Propagation of spectral functions and dilepton production at SIS energies

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- Motivation
- Dileptons
- Time evolution of spectral functions
- Dilepton production at HADES
- Summary

Why dileptons

- measured (DLS, HADES)
- without finalstate interaction
- vector mesons decay to dileptons $\rightarrow$ vector mesons in matter

Dilepton production in Heavy ion Collisions

Sources of dileptons

- $NN \rightarrow \ldots \rightarrow NNe^+e^-$ (measurable)
- $\pi^+\pi^-$ annihilation (measurable and theoretically well understood)
- other secondaries (πN, or NΔ $\rightarrow$NR$\rightarrow$ NNe^+e^-)

Strategy 1: put the measured $NN \rightarrow NNe^+e^-$, $\pi^+\pi^- \rightarrow e^+e^-$ and the estimated cross secton for the secondaries to a transport and obtain the HIC result.

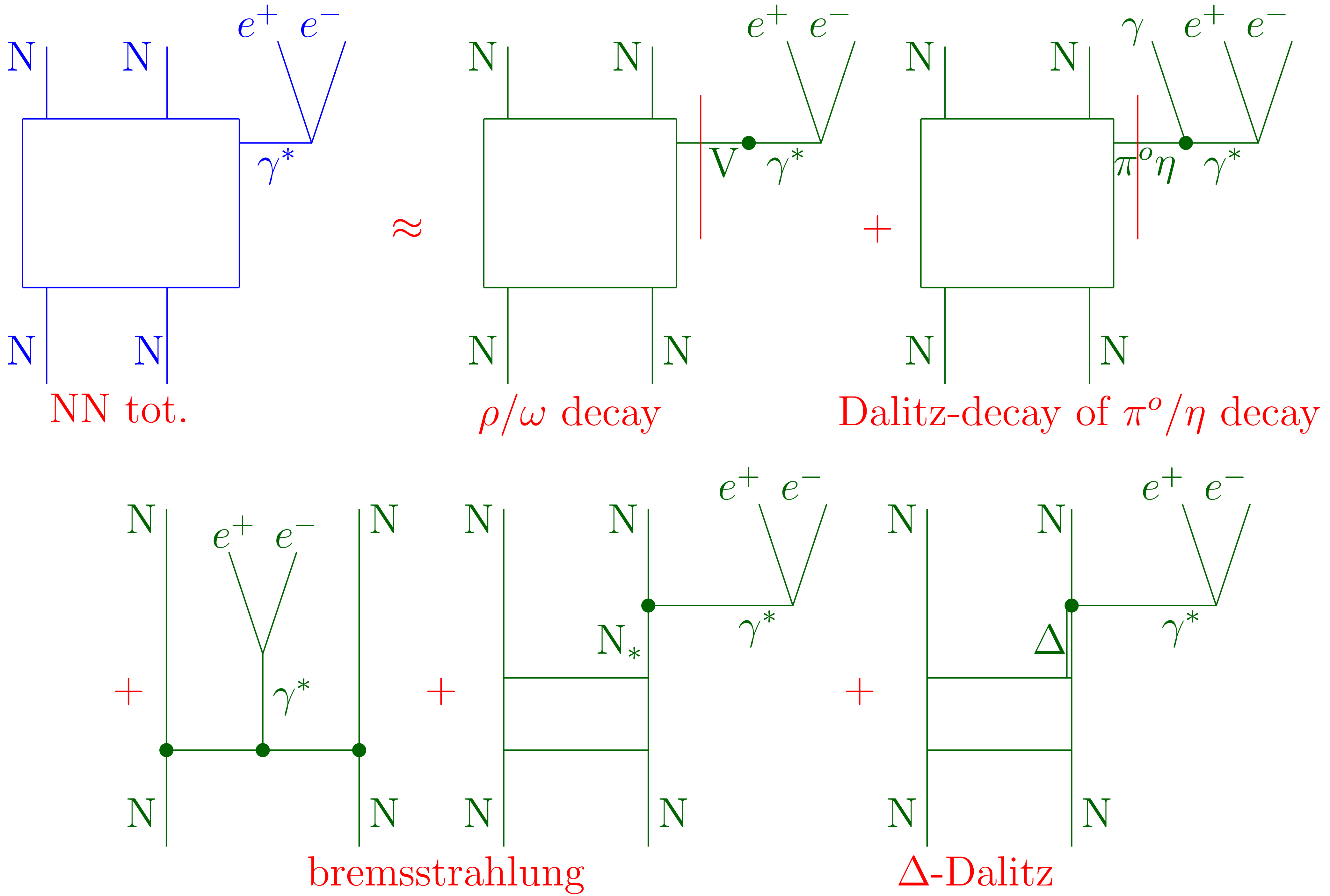
Problem:

Hunted in-medium effects are buried in the $NN \rightarrow NNe^+e^-$ cross section

Dilepton production in NN

- Direct decay of vector mesons and η
- Dalitz-decay of π, η and ω
- Dalitz-decay of baryon resonances
  Zetenyi, Wolf, Phys. Rev, C67 (2003) 044002;
  Heavy Ion Phys. 17 (2003) 27
- pn bremsstrahlung (not negligible)

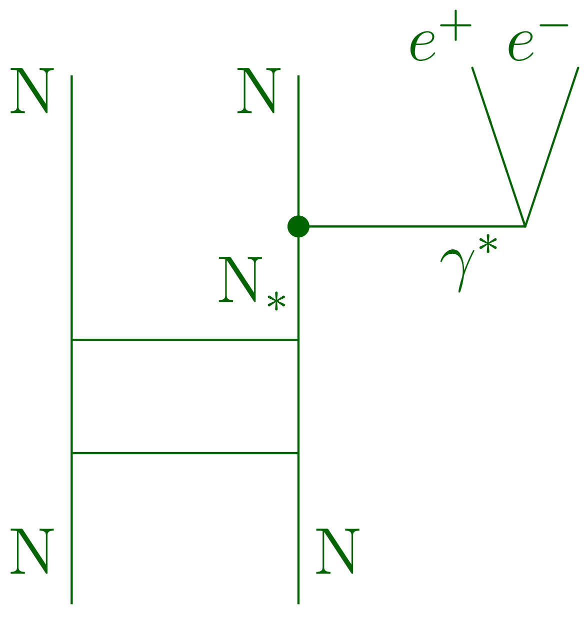
Dilepton Channels in NN



Bremsstrahlung calculations

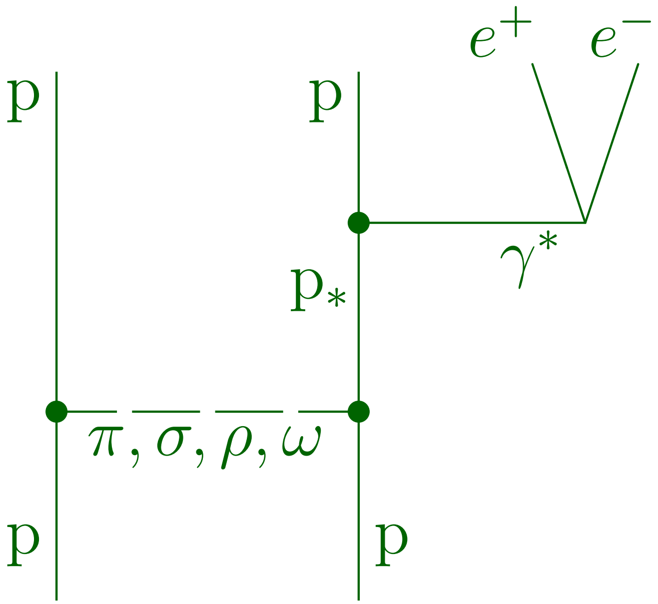
Soft photon approximation

$$\frac{d\sigma}{dM} = \frac{\sigma}{M}\,\frac{\alpha^2}{6\pi^3}\int\frac{d^3q}{q_0^3}\,\frac{R_2(\bar{s})}{R_2(s)}.$$

