PRD71(2005), JPG32(2006),
PRD77(2008), NPBPS186(2009),
arXiv:1106.4006 [hep-ph]

INCLUSIVE TAU LEPTON DECAY: THE EFFECTS DUE TO HADRONIZATION

A.V. Nesterenko

Bogoliubov Laboratory of Theoretical Physics

Joint Institute for Nuclear Research, Dubna, Russian Federation

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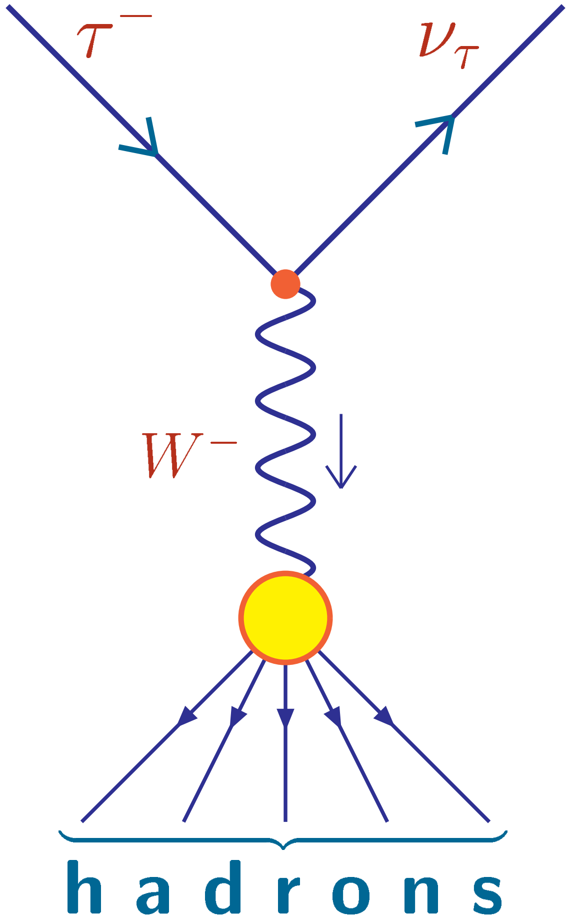
INTRODUCTION

The τ lepton is the only lepton which is heavy enough ($M_\tau \simeq 1.777\,\mathrm{GeV}$) to decay into hadrons. The interest to this process is primarily due to



- Tests of QCD and Standard Model
- Constraints on "New physics"
- Precise experimental data
- No need in phenomenological models
- Probes infrared hadron dynamics

The experimentally measurable quantity here is

$$R_\tau = \frac{\Gamma(\tau^- \to \text{hadrons}^- \, \nu_\tau)}{\Gamma(\tau^- \to e^- \, \bar{\nu}_e \, \nu_\tau)}$$

$$= R_{\tau,\mathrm{V}} + R_{\tau,\mathrm{A}} + R_{\tau,\mathrm{S}}$$

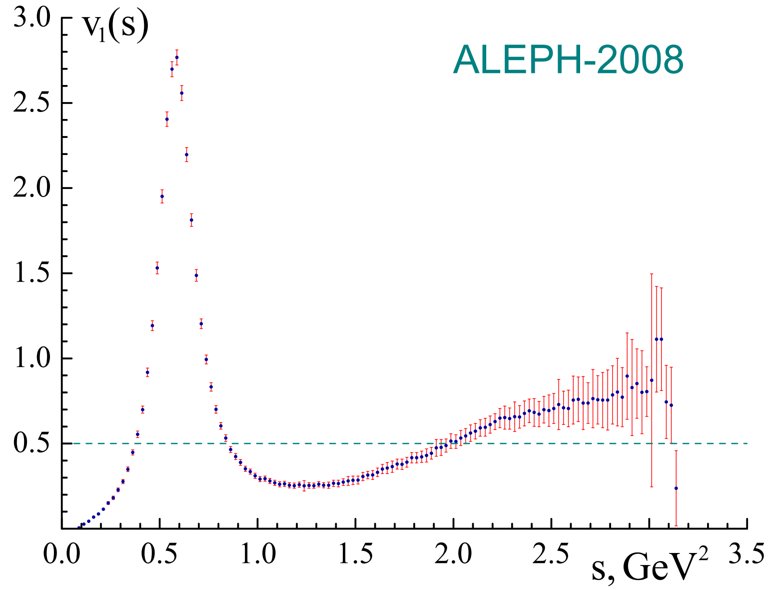
$$= 3.640 \pm 0.010,$$

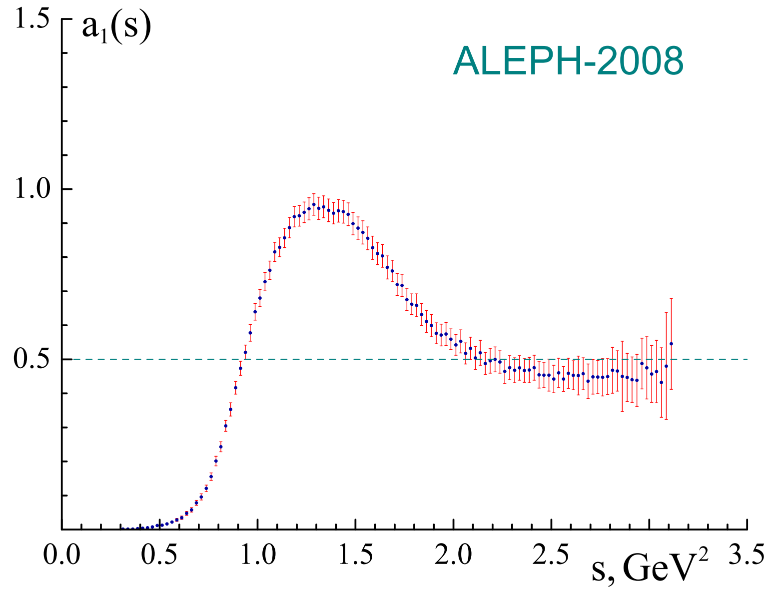
$$R_{\tau,\mathrm{V}} = R_{\tau,\mathrm{V}}^{J=0} + R_{\tau,\mathrm{V}}^{J=1}$$

$$= 1.783 \pm 0.011 \pm 0.002,$$

$$R_{\tau,\mathrm{A}} = R_{\tau,\mathrm{A}}^{J=0} + R_{\tau,\mathrm{A}}^{J=1}$$

$$= 1.695 \pm 0.011 \pm 0.002.$$





■ ***ALEPH Collab., EPJC4(1998), PR421(2005), RMP78(2006), EPJC56(2008).***

THEORETICAL DESCRIPTION

The theoretical prediction for the quantities on hand reads

$$R^{J=1}_{\tau,\mathrm{V/A}} = \frac{N_{\mathrm{c}}}{2}\,|V_{\mathrm{ud}}|^2\, S_{\mathrm{EW}}\Bigl(\Delta^{\mathrm{V/A}}_{\mathrm{QCD}} + \delta'_{\mathrm{EW}}\Bigr),$$

