Viscosity and thermal conductivity effects in the description of the I order phase transitions in HIC

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in nuclear systems:

- Liquid-gas and hadron-quark I order phase transitions in HIC

in condensed matter:

any 1-order phase transition of the liquid-gas type can be considered similarly

Hydrodynamics of the first order phase transition:

V.Skokov, D.V. , arXiv 0811.3868, JETP Lett. 90 (2009) 223;
Nucl. Phys. A828 (2009) 401; A846 (2010).

We solve the system of non-ideal hydro equations describing non-trivial fluctuations (droplets/bubbles, aerosol) in d=2 space +1 time dimensions numerically for van der Waals-like EoS, and for arbitrary d in the vicinity of the critical point analytically.

Non-ideal non-relativistic hydrodynamics

$$mn\left[\partial_t u_i + (\mathbf{u}\nabla)u_i\right] = -\nabla_i P$$

$$+\nabla_k\left[\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right) + \zeta\delta_{ik}\mathrm{div}\mathbf{u}\right] \quad (8)$$

$$\partial_t n + \mathrm{div}(n\mathbf{u}) = 0, \quad (9)$$

$$T\left[\frac{\partial s}{\partial t} + \mathrm{div}(s\mathbf{u})\right] = \mathrm{div}(\kappa\nabla T)$$

$$+\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right)^2 + \zeta(\mathrm{div}\mathbf{u})^2. \quad (10)$$

Here η and ζ are shear and bulk viscosities; $\mathbf{u}$ is the velocity of the element of the fluid; s is the entropy density; κ is the thermal conductivity; d is the dimensionality of space.

Dynamics of the phase transition is controlled by the slowest mode

provided u is small, in analytical treatment we neglect u^2 terms

Qualitative analysis and rough estimates

typical time for density fluctuation: $t_\rho \sim R$ (constant velocity)

$R(t)$ is the size of evolving seed

typical time for heat transport $t_T \sim R^2 c_V / \kappa$, c_V is specific heat density

We introduce R_{fog} -- typical seed size at which $t_\rho = t_T$

$t_\rho > t_T$ for $R(t) < R_{fog}$: **Density evolution stage**

(isothermal)

$t_T > t_\rho$ for $R(t) > R_{fog}$: **Heat transport stage**

Seeds with $R \sim R_{fog}$ are accumulated with passage of time: ⇨ **fog stage**

for H-QGP phase transition $R_{fog} \sim 0.1\text{-}1$ fm, for liquid-gas ~1-10 fm, fireball evolution time $t_{evol} \sim 10$ fm

Thermal conductivity effects should be incorporated in hydro simulations of HIC

Next is coalescence stage (occurs at still larger time scale), see Lifshiz, Pitaevsky, Physical Kinetics. v. X

Supercooled vapor; overheated liquid; aerosol-like mixture in spinodal region

Expand the Landau free energy in $\delta\rho = \rho - \rho_r$ and δ T near the reference point, close to ρ_{cr}, T_{cr}

$$\delta F = \int \frac{d^3x}{\rho_r}\left[\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda(\delta\rho)^4}{4} - \frac{\lambda v^2(\delta\rho)^2}{2} - \epsilon\delta\rho\right]$$

Surface term

$v^2 \sim (T_{cr} - T)$ in mean field treatment

ρ=mn

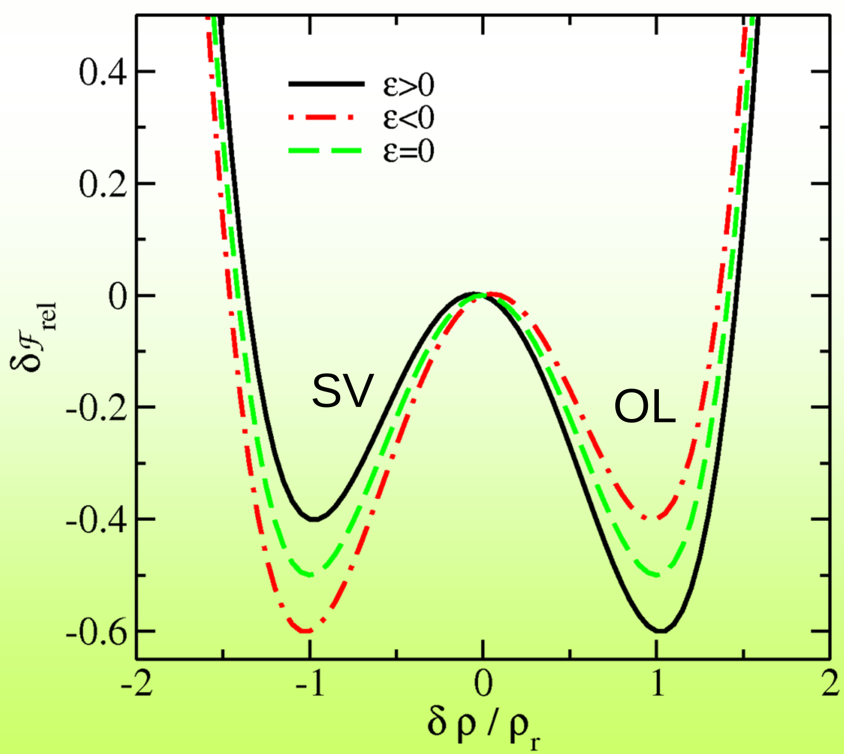
$$\delta P = \rho \frac{\delta[F_L(T,\delta\rho)]}{\delta(\delta\rho)}\Big|_T$$

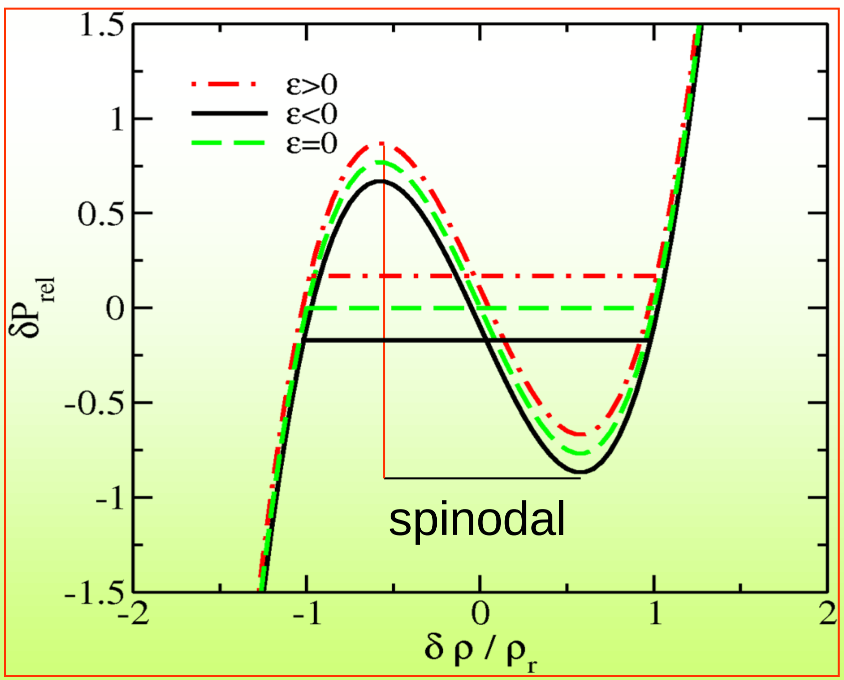
$$\delta\mathcal{F}_{rel} = \delta\mathcal{F}_L/\mathcal{F}_L(T_{cr},\rho_{cr})$$

