Angular Distributions of Drell-Yan Dileptons in the Parton Reggeization Approch

V. A. Saleev, M. A. Nefedov

Samara State University,

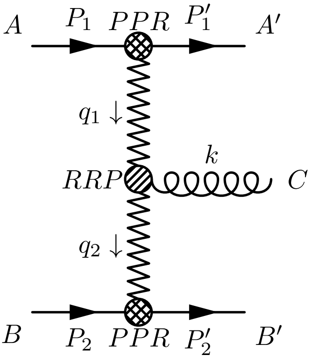
Samara, Russia

CALC-2012
Dubna, July 31, 2012

Outline.

- Introduction to the Parton Reggeization Approach.
- Quark reggeization. Derivation of Fadin-Sherman effective vertex.
- LO amplitude for Drell-Yan in the PRA. Formula for the cross-section.
- KMR prescription for the unintegrated PDFs.
- Transverse momentum and mass distributions in the photon exchange dominating region.
- Formalism for the angular distributions in the Collins-Soper frame. Angular coefficients.
- Comparation with NuSea and CDF data on angular coefficients.
- Predictions for the LHC.
- Lam-Tung relation breaking at small x.

The Multi-Regge Kinematics (MRK).



$$P_1' \approx P_1, \quad P_2' \approx P_2$$

$$k = x_1 P_1 + x_2 P_2 + k_\perp \ , \ s x_1 x_2 = k^2 + \mathbf{k}_T^2 \ ,$$
$$q_1 = x_1 P_1 + q_{1\perp} \ , \ q_1^2 = q_{1\perp}^2 = -t_1 \ ,$$
$$q_2 = x_2 P_2 + q_{2\perp} \ , \ q_2^2 = q_{2\perp}^2 = -t_2 \ ,$$
$$q_{1,2T} = (0, \mathbf{q}_{1,2T}, 0),$$
$$s = 2(P_1 P_2) \gg s_0,$$

MRK conditions:

$$x_1, \ x_2 \ll 1,$$
$$\frac{|t_1|}{s}, \ \frac{|t_2|}{s} \ll 1,$$
$$y_{B'} \ll y_H \ll y_{A'}$$

Amplitude with $t-$channel exchanges and MRK(QMRK) gives leading contribution for $s \to \infty$ and t-fixed.

Amplitude Reggeization.

Reggeized form of the amplitude:

$$\mathcal{A}_{AB}^{A'B'C} = 2s\gamma_{A'A}^{R_1}\left(\frac{s_1}{s_0}\right)^{\omega(t_1)}\frac{1}{t_1}\times\gamma_{R_1R_2}^{C}(q_1,q_2)\times\frac{1}{t_2}\left(\frac{s_2}{s_0}\right)^{\omega(t_2)}\gamma_{B'B}^{R_2}$$

$\gamma_{R_1R_2}^{C}(q_1,q_2)$ - RRP effective production vertex,

$\gamma_{A'A}^{R}$ - PPR effective scattering vertex,

$\omega(t)$ - Regge trajectory.

$\mathcal{Q}\bar{\mathcal{Q}}g(\gamma)$ Fadin-Sherman effective vertex

Effective vertex for vector boson production in the process $\mathcal{Q}(q_1) + \bar{\mathcal{Q}}(q_2) \to g(\gamma)$ (massless case):

$$\gamma_\mu^{(+-)}(q_1, q_2) = e\Gamma_\mu(q_1, q_2)$$

$$\Gamma_\mu(q_1, q_2) = \gamma_\mu + \hat{q}_2 \frac{n_\mu^+}{q_2^+} - \hat{q}_1 \frac{n_\mu^-}{q_1^-}$$

References:

- V. S. Fadin, V. E. Sherman, Sov. Phys. JETP **45**, 6, 1977
- L. N. Lipatov, M. I. Vyazovsky, Nucl.Phys. **B597** 399-409, arXiv:hep-ph/0009340v1, 2001

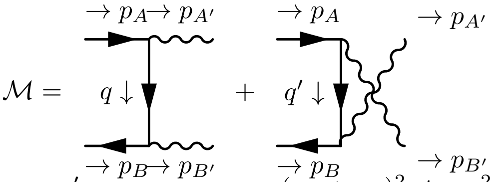
This vertex can be obtained in two steps:

Step 1: Derivation of the PPR ($q\mathcal{Q}\gamma$) vertexes $\gamma^{(+)}$ and $\gamma^{(-)}$.