$N_{\mathrm{c}}=3$, $|V_{\mathrm{ud}}|=0.9738\pm0.0005$, $S_{\mathrm{EW}}=1.0194\pm0.0050$, $\delta'_{\mathrm{EW}}=0.0010$,

$$\Delta^{\mathrm{V/A}}_{\mathrm{QCD}} = 2\int_{m^2_{\mathrm{V/A}}}^{M_\tau^2} f\!\left(\frac{s}{M_\tau^2}\right) R^{\mathrm{V/A}}(s)\,\frac{ds}{M_\tau^2}\,,$$

where $M_\tau = 1.777\,\mathrm{GeV}$, $f(x) = (1-x)^2\,(1+2x)$,

$$R^{\mathrm{V/A}}(s)=\frac{1}{2\pi i}\lim_{\varepsilon\to 0_+}\Bigl[\Pi^{\mathrm{V/A}}(s+i\varepsilon)-\Pi^{\mathrm{V/A}}(s-i\varepsilon)\Bigr]=\frac{1}{\pi}\,\mathrm{Im}\lim_{\varepsilon\to 0_+}\Pi^{\mathrm{V/A}}(s+i\varepsilon),$$

$$\Pi_{\mu\nu}(q^2)=i\int d^4x\, e^{iqx}\langle 0|\,T\bigl\{J_\mu(x)J_\nu(0)\bigr\}|0\rangle \equiv \frac{i}{12\pi^2}(q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2).$$

- ***Braaten, Narison, Pich, NPB373(1992); Le Diberder, Pich, PLB289(1992).***

For practical purposes it is convenient to use Adler function

$$D(Q^2) = -\frac{d\,\Pi(-Q^2)}{d\ln Q^2}, \qquad Q^2 = -q^2 = -s.$$

- *Adler, PRD10(1974).*

Its ultraviolet behavior can be approximated by

$$D(Q^2) \simeq D^{(\ell)}_{\rm pert}(Q^2) = 1 + \sum_{j=1}^{\ell} d_j \left[\alpha_{\rm s}^{(\ell)}(Q^2)\right]^j, \quad Q^2 \to \infty,$$

where $\alpha_{\rm s}^{(\ell)}(Q^2)$ is the ℓ–loop perturbative running coupling.

One–loop: $\alpha^{(1)}_{\rm pert}(Q^2) = 4\pi/\left[\beta_0 \ln(Q^2/\Lambda^2)\right]$, $\beta_0 = 11 - 2n_{\rm f}/3$, $d_1 = 1/\pi$.

- *Gorishny, Kataev, Larin, PLB259(1991); Surguladze, Samuel, PRL66(1991); Baikov, Chetyrkin, Kuhn, PRL101(2008).*

For functions $R(s)$ and $D(Q^2)$ the following relations hold:

$$R(s) = \frac{1}{2\pi i} \lim_{\varepsilon \to 0_+} \int_{s+i\varepsilon}^{s-i\varepsilon} D(-\zeta)\,\frac{d\zeta}{\zeta} \;\longleftrightarrow\; D(Q^2) = Q^2 \int_{m^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}\,ds.$$

- *Adler, PRD10(1974); Radyushkin (1982); Krasnikov, Pivovarov, PLB116(1982).*

MASSLESS LIMIT

The masses of all final state particles are neglected ($m = 0$).

In this analysis, it is convenient to handle the leading–order terms separately from the strong corrections:

$$R(s) = r^{(0)}(s) + r_{\mathrm{s}}^{(\ell)}(s), \qquad D(Q^2) = d^{(0)}(Q^2) + d_{\mathrm{s}}^{(\ell)}(Q^2).$$

The one–loop level ($\ell = 1$) with three active flavors ($n_{\mathrm{f}} = 3$) is assumed in what follows.

There are two equally justified methods of description Δ_{QCD}

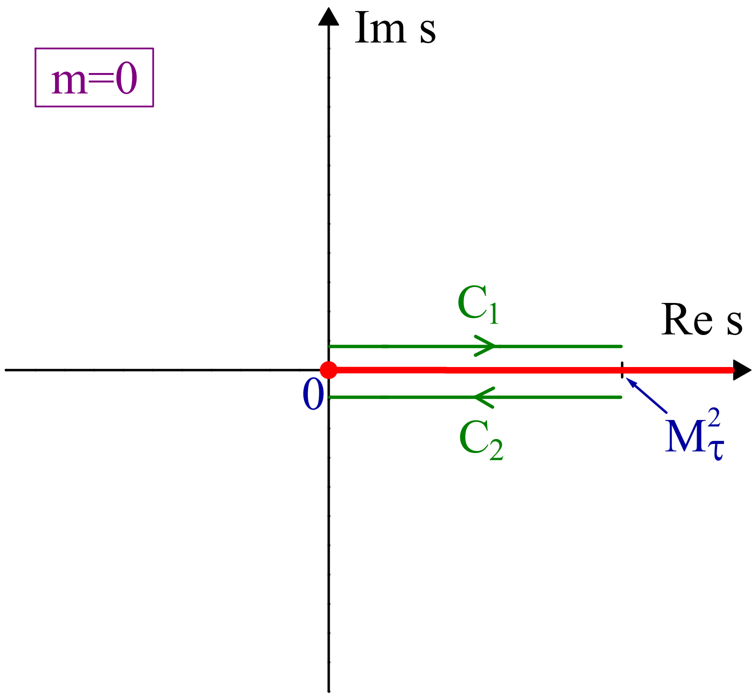
Method I: Use of definitions of $R(s)$ and $D(Q^2)$ only

The QCD contribution to $R_{\tau,\mathrm{V/A}}^{J=1}$

$$\Delta_{\mathrm{QCD}} = 2\int_0^{M_\tau^2} f\left(\frac{s}{M_\tau^2}\right) R(s)\, \frac{ds}{M_\tau^2}$$

can be represented as

$$\Delta_{\mathrm{QCD}} = \frac{1}{\pi i}\int_{0+i\varepsilon}^{M_\tau^2+i\varepsilon} f\left(\frac{s}{M_\tau^2}\right)\Pi(s)\,\frac{ds}{M_\tau^2} + \frac{1}{\pi i}\int_{M_\tau^2-i\varepsilon}^{0-i\varepsilon} f\left(\frac{s}{M_\tau^2}\right)\Pi(s)\,\frac{ds}{M_\tau^2}.$$