$$\delta P_{rel} = \delta P/P(T_{cr},\rho_{cr})$$

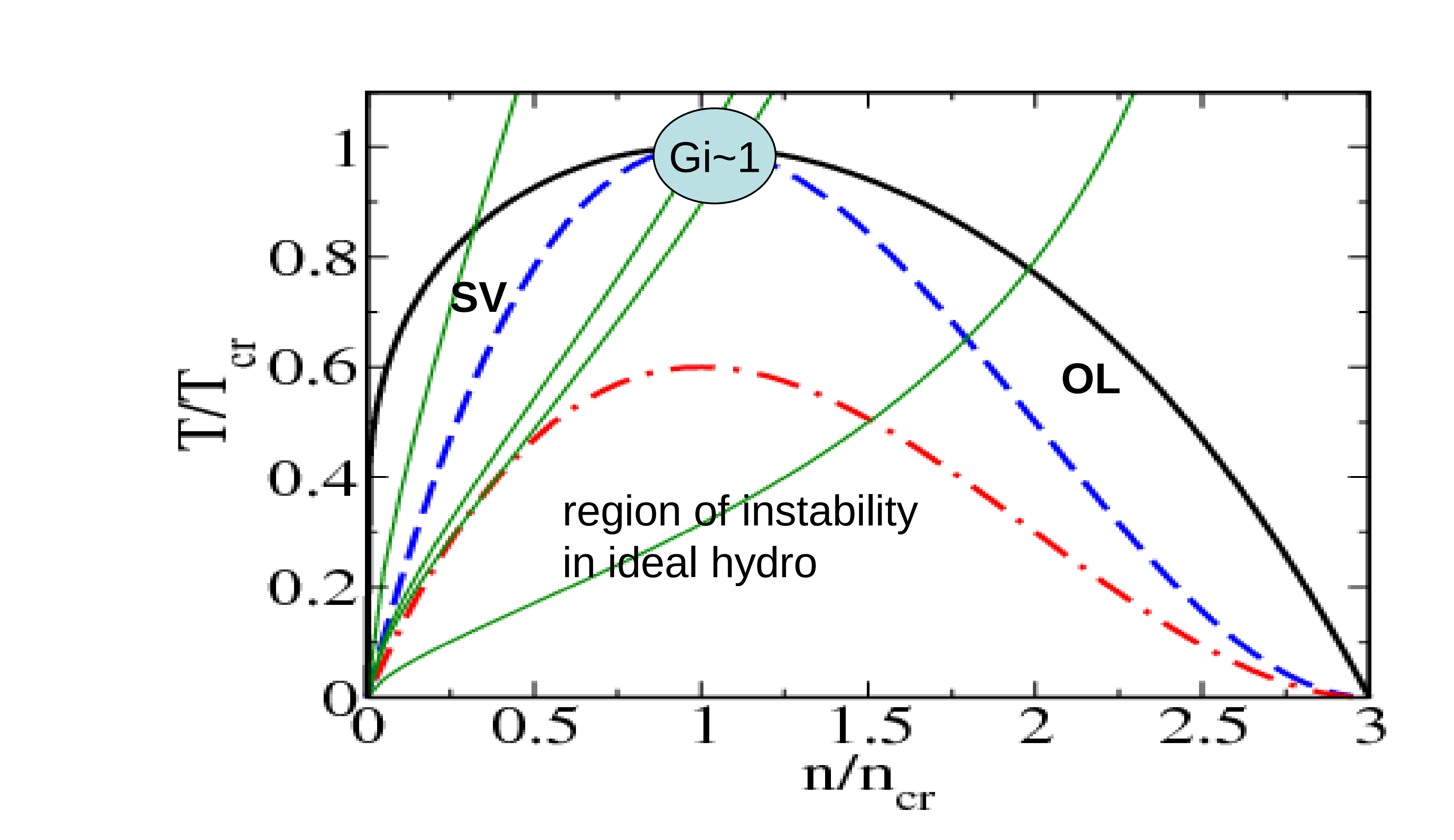
at critical point

isotherms





Constant entropy trajectories



- - - isothermal spinodal, - . - . - isoentropic spinodal, ▬ Maxwell construction

T_{max} =0.6 T_{cr} for van der Waals EoS

Is fluctuation region broad or narrow?

For the hadron quark phase transition we estimate

$$\mathrm{Gi} \gtrsim 1.4(100\ \mathrm{MeV\ fm}^{-2}/\sigma_0)^6$$

σ_0 is surface tension

For the liquid-gas transition

$$\mathrm{Gi} \sim 10(T_{cr}/18.6\ \mathrm{MeV})^6$$

in both cases fluctuation region might be very broad

In thermodynamical description fluctuation effects should be incorporated in EoS.

