Z bozon production in Drell-Yan process

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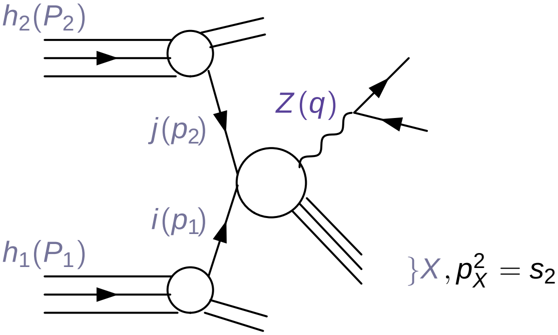
Drell-Yan process



$$\frac{d\sigma_{h_1h_2}}{dq_T^2 dy} = \sum_{i,j} \int dx_1 dx_2 f_i^{h_1}(x_1, \mu_F^2) f_j^{h_2}(x_2, \mu_F^2)$$

$$\frac{s d\bar{\sigma}_{ij}}{dt\, du}(\alpha_s(\mu_F^2), x_1 P_1, x_2 P_2)$$

Research goals and reasons

- The measurements of production of Z boson in Drell-Yan mechanism and jets are fundamental probes of the EW force and an essential starting point for searches of new physics beyond the SM.
- Detectors systematics should be under control. One of the ways to achieve it is the study very precisely the well-understood standard processes like W or Z boson production.
- The cross section of Z boson production with the high transverse momenta q_T and well-isolated leptons decay modes can be very easy triggered in detectors such as ATLAS and CMS. This provide a clean experimental signature with rather low background especially for Z boson production.

Previous Work

- The neutral boson production in a hadron-hadron collision was first invented by Sidney Drell and Tung-Mow Yan in 1970. Experimentally, this process was first observed by J.H. Christenson et al. in proton-uranium collisions at the Brookhaven National Laboratory.
- First theoretical calculation of the corrections to Z boson production in the Drell-Yan process was done in 1979 *Altarelli at al.*.
- The next-to-leading (NLO) QCD corrections for the large q_T vector boson production were calculated by *R. K. Ellis at al., 1983, R. J. Gonsalves at al., 1999; P. B. Arnold and M. H. Reno, 1999*
- The pure weak one-loop corrections and the leading logarithmic corrections were done by *J. H. Kuhn et al.,2005*

Outline

Steps Of Calculation

DIANA *M. Tentyukov and J. Fleischer, 2000*

$\rightarrow$ Set of Topologies of Diagrams

AIR *C. Anastasiou and A. Lazopoulos, 2004*

$\rightarrow$ Set of Master Integrals

FORM, FORTRAN, C++

$\rightarrow$ Analytical Formulae $\frac{s\,d\bar{\sigma}_{ij}}{dt\,du}$

PDFs:

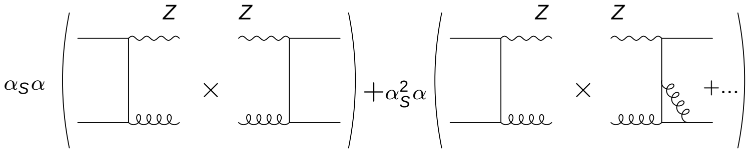
CTEQ6M *J. C. Collins et al.*, MRST *A. Martin, J. Stirling et al.*

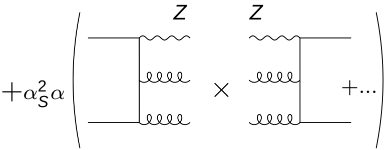
CUBA (VEGAS, SUAVE) *T. Hahn, 2006*

$\rightarrow$ Numerical Result

QCD corrections

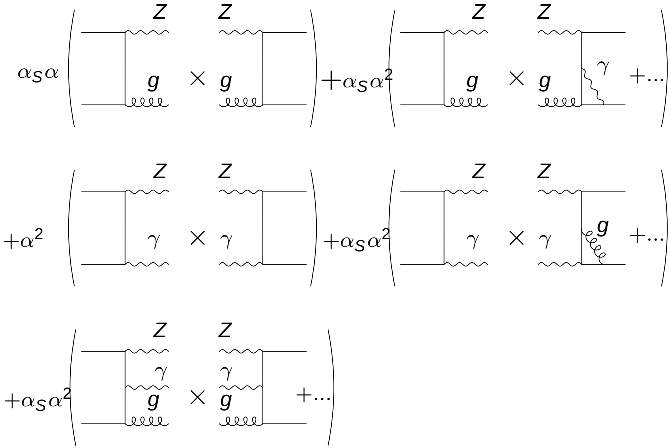
of the order $O(\alpha_S^2\alpha)$ in the perturbation theory: loops and bremsstrahlung corrections to the QCD $i + j \to Z + k$ Born process (here i, j and k are gluons, quarks or antiquarks)





