Relativistic description of the pair charmonium production at LHC

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Pair charmonium production

e^+e^- annihilation:

- G.T. Bodwin, J. Lee, C. Yu, Phys. Rev. D **77**, 094018 (2008).
- J.P. Ma, Z.G. Si, Phys. Rev. D **70**, 074007 (2004);
  A.E. Bondar, V.L. Chernyak, Phys. Lett. B **612**, 215 (2005);
  V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005).
- D. Ebert, A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006);
  D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Lett. B **672**, 264 (2009); E.N. Elekina, A.P. Martynenko, Phys. Rev. D **81**, 054006 (2010);
  A.P. Martynenko, A.M. Trunin, arXiv:1106.2741.

LHCb experimentally measured value:

$$\sigma^{exp}_{LHCb}[pp \to 2J/\psi + X] = 5.1 \pm 1.0 \pm 1.1 \text{ nb}\Big|_{\sqrt{S}=7 \text{ TeV}}$$

- R. Aaij *et al.* (LHCb Collaboration), Phys. Let. B **707**, 52 (2012).

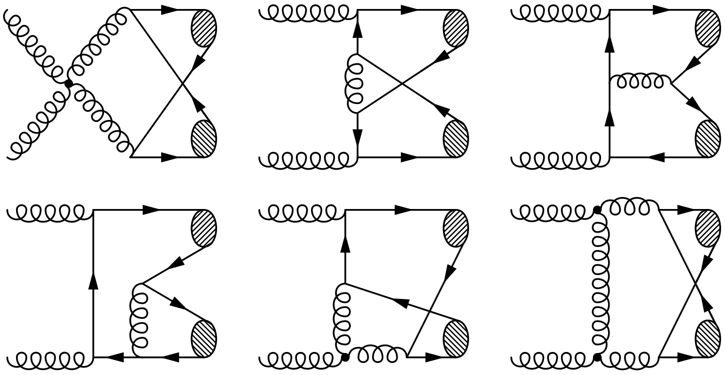
NRQCD predictions:

- R. Li, Y.-J. Zhang, K.-T. Chao, Phys. Rev. D **80**, 014020 (2009);
  S.P. Baranov, Phys. Rev. D **84**, 054012 (2011);
  A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A. Novoselov, Phys. Rev. D **84**, 094023 (2011).

31 LO α_s CSM gluon fusion diagrams

$$d\sigma[pp \to 2J/\psi + X] = \int dx_1 dx_2\, f_{g/p}(x_1) f_{g/p}(x_2)\, d\sigma[gg \to 2J/\psi], \qquad (1)$$

$$\mathcal{M}[gg \to 2J/\psi] = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3}\, \bar{\Psi}(p, P)\bar{\Psi}(q, Q) \otimes \mathcal{T}(p_1, p_2; q_1, q_2). \qquad (2)$$



Production amplitude

$$\mathcal{M}[gg \to 2J/\psi](k_1, k_2, P, Q) = \frac{1}{9} M_{J/\psi}\, \pi^2 \alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3}\, \mathrm{Tr}\, \mathfrak{M},$$

$$\begin{aligned} \mathfrak{M} = \mathcal{D}_1 \gamma_\beta \bar{\Psi}_{q,Q} \Gamma_1^\beta \bar{\Psi}_{p,P}\, \hat{\varepsilon}_2 \frac{m - \hat{k}_2 + \hat{p}_1}{(k_2 - p_1)^2 - m^2} + \\ \mathcal{D}_2 \gamma_\beta \bar{\Psi}_{q,Q} \Gamma_2^\beta \bar{\Psi}_{p,P}\, \hat{\varepsilon}_1 \frac{m - \hat{k}_1 + \hat{p}_1}{(k_1 - p_1)^2 - m^2} + \mathcal{D}_3 \bar{\Psi}_{q,Q} \Gamma_3^\beta \bar{\Psi}_{p,P}\, \gamma_\beta + \\ \mathcal{D}_4 \bar{\Psi}_{p,P} \Gamma_4^\beta \bar{\Psi}_{q,Q}\, \gamma_\beta + \mathcal{D}_1 \bar{\Psi}_{q,Q} \Gamma_5^\beta \bar{\Psi}_{p,P}\, \gamma_\beta \frac{m + \hat{k}_2 - \hat{q}_1}{(k_2 - q_1)^2 - m^2} \hat{\varepsilon}_2 + \\ \mathcal{D}_2 \bar{\Psi}_{q,Q} \Gamma_6^\beta \bar{\Psi}_{p,P}\, \gamma_\beta \frac{m + \hat{k}_1 - \hat{q}_1}{(k_1 - q_1)^2 - m^2} \hat{\varepsilon}_1, \end{aligned} \tag{3}$$