Mean field vs. fluctuations

For $\mathrm{Gi} \gtrsim 1$

stationary system is not uniform due to permanently creating and decaying fluctuations (it looks like a sup right before boiling)

For **dynamical system** (*like fireball in HIC*)
since typical time for developing of critical fluctuations is large, $t_0 \sim |T-T_{cr}|^{-1}$ (at least near critical point), fluctuations may have not sufficient time to appear ⇨

One can consider **mean field EoS** provided fireball evolution time $t_{evol} < t_0$

(argument by Zeldovich, Mikhailov UFN (1987) in description of explosion phenomena)

Dynamics of I order phase transition near critical point

From Navier-Stokes and continuity equations neglecting u^2 terms:

viscosities

$$-\frac{\partial^2\delta\rho}{\partial t^2} = \Delta\left[c\Delta\delta\rho + \lambda v^2\delta\rho - \lambda(\delta\rho)^3 + \epsilon - \rho_r^{-1}\left(\frac{4}{3}\eta_r + \zeta_r\right)\frac{\partial\delta\rho}{\partial t}\right]$$

See D.V. Phys.Scripta 47 (1993) 333 $\qquad \delta\rho = \rho - \rho_r$

In dimensionless variables

$$-\beta\frac{\partial^2\psi}{\partial\tau^2} = \Delta_\xi\left(\Delta_\xi\psi + 2\psi(1-\psi^2) + \widetilde{\epsilon} - \frac{\partial\psi}{\partial\tau}\right)$$

$$\delta\rho = v\,\psi, \quad \xi_i = x_i/l, \quad \tau = t/t_0$$

$$l = \left(\frac{2c}{\lambda v^2}\right)^{1/2}, \quad t_0 = \frac{2(\frac{4}{3}\eta_r + \zeta_r)}{\lambda v^2\rho_r}, \quad \widetilde{\epsilon} = \frac{2\epsilon}{\lambda v^3} \quad \beta = \frac{c\rho_r^2}{(\frac{4}{3}\eta_r + \zeta_r)^2}$$

$$v \propto |T - T_{cr}|^{1/2} \longrightarrow t_0 \propto |T - T_{cr}|^{-1}$$

processes in the vicinity of the critical point prove to be very slow

Peculiarities of hydro- description

Eq. is the 2-order in time derivatives -- beyond the standard Ginzburg-Landau description where:

$$-\rho_{\mathrm{r}}^{-2}\left(\tilde{d}\eta_{\mathrm{r}}+\zeta_{\mathrm{r}}\right)\frac{\partial\delta\rho}{\partial t}=\frac{\delta[F(T,\delta\rho)]}{\delta(\delta\rho)}|_{T}.$$

thermodynamical force drives system towards equilibrium

However for a produced fluctuation two initial conditions should be fulfilled

$$\delta\rho(t=0,\vec{r})=\delta\rho(0,\vec{r}),\qquad \frac{\partial\delta\rho(t,\vec{r})}{\partial t}\Big|_{t=0}\simeq 0$$

initial stage of fluctuation dynamics is not described in GL (mean field) approximation; at large t one can use the GL description

Flow-experiments at RHIC indicate on very low viscosity

Conformal theories show minimum η/s ~1/4π:
η/s ratio is under extensive discussion in the literature

However η/s does not appear in equations of motion for fluctuations

Dynamics of the density mode is controlled by another parameter β, which enters together with the second derivative in time. This parameter is expressed in terms of the **surface tension** and the **viscosity**

$$\beta = \frac{\sigma_0^2\, m}{32\, T_{\rm cr}\, [\frac{4}{3}\,\eta_{\rm r} + \zeta_{\rm r}]^2}$$

$$\sigma_0^2 = 32\, m\, \rho_{\rm cr}^2 T_{\rm cr}\, c$$

surface tension

The **larger viscosity** and the **smaller surface tension**,
the **more effectively viscous** is the fluidity of seeds.

β<<1 is the regime of effectively viscous fluidity
β >>1 is the regime of perfect fluidity

for liquid-gas phase transition β~0.01;
for H-QGP phase transition: β~0.02-0.2 , even for η/s ~1/4π:

Effectively very viscous fluidity of density fluctuations in the course of the phase transition!

Equation for the density fluctuation is supplemented by

the heat transport equation

for the variations of the entropy and temperature

For small u:

$$T_r\left[\partial_t \delta s - s_r (n_r)^{-1} \partial_t \delta n\right] = \kappa_r \Delta \delta T.$$

The variation of the temperature is related to the variation of the entropy density $s[n, T]$ by

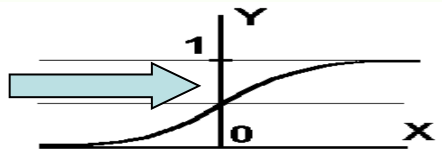
$$\delta T \simeq T_r (c_{V,r})^{-1} \left(\delta s - (\partial s / \partial n)_{T,r} \delta n\right),$$

Stage $t_\rho >> t_T$, limit of a large thermal conductivity, seeds evolve at almost constant T

$$\delta n(t,r) \simeq \frac{v(T)}{m}\left[\pm \mathrm{th}\frac{r-R_n(t)}{l} + \frac{\epsilon}{2\lambda_{cr} v^3(T)}\right] + (\delta n)_{cor}$$

$(\delta n)_{cor}$ is a small correction responsible for the baryon number conservation

stable seed interior



metastable exterior

$$\frac{\beta t_0^2}{2}\frac{d^2 R_n}{dt^2} = \frac{3\epsilon}{2\lambda_{cr} v^3(T)} - \frac{2l}{R_n} - \frac{t_0}{l}\frac{dR_n}{dt}$$