QED corrections

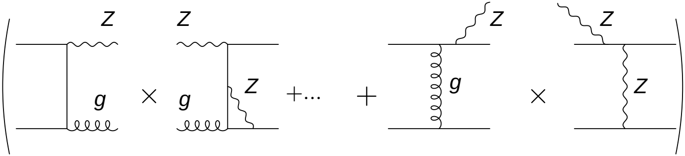
of the order $O(\alpha_S \alpha^2)$ in the perturbation theory: loops and bremsstrahlung *photonic* corrections to the QCD $i + j \to Z + k$ Born process and *gluon* corrections to QED $i + j \to Z + \gamma$ Born process



EW corrections

of the order $O(\alpha_S\alpha^2)$: loops diagrams in the exchange of weak bosons, and mixed EW-QCD corrections of the interference diagram $q + q \to q + q + Z$ with exchange of Z boson and the same diagram with exchange of gluon

$+\alpha_S\alpha^2$



Outline

Gamma Matrices

- *G. 't Hooft and M. J. G. Veltman, 1972*
- $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \quad Tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\tau\gamma_5) = -4i\varepsilon^{\mu\nu\rho\tau}$
- The diagrams have only one trace with γ_5:

$$Tr(...\gamma^\mu...\gamma^\sigma\gamma_5...\gamma^\nu\gamma_5)$$

$\rightarrow$ Using anticommutations relation $\{\gamma^\mu, \gamma_5\} = 0$, we cancel $\gamma_5\gamma^5 = 1$.
$\rightarrow$ And $A(p_1, p_2, q)\varepsilon^{\mu\nu\rho\tau} \rightarrow 0$ because of convolution with only 3 external momenta.

- The squared diagrams give us two traces with γ_5.

$$Tr(...\gamma^{\mu_1}...\gamma^{\sigma_1}\gamma_5...\gamma^{\nu_1}\gamma_5)Tr(...\gamma^{\mu_2}...\gamma^{\sigma_2}\gamma_5...\gamma^{\nu_2}\gamma_5)$$

$\varepsilon^{\mu\nu\rho\tau}\varepsilon^{\mu\nu\rho\tau} \neq 0$. In the case of Z boson production these diagrams are all finite and we calculated the traces in $d = 4$ dimensions.

Singularities

Soft or collinear limits in the parton kinematics correspond to

$$s_2 \to 0, \text{ where } s_2 = s + t + u - Q^2.$$

The factor $s_2^{-\varepsilon}$ in the phase space measure is used to separate explicitly the poles in the dimensional regulator ε and the finite integrable distribution in $1/\varepsilon$.

$$\frac{1}{s_2^{1+\varepsilon}} = -\frac{1}{\varepsilon}\delta(s_2)\left(1 - \varepsilon \ln A + \frac{\varepsilon^2}{2}\ln^2 A\right) + \left(\frac{1}{s_2}\right)_{A_+} - \varepsilon\left(\frac{\ln s_2}{s2}\right)_{A+} + O(\varepsilon^2)$$

$$\int_0^A ds_2 f(s_2)\left(\frac{1}{s_2}\right)_{A_+} = \int_0^A ds_2 \frac{f(s_2) - f(0)}{s_2}$$

$$\int_0^A ds_2 f(s_2)\left(\frac{\ln(s_2)}{s_2}\right)_{A_+} = \int_0^A ds_2 \frac{(f(s_2) - f(0))\ln(s_2)}{s_2}$$

Collinear Singularities

The calculation of the factorized cross-section $d\bar{\sigma}$:

$$\frac{sd\bar{\sigma}_{i,j}}{dtdu} = \frac{sd\sigma_{i,j}}{dtdu}$$

$$-\frac{\alpha_S}{2\pi}\sum_k\sum_{n=1,2}\int_0^1 dz_n R_{k\leftarrow i_n}(z_n, M^2)\frac{sd\sigma^{(1)}_{k,j_n}}{dt}|_{p_n\to z_n p_n}\delta(z_n(s+t-Q^2)+u)$$

$$\frac{sd\bar{\sigma}^{(2),QED}_{i,j}}{dtdu} = \frac{sd\sigma^{(2),QED}_{i,j}}{dtdu}$$

$$-\frac{\alpha_S}{2\pi}\sum_k\int_0^1 dz R_{QCD}(z, M^2)\frac{sd\sigma^{(1),QED}_{k,j}}{dt}$$

$$-\frac{\alpha}{2\pi}\sum_k\int_0^1 dz R_{QED}(z, M^2)\frac{sd\sigma^{(1),QCD}_{i,k}}{dt}$$