$k_{1,2} = x_{1,2}\sqrt{S}/2\,(1, 0, 0, \pm 1)$ — the initial gluon four-momenta;
P, Q — the total four-momenta of outcoming charmonia;
$p = L_P(0, \mathbf{p})$, $q = L_Q(0, \mathbf{q})$ — the relative four-momenta of (anti)quarks.
$\varepsilon_{1,2}$ — the polarization vectors of initial gluons.

Vertex functions

$$\Gamma_1^\beta = \hat{\varepsilon}_1 \frac{m - \hat{k}_1 + \hat{q}_2}{(k_1 - q_2)^2 - m^2} \gamma^\beta - 8\,\gamma^\beta \frac{m + \hat{k}_1 - \hat{p}_2}{(k_1 - p_2)^2 - m^2} \hat{\varepsilon}_1,$$

$$\begin{aligned}
\Gamma_3^\beta = {} & \hat{\varepsilon}_1 \frac{m - \hat{k}_1 + \hat{q}_2}{(k_1 - q_2)^2 - m^2} \Big[\gamma^\beta \frac{m + \hat{k}_2 - \hat{p}_2}{(k_2 - p_2)^2 - m^2} \hat{\varepsilon}_2 - 8\,\hat{\varepsilon}_2 \frac{m - \hat{p}_1 - \hat{p}_2 - \hat{q}_1}{(p_1 + p_2 + q_1)^2 - m^2} \gamma^\beta \Big] + \\
& \hat{\varepsilon}_2 \frac{m - \hat{k}_2 + \hat{q}_2}{(k_2 - q_2)^2 - m^2} \Big[\gamma^\beta \frac{m + \hat{k}_1 - \hat{p}_2}{(k_1 - p_2)^2 - m^2} \hat{\varepsilon}_1 - 8\,\hat{\varepsilon}_1 \frac{m - \hat{p}_1 - \hat{p}_2 - \hat{q}_1}{(p_1 + p_2 + q_1)^2 - m^2} \gamma^\beta \Big] - \\
& 8\,\gamma^\beta \frac{m + \hat{p}_1 + \hat{q}_1 + \hat{q}_2}{(p_1 + q_1 + q_2)^2 - m^2} \Big[\hat{\varepsilon}_2 \frac{m + \hat{k}_1 - \hat{p}_2}{(k_1 - p_2)^2 - m^2} \hat{\varepsilon}_1 + \hat{\varepsilon}_1 \frac{m + \hat{k}_2 - \hat{p}_2}{(k_2 - p_2)^2 - m^2} \hat{\varepsilon}_2 \Big] + \\
& 18\,\gamma_\alpha \Big[\mathcal{D}_1 \frac{m - \hat{k}_1 + \hat{q}_2}{(k_1 - q_2)^2 - m^2} \varepsilon_1^\alpha \gamma_\mu \mathfrak{E}_2^{\beta\mu}(p_1 + q_1) - \mathcal{D}_1 \frac{m + \hat{k}_1 - \hat{p}_2}{(k_1 - p_2)^2 - m^2} \hat{\varepsilon}_1 \mathfrak{E}_2^{\beta\alpha}(p_1 + q_1) + \\
& \mathcal{D}_2 \frac{m - \hat{k}_2 + \hat{q}_2}{(k_2 - q_2)^2 - m^2} \varepsilon_2^\alpha \gamma_\mu \mathfrak{E}_1^{\beta\mu}(p_1 + q_1) - \mathcal{D}_2 \frac{m + \hat{k}_2 - \hat{p}_2}{(k_2 - p_2)^2 - m^2} \hat{\varepsilon}_2 \mathfrak{E}_1^{\beta\alpha}(p_1 + q_1) \Big],
\end{aligned} \tag{4}$$

$$\mathfrak{E}_{1,2}^{\alpha\beta}(x) = \frac{1}{2} \big(2\,x\varepsilon_{1,2}\, g^{\alpha\beta} - (k_{1,2}^\beta + x^\beta)\varepsilon_{1,2}^\alpha + (2k_{1,2}^\alpha - x^\alpha)\varepsilon_{1,2}^\beta \big),$$

$\mathcal{D}_{1,2}^{-1} = (k_2 - p_{1,2} - q_{1,2})^2$,
$\mathcal{D}_{3,4}^{-1} = (p_{1,2} + q_{1,2})^2$ — inverse denominators of gluon propagators.