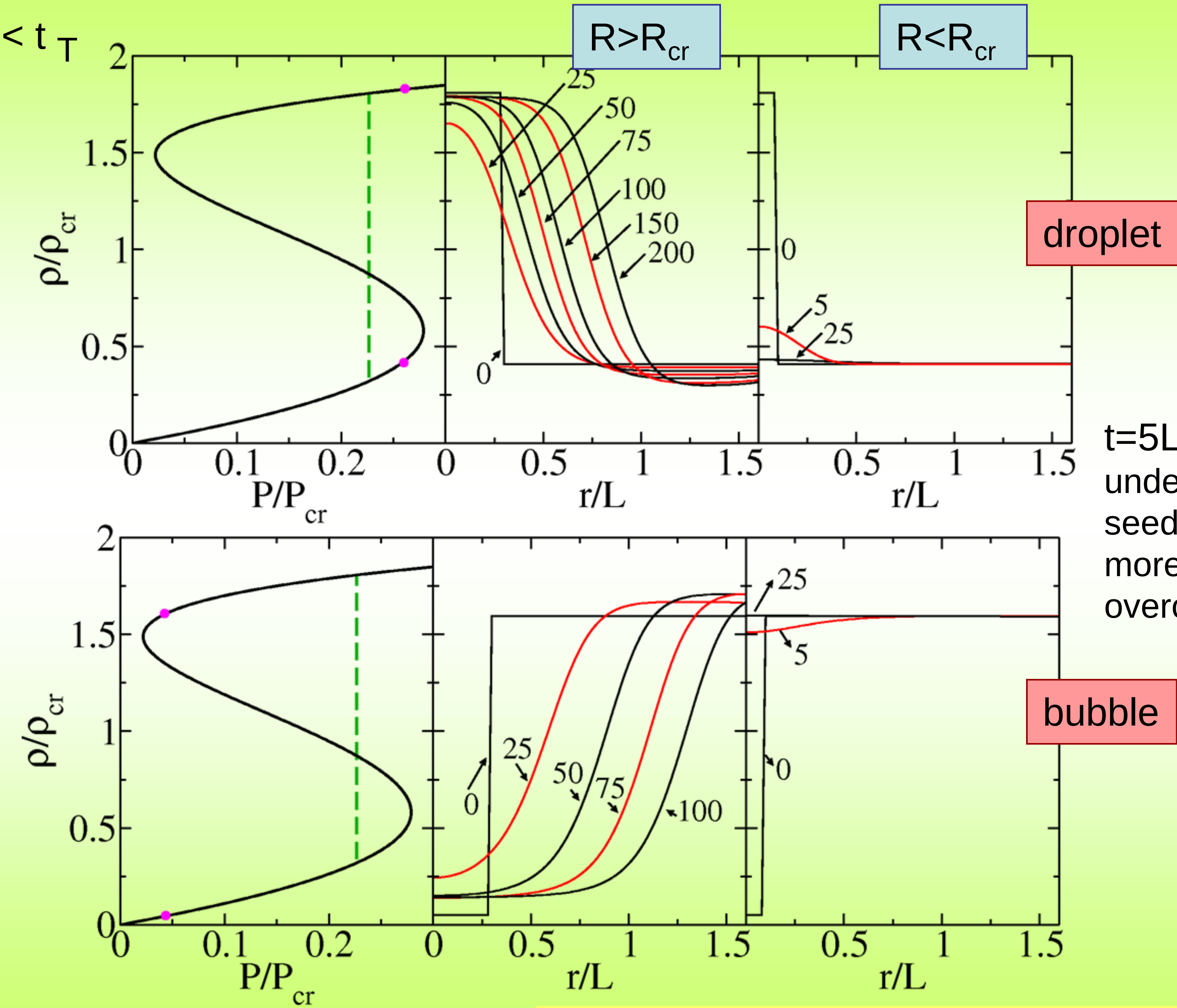
$R_{cr} = 4l\lambda_{cr} v^3(T)/(3\epsilon)$. First the bubble/droplet size $R_n(t) > R_{cr}$ grows with an acceleration and then it reaches a steady grow regime with a constant velocity $u_{as} = \frac{3\epsilon l}{\lambda_{cr} v^3(T) t_0} \propto \gamma_\epsilon |T_{cr} - T|^{1/2}$.

$$\delta s = \left(\frac{\partial s}{\partial n}\right)_T \left\{\frac{v(T)}{m}\left[\pm \mathrm{th}\frac{r-R_n(t)}{l} + \frac{\epsilon}{2\lambda_{cr} v^3(T)}\right] + (\delta n)_{cor}\right\}$$