Outline

Result

- Analytical formulae for QCD, QED and EW corrections

$$\frac{d\sigma}{dq_T^2\, dy} = \sum_{i,j} \int_B^1 dx_1 \int_0^A ds_2 \frac{f_i(x_1,\mu_F^2) f_j(x_2(s_2),\mu_F^2)}{x_1 S + U - Q^2} \frac{s\, d\hat{\sigma}_{i,j}}{dt\, du}(x_1 P_1, x_2(s_2) P_2, \mu_F^2)$$

- Numerical calculations
  - Total Cross Section
  - Transverse Momentum Distribution
  - Rapidity Distribution
- Tevatron($\sqrt{S} = 1.96 TeV$) and LHC ($\sqrt{S} = 14 TeV$)

The total cross section of Z boson production

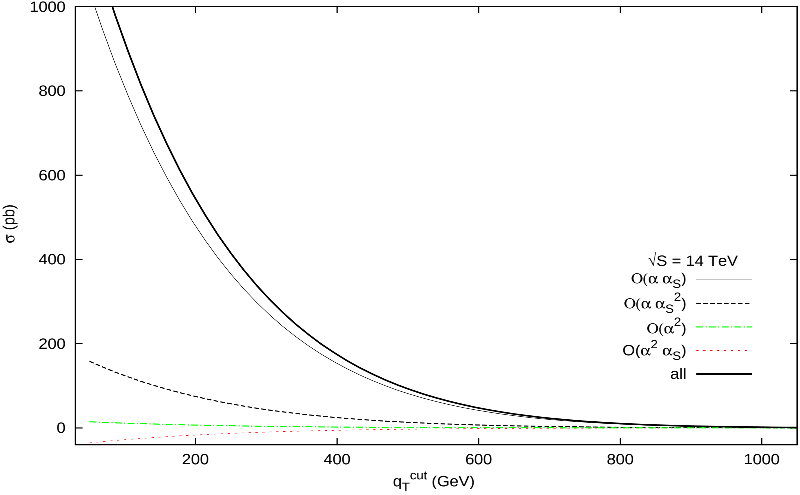


Figure: $\sqrt{S} = 14\,\mathrm{TeV}$ (LHC energy).

$K - 1$ factor to the total σ for $p + p \longrightarrow Z + X$

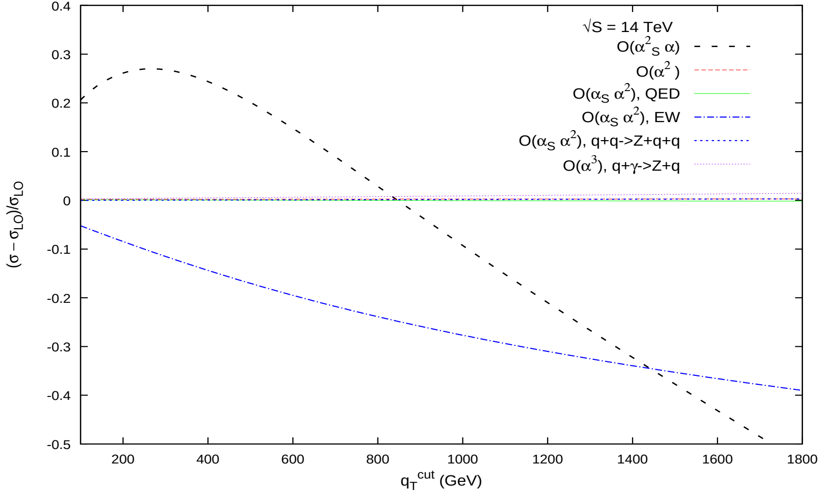


Figure: NLO QCD and EW corrections. $\sqrt{S} = 14TeV$ (LHC). All NLO contributions included.

$K - 1$ factor to the total σ for $p + p \longrightarrow Z + X$

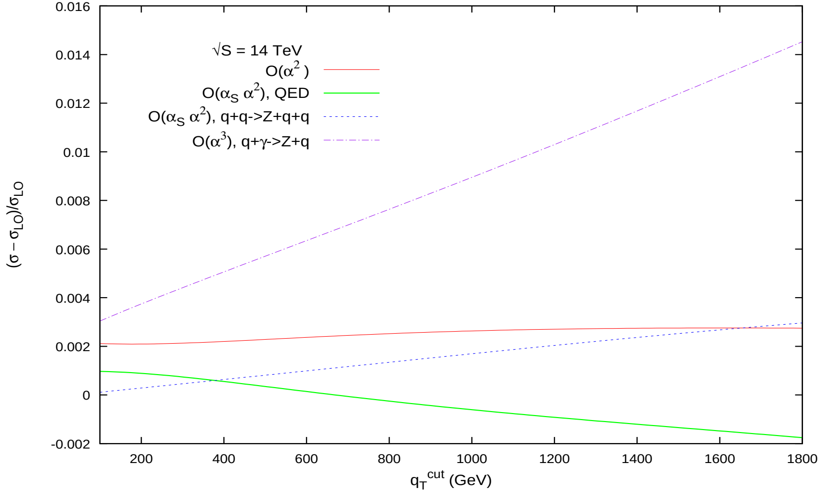


Figure: NLO QCD and EW corrections. $\sqrt{S} = 14 TeV$ (LHC). Close up without NLO QCD and EW.

