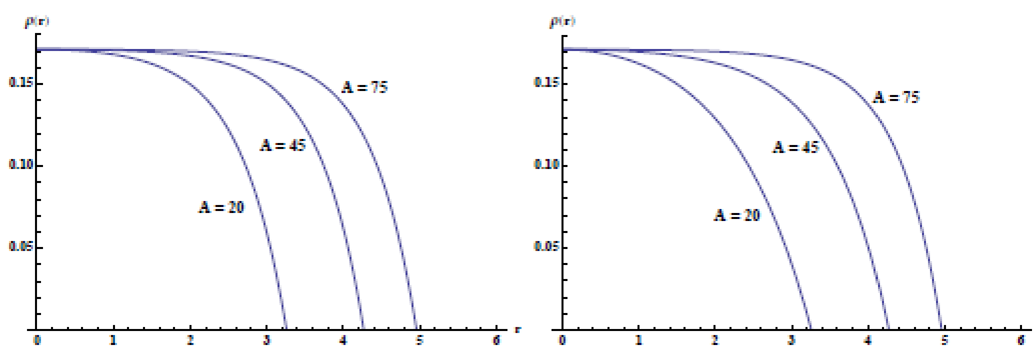


Example: self-bound dense objects in hQCD

K. K. Kim, Y. Kim, Y. Ko, e-Print: arXiv:1007.2470 [hep-ph]



Nucleon density distribution in nuclei obtained from holographic QCD with z_m fixed in the hard wall model, where $1/z_m \sim 320$ MeV.



Nucleon density as a function of the distance to the center of the nucleus obtained from holographic QCD: on the left with fixed z_m, $1/z_m \sim 100$ MeV, on the right $1/z_m \sim 60$ MeV for $A = 20$ and $1/z_m \sim 80$ MeV for $A = 45$ and $1/z_m \sim 100$ MeV for $A = 70$.

Summary

- In-medium AdS/QCD (or holographic QCD) seems fine with large N_c QCD.
- To be more QCD-like, collecting all large N_c corrections consistently is essential.