seeds with $R<R_{cr}$ disolve

Hadron-QGP phase transition: droplet/bubble evolution from metastable phases

For $t_\rho < t_T$

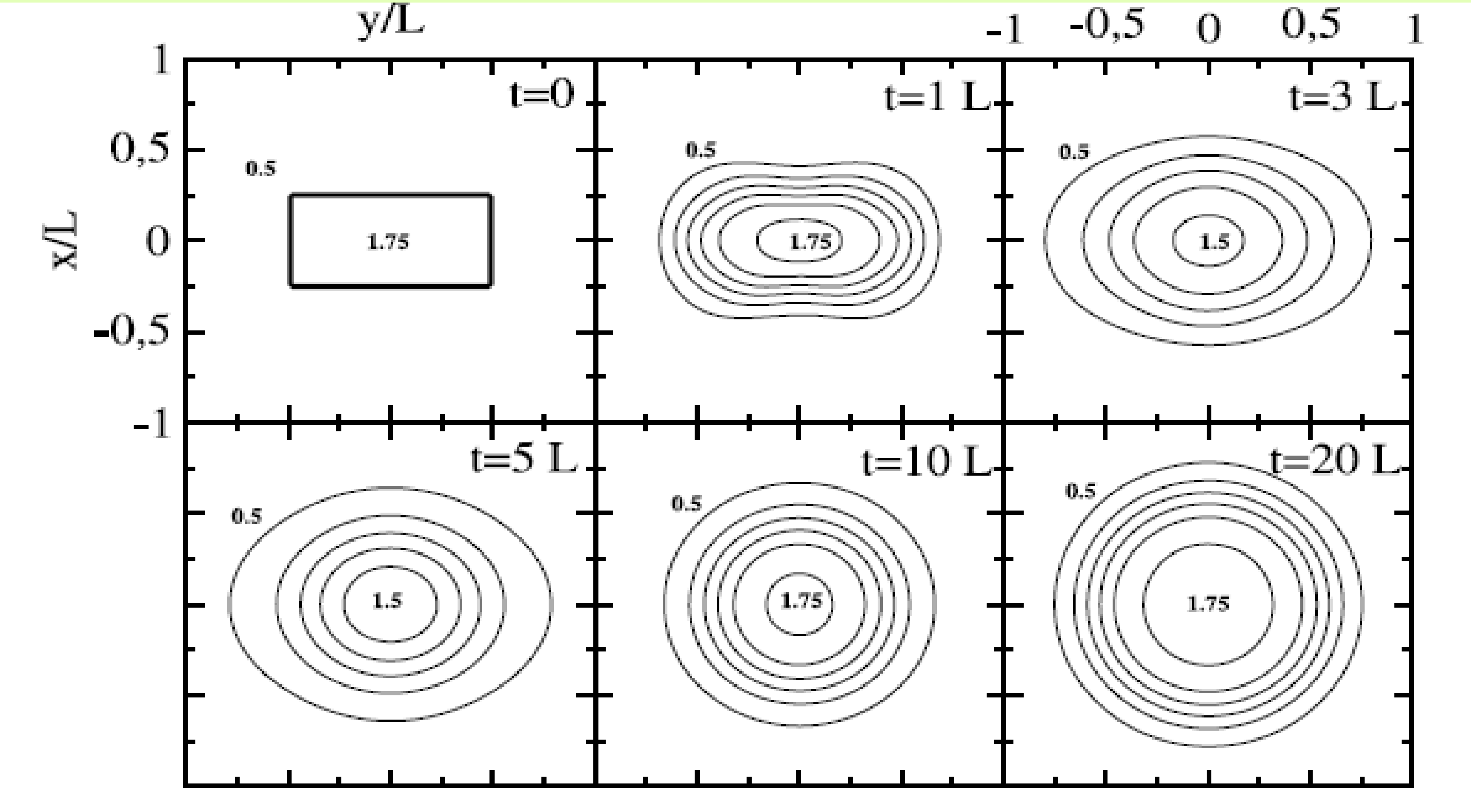


t=5L=25 fm, undercritical seeds dissolve more rapidly, overcritical-slowly

$(T_{cr} - T)/T_{cr} = 0.15$; T_{cr}=162 MeV; L=5 fm; β =0.2

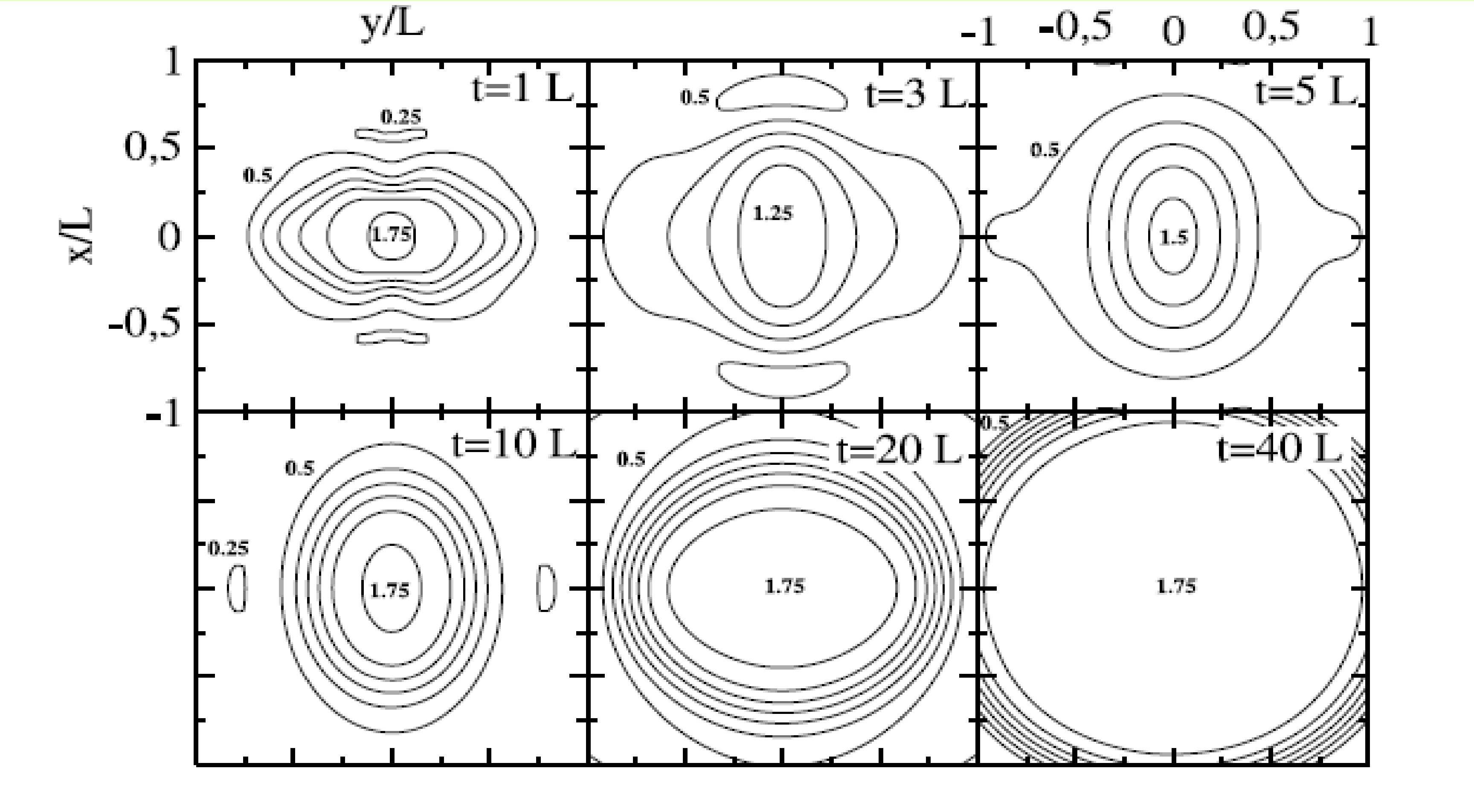
Change of the seed shape with time

Iso-lines of the density n/n_{cr} with increment 0.25



Initially anisotropic droplet slowly acquires spherical form $\beta = 0.1 \ll 1$

Change of the seed shape with time



For almost perfect fluid the process is more peculiar and still more slow $\beta=1000>>1$

Limit of zero thermal conductivity

$$\delta n(t,r) \simeq \frac{v(\tilde{s})}{m}\left[\pm \operatorname{th}\frac{r-R_n(t)}{l} + \frac{\epsilon}{2\lambda_{P,max}v^3(\tilde{s})}\right] + (\delta n)_{cor},$$

but now at fixed entropy per baryon rather than at fixed T

$$\delta\tilde{s} = 0 = (\delta s n_{P,max} - s_{P,max}\delta n)/n^2_{P,max}$$

Instabilities in spinodal region

aerosol-like mixture of bubbles and droplets (**mixed phase**)

$$\delta n = \delta n_0 \exp[\gamma t + i\,\mathbf{pr}],$$

$$\delta s = \delta s_0 \exp[\gamma t + i\,\mathbf{pr}],$$

$$T = T_> + \delta T_0 \exp[\gamma t + i\,\mathbf{pr}]$$

$T_>$ is the temperature of the uniform matter

From equations of non-ideal hydro:

$$\gamma^2 = -p^2\left[u_T^2 + \frac{(\tilde{d}\eta + \zeta)\gamma}{mn} + cp^2 + \frac{u_{\tilde{s}}^2 - u_T^2}{1 + \kappa p^2/(c_V\gamma)}\right]$$

$u_{\tilde{s}}^2 = m^{-1}(\partial P/\partial n)_{\tilde{s}}$ and $u_T^2 = m^{-1}(\partial P/\partial n)_T$ are speeds of sound

V. Skokov, D.V. , arXiv 0811.3868, JETP Lett. 90 (2009) 223; Nucl. Phys. A828 (2009) 401; A846 (2010); J. Randrup, PRC79 (2009) 024601, arXiv arXiv:1007.1448