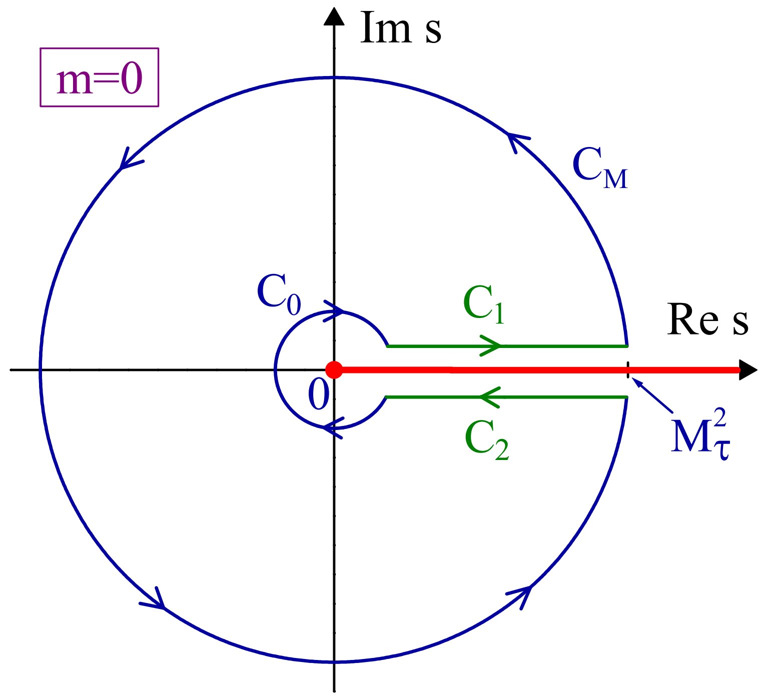
Integration by parts eventually results in

$$\Delta_{\mathrm{QCD}} = g(1)\,R(M_\tau^2) + \int_0^{M_\tau^2} g\left(\frac{\sigma}{M_\tau^2}\right)\varrho(\sigma)\,\frac{d\sigma}{\sigma}, \qquad g(x) = x(2-2x^2+x^3),$$

$$\varrho(\sigma) = \frac{1}{2\pi i}\lim_{\varepsilon\to 0_+}\left[d_{\mathrm{s}}(-\sigma-i\varepsilon) - d_{\mathrm{s}}(-\sigma+i\varepsilon)\right].$$

Method II: Method I + deformation of integration contour

Identically to "Method I", Δ_{QCD} is rewritten here as the sum of two integrals along the edges of physical cut of $\Pi(q^2)$:



$$\Delta_{\text{QCD}} = \frac{1}{\pi i} \int_{0+i\varepsilon}^{M_\tau^2+i\varepsilon} f\left(\frac{s}{M_\tau^2}\right) \Pi(s) \frac{ds}{M_\tau^2} + \frac{1}{\pi i} \int_{M_\tau^2-i\varepsilon}^{0-i\varepsilon} f\left(\frac{s}{M_\tau^2}\right) \Pi(s) \frac{ds}{M_\tau^2}.$$

Additional deformation of contour: $C_1 + C_2 \rightarrow -(C_0 + C_{\text{M}})$

$$\Delta_{\text{QCD}} = \frac{i}{\pi}\left[\int_{C_0} f\left(\frac{s}{M_\tau^2}\right) \Pi(s) \frac{ds}{M_\tau^2} + \int_{C_{\text{M}}} f\left(\frac{s}{M_\tau^2}\right) \Pi(s) \frac{ds}{M_\tau^2}\right]$$

$$= \frac{1}{2\pi} \lim_{\varepsilon \to 0_+} \int_{-\pi+\varepsilon}^{\pi-\varepsilon} D\left(M_\tau^2\, e^{i\theta}\right)\left(1 + 2e^{i\theta} - 2e^{i3\theta} - e^{i4\theta}\right) d\theta.$$

PERTURBATIVE APPROACH

All the presented above is valid only for "genuine physical" functions $\Pi_{\text{phys}}(q^2)$ and $D_{\text{phys}}(Q^2)$ in the massless limit.

However, one has to deal with their perturbative approximations, which are inconsistent with dispersion relations for these functions.

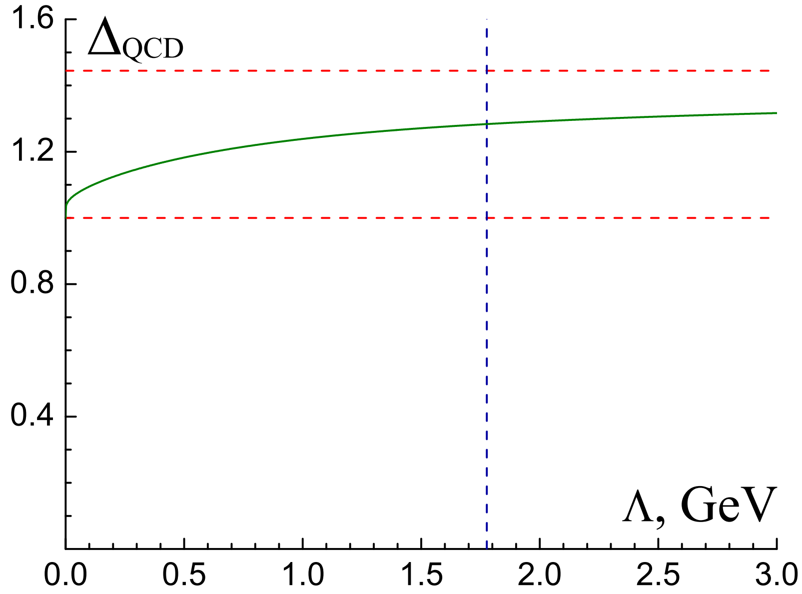
Hadronic decay of τ lepton within perturbative approach: the direct use of perturbative approximations $\Pi_{\text{pert}}(q^2)$ and $D_{\text{pert}}(Q^2)$ in the aforementioned expressions for Δ_{QCD}.

Method I + one–loop pQCD:

$$\Delta_{\text{QCD}} = 1 + \int_0^{\infty} h\left(\frac{\sigma}{M_\tau^2}\right) \rho_{\text{pert}}^{(1)}(\sigma) \frac{d\sigma}{\sigma},$$

$$h(x) = g(x)\,\theta(1-x) + g(1)\,\theta(x-1),$$

$$\rho_{\text{pert}}^{(1)}(\sigma) = \frac{4}{\beta_0} \frac{1}{\ln^2(\sigma/\Lambda^2) + \pi^2}.$$



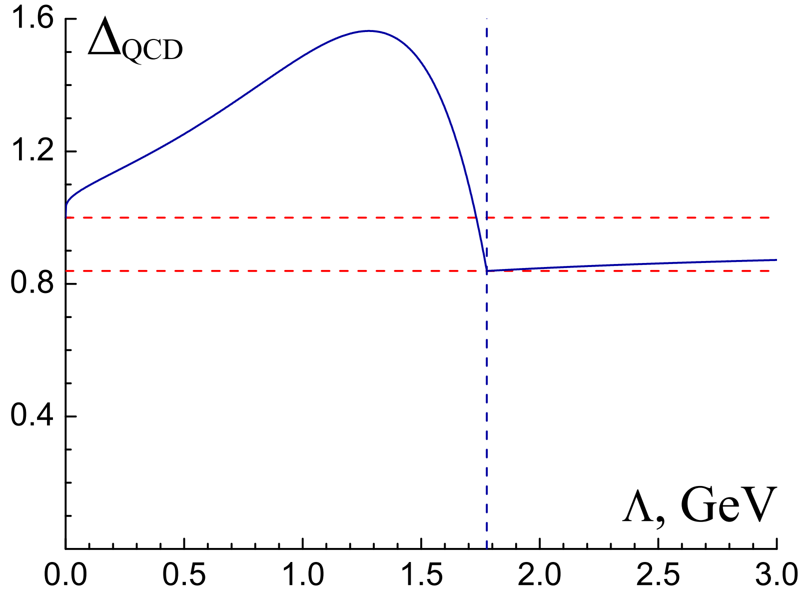
Method II + one–loop pQCD:

$$\Delta_{\text{QCD}} = 1 + \frac{4}{\beta_0} \int_0^{\pi} \frac{\lambda A_1(\theta) + \theta A_2(\theta)}{\pi(\lambda^2 + \theta^2)}\, d\theta,$$