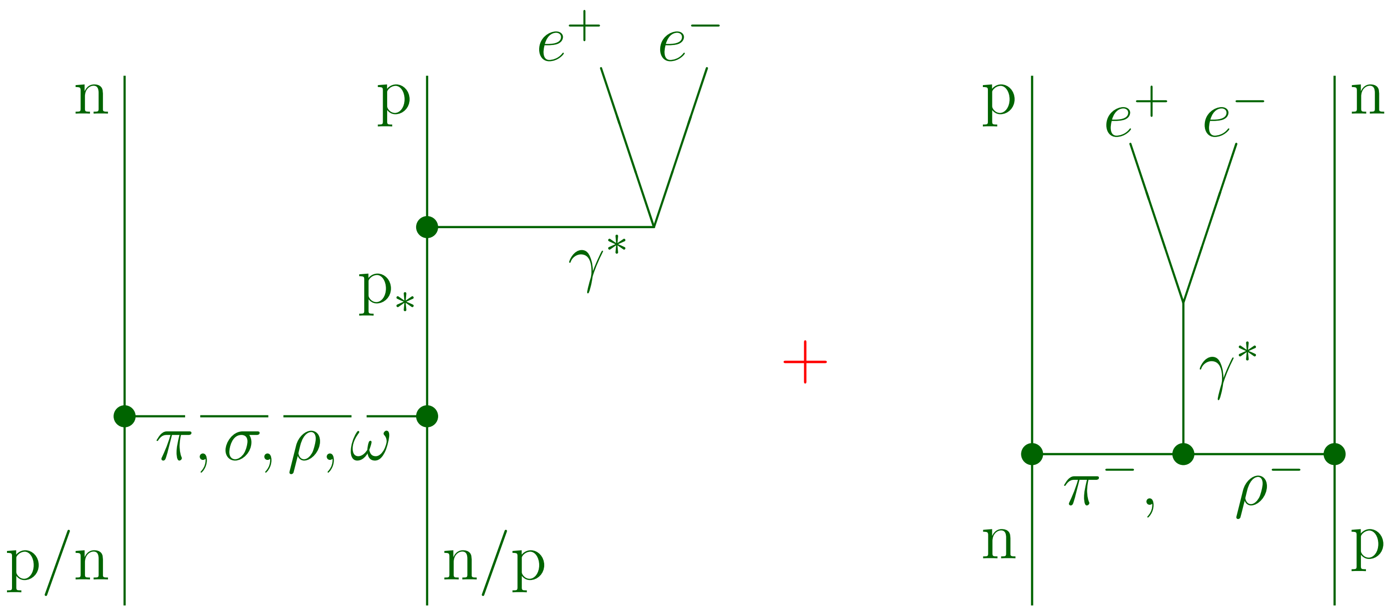


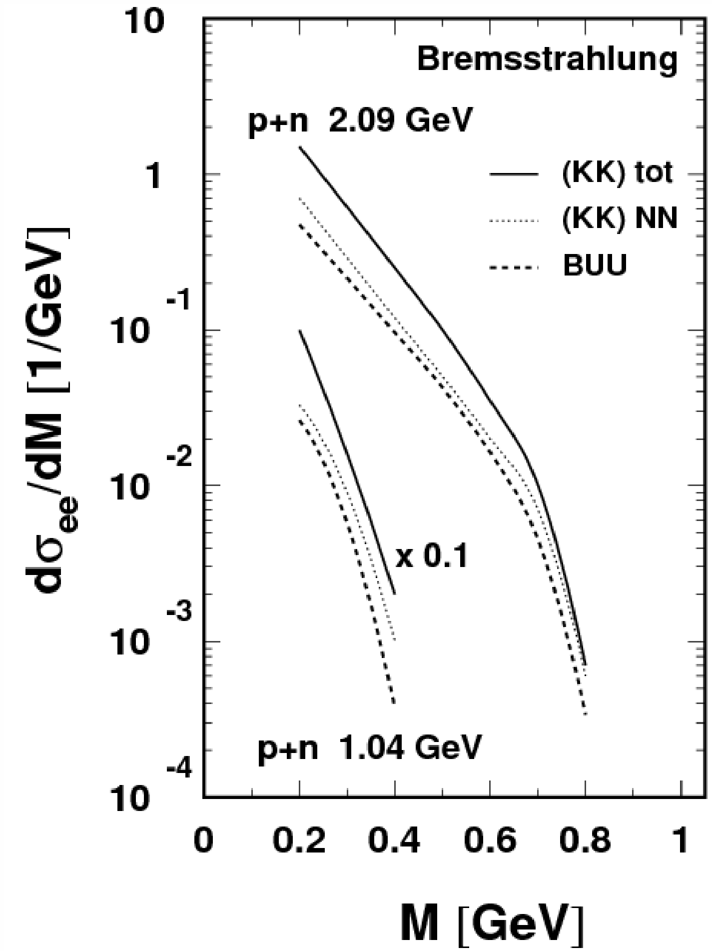
T-matrix calculations

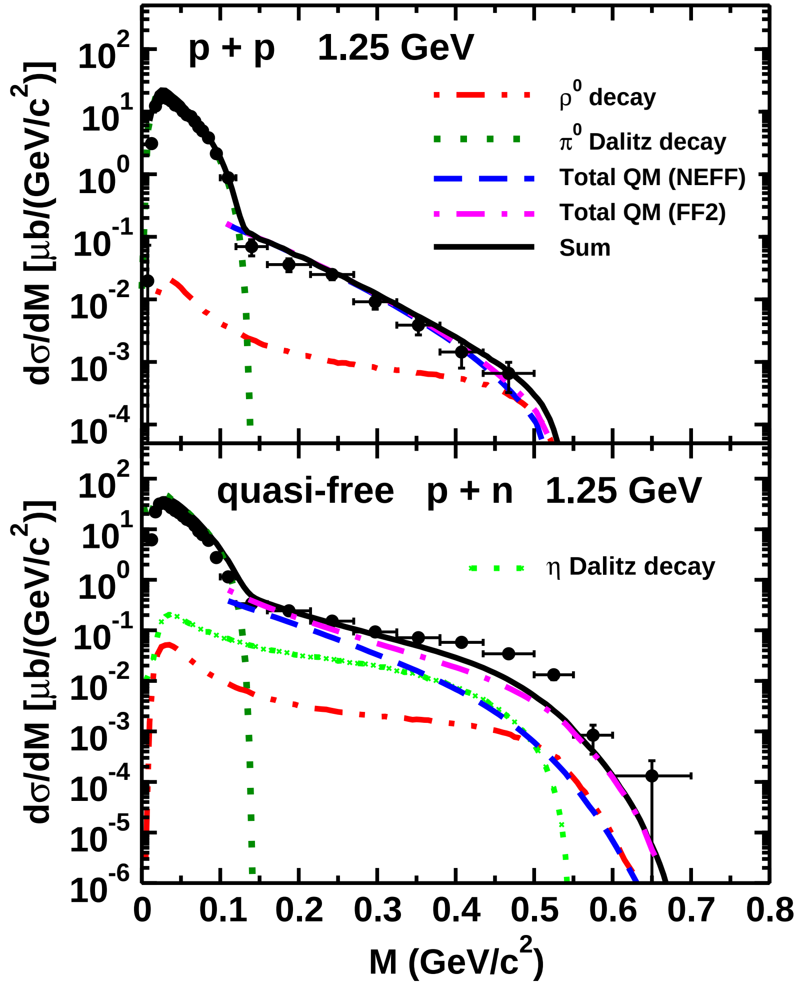
pp:



pn:







- L.P. Kaptari, B. Kämpfer, Nucl. Phys. A **764** (2006) 338.
- R. Shyam, U. Mosel, Phys. Rev. C**67** (2003) 065202, C**79** (2009) 035203, nucl-th:1006.3873

Dalitz-decay of baryon resonances

$$\text{QED:} \qquad \frac{d\Gamma_{R\to Ne^+e^-}}{dM^2} = \frac{\alpha}{3\pi}\frac{1}{M^2}\Gamma_{R\to N\gamma}(M).$$

$$\Gamma_{R\to N\gamma}(M) = \frac{\sqrt{\lambda(m_*^2, m^2, M^2)}}{16\pi m_*^3}\frac{1}{n_{pol,R}}\sum_{pol}|\langle N\gamma|T|R\rangle|^2,$$

- spin-J fermion, $J \geq 3/2$: Rarita-Schwinger spinor-tensor field

$$\begin{aligned} u^{\cdots\rho_i\cdots\rho_k\cdots}(p_*, \lambda_*) &= u^{\cdots\rho_k\cdots\rho_i\cdots}(p_*, \lambda_*), \\ u^{\cdots\sigma\cdots}{}_{\sigma}{}^{\cdots}(p_*, \lambda_*) &= u^{\cdots\sigma\cdots}(p_*, \lambda_*)p_{*\sigma} = u^{\cdots\sigma\cdots}(p_*, \lambda_*)\gamma_\sigma = 0, \end{aligned}$$

EM coupling of baryon resonances

- There are 3 independent tensor structures (for S$\geq 3/2$) for coupling of nucleon and Rarita-Schwinger spinors ($G = 1$ or γ_5):

$$\Gamma_{\mu\rho_1\cdots\rho_n} = \sum_{i=1}^{3} f_i(q^2 = M^2)\chi^i_{\mu\rho_1} p_{\rho_2}\cdots p_{\rho_n} G,$$