$K - 1$ factor to the total σ for $p + \bar{p} \longrightarrow Z + X$

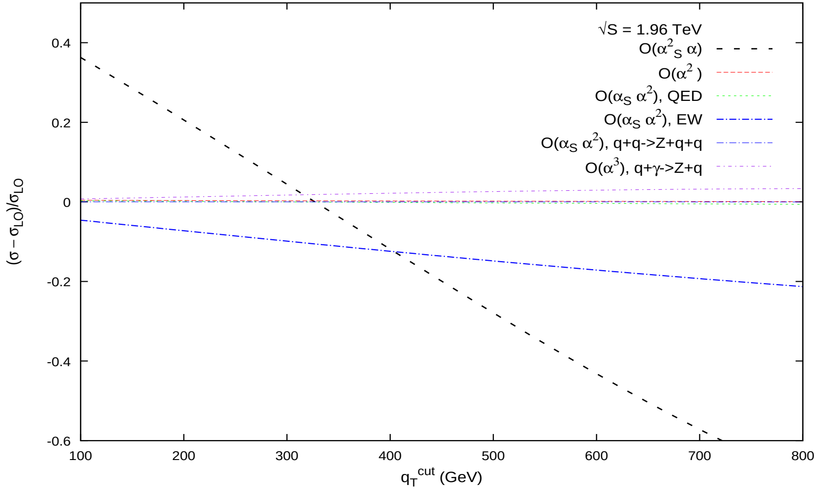


Figure: NLO QCD and EW corrections. $\sqrt{S} = 1.96 TeV$ (Tevatron). All NLO contributions included.

$K - 1$ factor to the total σ for $p + \bar{p} \longrightarrow Z + X$

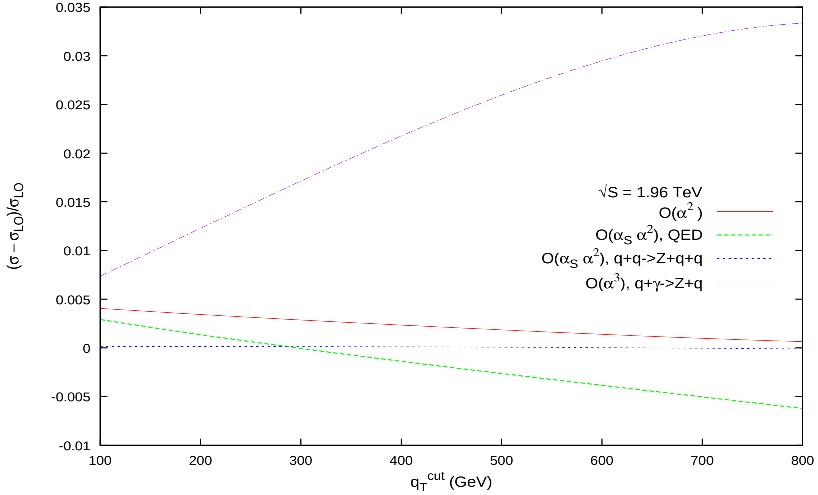


Figure: NLO QCD and EW corrections. $\sqrt{S} = 1.96 TeV$ (Tevatron). Close up without NLO QCD and EW.

The q_T distributions

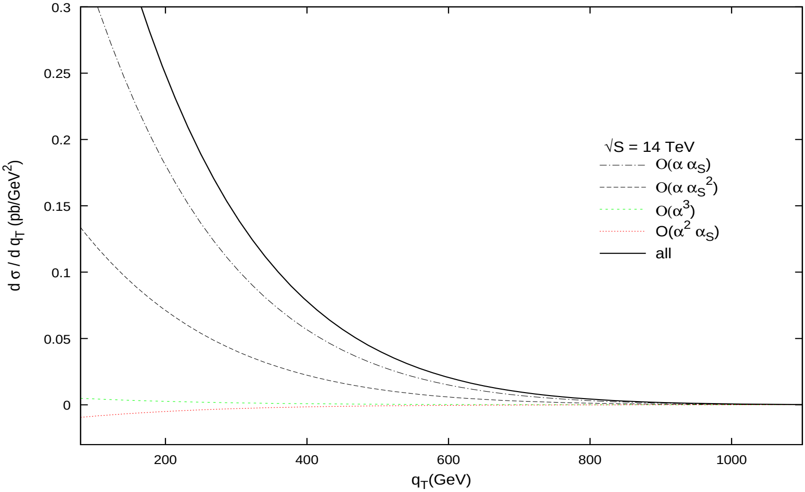


Figure: $\sqrt{S} = 14 TeV$ (LHC). The full value of cross section.