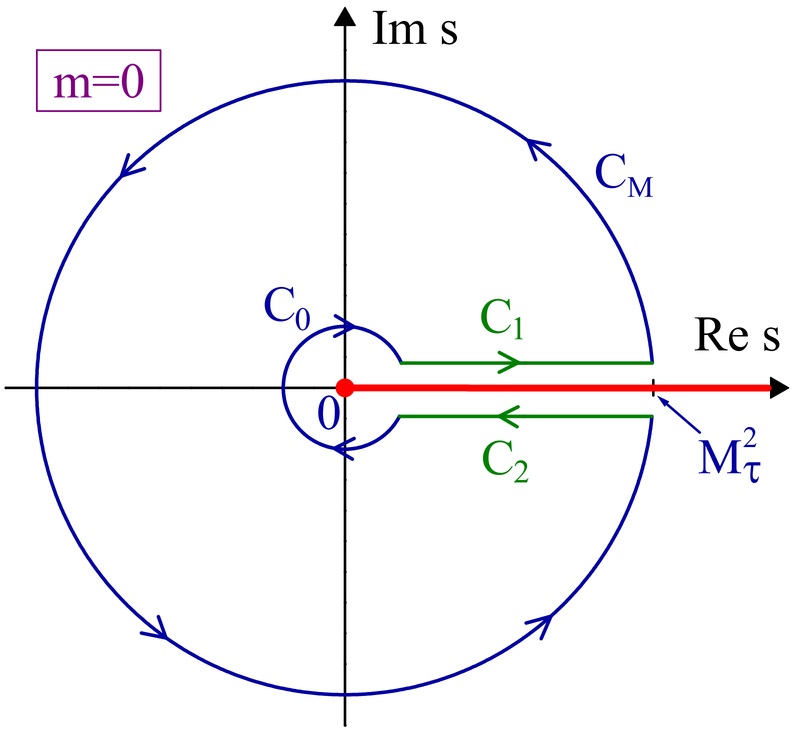
$$A_1(\theta) = 1 + 2\cos(\theta) - 2\cos(3\theta) - \cos(4\theta),$$

$$A_2(\theta) = 2\sin(\theta) - 2\sin(3\theta) - \sin(4\theta),$$

$$\lambda = \ln\left(M_\tau^2/\Lambda^2\right).$$



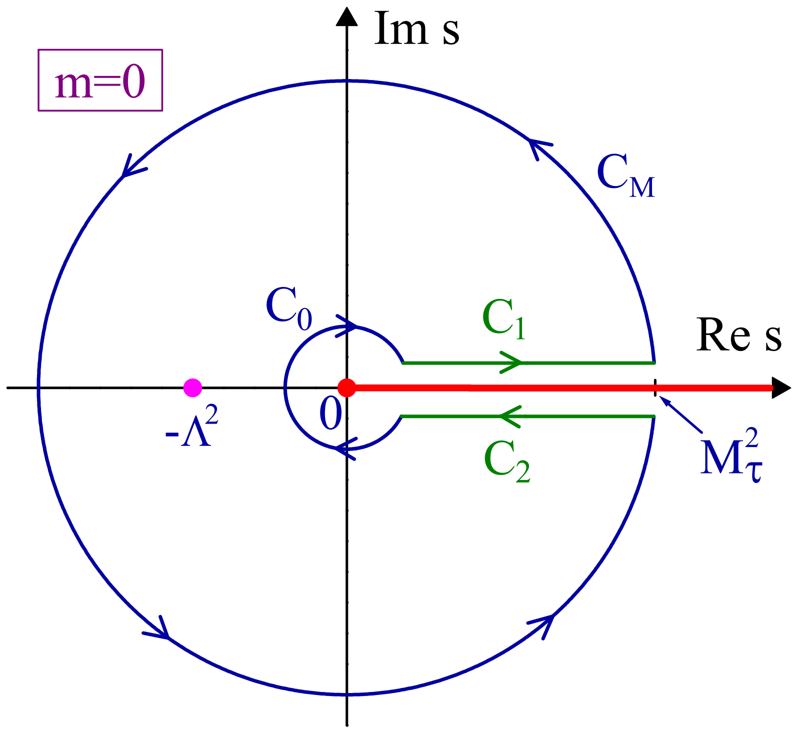
Unknown "genuine physical" Adler function $D_{\mathrm{phys}}(Q^2)$:



$$C_1 + C_2 = -\,(C_0 + C_{\mathrm{M}})$$

The use of either of two integration contours would have led to the same result.

One–loop perturbative Adler function $D^{(1)}_{\mathrm{pert}}(Q^2)$:



$$C_1 + C_2 \neq -\,(C_0 + C_{\mathrm{M}})$$

The integration contours used within methods I and II are not equivalent.

The leading–order perturbative term:

The massless parton model prediction

$$\Pi^{(0)}_{\text{pert}}(q^2) = -\ln\left(\frac{-q^2}{\mu^2}\right) \longrightarrow \left\{d^{(0)}_{\text{pert}}(Q^2)=1,\ r^{(0)}_{\text{pert}}(s)=1\right\}, \qquad |q^2|\to\infty$$

gives the same result for Δ_{QCD} within either of two methods:

$$\Delta^{(0)}_{\text{QCD}} = 1.$$

The one–loop perturbative correction:

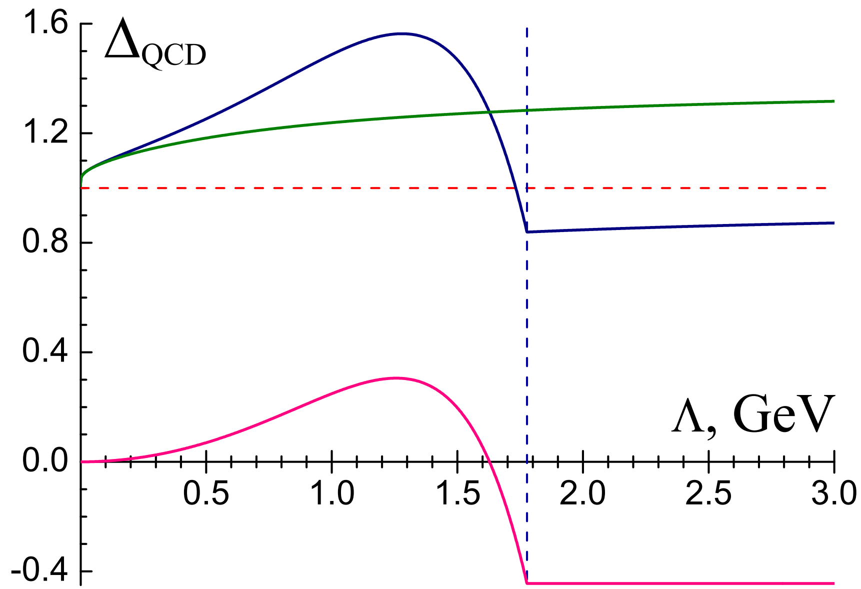
In Method II the residue term

$$\Delta^{(1)}_{\text{res}} = \frac{4}{\beta_0}\, h_1\!\left(\frac{\Lambda^2}{M_\tau^2}\right), \quad \textbf{where}$$

$$h_1(x) = h_2(x)\theta(1-x) + h_2(1)\theta(x-1),$$

$$h_2(x) = x(2-2x^2-x^3),$$

is additionally accounted for.

