New Perturbation Theory for Gauge Theories

Grigorii Pivovarov

Institute for Nuclear Research
Russian Academy of Sciences

CALC, July 2012, Dubna

EFFECTIVE, CAUSAL, INACTION

EFFECTIVE FIELD THORY

Polchinski, 1984

$$\frac{dV_\Lambda}{d\Lambda} = \frac{1}{2}\frac{\delta V_\Lambda}{\delta\phi}\frac{dD_\Lambda}{d\Lambda}\frac{\delta V_\Lambda}{\delta\phi} - \frac{1}{2}\mathbf{Tr}\left(\frac{\delta^2 V_\Lambda}{\delta\phi\delta\phi}D_\Lambda\right)$$

THERE IS A GAP HERE

CAUSAL FORMULATION

Epstein&Glaser, 1973

$$\frac{\delta}{\delta g(x)}\left[\frac{\delta S(g)}{\delta g(y)}S^+(g)\right] = 0 \text{ at } x \leq y$$

INACTION APPROACH

G.P., 2010

$$W(J,K) = L_q^{-1} \circ P_\mu \circ L_q[W](J,K)$$

EFT: PRO & CON

- Pro side
  - Intuitive and geometrical picture for evolution of the couplings (due to Wilson)
  - Analogy with condensed matter (lattice formulation)
  - Interpretation of UV divergences
- Con side
  - Requires regularization: cutoff or dimesional
  - Cutoff breaks gauge invariance
  - DR is problematic for scalar fields (see below)
  - Provides an escape for theorists relegating real theory to high energy scales

Physical Consequences of Quadratic Divergencies

Wilson, 1971; Susskind, 1979; 't Hooft, 1980

- $M^2 = M_0^2 - \Lambda^2 P(g_0) + \dots$
- Either $P(g_0) = 0$ (supersymmetry), or ...
- $\frac{M_0^2}{\Lambda^2} \approx P(g_0)$, which means that
- Parameters of the effective high-energy theory should be fine tuned...

Naturalness Problem

Scalar mass is oversensitive to tiny changes in the strength of scalar self coupling measured at high energies
Wilson: **Scalar fields are forbidden**

MS Scheme for Scalar Propagator

Collins, 1974

- Nothing "unnatural" in RG functions of scalar field withing MS scheme
- UV asymptotics of the scalar propagator:
  $\frac{1}{Q^2} \to \frac{1}{(Q^2)^{1-\gamma_\phi} \mu^{2\gamma_\phi}}$
- anomalous dimension of scalar field within ϕ^4
  $\gamma_\phi = \frac{g^2}{12(16\pi^2)^2}$

MS Scheme Renormalization Group

Naturalness problem is inexistent

Where Are the Quadratic Divergencies within Dimensional Regularization?

Veltman, 1981

- For a diagram with m loops, quadratic divergence is related to a pole near dimension $4 - 2/m$
- Vanishing of the pole near dimension 2 is "Veltman condition": $2M_W^2 + M_Z^2 + M_H^2 - 4M_t^2 = 0$

Al-sarhi, Jack & Jones, 1992

- Quadratic divergencies poles computed up to four loops

The quadratic divergence poles are accumulated towards the physical dimension

Dimesional Regularisation and Minimal Subtractions

Unable to treat naturalness problem conclusively

Preparing for Light Higgs

Effective Field Theory paradigm

- **NO SCALARS or**
- **Dangerous use of dimensional regularization and minimal subtractions. Pessimistic conclusions.**

Fallback

Modified Epstein-Glaser$=$Inaction approach

Wish List

New Perturbation Theory Wish List:

- No divergences
- No Regularization
- STI (unitarity) built in
- Various evolution equations (like rg equations) built in

Epstein-Glaser Approach

All but the last wishes are fulfilled

$$S(g) = 1 + \sum_n \int dx_1 \ldots dx_n T_n(x_1, \ldots, x_n) g(x_1) \ldots g(x_n)$$

Causality Equation

Bogoliubov, 1955

$$\frac{\delta}{\delta g(x)} \left[\frac{\delta S(g)}{\delta g(y)} S^+(g) \right] = 0 \text{ at } x \leq y$$

- Epstein & Glaser, 1973: Perturbative solution to Bogoliubov equation
- G. Scharf, 2001: Generalization to gauge theories

UPDATING EPSTEIN-GLASER

- Get rid of couplings depending on space-time point
- S-matrix $\to$ Green functions
- Intuitive (geometric) evolution equations for Green functions

Inaction Equation

Action—Connected Green Functions Duality

Dominicis-Englert, 1967

- $e^{iW(J)} = \int D\phi e^{-iS(\phi)+i\phi J}$
- $e^{-iS(\phi)} = \int DJ e^{iW(J)-i\phi J}$