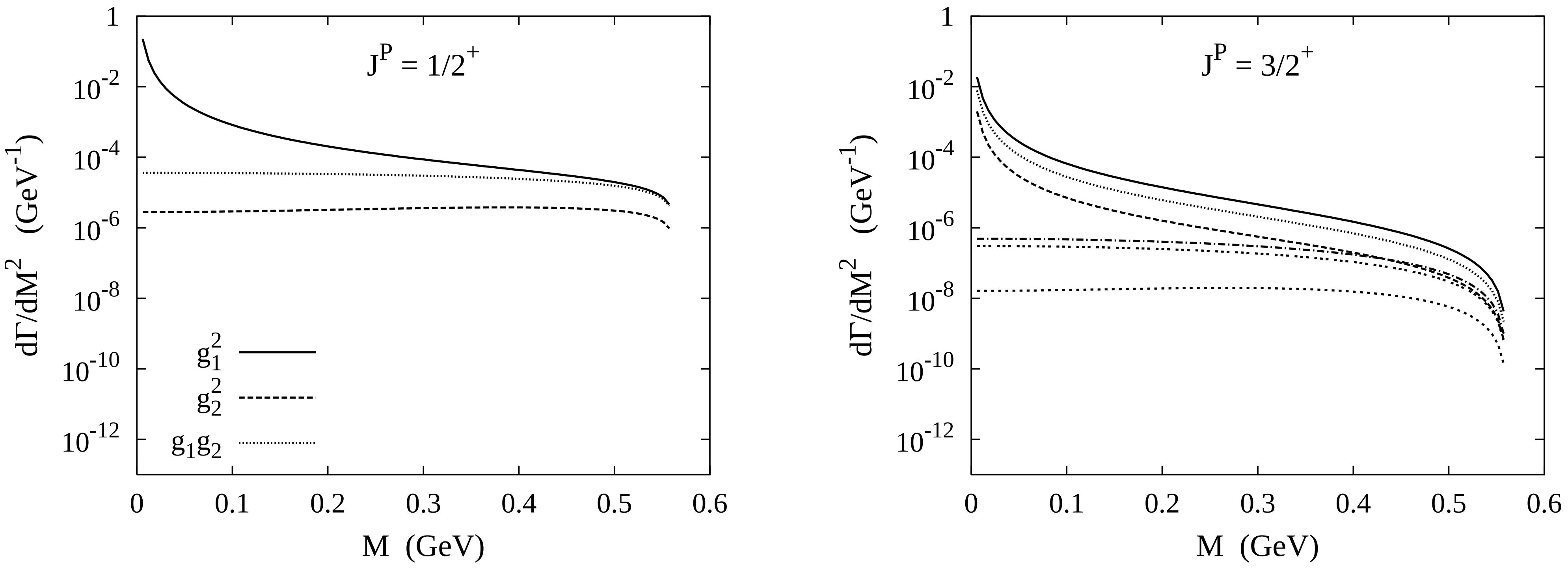
with

$$\chi^1_{\mu\rho} = \gamma_\mu q_\rho - \not{q} g_{\mu\rho},$$

$$\chi^2_{\mu\rho} = P_\mu q_\rho - (P\cdot q) g_{\mu\rho},$$

$$\chi^3_{\mu\rho} = q_\mu q_\rho - q^2 g_{\mu\rho},$$

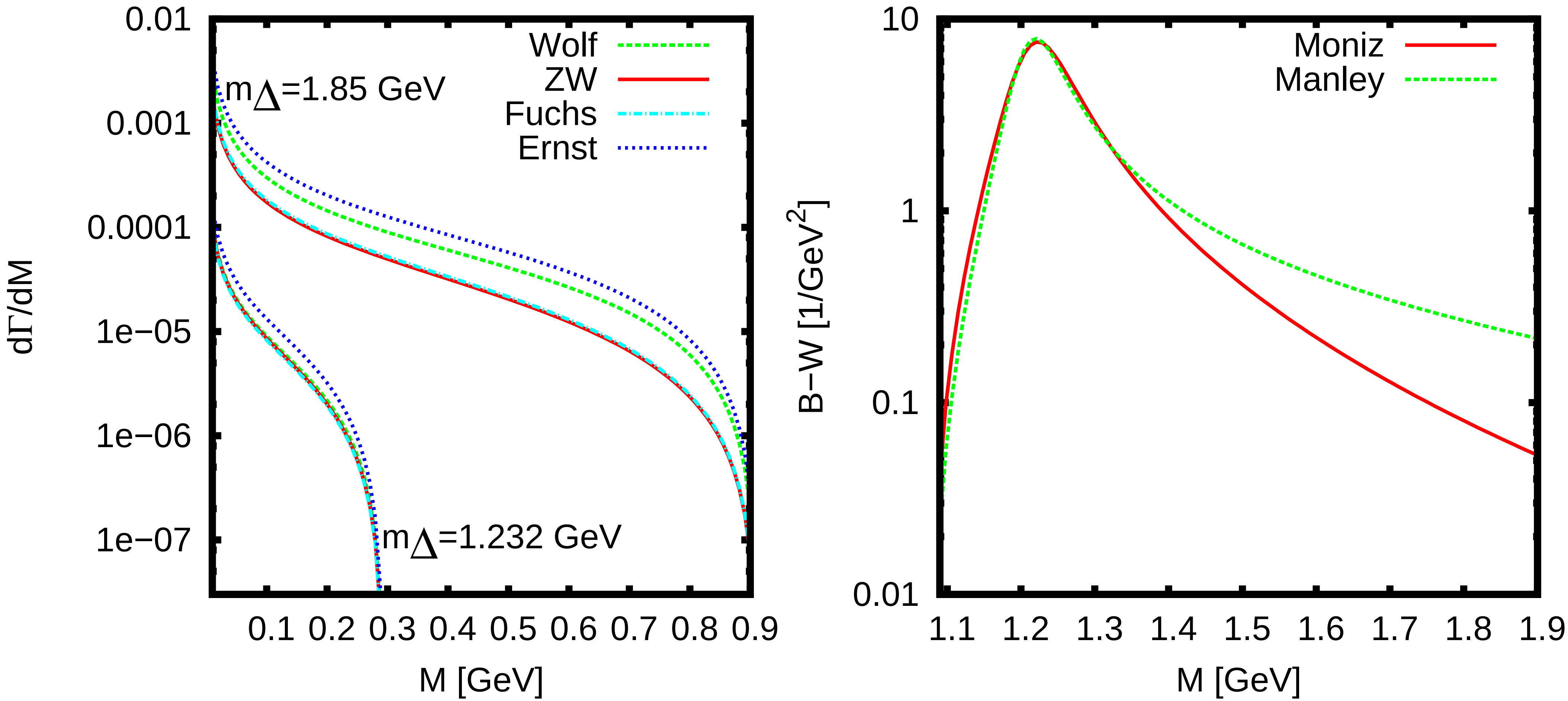
Dalitz-decay contributions



$m_* = 1.5$ GeV. Dimensionless coupling constants are set to 1.

In the S$= 1/2$ case g_2 and in the S$\geq 3/2$ case g_3 cannot be fixed at M=0, since their contributions there are identically 0.

$\Delta(1232)$



Δ properties are fixed around the resonance region, but because of its very strong electromagnetic coupling it dominates the Dalitz-decay spectrum at high masses (~ 1.7 GeV), although for pion production its effect already negligible.

Summary of elementary dilepton production

- There is no good bremsstrahlung calculation (describes pp end pn at the same time).
- Delta-Dalitz decay contribution is very uncertain, too
- Complete NN→ NNe^+e^- calculation is needed with angular dependence and compare with experimental data for pp and pn. Deduce the relative strengths. Then put into transport.

Why off-shell transport

- medium effects on the spectrum of vector mesons
  — indication of mass shift of longliving ω's
- how they get on-shell (energy-momentum conservation)
- if it is broad, even the local density approximation has no precise meaning

Off-shell transport

- Kadanoff-Baym equation for retarded Green-function
  Wigner-transformation, gradient expansion
- transport equation for $F_\alpha = f_\alpha(x,p,t)A_\alpha$
  $A(p) = -2ImG^{ret} = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \mathrm{Re}\Sigma^{ret})^2 + \frac{1}{4}\hat{\Gamma}^2}$,