The q_T distributions

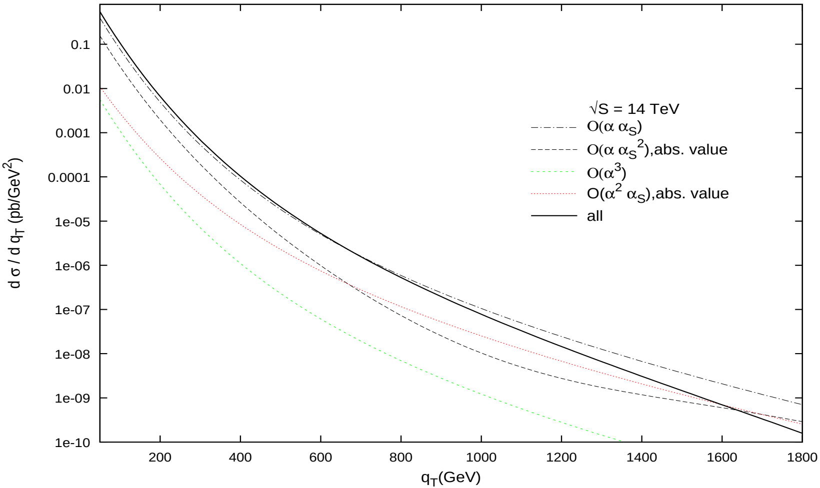


Figure: $\sqrt{S} = 14 TeV$ (LHC).The logarithmic values.

The rapidity distributions.

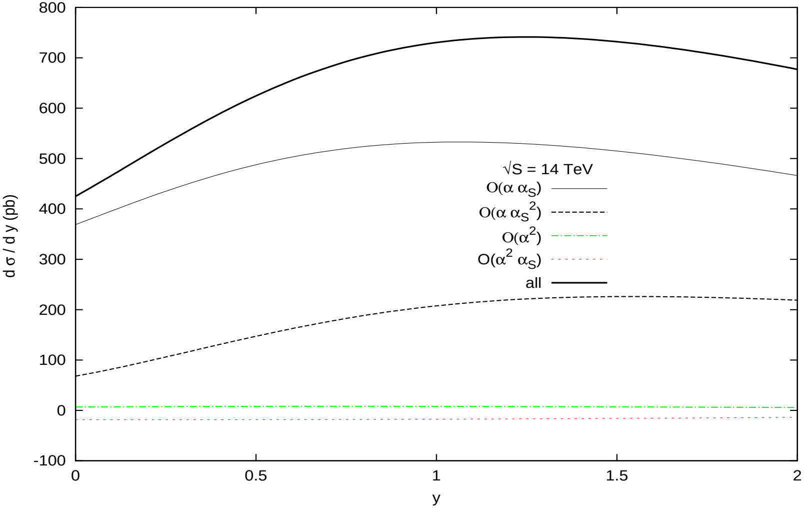


Figure: $\sqrt{S} = 14 TeV$ (LHC)

The rapidity distributions.

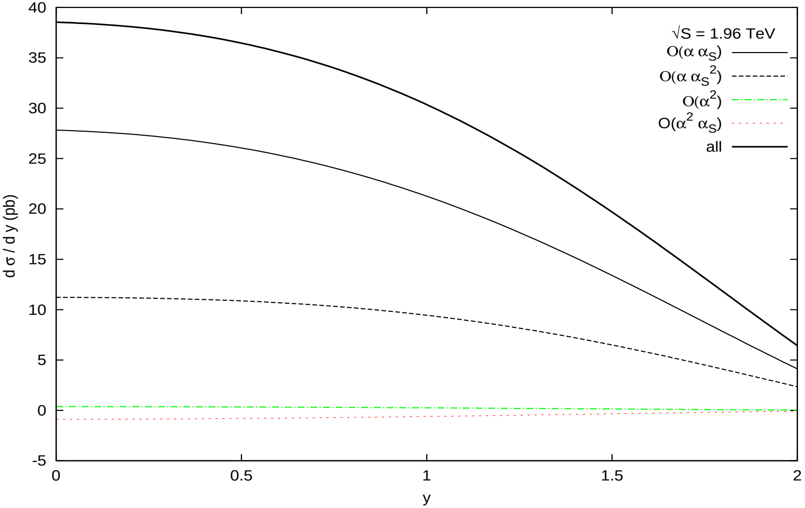


Figure: $\sqrt{S} = 1.96 TeV$ (Tevatron).

