Excited Nuleon as a van der Waals system of Partons

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Saturation in deep inelastic scattering (DIS) and deeply virtual Compton scattering (DVCS) is associated with a phase transition between the partonic gas, typical of moderate x and Q^2, and partonic fluid appearing at increasing Q^2 and decreasing Bjorken x. We suggest the Van Der Waals equation of state to describe properly this phase transition.

1. INTRODUCTION

The thermodynamic approach to high-energy hadronic collisions was initiated by the paper of Fermi [1], subsequently successfully applied to high-energy nuclear collisions. Less familiar, although numerous, are the applications of the thermodynamical approach to deep inelastic scattering (DIS), see [2] and [3] and references therein. While the creation of a large number of particles, justifying the applicability of the statistical mechanics, is typical of both classes of reactions, there are subtleties that make the thermodynamics of DIS different from that of hadronic or nuclear collisions. One is connected with the reference frame: while thermalization and possible creation of "quark-gluon plasma" is considered in the rest frame, the partonic picture of DIS is realized in the infinite momentum frame, $p_z \to \infty$. The second point is the role and treatment of temperature. Any statistical distribution in a gas of quasi-free particles (nucleons or partons) implies the existence of a "temperature" T, as e.g. in the Boltzmann distribution $e^{-E/T}$, although its physical interpretation is not unique. A limited temperature T is typical of the hadronic phase [4] but not of the partons, that can be heated indefinitely.

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In Ref. [5] arguments based on parton-hadron duality lead the concept of "pseudo-thermalization", according to which, after the hadron is broken by the interaction, the inclusive spectrum reflects the "thermal" distribution holding at the parton level.

A related problem in DIS is the choice of the variable in the statistical distribution. The use of the Bjorken variable x, instead of the energy (or momentum), in the above (or similar) distributions needs the introduction of a proper dimensional parameter, related to the change of the coordinate systems (see, e.g. [2]), $\exp(-x/\bar{x})$, $\bar{x} = x(1 + k^2/x^2m^2)$, where k is the quark transverse momentum and m is the proton mass. Note that the "temperature" T here is that of the partonic gas (or liquid/fluid) inside the nucleon and hence, unlike the hadronic systems [4], it must not be limited.

It was suggested in Ref. [3] that saturation in DIS, predicted by QCD and observed experimentally, corresponds to the condensation of partonic gas to a fluid. The interior of the nucleon excited in DIS or DVCS undergoes a phase transition from a partonic gas (high- and intermediate x, $x \sim 0.05$) to a partonic fluid. The division line is located roughly at those values of x and Q^2 where Bjorken scaling is broken, as shown in Figs. 1 and 2.

Our idea, based on the observed behavior of the DIS structure functions, is that the partonic matter in a nucleon (or nucleus) as seen in DIS, undergoes a change of phase from a nearly perfect gas, typical of the Bjorken scaling region, to a liquid, where the logarithmic scaling violation is replaced by a power (see Figs. 1 and 2). The presence of two regions, namely that of Bjorken scaling and beyond it (call them for the moment "dilute" and "dense"), via an intermediate mixed phase, are visible in this figure. They can be quantifies in various ways that will be discussed in the next section. The relevant variable here are the fraction of the nucleon momentum, or the Bjorken variable x and the incident photon's virtuality Q^2.

The co-existence of two phases, gaseous and fluid, can be described, e.g. by the van der Waals equation, valid in a tremendous range of its variables and applicable to any system (e.g. molecular, atomic or nuclear) with short range repulsion and long range attraction between the constituents.

We assume that the interior of a nucleon (the idea can be extended to nuclei as well), as seen in (inclusive) deep inelastic scattering (DIS) or (exclusive) deeply virtual Compton scattering (DVCS), is a thermodynamic system, that, similar to the case of nuclear or heavy-ion collisions, bear collective (thermodynamic) properties governed by a relevant equation of state (EoS).

2. THE EXCITED NUCLEON AS A VAN DER WAALS PARTONIC SYSTEM

Having introduced the statistical properties of the SF, we now proceed to an equation of state (EoS) describing the transition between a parton gas, via a mixed foggy phase, to the partonic liquid. To this end, we use the van der Waals equation

$$(P + N^2 a/V^2)(V - Nb) = NT, \tag{1}$$

see, e.g. [8, 12], where a and b are parameters depending on the properties of the system, N is the number of particles and V is the volume of the "container", $V(s) = \pi R^3(s)$, $R(s) \sim \ln s$ is the nucleon radius in our case. For point-like particles (perfect gas), $a = b = 0$, and Eq. (1) reduces to $pV = NT$, and, since $N/V \sim T^3$, we get in this approximation $p \sim T^4$.