  Cassing, Juchem (2000) and Leupold (2000)
- testparticle approximation

Transport equations

- $$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right]$$

  $$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right]$$

  $$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right]$$

- where $C_{(i)}$ renormalization factor

  $$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

  dangerous, $C_{(i)}$ can be 1

  if $C_{(i)} > 0.5$ we use $\frac{1}{1-C_{(i)}} = 1.33(1 + C_{(i)})$

  However $C_{(i)} = 0$ do not change the results substantially

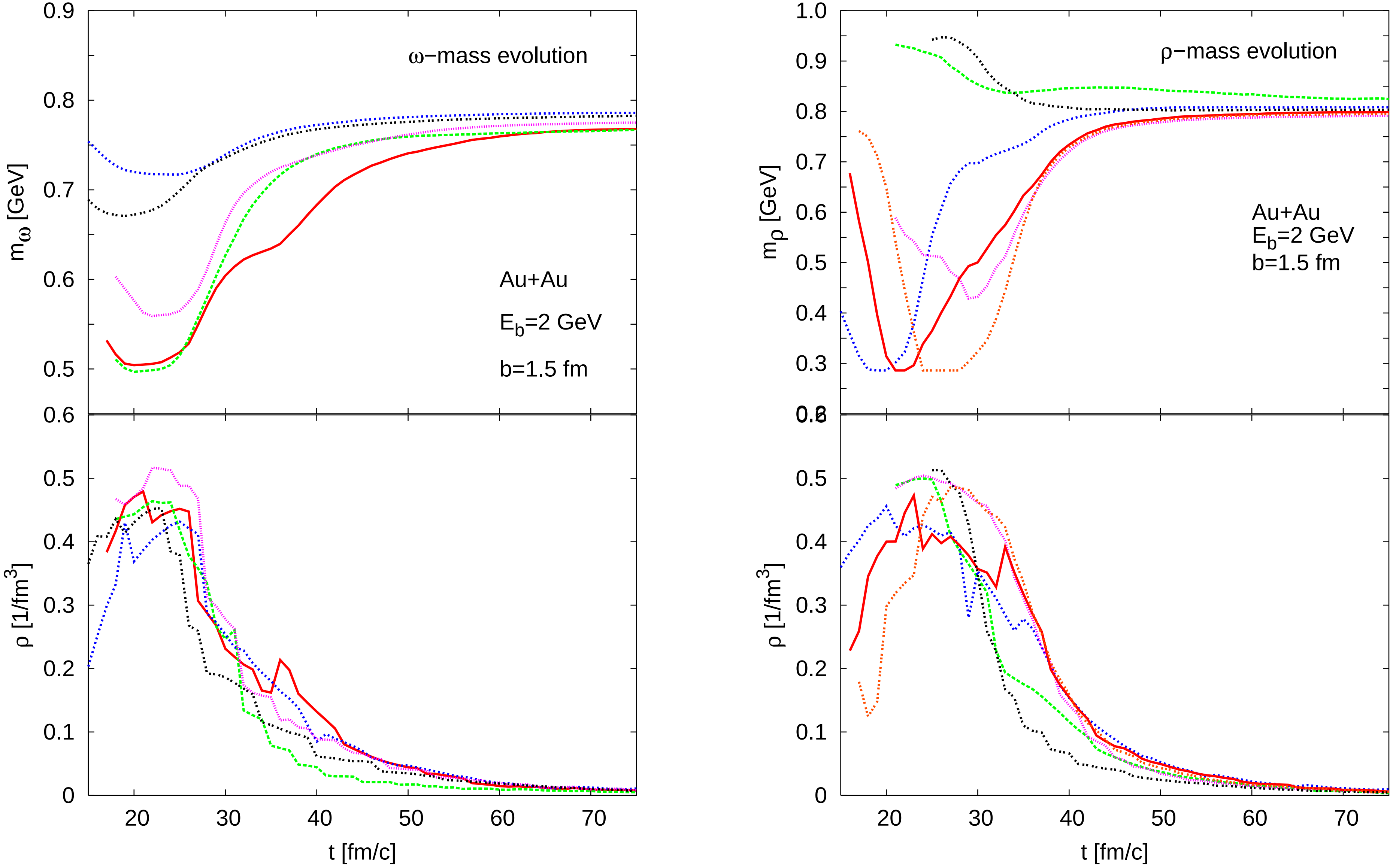
- the last equation can be rewritten as

  $$\frac{dM_i^2}{dt} = \frac{M_i^2 - M_0^2}{\Gamma_{(i)}} \frac{d\Gamma_{(i)}}{dt}$$

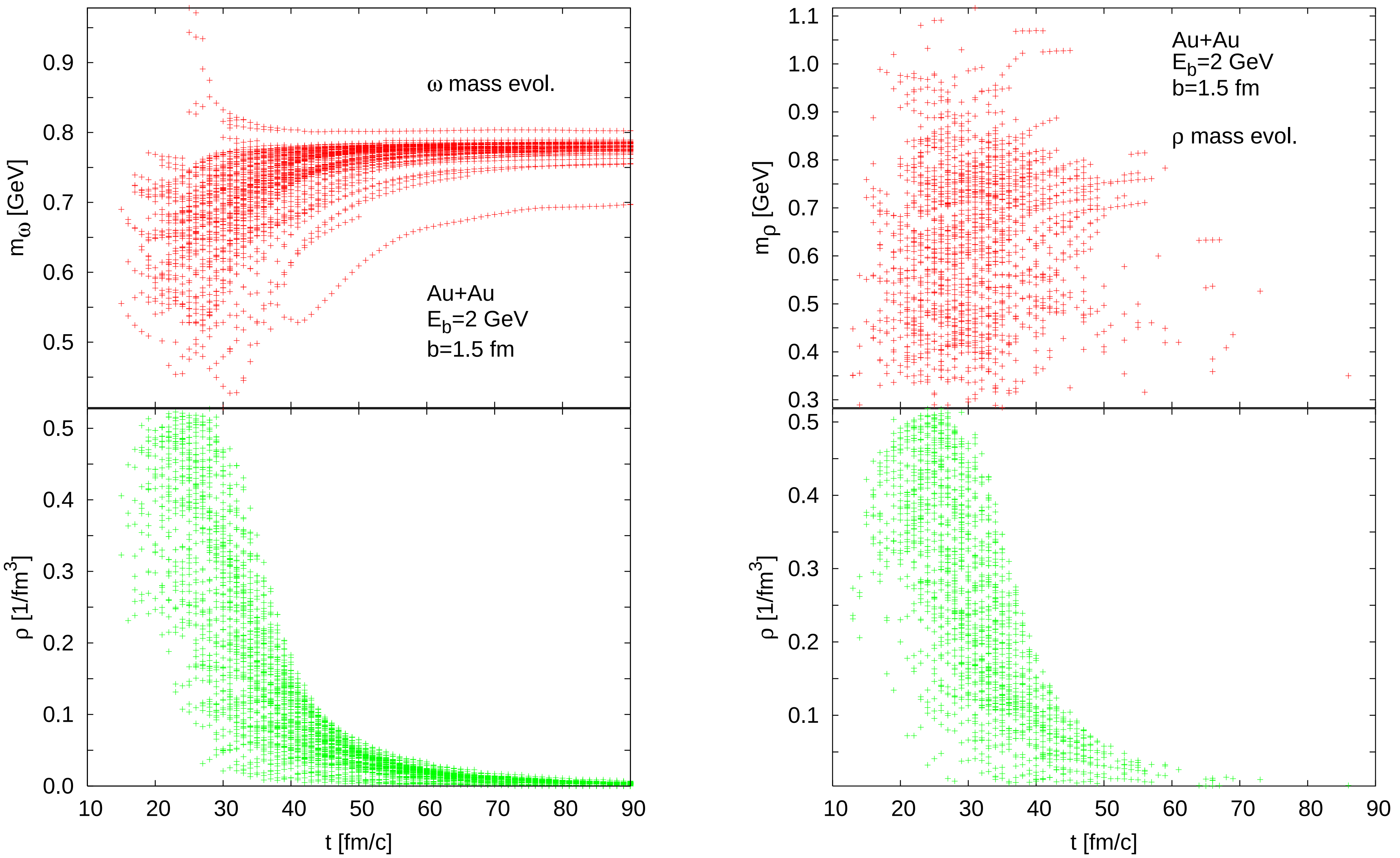
Medium effects

- imaginary part (collisional broadening):
  $\Gamma = \Gamma_{vac} + nv\sigma\gamma$
- real part (mass shift)
  $M = M_{vac} + n/n_o \Delta M$
  $\Delta M_\omega = -50$ MeV, $\Delta M_\rho = -120$ MeV
- danger of double counting
  collision term already contains partly the mixing of mesons with resonance-hole excitations
  but sum up only to finite order