Three solutions

For small momenta:

$$\gamma_{1,2} = \pm i u_{\tilde{s}} p + \left[\frac{\kappa}{c_V} \left(\frac{u_T^2}{u_{\tilde{s}}^2} - 1 \right) - \frac{\tilde{d}\eta + \zeta}{mn} \right] \frac{p^2}{2},$$

Density mode

$$\gamma_3 = -\frac{\kappa u_T^2 p^2}{u_{\tilde{s}}^2 c_V} \left[1 - \frac{u_T^2 - u_{\tilde{s}}^2}{u_{\tilde{s}}^2 u_T^2} \left(c + \frac{\kappa u_T^4}{u_{\tilde{s}}^2 c_V^2} - \frac{(\tilde{d}\eta + \zeta)\kappa u_T^2}{mnc_V u_{\tilde{s}}^2} \right) p^2 \right]$$

Thermal mode

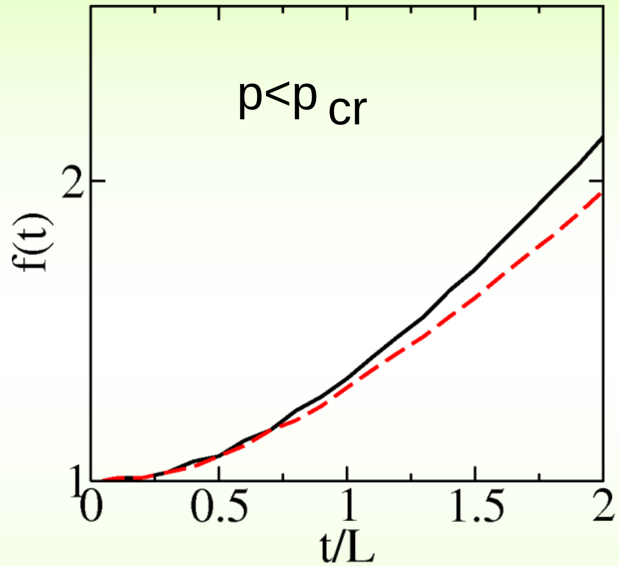
Does instability arise after the trajectory crosses the isothermal spinodal line or adiabatic one?

Limit of large thermal conductivity

$$\kappa \gg \nu c_V \sqrt{c} \quad , \nu = (u_{\tilde{s}}^2 - u_T^2)/(-u_T^2)$$

instability arises for the density mode, when trajectory crosses isothermal spinodal line

amplitude of the growing modes



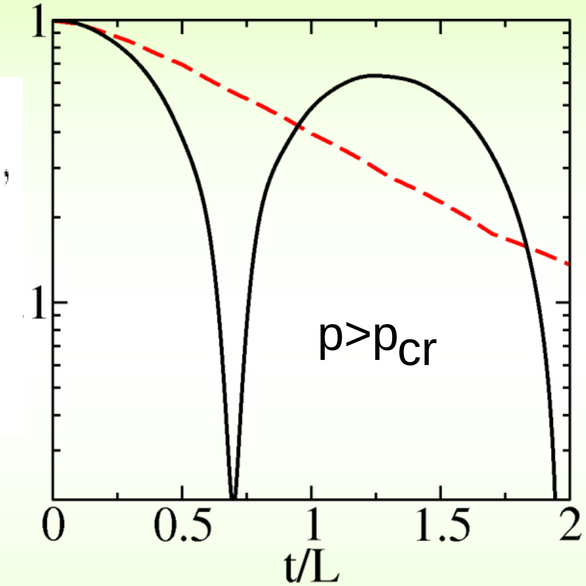
for most rapidly growing modes:

$$\gamma_m = \frac{(-u_T^2) m n_{cr}}{(2\sqrt{\beta}+1)(\tilde{d}\eta + \zeta)},$$

$$p_m^2 = \frac{(-u_T^2)\sqrt{\beta}}{(2\sqrt{\beta}+1)c}.$$

⇒ $R_m \sim 1/p_m$

oscillating modes for β>>1



β=0.1 dash line, β=10 solid line

t=2L=10 fm

$$\delta T_0 = \delta n_0 \frac{T s[1 - n(\partial s/\partial n)_T / s]}{c_V n[1 + \kappa/(\sqrt{c} c_V)]}$$

Far from critical point time evolution is rapid –effect of warm Champagne

Limit of small thermal conductivity $\kappa \ll \nu c_V \sqrt{c}$.

Instability arises when trajectory crosses isothermal spinodal line, but now for the thermal mode

$$p_m^2 \simeq -u_T^2/(2c), \qquad \gamma_{3m} = \gamma_3(p_m) \simeq \frac{\kappa u_T^4}{4 c c_V u_{\tilde{s}}^2}.$$

Limit of $\kappa = 0$ (like in ideal hydro. calculations) **is special:**
no thermal mode

Instability arises for the density mode far below T_{cr}, only when trajectory crosses **adiabatic spinodal line**

$$\gamma^2 = -p^2\left[u_{\tilde{s}}^2 + \frac{(\tilde{d}\eta + \zeta)\gamma}{mn} + cp^2\right].$$

Solution is similar to that for the density modes at large κ, but now the entropy per baryon is fixed rather than the temperature.

ideal hydro *(at least without taking special care)* **cannot correctly describe dynamics of the first-order phase transition**

Conclusions

The larger viscosity and the smaller surface tension the effectively more viscous is the fluidity

Anomalies in thermal fluctuations near CEP (which are under extensive discussion) may have not sufficient time to develop

⇨ argument in favor of mean field EoS

Thus T_{cr} calculated in thermal models with fluctuations included might be different from the value which may manifest in fluctuations in HIC

Heat transport effects play important role

Effects of spinodal decomposition can be easier observed since they require a shorter time to develop

Since in reality κ is not zero, spinodal instabilities start to develop when the trajectory crosses the isothermal spinodal line rather than the adiabatic one as it were in ideal hydro, i.e. at much higher T. This favors observation of manifestation of spinodal decomposition in the H-QGP phase transition in HIC

Concluding:

- One may hope to observe non-monotonous behavior of different observables in HIC due to manifestation of non-trivial fluctuation effects (especially of **spinodal decomposition** at I order hadron-quark phase transition) at monotonous increase of collision energies:

collision energy increase with a certain energy step will be possible at FAIR and NICA

Values of viscosities and thermal conductivity

There exist many (although very different) estimates of viscosities in hadron and quark matter and almost no appropriate estimates of the heat conductivity

Viscosities in SHMC model: hadron phase

A.Khvorostukhin, V.Toneev, D.V. Nucl.Phys. A845:106 (2010)

(From V.Toneev presentation)