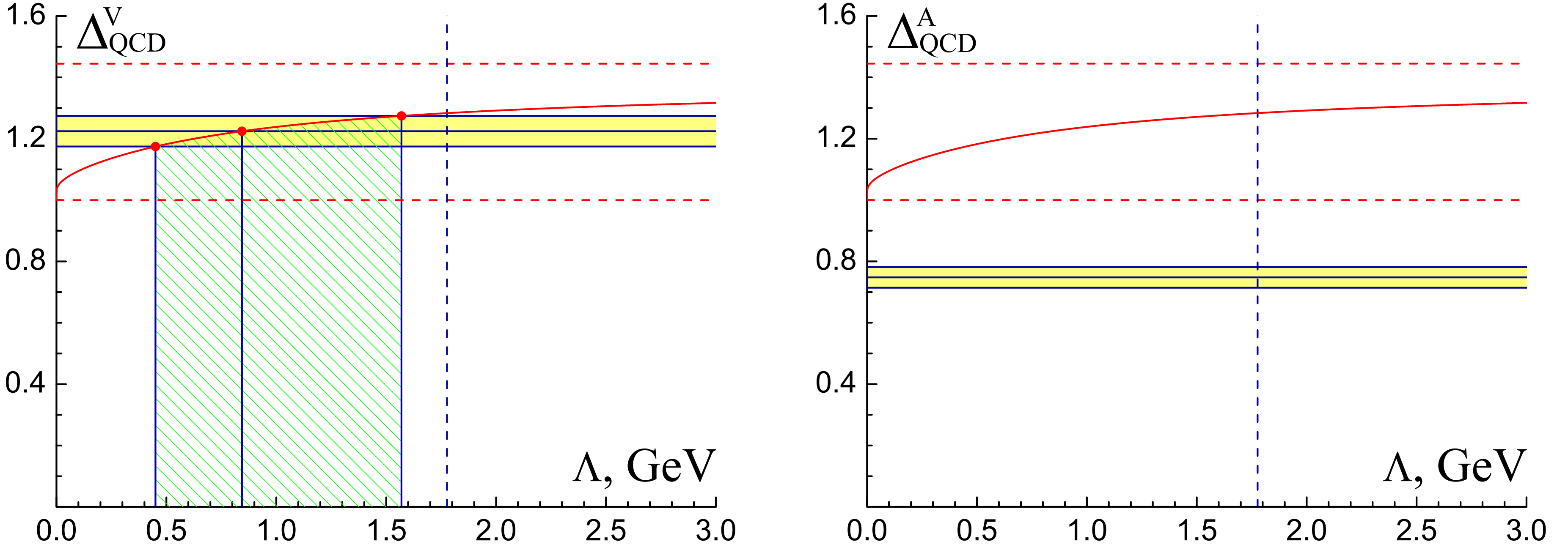


COMPARISON WITH EXPERIMENTAL DATA

ALEPH–2008 data: $\Delta^{\rm V}_{\rm exp} = 1.224 \pm 0.050$, $\Delta^{\rm A}_{\rm exp} = 0.748 \pm 0.034$.

However, within perturbative approach $\Delta^{\rm V}_{\rm pert} \equiv \Delta^{\rm A}_{\rm pert}$.

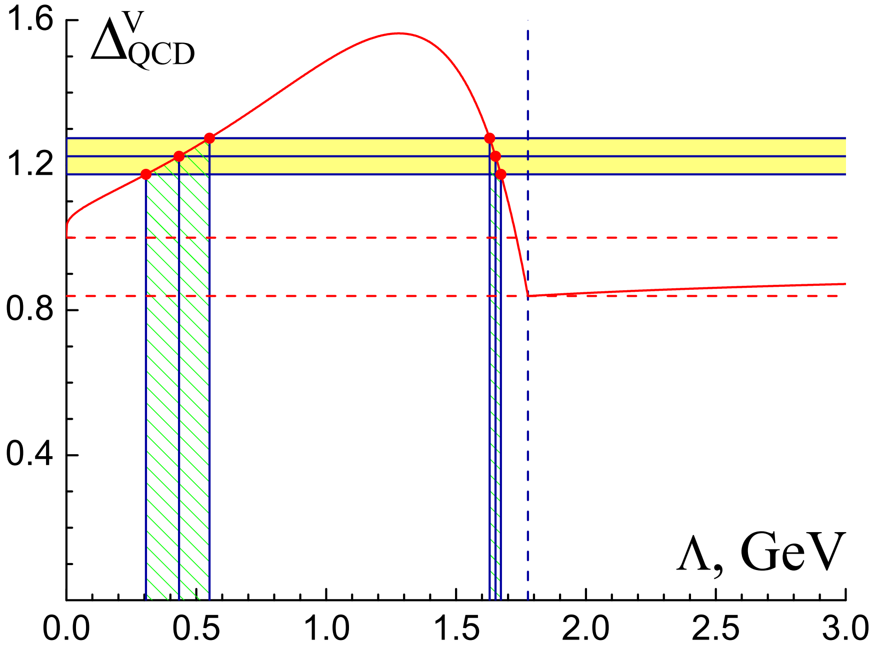
Method I: One solution for V-channel, none for A-channel

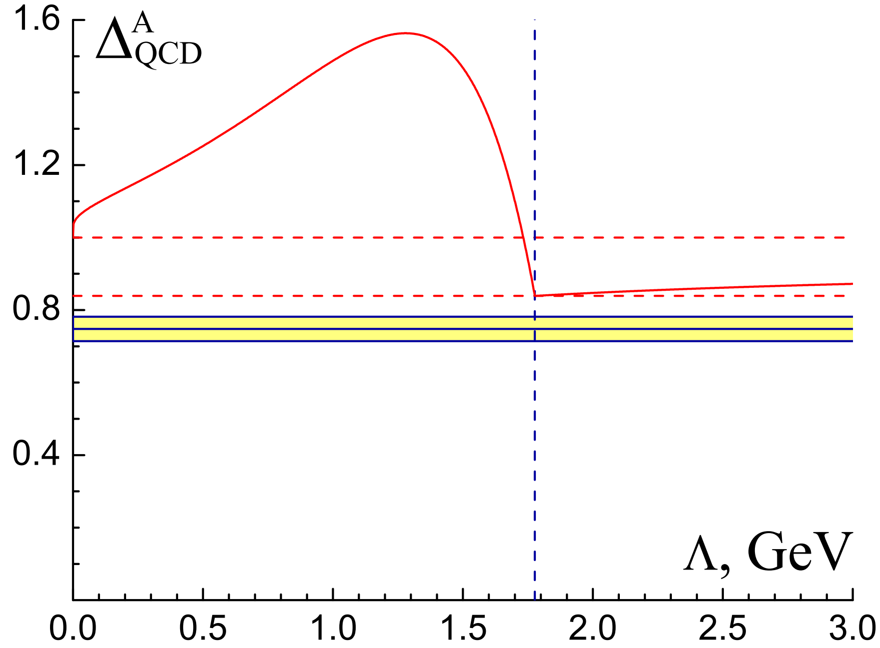


$\Lambda = \left(844^{+726}_{-393}\right)$ MeV

no solution

Method II: Two solutions for V-channel, none for A-channel





$$\Lambda = \left(434^{+117}_{-127}\right)\,\mathrm{MeV}$$

$$\Lambda = \left(1652^{+21}_{-23}\right)\,\mathrm{MeV}$$

no solution

Perturbative approach, one-loop level, $n_f = 3$ active flavors:

	V-channel	A-channel
Method I	$\Lambda = \left(844^{+726}_{-393}\right)$ MeV	**no solution**
Method II	$\boxed{\Lambda = \left(434^{+117}_{-127}\right) \text{ MeV}}$ $\Lambda = \left(1652^{+21}_{-23}\right)$ MeV	**no solution**

V-channel: perturbative approach gives three equally justified solutions, but only highlighted one is usually retained.