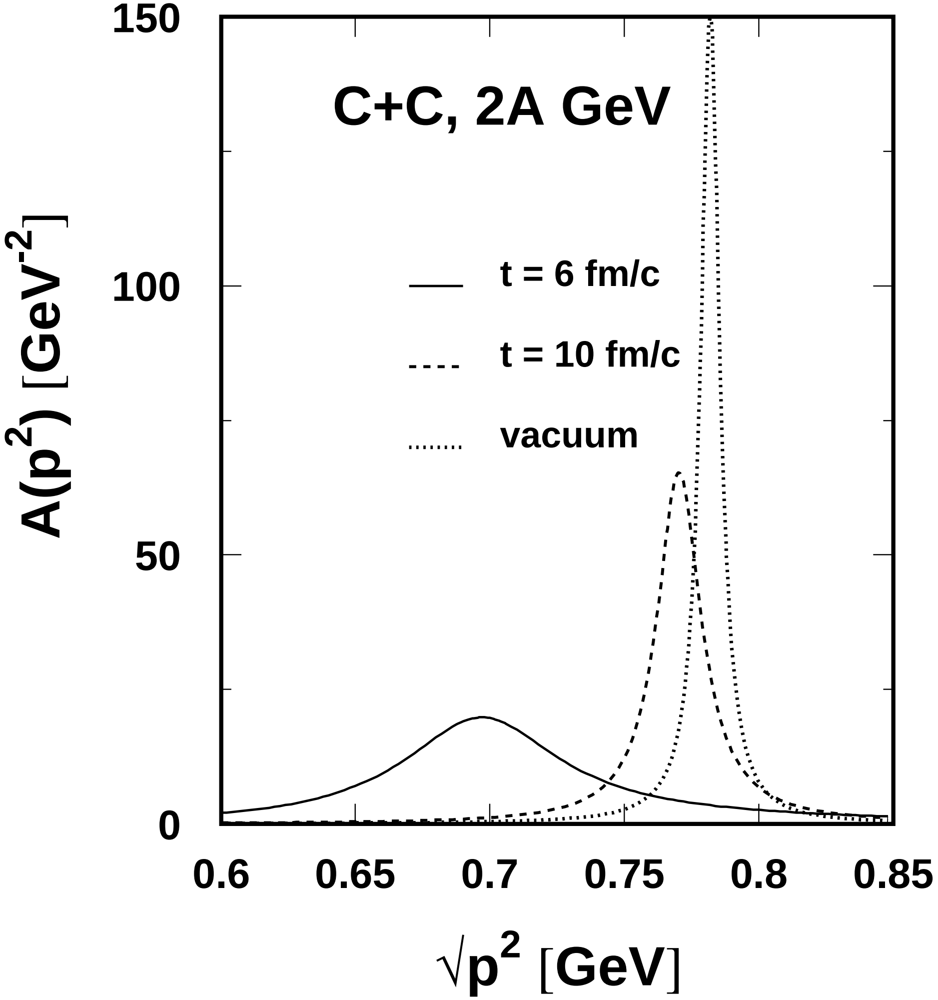
Evolution of masses



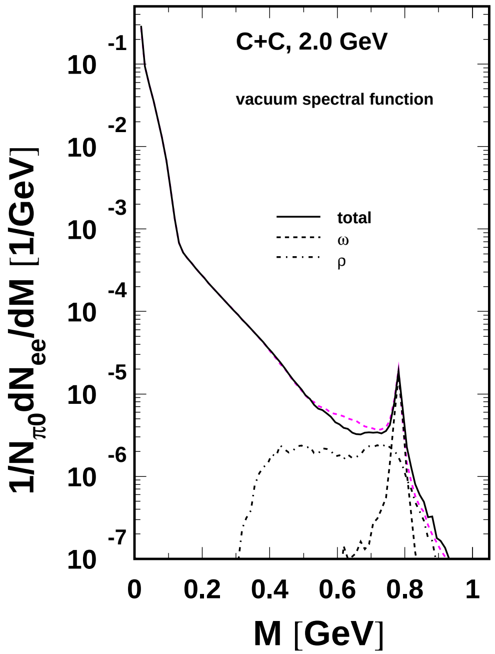
Evolution of masses



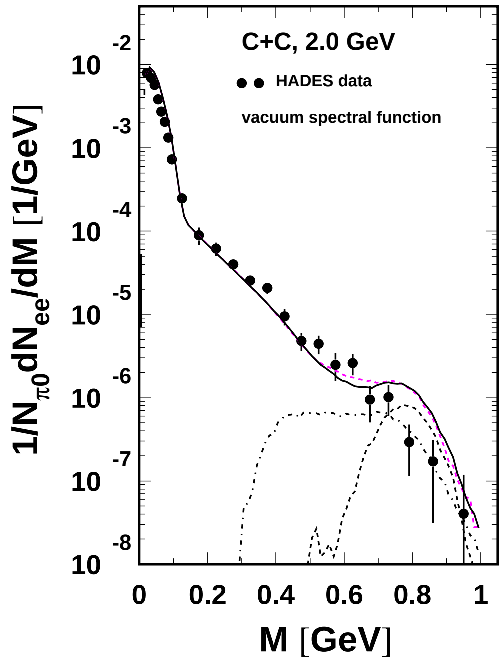
Evolution of the ω spectrum



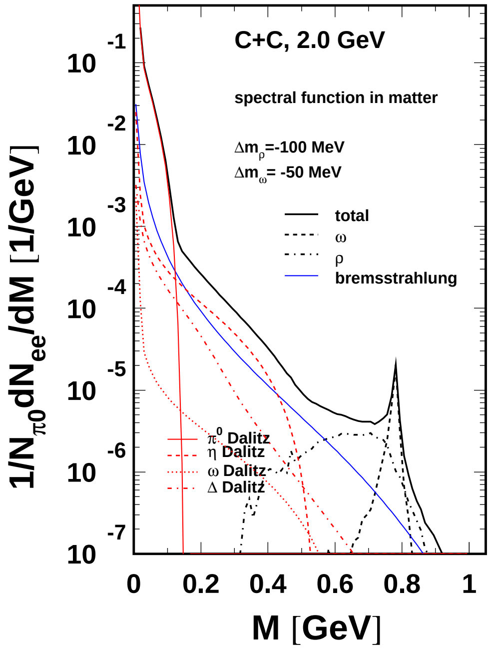
C + C 2 GeV



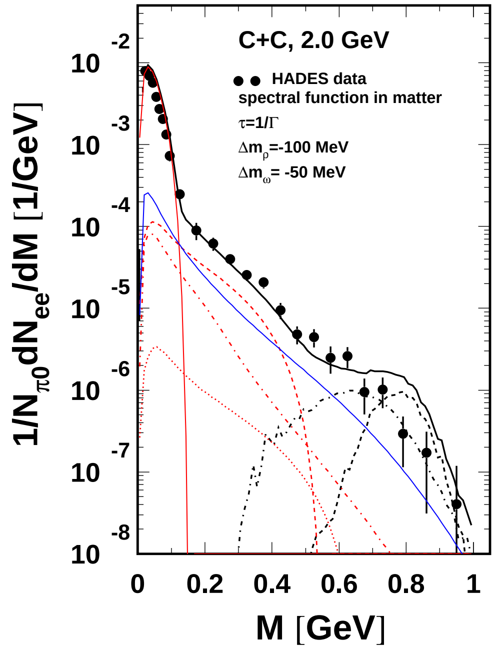
C+C, 2.0 GeV
vacuum spectral function
total
ω
ρ
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]



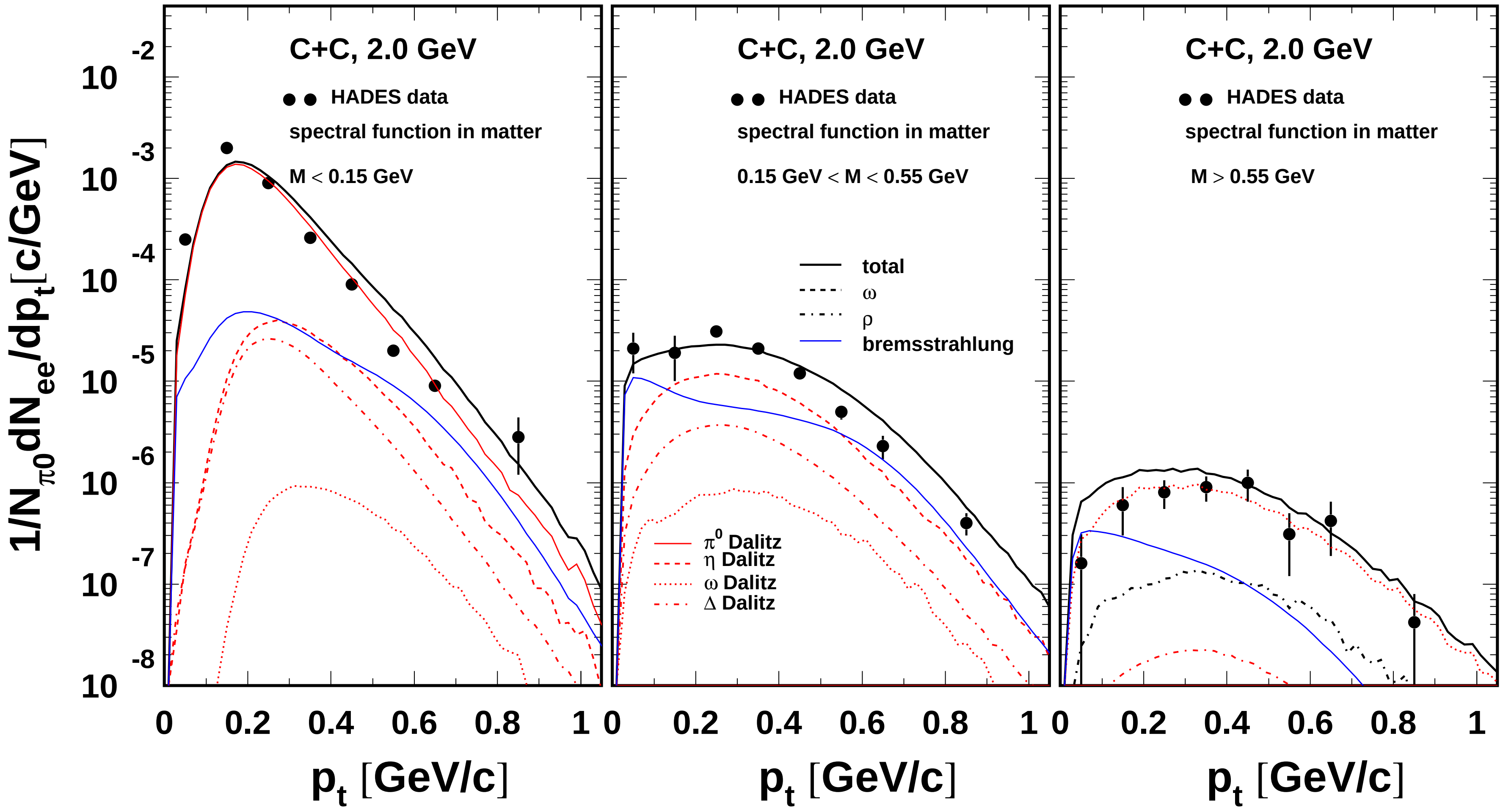
C+C, 2.0 GeV
HADES data
vacuum spectral function
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]



C+C, 2.0 GeV
spectral function in matter
Δm$_\rho$=-100 MeV
Δm$_\omega$= -50 MeV
total
ω
ρ
bremsstrahlung
π^0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]