Step 2: Derivation of the RRP ($\mathcal{Q}\overline{\mathcal{Q}}\gamma$) vertex $\gamma^{(+-)}$.

Step 1: Derivation of the PPR vertexes. Extraction of the pole part.

In the LO on α_s, this vertex can be obtaibed by determinins the s-asympthotics of the tree-level amplitude of $2 \to 2$ process:



Where, $q = p_A - p_{A'}$, $q' = p_A - p_{B'}$, $s = (p_A + p_B)^2$, $t = q^2$, $u = (q')^2$.
The $s \to \infty$ asympthotics of the amplitude, corresponds to the t-channel pole part:

$$\mathcal{P}_t \mathcal{M} = e^2 \bar{v}(p_B)\gamma^\mu \frac{i}{\hat{q} - m}\gamma^\nu u(p_A)\epsilon^*_\mu(p_{B'})\epsilon^*_\nu(p_{A'})$$

But this expression is not GI, if $\epsilon^*_\mu(p_{A'}) \to (p_{A'})_\mu$ or $\epsilon^*_\mu(p_{B'}) \to (p_{B'})_\mu$
$\Rightarrow \ \mathcal{P}_t \mathcal{A} \neq 0$.

Step 1: Derivation of the PPR vertexes. Restoration of the gauge invariance.

To restore gauge invariance, let's make the substitution:

$$\gamma_\mu \to \gamma_\mu - \hat{p}_{B'} \frac{(n^-)^\mu}{p^-_{B'}}, \; \gamma_\nu \to \gamma_\nu - \hat{p}_{A'} \frac{(n^+)^\mu}{p^-_{A'}}$$

Where: $n^+ n^- = 2$, $(n^\pm)^2 = 0$, $\forall k, k^\pm = kn^\pm$ and at $s \to \infty$: $p_A \sim \frac{\sqrt{s}}{2} n^-$, $p_B \sim \frac{\sqrt{s}}{2} n^+$. So, after some transformations, using the Dirac equation, the reggeised amplitude can be represented as follows:

$$\mathcal{A} = e^2 \bar{v}(p_B) \gamma_\mu^{(-)}(-p_{B'}, q) \frac{i}{\hat{q} - m} \gamma_\nu^{(+)}(p_{A'}, q) u(p_A) \epsilon^{*\mu}(p_{B'}) \epsilon^{*\nu}(p_{A'})$$

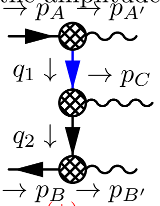
Where the PPR vertexes are:

$$\gamma_\mu^{(\pm)}(p, q) = \gamma_\mu + (\hat{q} - m) \frac{n^\pm_\mu}{p^\pm}$$

Step 2: Derivation of the RRP ($\mathcal{Q}\overline{\mathcal{Q}}\gamma$) vertex $\gamma^{(+-)}$.

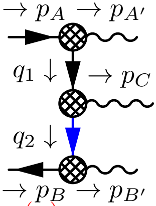
Let us consider the $\mathcal{Q}\overline{\mathcal{Q}} \to \gamma\gamma\gamma$ process in MRK. So, the t_1 and t_2 pole parts of the amplitude can be written as follows:

$\mathcal{P}_{t_1}\mathcal{M} =$



$$= e^3 \bar{v}(p_B)\gamma_\mu^{(-)}(-p_{B'}, q_2)\frac{i}{\hat{q}_2 - m} \times \gamma_\nu^{(+)}(p_C, q_2)\frac{i}{\hat{q}_1 - m}\gamma_\lambda^{(+)}(p_A, q_1)u(p_A)\epsilon^{*\mu}\epsilon^{*\nu}\epsilon^{*\lambda}$$