A-channel: perturbative approach fails to describe experimental data on inclusive τ lepton hadronic decay.

DISPERSIVE APPROACH TO QCD

Dispersion relations impose stringent physical nonperturbative constraints on the quantities on hand:

$$D(Q^2) = Q^2 \int_{m^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}\, ds \quad \longrightarrow \quad \begin{cases} D(Q^2) = 0 \text{ at } Q^2 = 0; \\ \text{the only cut } Q^2 \le -m^2 \end{cases}$$

BASIC IDEA: merge perturbative approximation for Adler function with such nonperturbative constraints

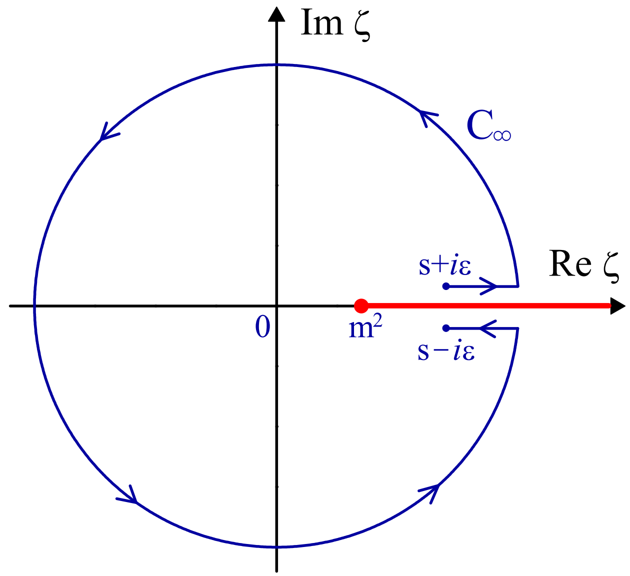
$$R(s) \overset{①}{=} \frac{1}{2\pi i} \lim_{\varepsilon \to 0_+} \int_{s+i\varepsilon}^{s-i\varepsilon} D(-\zeta)\, \frac{d\zeta}{\zeta} \quad \longleftrightarrow \quad D(Q^2) \overset{②}{=} Q^2 \int_{m^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}\, ds$$

$$D(Q^2) = d^{(0)}(Q^2) + d_{\mathrm{s}}(Q^2)$$

↓ ①

$$R(s) = r^{(0)}(s) + \theta\left(1 - \frac{m^2}{s}\right) \int_s^{\infty} \varrho(\sigma)\, \frac{d\sigma}{\sigma},$$

$$\varrho(\sigma) = \frac{1}{2\pi i} \lim_{\varepsilon \to 0_+} \left[d_{\mathrm{s}}(-\sigma - i\varepsilon) - d_{\mathrm{s}}(-\sigma + i\varepsilon)\right]$$



↓ ②

$$D(Q^2) = d^{(0)}(Q^2) + \frac{Q^2}{Q^2 + m^2} \int_{m^2}^{\infty} \varrho(\sigma)\, \frac{\sigma - m^2}{\sigma + Q^2}\, \frac{d\sigma}{\sigma}$$

- ***Nesterenko, Papavassiliou, PRD71(2005); JPG32(2006).***

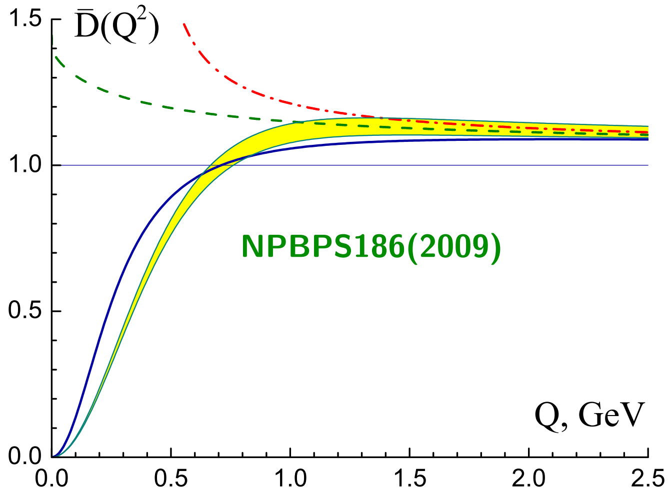
This derivation requires only dispersion relations for $D(Q^2)$ and $R(s)$ and the fact that $d_{\mathrm{s}}(Q^2) \to 0$ for $Q^2 \to \infty$.

Neither additional approximations nor model–dependent assumptions were involved.

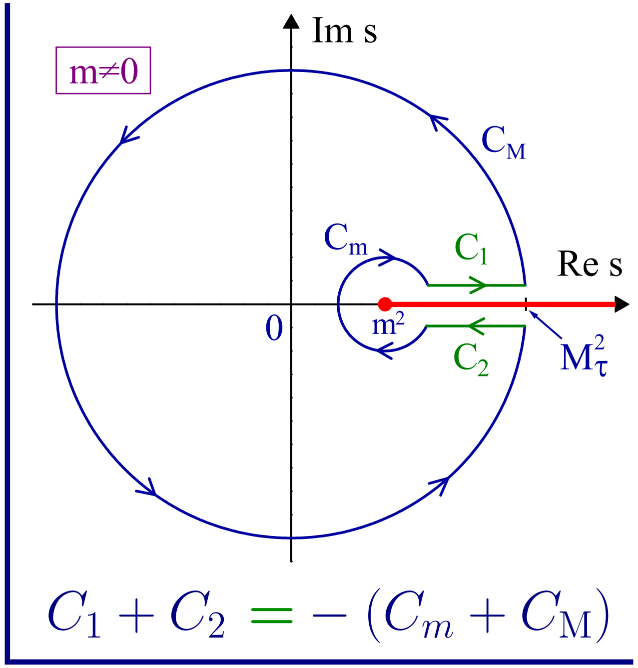
The derived expression for $D(Q^2)$:

- **no unphysical singularities**
- **correct analytic properties in Q^2**
- **applicable in entire $0 \le Q^2 < \infty$**

■ ***Nesterenko, Papavassiliou, JPG32(2006).***



Obtained $D(Q^2)$ leads to the same result for $\Delta_{\rm QCD}$ for either choice of the integration contour