Outline

Summary

- The production of Z boson in the hadron-hadron interactions plays a very important role in the modern physics with relation to the development of the colliders technique and the building of new high energies hadron-hadron colliders like LHC.
- The accuracy of the calculated cross section give us a good approximation to the knowledge of the value of background and help us to separate important events.
- Our result includes the full QCD, QED and EW corrections of Z boson production in Drell-Yan mechanism up to general orders $O(\alpha_S \alpha, \alpha_S^2 \alpha, \alpha_S \alpha^2)$.
- The numerical results are presented by plots of the total cross section σ^{tot} and of the distributions in the transverse momentum $\frac{d\sigma}{dq_T}$ and in the rapidity $\frac{d\sigma}{dy}$ of Z boson.

Summary

- The contribution of the QCD corrections of order $O(\alpha_S^2\alpha)$ is most important at small q_T^{cut}.

  At large values of the momentum q_T^{cut} (for LHC energies $q_T^{cut} > 1200GeV$ and for Tevatron energies $q_T^{cut} > 400GeV$) the EW contributions of the order $O(\alpha_S\alpha^2)$ play a significant role and achieve up to 30% for LHC and 20% for Tevatron energies.

- The results will be applied to data analysis of modern and future experiments at hadrons colliders.

Thank You!!!

$$d\sigma_{i,j} = \frac{1}{2E_i 2E_j |v_i - v_j|} |M(p_i, p_j \to \{p_f\})|^2 d\Phi^{(n)}$$

$2E_i 2E_j |v_i - v_j|$ is the flux factor
$|M(p_i, p_j \to \{p_f\})|^2$ is the partonic matrix elements of interaction particles p_i and p_j with production of a set of final particles $\{p_f\}$
E_i and E_j are the energies of incoming particles.

$$\int d\Phi^{(n)} = \Big(\prod_f \int \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f}\Big)(2\pi)^4 \delta^{(4)}\big(p_i + p_j - \sum_f (p_f)\big).$$

SM

$$\mathcal{L}_{SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm fermion} + \mathcal{L}_{\rm higgs} + \mathcal{L}_{\rm yukawa}.$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} B^{\mu\nu} B^{\mu\nu}.$$

$$\mathcal{L}_{\rm fermions} = \sum_L \bar{L} i \gamma^\mu D_\mu L + \sum_r \bar{r} i \gamma^\mu D_\mu r,$$

$$\mathcal{L}_{\rm higgs} = |D\Phi|^2 - V(\Phi^\dagger \Phi),$$

$$V(\Phi^\dagger\Phi) = \frac{\lambda^2}{4}\left(\Phi^\dagger\Phi - v^2\right)^2,$$

$$\mathcal{L}_{\rm yukawa} \approx -g_Y \bar{L} \Phi r + h.c.$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D_\mu = i\partial_\mu - g_s G^a_\mu t^a - g' \frac{1}{2} Y_W B_\mu - g \frac{1}{2} \vec{\tau}_{\mathrm{L}} \vec{W}_\mu$$

$$\mathcal{L}_{\mathrm{NC}} = e j^{\mathrm{EM}}_\mu A^\mu + \frac{g}{\cos\theta} j^{\mathrm{Z}}_\mu Z^\mu,$$

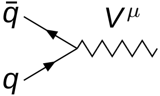
where electromagnetic j^{EM} and neutral j^{Z}_μ weak boson currents:

$$\begin{aligned} j^{\mathrm{EM}}_\mu &= j^3_\mu + \frac{1}{2} j^{\mathrm{Y}}_\mu, \\ j^{\mathrm{Z}}_\mu &= j^3_\mu - \sin^2\theta j^{\mathrm{Y}}_\mu, \\ j^3_\mu &= \bar{L}\gamma_\mu \frac{\tau_3}{2} L, \\ j^{\mathrm{Y}}_\mu &= -\bar{L}\gamma_\mu L - 2\bar{r}\gamma_\mu r. \end{aligned}$$

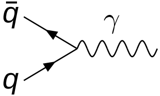
Propagators

quark	p 	: $-i\dfrac{\gamma^{\mu}p_{\mu}+m}{p^2-m^2+i\epsilon}$
gluon	p 	: $-i\dfrac{g_{\mu\nu}t^{ab}}{p^2+i\epsilon}\left(g_{\mu\nu}+(\xi-1)\dfrac{p_{\mu}p_{\nu}}{p^2}\right)$
photon	p 	: $-i\dfrac{1}{p^2+i\epsilon}\left(g_{\mu\nu}+(\xi-1)\dfrac{p_{\mu}p_{\nu}}{p^2}\right)$
gauge boson	p 	: $-i\dfrac{1}{p^2-m^2+i\epsilon}\left(g_{\mu\nu}-(\xi-1)\dfrac{p^{\mu}p^{\nu}}{p^2-\xi m^2}\right)$
ghost	p	: $-i\dfrac{1}{p^2-\xi m^2+i\epsilon}$