Alternatively, Eq. (1) can be written as [8]

$$(P + a/V^2)(V - b) = RT,$$

or, equivalently

$$P = \frac{RT}{V-b} - \frac{a}{V^2}.$$

The parameter b is responsible for the finite dimensions of the constituents, related to $1/Q$ in our case, and the term a/V^2 is related to the (long-range) forces between the constituents. From this cubic equation in V one finds [8] the following values for the critical values $V = V_c$, $P = P_c$, and $T = T_c$ in terms of the parameters a and b:

$$V_c = 3b, \quad p_c = a/(27b^2) \quad T_c = 8a/(27Rb).$$

The particle number $N(s)$ can be calculated as [13]

$$N(s) = \int_0^1 dx F_2(x, Q^2),$$

where $F_2(x, Q^2)$ is the nucleon structure function, measured in deep inelastic scattering (see, e.g. [10, 11]). We remind the kinematics: $s = Q^2(1-x)/x + m^2$, which at small x reduces to $s \approx Q^2/x$. The radius of the constituent as seen in DIS is $r_0 \sim 1/Q$, hence its two-dimensional volume is $\sim Q^{-2}$.

By introducing "reduced" volume, pressure and temperature,

$$\mathcal{P} = P/P_c, \quad \mathcal{V} = V/V_c = \rho_c/\rho, \quad \mathcal{T} = T/T_c,$$

the van der Waals equation (1) can be rewritten as

$$\Big(\mathcal{P} + 3/\mathcal{V}^3\Big)\Big(\mathcal{V} - 1/3\Big) = 8\mathcal{T}/3. \tag{2}$$

Note that Eq. (2) contains only numerical constants, and therefore it is universal. States of various substances with the same values of $\mathcal{P}$, $\mathcal{V}$, and $\mathcal{T}$ are called "corresponding states"; Eq. (2) is called the "van der Waals equation for corresponding states". The universality of the liquid-gas phase transition and the corresponding principle are typical if any system with short-range repulsive and long-range attractive forces. This property is shared both by ordinary liquids and by nuclear matter. This was demonstrated, in particular, in Ref. [9], where typical van der Waals curves were shown to be similar to those derived by means of the Skyrme effective interaction.

Following Refs. [2, 9], we present two example of EoS, one based on the Skyrme effective interaction and finite-temperature Hartree-Fock theory, and the other one is the van der Waals EoS. Jaqaman et al. [9] start with the EoS

$$P = \rho kT - a_0\rho^2 + a_3(1+\sigma)\rho^{(2+\sigma)}, \tag{3}$$

where $\rho = N/V$ is the density and a_0, a_3 and σ are parameters, $\sigma = 1$ corresponding to the usual Skyrme interaction. Particular values of the above parameters, corresponding to various options (degenerate and non-degenerate Fermi gas) as well as to finite and infinite nuclear matter are quoted in Table 1 of Ref. [9].

According to the law of the corresponding states, Eq. (3) is universal for scaled (reduced) variables, for which, with $\sigma = 1$, it becomes

$$P = 3\mathcal{T}/\mathcal{V} - 3/\mathcal{V}^2 + 1/\mathcal{V}^3,$$

to be compared with the van der Waals EoS

$$P = 8\mathcal{T}/(3\mathcal{V} - 1) - 3/\mathcal{V}^2.$$

Let us now write the van der Waals EoS in the form

$$P(T;N,V) = -\Big(\frac{\partial F}{\partial V}\Big)_{TN} = \frac{NT}{V - bN} - a\Big(\frac{N}{V}\Big)^2 = \frac{nT}{1 - bn} - an^2, \tag{4}$$

where $n = N/V$ is the particle number density, a is the strength of the mean-field attraction, and b governs the short-range repulsion. We identify the particle number density with the SF $F_2(x, Q^2)$. Fig. 3 shows the pressure-density dependence calculated from Eq. (4) with $a = 5\ GeV^{-2}$ and $b = 0.2\ GeV^{-3}$. Notice that while Eq. (4) is sensitive to b (short-range repulsion), it is less so to a (long-range attraction). Representative isotherms are shown in this figure: line C (second from the top) is the critical one, $T_c = 8a/(27b)$. Above this temperature (top line, B), the pressure rises uniformly with density, corresponding to a single thermodynamical state for each P and T. By contrast, for subcritical temperatures ($0 < T < T_c$) (lines D-G) the function $P(n)$ has a maximum followed by a minimum, see Fig. 3. Below the critical value P_c, three density regimes exist. The smallest density region lies in the the gaseous phase below the spinodal region, while the highest densities lie in the liquid phase, above the spinodal region. The coexistence phase can be determined by a Maxwell construction.