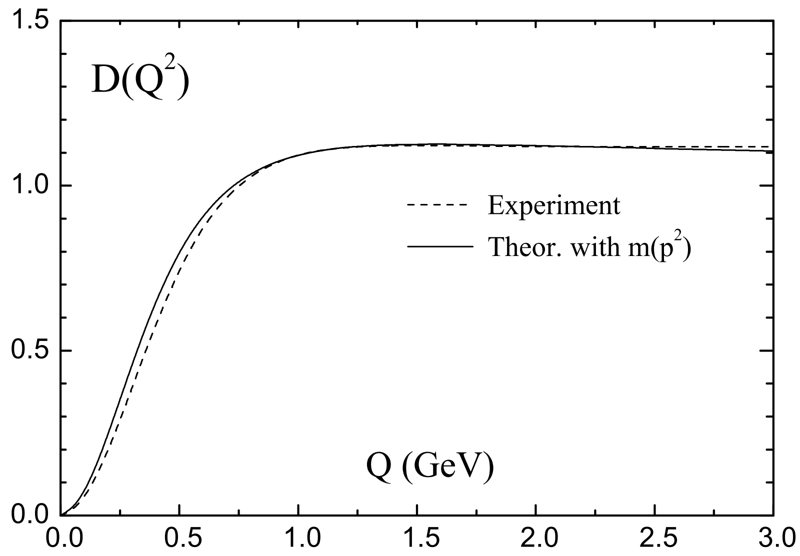
In the massless limit ($m = 0$) derived representations become identical to those of the "Analytic Perturbation Theory" (APT).

■ ***Shirkov, Solovtsov, Milton, PRL79(1997); PRD55(1997); TMP150(2007).***

But it is essential to keep $m \neq 0$ within approach on hand.

Some attempts to improve massless APT behavior of $D(Q^2)$:

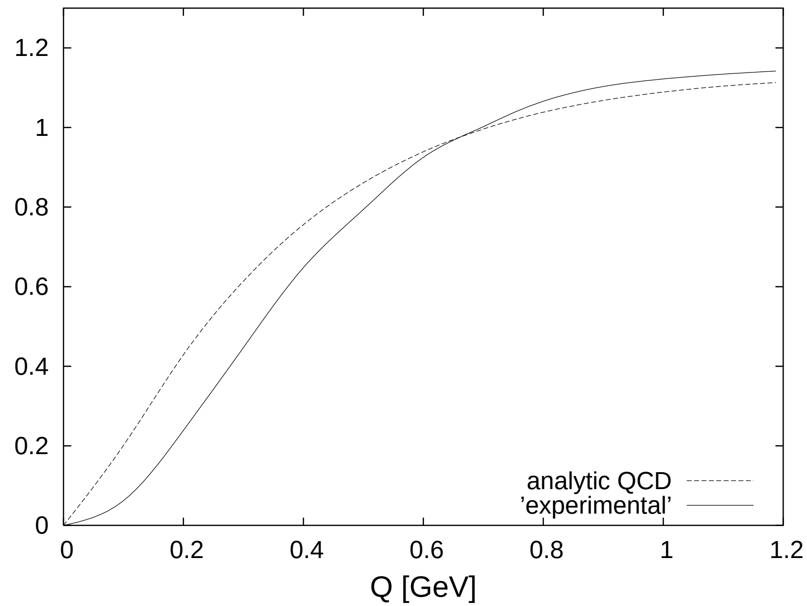
APT + relativistic quark mass threshold resummation:



[plot taken from MPLA21(2006)]

- ***Milton, Solovtsov, Solovtsova (2001)–(2006)***

APT + vector meson dominance assumption:



[plot taken from NPBPS164(2007)]

- ***Cvetic, Valenzuela, Schmidt (2005)–(2007)***

TAU DECAY WITHIN DISPERSIVE APPROACH

- **The effects due to hadronization are retained** $(m \neq 0)$

- **Smooth kinematic threshold for the leading term of** $R(s)$**:**

$$r^{(0)}_{\mathrm{V/A}}(s) = \left(1 - \frac{m^2_{\mathrm{V/A}}}{s}\right)^{3/2} \longleftrightarrow d^{(0)}_{\mathrm{V/A}}(Q^2) = 1 + \frac{3}{\xi}\left\{1 + \frac{u(\xi)}{2}\ln\left[1 + 2\xi\left(1 - u(\xi)\right)\right]\right\}$$

where $u(\xi) = \sqrt{1 + \xi^{-1}}$, $\xi = Q^2/m^2_{\mathrm{V/A}}$

- **Nonperturbative model for one–loop spectral density:**

$$\rho(\sigma) = \frac{4}{\beta_0}\frac{1}{\ln^2(\sigma/\Lambda^2) + \pi^2} + \frac{\Lambda^2}{\sigma}$$

■ ***Nesterenko, PRD62(2000); PRD64(2001); (2011).***

In turn, expression for $\Delta_{\rm QCD}^{\rm V/A}$ has taken the following form:

$$\Delta_{\rm QCD}^{\rm V/A} = \sqrt{1-\zeta_{\rm V/A}}\left(1 + 6\zeta_{\rm V/A} - \frac{5}{8}\zeta_{\rm V/A}^{2} + \frac{3}{16}\zeta_{\rm V/A}^{3}\right)$$
$$- 3\zeta_{\rm V/A}\left(1 + \frac{1}{8}\zeta_{\rm V/A}^{2} - \frac{1}{32}\zeta_{\rm V/A}^{3}\right) \ln\left[\frac{2}{\zeta_{\rm V/A}}\left(1 + \sqrt{1 - \zeta_{\rm V/A}}\right) - 1\right]$$
$$+ \int_{m_{\rm V/A}^2}^{\infty} H\left(\frac{\sigma}{M_\tau^2}\right)\rho(\sigma)\,\frac{d\sigma}{\sigma},$$

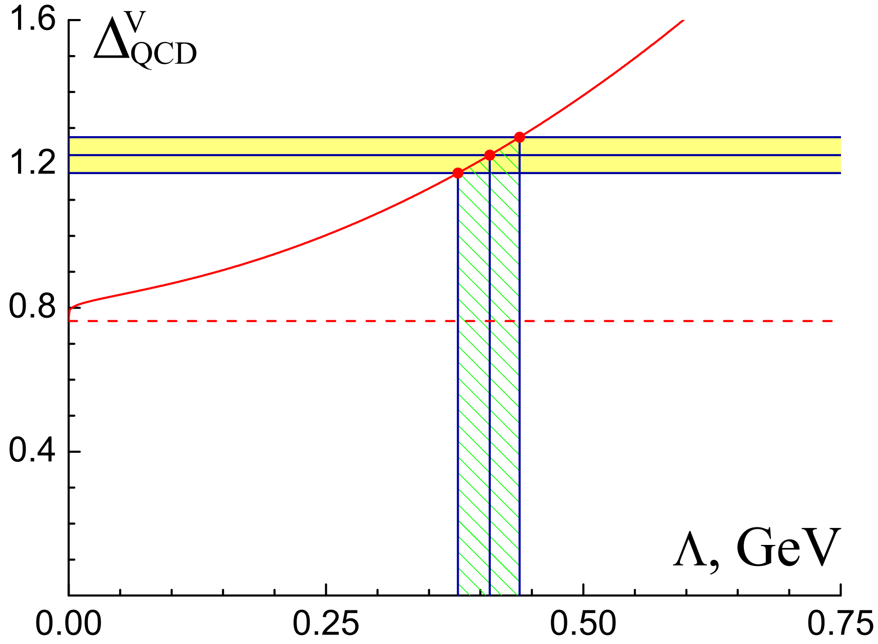
where $\zeta_{\rm V/A} = m_{\rm V/A}^2/M_\tau^2$**,**