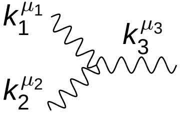
Couplings of interaction

a) 

$$: \quad -ie\gamma^{\mu}\left(L_{f_if_j}\frac{1-\gamma_5}{2}+R_{f_if_j}\frac{1+\gamma_5}{2}\right),$$

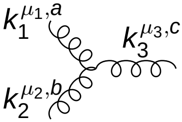
b) 

$$: \quad -ig\gamma^{\mu}t_c,$$

c) 

$$: \quad -ieK\Big(g^{\mu_1\mu_2}(k_1-k_2)^{\mu_3}$$

$$+\, g^{\mu_2\mu_3}(k_2-k_3)^{\mu_1}+g^{\mu_3\mu_1}$$

d) 

$$: \quad gf^{abc}\Big(g^{\mu_1\mu_2}(k_1-k_2)^{\mu_3}$$

$$i\gamma^{\mu}(v_f - \gamma_5 a_f),$$

where v_f and a_f are

$$v_f = \frac{I_3 - 2Q_f \sin^2\theta_W}{2\sin\theta_W\cos\theta_W}, \qquad a_f = \frac{I_3}{2\sin\theta_W\cos\theta_W}$$

with I_3 being isospin of a quark.

Generators of $SU(3)$ groups

$$\left[t^a, t^b\right] = if^{abc} t^c,$$
$$t^a_{i,j} t^a_{j,k} = C_F \delta_{ik},$$
$$f^{acd} f^{bcd} = C_A \delta_{ab},$$
$$t^a_{i,j}, t^b_{i,j} = T_R \delta_{ij},$$

f^{abc} are structure constants of $SU(3)$ groups
$C_F = \frac{N_C^2 - 1}{2N_C}$ is the "Casimir" color factor associated with gluon emission from the quark
$C_A \equiv N_C = 3$ is the color factor associated with gluon emission from a gluon
$T_R = \frac{1}{2}$ is the color factor for a gluon to split to quark-antiquark pair

Running Constant α_S

$$\alpha_S(\mu_R) \equiv \frac{g^2(\mu_R)}{4\pi} = \frac{4\pi}{\beta_0 \ln(\mu_R^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln[\ln(\mu_R^2/\Lambda^2)]}{\ln(\mu_R^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu_R^2/\Lambda^2)i}\left(\left(\ln[\ln(\mu_R^2/\Lambda^2)] - 1/2\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right].$$

$$\beta_0 = 11 - \frac{2}{3}n_f, \quad \beta_1 = 51 - \frac{19}{3}n_f, \quad \beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2.$$

Running Constant α

$$\alpha(\mu) = \frac{e^2(\mu)}{4\pi}(\delta\alpha_{bos} + \delta\alpha_{lep} + \delta\alpha_{top} + \delta\alpha^{(5)}_{hadrons}(M_Z^2) - \delta\alpha_{udscb}(M_Z^2)),$$

$$\begin{aligned}
\delta\alpha_{bos} &= \frac{\alpha}{4\pi}(7\ln\frac{M_W^2}{\mu^2} - \frac{2}{3}), \\
\delta\alpha_{lep} &= -\frac{\alpha}{3\pi}\sum_{l=e,\mu,\tau}\ln\frac{m_l^2}{\mu^2}, \\
\delta\alpha_{top} &= -\frac{4\alpha}{9\pi}\ln\frac{m_t^2}{\mu^2}, \\
\delta\alpha^{(5)}_{hadrons}(M_Z^2) &= 0.027572 \pm 0.000359, \\
\delta\alpha_{udscb}(M_Z^2) &= \frac{11\alpha}{9\pi}\left(\ln\frac{m_Z^2}{\mu^2} - \frac{5}{3}\right).
\end{aligned}$$

Integration By Parts

Constructing algorithms which reduce the number of all integrals of the process to a few master integrals.

$$0 = \int \mathrm{d}^d k \frac{\partial}{\partial k_\mu} \frac{\eta^\mu}{[k^2]^{\nu_1}[(k+p_1)^2]^{\nu_2}[(k+p_{12})^2]^{\nu_3}[(k+p_{123})^2]^{\nu_4}},$$

where sum of momenta $p_{ij\dots k} = p_i + p_j + \dots + p_k$,
$\eta^\mu = k,\ k+p_1,\ k+p_{12},\ k+p_{123}$.