In a forthcoming paper we intend to specify further the microscopic properties of the VDW equation by appending the above semi-quantitative considerations with the details of the interquark forces. To this end, as a first approximation, potential models of quark interactions will be used. In the simplest one, the potential of interquark forces is approximated by

$$P(r) = -A/(r - r_0) + Br + C,$$

where $A,\ B$ and C are parameters and $r_0 = 1/Q$ is the Q^2- dependent quark radius. The above potential has the salient feature of the VDW EoS, namly as quark-partons come close, the repulsive barrier, due to their finite volume comes into play. At large distances, $r \sim R(s)$ the partons become quasi-free.

3. CONCLUSIONS

Below we list our (temporary) conclusions and mention some open questions left behind this paper:

For simplicity, we have concentrated on the singlet component of the SF (gluons and sea quarks), dominating the low x region, where saturation takes place. The inclusion the non-singlet and higher x components (valence quarks), according to the prescriptions given, e.g. in Ref. [10], are straightforward. We omitted any discussion of the large x $(1 - x)^n$

factor in the SF. This is because the statistical approach is not valid in the $x \to 1$ limit, on the one hand, and for simplicity, on the other hand.

We did not consider the possible (a) symmetry between "heating" and "cooling" of the excited nucleon. During deep inelastic scattering, the nucleon (nucleus) gets excited (heated), whereafter it releases its energy (heat) by producing secondaries. The energy distribution of these secondaries may reveal the temperature of the system, however this "ultimate" temperature [4], created after the confinement transition, will be different from that of the partonic system inside the proton, which may increase indefinitely.

Any phase transition may produce fluctuations in the observed spectra of produced particles. These fluctuations may originate either from the gas-fluid-gas phase transition under discussion, or from the (de)confinement transition, beyond the scope of the present paper.

The gas-fluid transition does not necessarily follow the van der Waals EoS. Possible alternatives are e.g. percolation or clustering of partons similar to the case of the molecule, see [3]. Quark recombination or coalescence, extending the concept of single parton fragmentation, which has been used in elementary collisions since the '70s, was studied recently in Ref. [14] in the context of heavy ion collisions. It should be remembered however, that the above models include (de)confinement transition, absent from our approach.

Experimentally, the onset of the gas-fluid phase transition may be verified by the observed spectrum of the produced particles. The $p_\perp$ distribution of produced from the dilute gaseous state, i.e. below the saturation region, can be computed perturbatively, while beyond the saturation border line they result from the collision of a very large number of constituent, as in the color glass condensate [15]. The observation of any correlation between the transverse distribution of the particles produced in DIS (or DVCS) below- or beyond the saturation border line will (dis)favor the picture presented in this paper. We intend to come back to this point in our future investigations.

Whatever the details of the transition, the important point to realize is the existence of a dense partonic substance, different from the perfect partonic gas, associated with Bjorken scaling in DIS or the so-called quark-gluon plasma, predicted by perturbative QCD and expected in high-energy hadronic and/or nuclear collisions. Instead, experimental data on DIS and on high-energy heavy ion collisions show that the partonic matter may appear as a fluid, called by L. McLerran *et al.* "color glass condensate" [15]. The nature of the strongly interacting matter under extreme conditions should be universal, be it produced in hadron-hadron, heavy nuclei or in deep inelastic lepton-hadron scattering.

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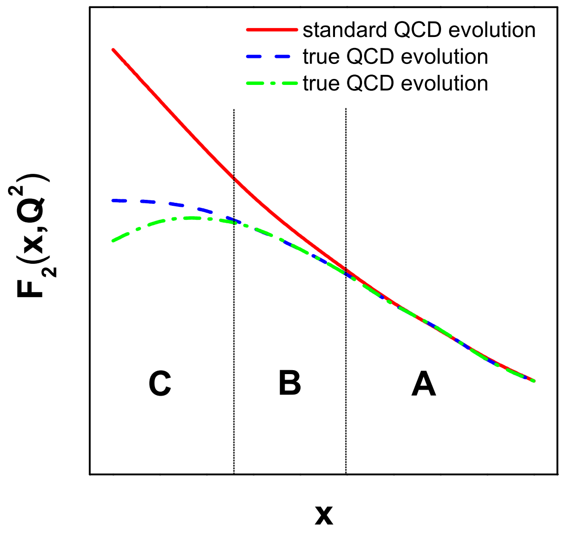

Figure 1. Small-x behavior of the structure function: standard QCD evolution versus "true QCD" evolution. A is the perturbative region, B is the transition region, C is the nonperturbative region.