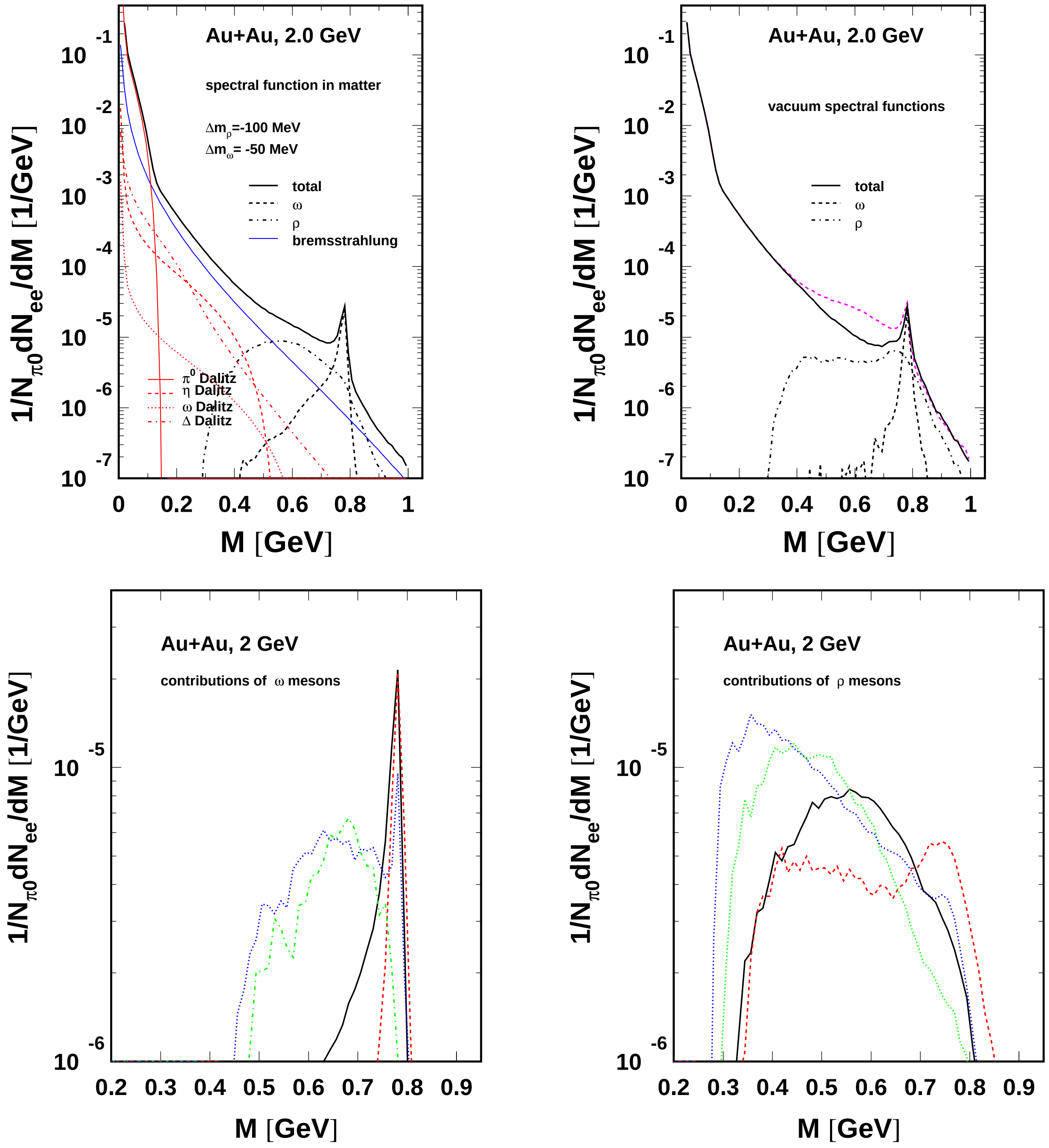


C+C, 2.0 GeV
HADES data
spectral function in matter
τ=1/Γ
Δm$_\rho$=-100 MeV
Δm$_\omega$= -50 MeV
1/N$_{\pi0}$dN$_{ee}$/dM [1/GeV]
M [GeV]



C+C, 2.0 GeV
HADES data
spectral function in matter
M < 0.15 GeV
0.15 GeV < M < 0.55 GeV
M > 0.55 GeV
total
ω
ρ
bremsstrahlung
π0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/Nπ0 dNee/dpt [c/GeV]
pt [GeV/c]

Au + Au 2 GeV

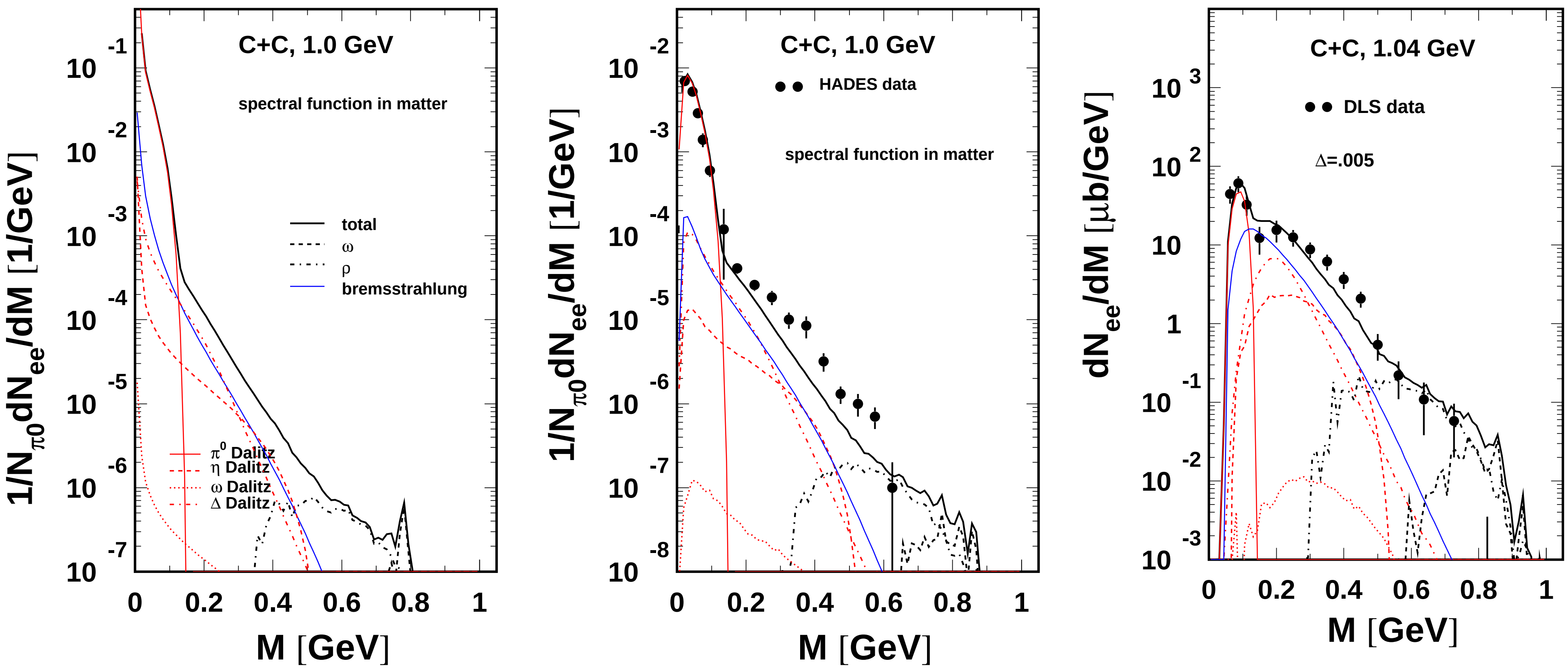


Vacuum

Matter

Static

C + C 1 AGeV



C+C, 1.0 GeV
spectral function in matter
total
ω
ρ
bremsstrahlung
π0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/Nπ0dNee/dM [1/GeV]
M [GeV]
C+C, 1.0 GeV
HADES data
spectral function in matter
1/Nπ0dNee/dM [1/GeV]
M [GeV]
C+C, 1.04 GeV
DLS data
Δ=.005
dNee/dM [μb/GeV]
M [GeV]

Summary

- BUU with off-shell propagation
- several theoretical uncertainties
- needs of precise data in
  - pp, pn collision (bremsstrahlung, resonance-Dalitz decay)
  - Au+Au 1 GeV and at the highest available energy

BUU

- Boltzmann-Ühling-Uhlenbeck equation

$$\frac{\partial F}{\partial t} + \frac{\partial H}{\partial \mathbf{p}} \frac{\partial F}{\partial \mathbf{x}} - \frac{\partial H}{\partial \mathbf{x}} \frac{\partial F}{\partial \mathbf{p}} = \mathcal{C}, \quad H = \sqrt{(m_0 + U(\mathbf{p}, \mathbf{x}))^2 + \mathbf{p}^2}$$

- potential: momentum dependent, soft: K=215 MeV

$$U^{nr} = A \frac{n}{n_0} + B \left(\frac{n}{n_0} \right)^{\tau} + C \frac{2}{n_0} \int \frac{d^3 p'}{(2\pi)^3} \frac{f_N(x, p')}{1 + \left(\frac{\mathbf{p} - \mathbf{p}'}{\Lambda} \right)^2},$$