Inaction Equation

G.P., 2010

$$W(J) = L_q^{-1} \circ P_\mu \circ L_q[W](J)$$

- Quantum Legendre transform L_q
- Projector onto space of local functionals P_μ

Quantum Legendre Transform

$$L_q[W](\phi) \equiv i\hbar \log \left[\int DJ e^{\frac{i}{\hbar}(W(J)-\phi J)} \right]$$

Expansion in $\hbar$

- $L_q[W](\phi) = L[W](\phi) +$ quantum corrections
- $L[W](\phi) = \sup_J \left[\phi J - W(J) \right]$

P_μ EXPLICIT

- $\mu = \{p, k_1, k_2, q_1, q_2, q_3\}$
- Analogy with BPHZ: operator t_p^d
- $P_\mu\Big(\int dl_1 dl_2 \delta(l_1 + l_2) G(l_1)\phi_1(l_1)\phi_2(l_2)\Big) =$
  $\int dl_1 dl_2 \delta(l_1 + l_2)(t_p^d G)(l_1)\phi_1(l_1)\phi_2(l_2)$
- Similarly for cubic and quartic operators

BREAKS LORENTZ INVARIANCE

P_μ may break all sorts of invariances, and we will keep it under control (see below)

Perturbative Solution

Inaction Equation

$$W(J) = L_q^{-1} \circ P_\mu \circ L_q[W](J)$$

Linearization at W_F

$$W(J) = \tilde{P}_\mu[W](J)$$

- In linear approximation W belongs to a finite dimensional linear space

$$W = W_F + W_\mu + (1 - \tilde{P}_\mu) L_q^{-1} \circ P_\mu \circ L_q[W]$$

Slavnov-Taylor Identities

BRST-antiBRST

Baulieu & Thiery-Mieg, 1982

$$J_{0,0}A + J_{1,0}D(A)c + J_{0,1}D(A)\bar{c} + J_{1,1}(D(D(A)c)\bar{c} + D(A)b)$$

Linear Representation of BRST-antiBRST

- $\delta J_{1,0} = J_{0,0}$, $\delta J_{1,1} = J_{0,1}$
- $\bar{\delta} J_{0,1} = J_{0,0}$, $\bar{\delta} J_{1,1} = -J_{1,0}$

Slavnov-Taylor Identities

- $\delta W(J) = 0$
- $\bar{\delta} W(J) = 0$

LINEAR TRANSFORMATION OF SOURCES...

- complete $L_q[.]$ vs. partial $L_q[s \subset \{J\}; .]$:
- $L_q[s \subset \{J\}, W](\phi) \equiv i\hbar \log \left[\int \Pi_{J \in s} DJ e^{\frac{i}{\hbar}(W(J) - \sum_{J \in s} \phi J)} \right]$
- Symmetry of W with respect to linear transformations translates to a linear symmetry of action for the complete L_q
- Symmetry of W with respect to linear transformations translates to a non-linear symmetry of action for the partial $L_q[s, .]$
- $\delta\phi = \delta_L \phi + \frac{\delta S}{\delta K}$

EXAMPLE

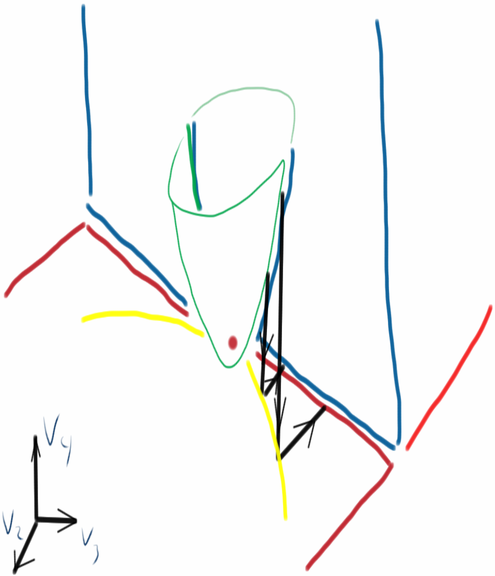
HIGGS-KIBBLE MODEL

list of sources:

- $J_{\alpha,\beta}$ vector sources
- $\xi_{\alpha,\beta}$ ghost sources
- $\eta_{\alpha,\beta}$ anti-ghost sources
- $\Phi_{\alpha,\beta}$ scalar sources

- $J_{0,0}, \xi_{0,0}, \eta_{0,0}, \eta_{1,0}, \Phi_{0,0}$ active sources
- $\eta_{1,0}$ is the source for the auxilary scalar boson b