Transformation of relativistic wave functions

Quasipotential wave functions are calculated in the meson rest frame and then transformed to the reference frames moving with the four-momenta $P(Q)$:

$$\bar{\Psi}_{p,P} = \frac{\bar{\Psi}_0^{J/\psi}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]} \left[\frac{\hat{v}_1 - 1}{2} + \hat{v}_1 \frac{\mathbf{p}^2}{2m(\epsilon(p)+m)} - \frac{\hat{p}}{2m}\right] \times$$
$$\hat{\varepsilon}_P^*(P, S_z)\,(1+\hat{v}_1)\left[\frac{\hat{v}_1 + 1}{2} + \hat{v}_1 \frac{\mathbf{p}^2}{2m(\epsilon(p)+m)} + \frac{\hat{p}}{2m}\right], \tag{5}$$

$$\bar{\Psi}_{q,Q} = \frac{\bar{\Psi}_0^{J/\psi}(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{\epsilon(q)+m}{2m}\right]} \left[\frac{\hat{v}_2 - 1}{2} + \hat{v}_2 \frac{\mathbf{q}^2}{2m(\epsilon(q)+m)} + \frac{\hat{q}}{2m}\right] \times$$
$$\hat{\varepsilon}_Q^*(Q, S_z)\,(1+\hat{v}_2)\left[\frac{\hat{v}_2 + 1}{2} + \hat{v}_2 \frac{\mathbf{q}^2}{2m(\epsilon(q)+m)} - \frac{\hat{q}}{2m}\right]. \tag{6}$$

$v_1 = \frac{P}{M_{J/\psi}}$, $v_2 = \frac{Q}{M_{J/\psi}}$;
$\epsilon(p) = \sqrt{m^2 + \mathbf{p}^2}$;
m — c-quark mass.

$\varepsilon_{P,Q}$ — polarizations of outcoming charmonia with four-momenta P(Q).

Expansion of quark and gluon propagators

$$\frac{1}{(p_1+q_1)^2} = \frac{4}{s} - \frac{16}{s^2}\left[(p+q)^2 + pQ + qP\right] + \cdots\,,$$
$$\frac{1}{(k_2-q_2)^2 - m^2} = \frac{2}{t-M^2} - \frac{4}{(t-M^2)^2}\left[q^2 + 2\,qk_2\right] + \cdots\,, \tag{7}$$

where $s = x_1 x_2 S$ and $t = (P-k_1)^2 = (Q-k_2)^2$ — the Mandelstam variables for the gluonic subprocess $gg \to 2J/\psi$.

$$4M^2 \leq s, \quad \left|t + \frac{s}{2} - M^2\right| \leq \frac{s}{2}\sqrt{1 - \frac{4M^2}{s}}. \tag{8}$$

In the case of the most unfavourable values of the variables $x_{1,2}$ and t the expansion parameters in (7) can be roughly assessed as $2p^2/M^2$ and $2q^2/M^2$.

$$\int \frac{\Psi_0^{\mathcal{S}}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\frac{d\mathbf{p}}{(2\pi)^3} = \frac{1}{\sqrt{2}\,\pi}\int\limits_0^\infty \frac{p^2 R_{\mathcal{S}}(p)}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}dp,$$
$$\int p_\mu p_\nu \frac{\Psi_0^{\mathcal{S}}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\frac{d\mathbf{p}}{(2\pi)^3} = -\frac{1}{3\sqrt{2}\,\pi}(g_{\mu\nu} - v_{1\mu}v_{1\nu})\int\limits_0^\infty \frac{p^4 R_{\mathcal{S}}(p)}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}dp. \tag{9}$$

The differential cross-section

$$\frac{d\sigma}{dt}[gg \to 2J/\psi](t,s) = \frac{\pi m^2 \alpha_s^4}{2304\, s^2} |\tilde{R}(0)|^4 \sum_{i=0}^{3} \omega_i F^{(i)}(t,s), \qquad (10)$$

$$\omega_0 = 1, \quad \omega_1 = \frac{I_1}{I_0}, \quad \omega_2 = \frac{I_2}{I_0}, \quad \omega_3 = \omega_1^2. \qquad (11)$$

The relativistic generalization of the radial wave function at the origin:

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{m + \epsilon(p)}{2\epsilon(p)} R(p) p^2 dp. \qquad (12)$$

$$I_0 = \int_0^\infty \frac{m + \epsilon(p)}{2\epsilon(p)} R(p) p^2 dp, \quad I_{1,2} = \int_0^m \frac{m + \epsilon(p)}{2\epsilon(p)} \left(\frac{m - \epsilon(p)}{m + \epsilon(p)} \right)^{1,2} R(p) p^2 dp. \qquad (13)$$

The definition of the relativistic parameters $I_{1,2}$ contains cutoff $\Lambda = m$ due to uncertainty of the relativistic wave function in the region $p \gtrsim m$.