  Teis et al., Z. Phys. 1997

- testparticle method

$$F = \sum_{i=1}^{N_{test}} \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t)) \delta^{(4)}(p - p_i(t)).$$

Collision term

- NN $\leftrightarrow$ NR, NN $\leftrightarrow$ $\Delta\Delta$
- baryon resonance can decay via 9 channels
  R $\leftrightarrow$ Nπ, Nη, Nσ, Nρ, Nω, $\Delta\pi$, N(1440)π, KΛ, KΣ
- 24 baryon resonances + Λ and Σ baryons
  $\pi, \eta, \sigma, \rho, \omega$ and kaons
- $\pi\pi \leftrightarrow \rho$, $\pi\pi \leftrightarrow \sigma$, $\pi\rho \leftrightarrow \omega$
- for resonances: energy dependent with
- $\frac{d\sigma^{X\rightarrow NR}}{dM_R} \sim A(M_R)\lambda^{0.5}(s, M_R^2, M_N^2)$

Cross sections

Elastic baryon-baryon cross section is fitted to the elastic pp data

Meson absorption cross sections are given by

$$\sigma_{\pi N \to R} = \frac{4\pi}{p^2}(spin factors)\frac{\Gamma_{in}\Gamma_{tot}}{(s - m_R^2) + s\Gamma_{tot}^2}$$

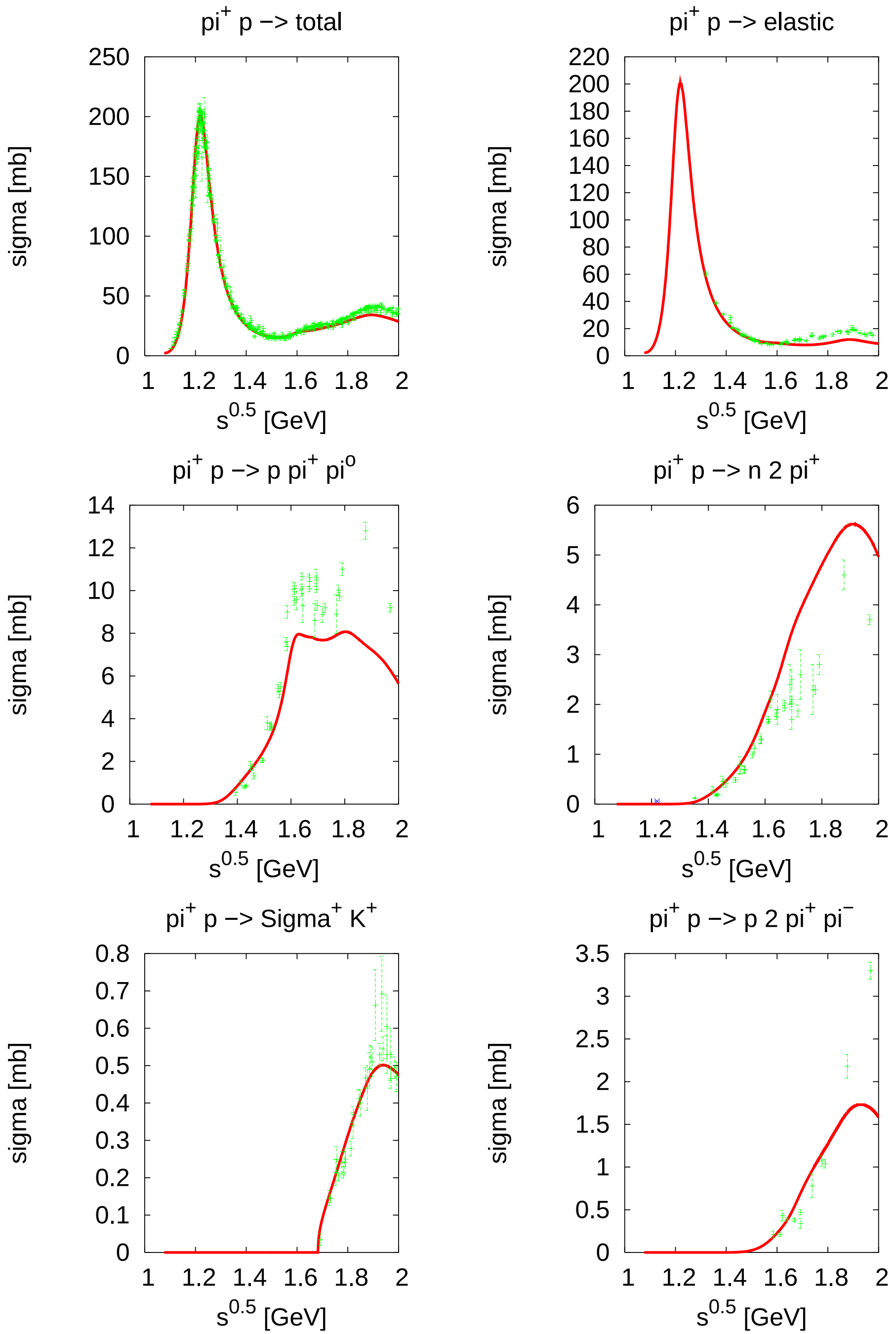
Baryon resonance parameters: mass, width, branching ratios are fitted by describing the meson production channels in πN collisions:

$$\sigma_{\pi N \to NM} = \sum_R \sigma_{\pi N \to R} \frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

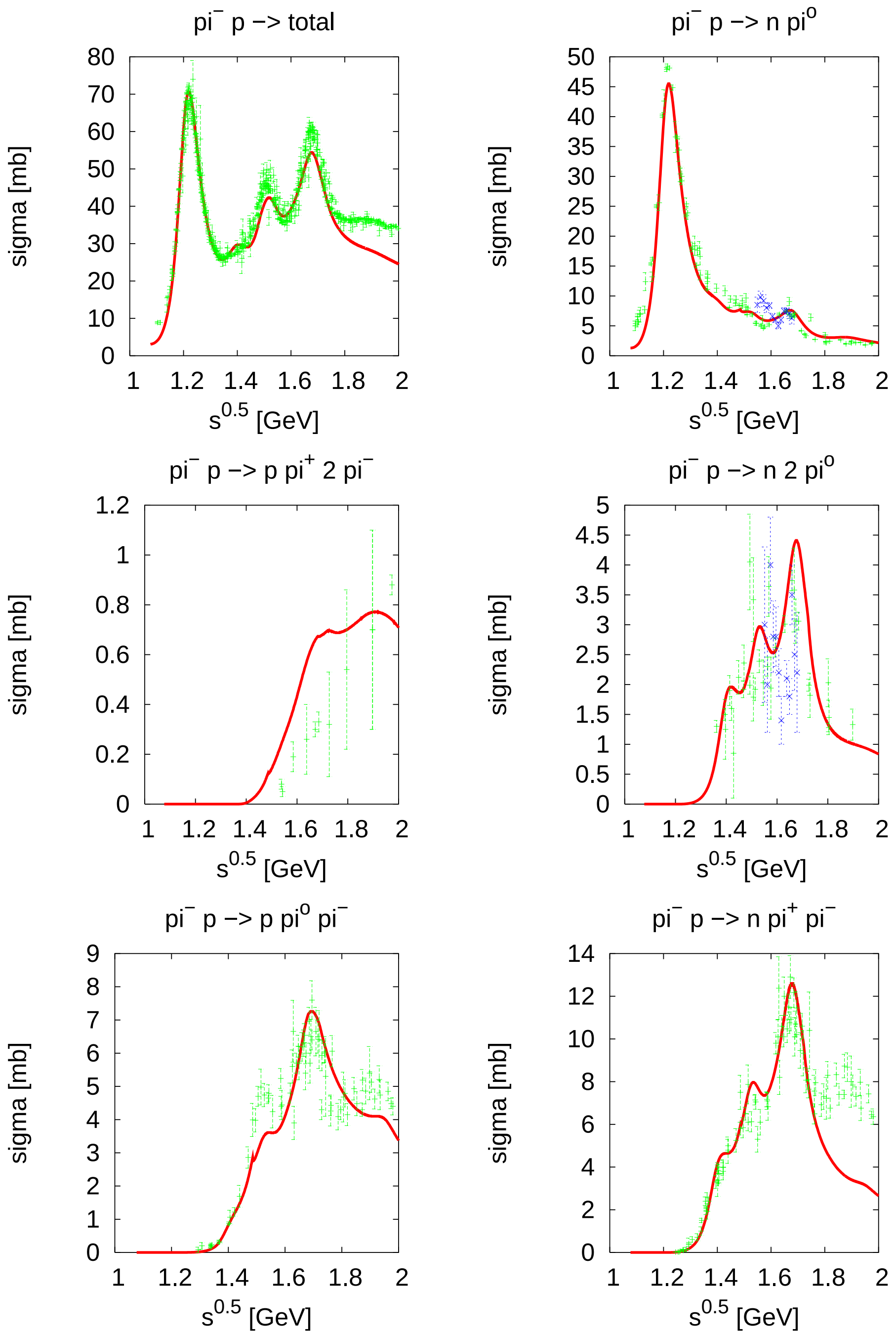
Resonance production cross section $NN \to NR$ is given by the fit of

$$\sigma_{NN \to NM} = \sum_R \sigma_{NN \to NR} \frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