SKETCH



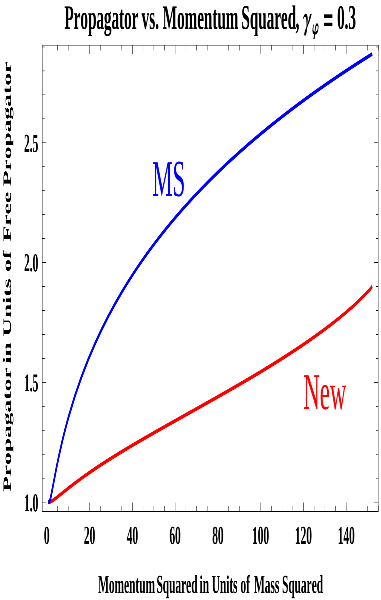
- Green = Inaction equation
- Red dot = Free Theory
- Red plane = tangent to green
- Blue = Slavnov-Taylor
- Black lines with arrows = Projectors
- Yellow = dependent couplings
- Changing projectors = evolution equations for couplings

COMPARISON

G.P., 2010

- Coincides with BPHZ in lowest nontrivial order
- Tadpoles never appear
- Standard coupling evolution
- New mass evolution

Evolution of Scalar Propagator

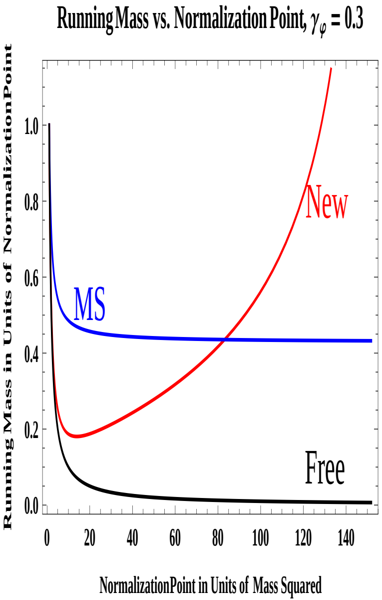


- $D_{MS} = \frac{1}{(aQ^2+bM^2)^{1-\gamma_\phi}}$
- $D_{MS} > D_{New}$
- $\frac{D_{MS}(Q^2)}{D_F(Q^2)} \sim \left(\frac{Q^2}{M^2}\right)^{\gamma_\phi}$
- $\frac{D_{New}(Q^2)}{D_F(Q^2)} \sim \left(\frac{Q^2}{M^2}\right)$
- $R'(Q^2) \to 0$

Scalar Propagator

is a nonzero constant at infinite momentum

Evolution of the Scalar Mass



- $m^2_{Free} = \frac{M^2}{Q^2}$
- $m^2_{MS} \sim \frac{\gamma_\phi}{1-\gamma_\phi}$
- m^2_{New} shoots up when R' becomes small
- m^2_{New} has a minimum
- At the minimum $m^2_{New} \approx \gamma_\phi$

Running Mass of a Scalar Field

in units of the normalization point has a minimum

SECOND ORDER EQUATION

- $R'' = -\frac{8\gamma_\phi}{(R')^3 Q^2} \int_0^\infty J_3(x)[mK_1(mx)]^3 x dx + \ldots$
- $m^2 \equiv R/(Q^2 R') - 1$
- J_3 Bessel, K_1 modified Bessel
- Initial conditions
  $R(M^2) = 2M^2$, $R'(M^2) = 1$

System of First Order Equations

- First order equations
  $\frac{d}{dt}m^2 = -m^2 + \frac{\gamma_\phi}{n}(1+m^2)\Phi(m)$,
  $\frac{d}{dt}n = -4\gamma_\phi\Phi(m)$,
  where $n = (R')^4$, $t = \log(Q^2/M^2)$
- Initial conditions
  $m^2(0) = 1$, $n(0) = 1$
- $\Phi(m) \approx \frac{0.3609}{6m^2+0.3609}$

THE RUNNING MASS

- for $M^2/\gamma_\phi < Q^2 \ll M^2 \exp(1/(4\gamma_\phi))$
- $M^2(Q^2) \approx \frac{\gamma_\phi Q^2}{1-4\gamma_\phi \log(Q^2/M^2)}$

For high normalization points, running mass is independent of the physical mass

THE LANDAU POLE

in the running mass invalidates perturbation theory

SUMMARY

- Inaction approach=Inaction equation $+$ Linear STI $+$ Incomplete quantum Legendre
- Within inaction approach geometric and intuitive evolution equations for Green functions are available
- Geometry of theory surface instead of RG flow in the space of effective actions of EFT

OUTLOOK

- Evolution equations for Higgs-Kibble model
- Implications for standard model
- Consequences of changing the set of active sources
- Why this particlular mapping $L_q^{-1} \circ P_\mu \circ L_q$?