$$\begin{aligned}
0 &= [s\nu_1\mathbf{1}^+ + (\mathbf{d} - \nu_{\mathbf{12334}}) - (\nu_{\mathbf{1}}\mathbf{1}^+ + \nu_{\mathbf{2}}\mathbf{2}^+ + \nu_{\mathbf{4}}\mathbf{4}^+)\mathbf{3}^-]\mathbf{B} \\
0 &= [t\nu_2\mathbf{2}^+ + (\mathbf{d} - \nu_{\mathbf{12344}}) - (\nu_{\mathbf{1}}\mathbf{1}^+ + \nu_{\mathbf{2}}\mathbf{2}^+ + \nu_{\mathbf{3}}\mathbf{3}^+)\mathbf{4}^-]\mathbf{B} \\
0 &= [s\nu_3\mathbf{3}^+ + (\mathbf{d} - \nu_{\mathbf{11234}}) - (\nu_{\mathbf{2}}\mathbf{2}^+ + \nu_{\mathbf{3}}\mathbf{3}^+ + \nu_{\mathbf{4}}\mathbf{4}^+)\mathbf{1}^-]\mathbf{B} \\
0 &= [t\nu_4\mathbf{4}^+ + (\mathbf{d} - \nu_{\mathbf{12234}}) - (\nu_{\mathbf{2}}\mathbf{1}^+ + \nu_{\mathbf{3}}\mathbf{3}^+ + \nu_{\mathbf{4}}\mathbf{4}^+)\mathbf{2}^-]\mathbf{B}
\end{aligned}$$

where $\nu_{iijk\dots} = \nu_i + \nu_i + \nu_j + \nu_k + \dots$, $\mathbf{3}^\pm\mathbf{B} = \mathbf{B}(\nu_{\mathbf{1}}, \nu_{\mathbf{2}}, \nu_{\mathbf{3}} \pm \mathbf{1}, \nu_{\mathbf{4}})$

K. G. Chetyrkin, F. V. Tkachov, 1981 , C. Anastasiou at al., 2004

Splitting Functions *DGLAP*

$$R_{k\leftarrow i}(z,M^2) = -\frac{1}{\varepsilon}P_{k\leftarrow i}(z)\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} + C_{k\leftarrow i}(z).$$

$$\begin{aligned}
P_{qq}(y) &= C_F\left(\frac{1+y^2}{(1-y)_+} + \frac{3}{2}\delta(y-1)\right), \\
P_{gq}(y) &= C_F\frac{1+(1-y)^2}{y}, \\
P_{gg}(y) &= 2C_A\left(\frac{1}{(1-y)_+} + \frac{1}{y} + y(1-y) - 2\right) \\
&\quad + \delta(y-1)\left(\frac{11}{6}C_A - \frac{2}{3}T_F\right), \\
P_{qg}(y) &= \frac{y^2+(1-y)^2}{2},
\end{aligned}$$

D-dimensional Integrals

$$\int \mathrm{d}^4 k \to \mu^{4-d} \int \mathrm{d}^d k, \qquad d = 4 - 2\varepsilon$$

$$\mathrm{d}^d k = \mathrm{d}k_0 |k|^{d-2} \mathrm{d}|k| \mathrm{d}\phi \prod_{k=1}^{d-3} \sin^k \theta_k \mathrm{d}\theta_k$$

$$\frac{\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2}(m^2)^{2-d/2}} = \frac{1}{(4\pi)^2}\left(\frac{2}{\varepsilon} - \gamma + \ln(4\pi) - \ln(m^2)\right)$$
$$\to \frac{1}{(4\pi)^2}\left(-\ln(m^2/M^2)\right),$$

G. Passarino and M. J. G. Veltman, 1979

T. Matsuura, S. C. van der Marck and W. L. van Neerven, 1989

Bremmstrahlung Integrals

$$\int \frac{d^{d-1}p_3 d^{d-1}p_4}{(2\pi)^{2d-2}4E_3E_4}(2\pi)^d \delta(p_1+p_2-q-p_3-p_4)|M^2|$$

van Neerven way $\rightarrow$

$$\begin{aligned} I_n^{(k,l)} &= \int_0^{\pi} d\beta_1 \sin^{d-3}\beta_1 \int_0^{\pi} d\beta_2 \sin^{d-4}\beta_2 \\ &\quad (a+b\cos\beta_1)^{-k}(A+B\cos\beta_1+C\sin\beta_1\cos\beta_2)^{-l} \\ &= 2^{1-i-j}\pi\frac{\Gamma(d/2-1-i)\Gamma(d/2-1-j)\Gamma(d-3)}{\Gamma(d-2-i-j)\Gamma^2(d/2-1)} \\ &\quad {}_2F_1\left(\begin{matrix} i,j \\ d/2-1 \end{matrix};\cos^2\frac{\chi}{2}\right) \end{aligned}$$