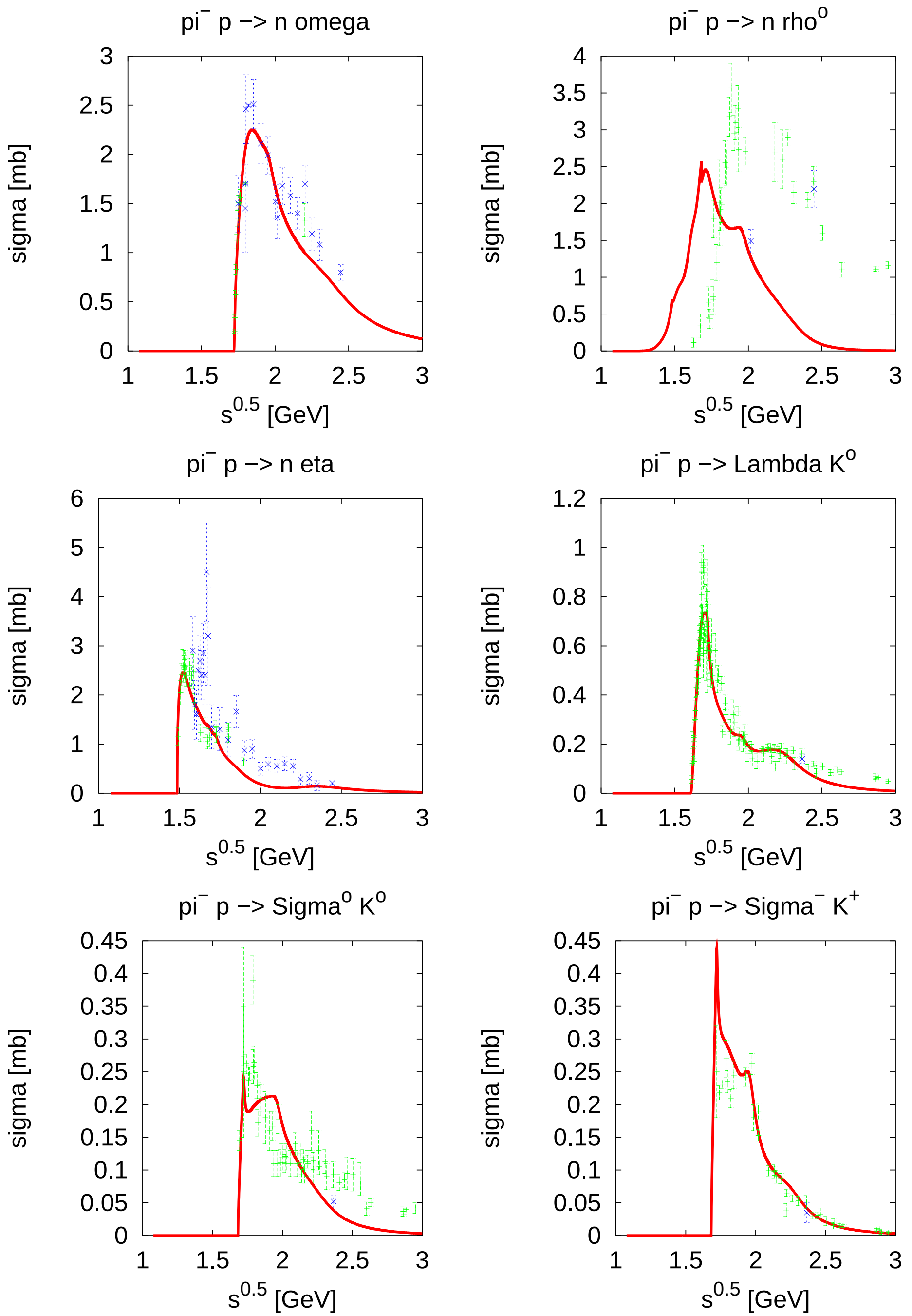
27 baryons, 6 mesons. Fit is done by the Minuit package (CERN)



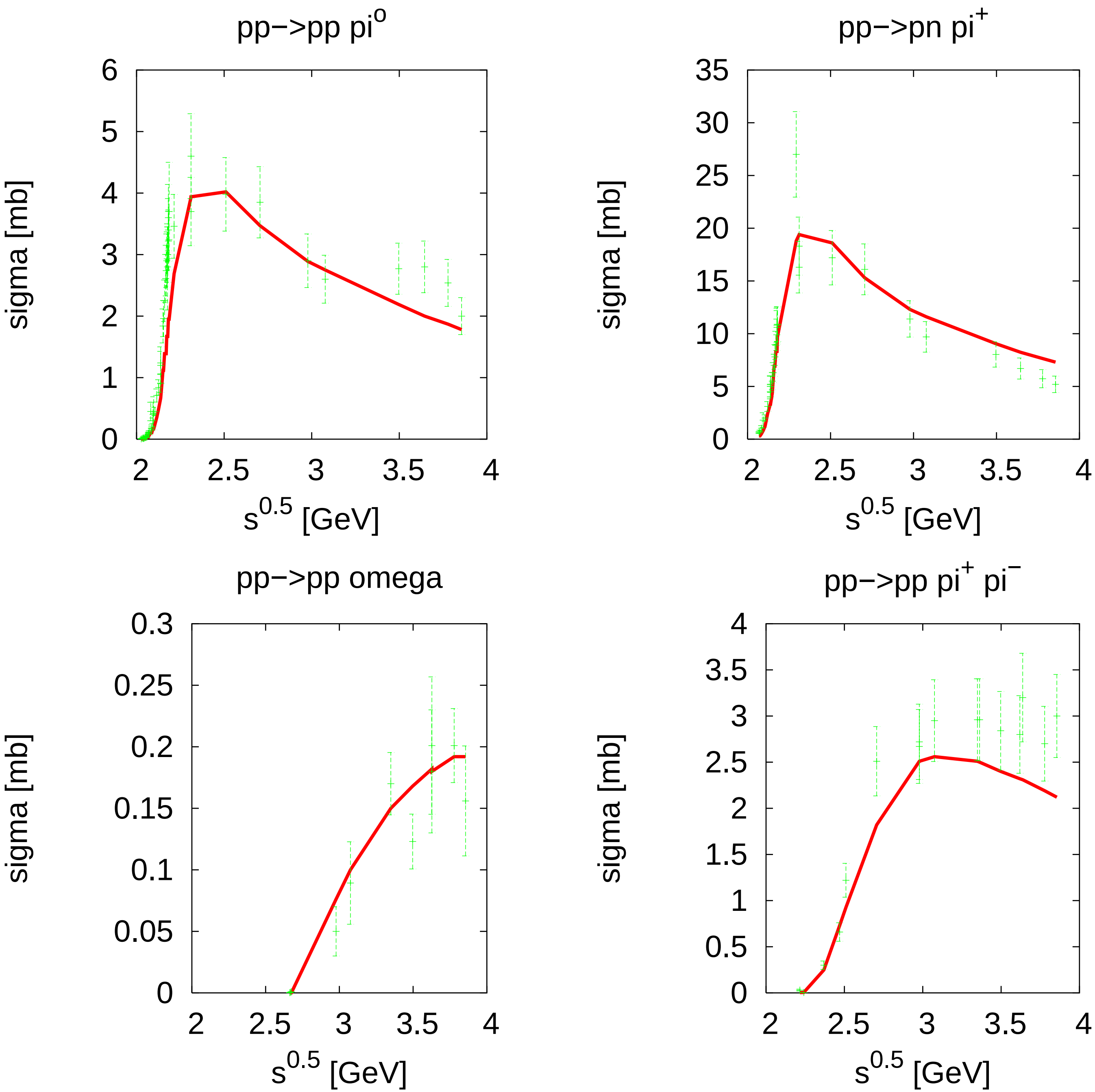
pi+ p -> total
pi+ p -> elastic
pi+ p -> p pi+ pio
pi+ p -> n 2 pi+
pi+ p -> Sigma+ K+
pi+ p -> p 2 pi+ pi−
sigma [mb]
s0.5 [GeV]



pi^{-} p −> total
pi^{-} p −> n pi^{0}
pi^{-} p −> p pi^{+} 2 pi^{-}
pi^{-} p −> n 2 pi^{0}
pi^{-} p −> p pi^{0} pi^{-}
pi^{-} p −> n pi^{+} pi^{-}
sigma [mb]
s$^{0.5}$ [GeV]



pi$^-$ p -> n omega
pi$^-$ p -> n rho^o
pi$^-$ p -> n eta
pi$^-$ p -> Lambda K^o
pi$^-$ p -> Sigma^o K^o
pi$^-$ p -> Sigma$^-$ K$^+$
sigma [mb]
s$^{0.5}$ [GeV]



pp−>pp pi^o
pp−>pn pi^+
pp−>pp omega
pp−>pp pi^+ pi^−
sigma [mb]
s^0.5 [GeV]
0 1 2 3 4 5 6
0 5 10 15 20 25 30 35
0 0.05 0.1 0.15 0.2 0.25 0.3
0 0.5 1 1.5 2 2.5 3 3.5 4
2 2.5 3 3.5 4