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm MF} + \mathcal{L}_{\rm ex}$$

$$\mathcal{L}_{\rm bar} = \sum_{b\in\{\rm bar\}} \left[i\bar{\Psi}_b \left(\partial_\mu + i\, g_{\omega b}\, \chi_\omega\, \omega_\mu\right)\gamma^\mu \Psi_b - m_b^* \bar{\Psi}_b \Psi_b \right].$$

$\{b\} = N(938),\ \Delta(1232),\ \Lambda(1116),\ \Sigma(1193),\ \Xi(1318),\ \Sigma^*(1385),\ \Xi^*(1530),$ and $\Omega(1672),$

$$m_b^*/m_b = \Phi_b(\chi_\sigma \sigma) = 1 - g_{\sigma b}\, \chi_\sigma\, \sigma/m_b\,,\ b\in\{b\}$$

$$\mathcal{L}_{\rm MF} = \frac{\partial^\mu\sigma\ \partial_\mu\sigma}{2} - \frac{m_\sigma^{*2}\,\sigma^2}{2} - U(\sigma) - \frac{\omega_{\mu\nu}\,\omega^{\mu\nu}}{4} + \frac{m_\omega^{*2}\,\omega_\mu\omega^\mu}{2}$$

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\ .$$

isospin-symmetric hadronic matter

$$m_m^*/m_m = |\Phi_m(\chi_\sigma\sigma)|\,,\quad \{m\} = \sigma, \omega\,.$$

Brown-Rho scaling ansatz

$$\Phi = \Phi_N = \Phi_\sigma = \Phi_\omega = \Phi_\rho = 1 - f,\quad f = g_{\sigma N}\ \chi_\sigma\ \sigma/m_N$$

$$U = m_N^4 \left(\frac{b}{3} f^3 + \frac{c}{4} f^4\right).$$

$$\mathcal{L}_{\rm ex} = \sum_{\rm bos\in\{ex\}} \mathcal{L}_{\rm bos}$$

$$\{\rm ex\} = \pi; K, \bar{K}; \eta(547); \sigma', \omega', \rho'; K^{*\pm,0}(892), \eta'(958), \phi(1020)$$

Two phase model

Quark-gluon phase, HQB model: the IG of the massive quarks, antiquarks and gluons

$$\varepsilon^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str}) = \sum_{a\in\{q\}} \varepsilon_a^{\rm IG}(T,\mu_a) + B$$

$$P^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str}) = \sum_{a\in\{q\}} P_a^{\rm IG}(T,\mu_a) - B$$

$$n_{\rm bar}^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str}) = \sum_{a\in\{q\}} b_a\; n_a^{\rm IG}(T,\mu_a)$$

$$n_{\rm str}^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str}) = \sum_{a\in\{q\}} s_a\; n_a^{\rm IG}(T,\mu_a)$$

$\{q\} = q,\bar{q},g$

Gibbs conditions:

$$P^{\rm SHMC}(T,\mu_{\rm bar},\mu_{\rm str}) = P^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str})\ ,$$

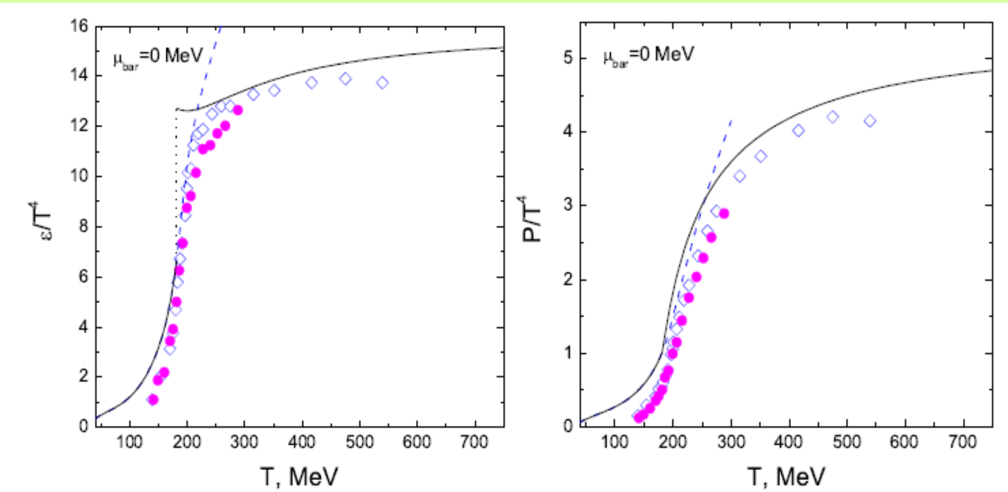
$$n_{\rm bar}(T,\mu_{\rm bar},\mu_{\rm str}) = \alpha\; n_{\rm bar}^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str}) + (1-\alpha)\; n_{\rm bar}^{\rm SHMC}(T,\mu_{\rm bar},\mu_{\rm str})$$

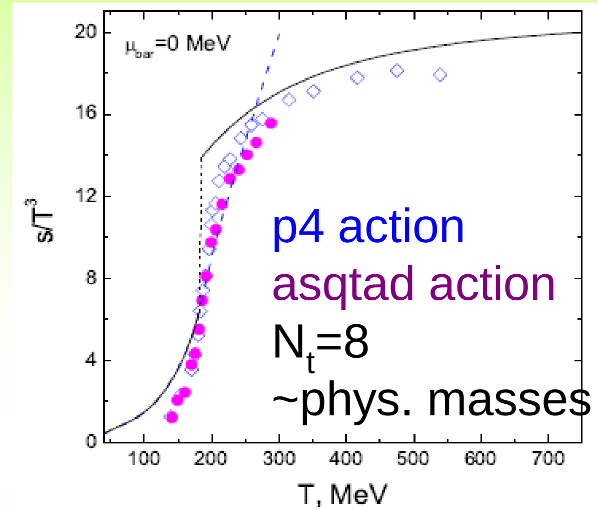
$$0 = \alpha\; n_{\rm str}^{\rm HQB}(T,\mu_{\rm bar},\mu_{\rm str}) + (1-\alpha)\; n_{\rm str}^{\rm SHMC}(T,\mu_{\rm bar},\mu_{\rm str})\ ,$$