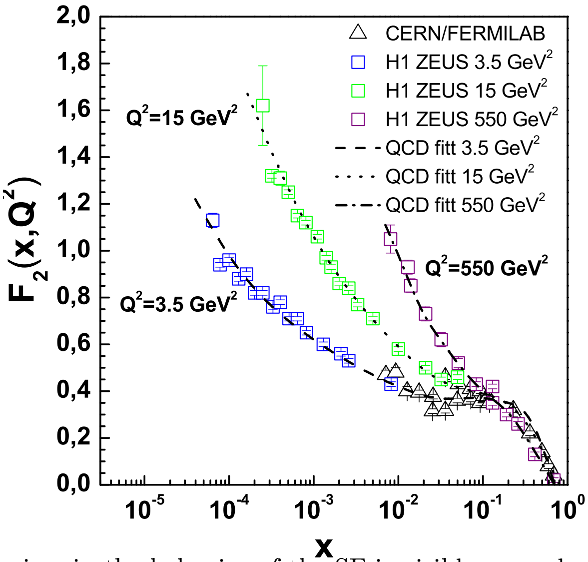

Figure 2. Change of regime in the behavior of the SF is visible around $x \sim 10^{-2}$. Beyond that value, Bjorken scaling holds (modulus $\ln Q^2$), while at lower x, the Q^2 dependence changes drastically and it cannot be considered as a small scaling violating effect anymore (see Ref. [6]).

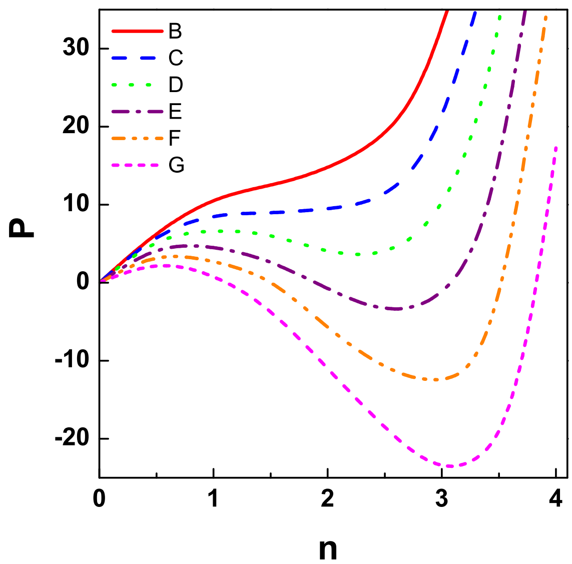

Figure 3. The pressure-to-density dependence calculated, in arbitrary units, from Eq.(4). The second curve from the top, C, is the critical one; beyond and below are the subcritical (curve B) and critical $(D - G)$ ones.

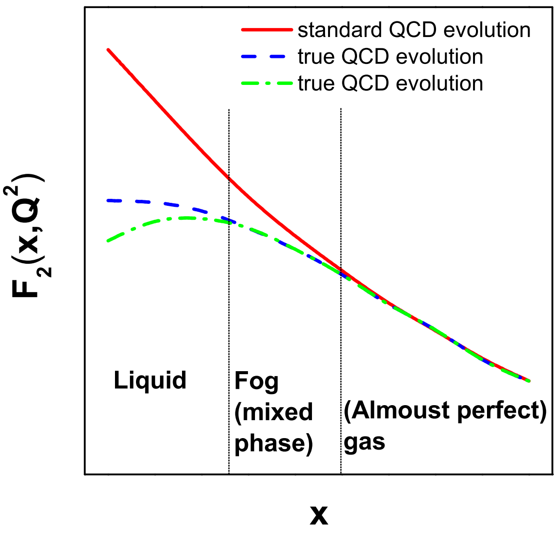

Figure 4. Gaseous, foggy and liquid states in various kinematical regions of DIS (cf. Fig. 1).

FIGURE CAPTIONS

Fig.1: Small-x behavior of the structure function: standard QCD evolution versus "true QCD" evolution. A is the perturbative region, B is the transition region, C is the nonperturbative region.

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Fig.4: Gaseous, foggy and liquid states in various kinematical regions of DIS (cf. Fig. 1).