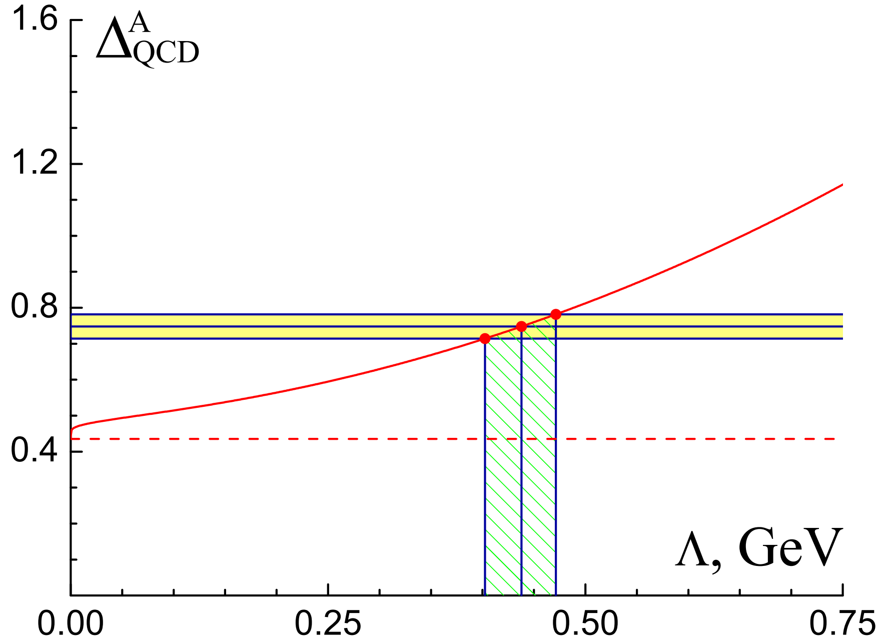
$$H(x) = g(x)\,\theta(1 - x) + g(1)\,\theta(x - 1) - g\left(\zeta_{\rm V/A}\right),$$

$$g(x) = x(2 - 2x^2 + x^3)$$

- ***Nesterenko, NPBPS186(2009); (2011).***

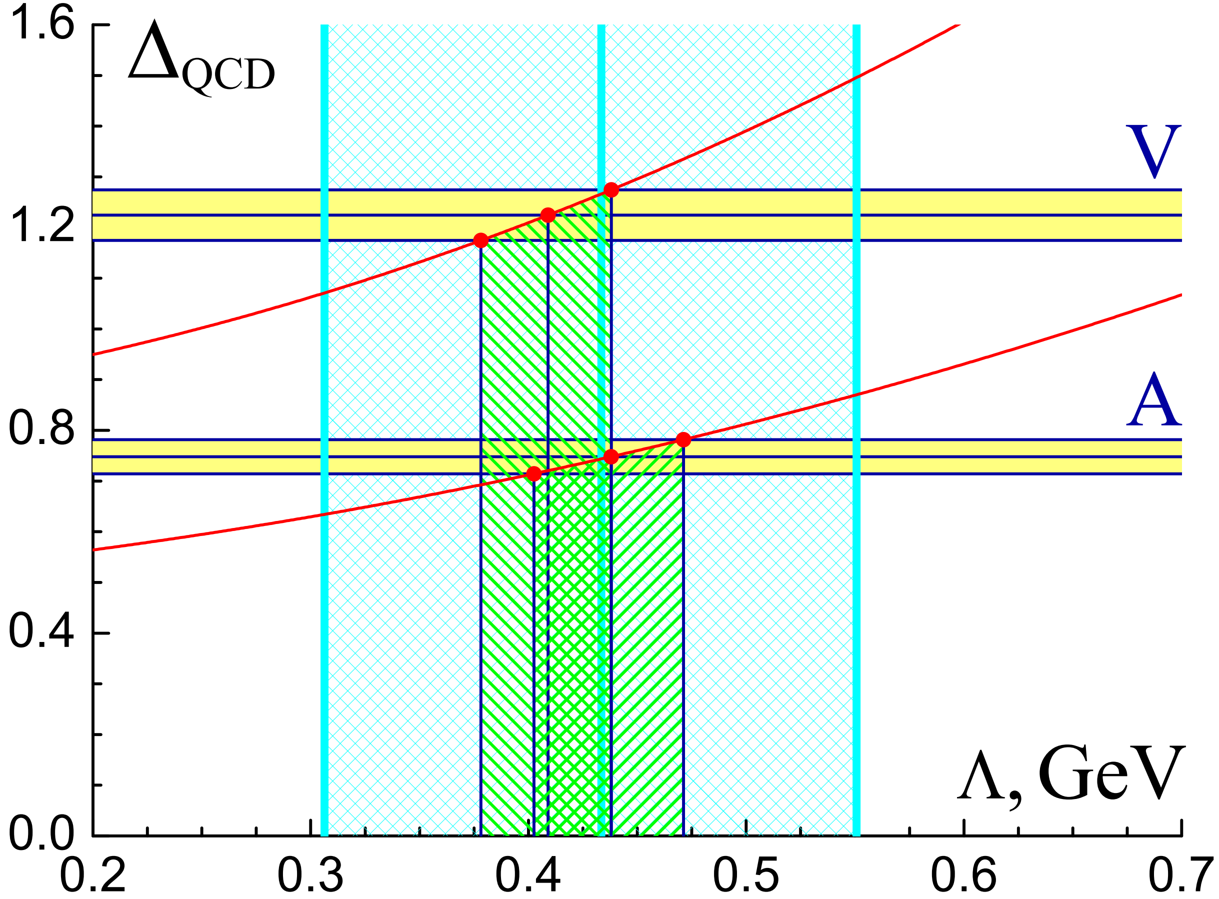
This results in nearly identical solutions for QCD scale parameter Λ in both vector and axial–vector channels:





$$\Lambda = \big(408 \pm 30\big)\ \mathrm{MeV} \qquad \Lambda = \big(437 \pm 34\big)\ \mathrm{MeV}$$

The obtained solutions also agree with perturbative (V) one:



	Perturbative approach	Dispersive approach
V-channel	$\Lambda = \left(434^{+117}_{-127}\right)$ MeV	$\Lambda = \left(408 \pm 30\right)$ MeV
A-channel	**no solution**	$\Lambda = \left(437 \pm 34\right)$ MeV

SUMMARY

- **Theoretical description of τ lepton hadronic decay is performed in the framework of Dispersive approach to QCD**

- **The significance of effects due to hadronization is argued**

- **The approach on hand is capable of describing experimental data on inclusive τ lepton hadronic decay in both vector and axial–vector channels**

- **The vicinity of obtained values of QCD scale parameter Λ testifies to the self–consistency of developed approach**