$$\alpha = V^{\rm HQB}/V$$

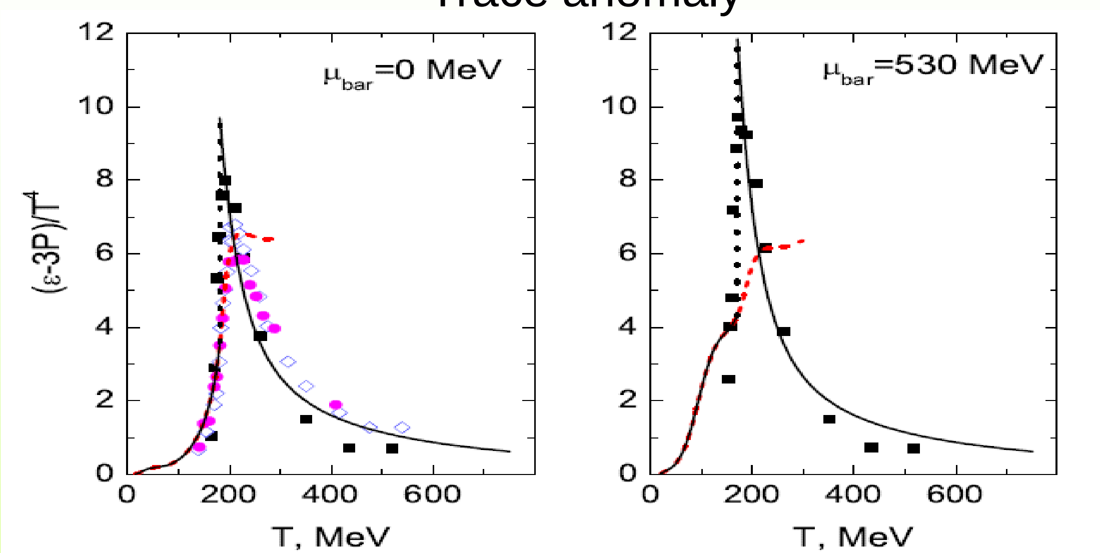
Comparison of EoS with lattice data

energy density pressure entropy





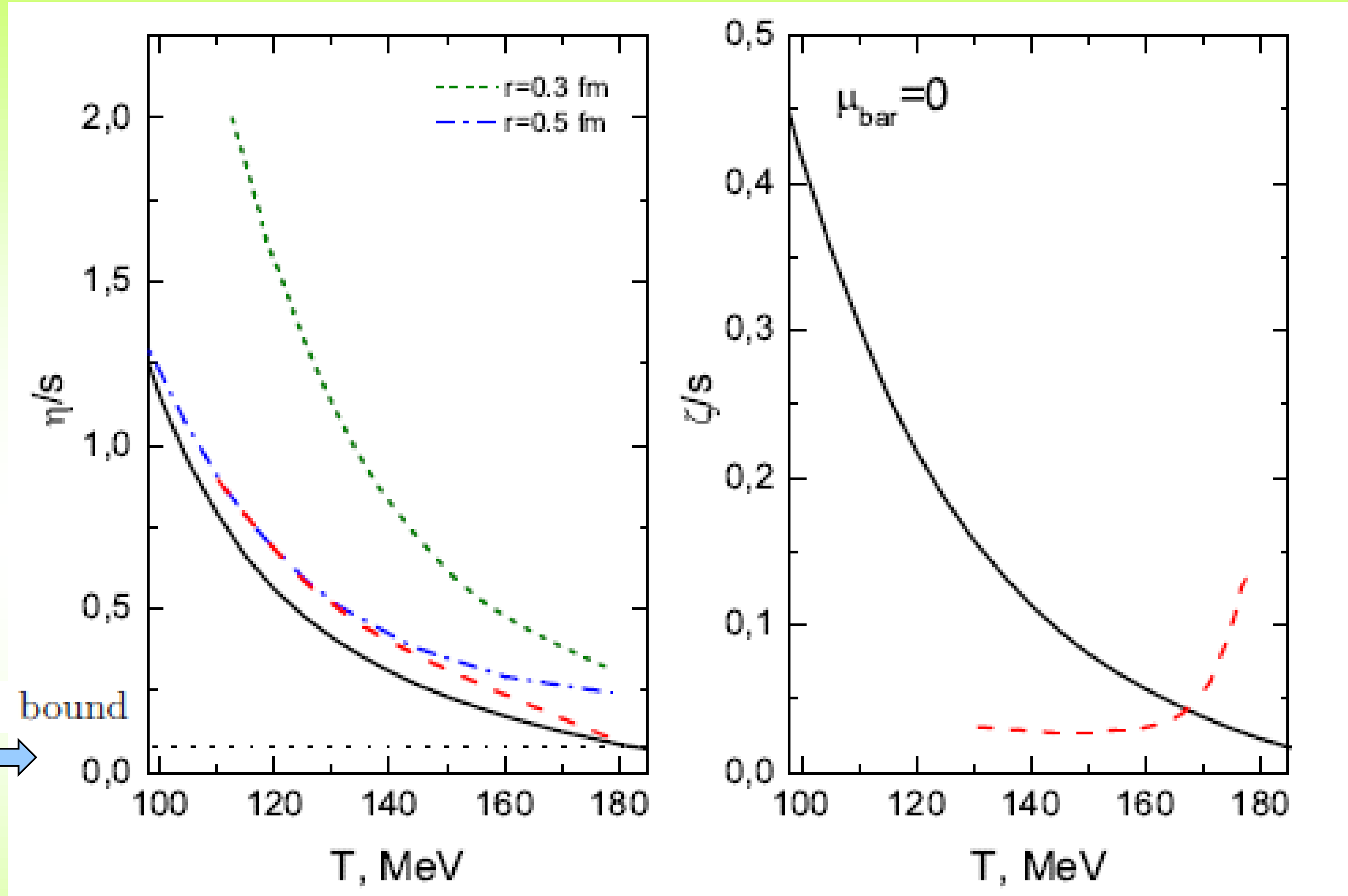
Trace anomaly



A.Bazavov et al., Phys. Rev. **D80,** 014504 (2009)

Z.Fodor et al., Phys. Lett. **B568**, 73 (2003)

Viscosity behavior for $\mu_{bar}=0$



lower AdS/CFT bound

$\eta/s = 1/4\pi$

Excluded-volume hadron gas model: M. Gorenstein et al., Phys. Rev. **C77**, 024911 (2008)

$$\eta = \frac{5}{64\sqrt{\pi}} \frac{\sqrt{mT}}{r^2}$$

Resonance gas with Hagedorn states: J.Naronha-Hostler et al., Phys. Rev. Lett. **103**, 172302 (2009)

$$\rho(m) = m^{-a} \ \exp(m/T_{\mathrm{H}})$$

Viscosities in SHMC model for baryon enriched matter