Effective relativistic Hamiltonian

$$H = H_0 + \Delta U_1 + \Delta U_2 + \Delta U_3, \quad H_0 = 2\sqrt{\mathbf{p}^2 + m^2} - 2m - \frac{C_F \tilde{\alpha}_s}{r} + Ar + B, \quad (14)$$

$$\Delta U_1(r) = -\frac{C_F \alpha_s^2}{4\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E \beta_0\right], \quad a_1 = \frac{31}{3} - \frac{10}{9} n_f,$$

$$\Delta U_2(r) = -\frac{C_F \alpha_s}{2m^2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{\pi C_F \alpha_s}{m^2}\delta(\mathbf{r}) + \frac{3 C_F \alpha_s}{2m^2 r^3}(\mathbf{S}\mathbf{L}) -$$

$$\frac{C_F \alpha_s}{2m^2}\left[\frac{\mathbf{S}^2}{r^3} - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^5} - \frac{4\pi}{3}(2\mathbf{S}^2 - 3)\delta(\mathbf{r})\right] - \frac{C_A C_F \alpha_s^2}{2mr^2},$$

$$\Delta U_3(r) = f_V\Big[\frac{A}{2m^2 r}\Big(1 + \frac{8}{3}\mathbf{S}_1\mathbf{S}_2\Big) + \frac{3A}{2m^2 r}\mathbf{L}\mathbf{S} + \frac{A}{3m^2 r}\Big(\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\Big)\Big] -$$

$$(1 - f_V)\frac{A}{2m^2 r}\mathbf{L}\mathbf{S}, \quad (15)$$

$$A = 0.18\text{GeV}^2, \quad B = -0.16\text{GeV}, \quad f_v = 0.7, \quad \alpha_s(m^2) \approx 0.314, \quad \tilde{\alpha}_s(m^2) \approx 0.242,$$
$$m = 1.55 \text{ GeV}. \quad (16)$$

Comparison between the numerical and experimental $\mathcal{S}$-wave charmonium masses:

$$M_{J/\psi}^{num} = 3.072 \text{ GeV} \quad \text{and} \quad M_{\eta_c}^{num} = 2.988 \text{ GeV},$$
$$M_{J/\psi}^{exp} = 3.097 \text{ GeV} \quad \text{and} \quad M_{\eta_c}^{exp} = 2.980 \text{ GeV}.$$

- K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

Numerical results

Our results (CTEQ5L pdf):

$$\sigma_{rel}^{total} = 9.6 \text{ nb}, \qquad \sigma_{rel}^{LHCb} = \mathbf{1.6} \text{ nb}.$$

Experimentally measured value ($2 < y_{P,Q} < 4.5$):

$$\sigma_{exp}^{LHCb}[pp \to 2J/\psi + X] = \mathbf{5.1} \pm \mathbf{1.0} \pm \mathbf{1.1} \text{ nb}.$$

NRQCD predictions:

$$\sigma_{NRQCD}^{total} = 18 \text{ nb}, \quad \sigma_{NRQCD}^{LHCb} = \mathbf{3.2} \text{ nb}.$$

- A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, and A.A. Novoselov, Phys. Rev. D **84**, 094023 (2011).

Double parton scattering contribution:

$$\sigma_{DPS}^{LHCb} = \mathbf{2} \text{ nb}.$$

- S.P. Baranov, A.M. Snigirev, and N.P. Zotov, Phys. Lett. B **705**, 116 (2011);
- A. Novoselov, arXiv:1106.2184.

Different sources of relativistic corrections (nb):

full nonrel.	+ relativistic normalization	+ relativistic wave functions	+ amplitude expansions
23.1	6.3	3.1	**9.6**
	$\times 3.7^{-1}$	$\times 2.0^{-1}$	$\times 3.1$

Thank you for attention!