$\mathcal{P}_{t_2}\mathcal{M} =$



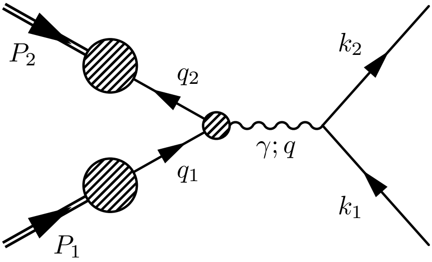
$$= e^3 \bar{v}(p_B)\gamma_\mu^{(-)}(-p_{B'}, q_2)\frac{i}{\hat{q}_2 - m} \times \gamma_\nu^{(-)}(-p_C, q_1)\frac{i}{\hat{q}_1 - m}\gamma_\lambda^{(+)}(p_A, q_1)u(p_A)\epsilon^{*\mu}\epsilon^{*\nu}\epsilon^{*\lambda}$$

Step 2: Derivation of the RRP ($\mathcal{Q}\overline{\mathcal{Q}}\gamma$) vertex $\gamma^{(+-)}$.

After collecting all singular terms in t_1 and t_2 channels witout double counting, we can obtain the RRP vertex:

$$\gamma_\nu^{(+-)}(q_1, q_2, p) = e\left(\gamma_\nu^{(+)}(p, q_2) + \gamma_\nu^{(-)}(-p, q_1) - \gamma_\nu\right) =$$
$$= e\left(\gamma_\nu - (\hat{q}_1 - m)\frac{n_\nu^-}{p^-} + (\hat{q}_2 - m)\frac{n_\nu^+}{p^+}\right)$$

Drell-Yan LO Amplitude in the PRA



$$\mathcal{A}(\mathcal{Q}\bar{\mathcal{Q}} \to \gamma^* \to l^+ l^-) = \frac{e^2 e_q}{Q^2} \left(\bar{v}(x_2 P_2)\Gamma^\mu(q_1, q_2) u(x_1 P_1)\right) \times \left(\bar{u}(k_1)\gamma_\mu v(k_2)\right),$$

$$Q^2 = (q_1 + q_2)^2 = (k_1 + k_2)^2$$

In the Collinear PM the LO subprocesses are $2 \to 2$:

$$q + g \to q + \gamma^*, \qquad q + \bar{q} \to g + \gamma^*.$$

Partonic tensors in PM and PRA.

$$\overline{|\mathcal{A}|^2}(\mathcal{Q}\bar{\mathcal{Q}} \to \gamma^* \to l^+ l^-) = \frac{16\pi^2}{3Q^4}\alpha^2 e_q^2 w_{PRA}^{\mu\nu} L_{\mu\nu}$$

Parton Reggeization Approach (PRA):

$$w_{PRA}^{\mu\nu} = x_1 x_2 \left[-S g^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu) \frac{2x_1 x_2 S - Q^2 - t_1 - t_2}{x_1 x_2 S} + \right. \\ + \frac{2}{x_2}(q_1^\mu P_1^\nu + q_1^\nu P_1^\mu) + \frac{2}{x_1}(q_2^\mu P_2^\nu + q_2^\nu P_2^\mu) + \\ \left. + \frac{4(t_1 - x_1 x_2 S)}{S x_2^2} P_1^\mu P_1^\nu + \frac{4(t_2 - x_1 x_2 S)}{S x_1^2} P_2^\mu P_2^\nu \right]$$

Collinear Parton Model (PM):

$$w_{PM}^{\mu\nu} = x_1 x_2 \left[-S g^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu) \right], \text{ with } q_{1,2}^\mu = x_{1,2} P_{1,2}^\mu$$

Drell-Yan cross section.

Master formula for the $2 \to 1$ cross-section for $pp \to \gamma^* \to l^+ l^-$:

$$\frac{d^3\sigma}{dQ^2 dy dq_T} = \frac{q_T}{Q_T^4} \sum_{f_1, f_2} \int dt_1 d\phi_1 \Phi_{f_1}(x_1, t_1, \mu_F) \Phi_{f_2}(x_2, t_2, \mu_F) \overline{|\mathcal{A}|^2}$$

Where $\Phi(x, t, \mu_F)$-unintegrated PDFs, $Q_T = \sqrt{Q^2 + q_T^2}$.
Factorization scale $\mu_F = \xi Q_T$, $1/2 < \xi < 2$.
ME, integrated over momentum $\mathbf{k}_{1,2}$:

$$\overline{|\mathcal{A}|^2} = \frac{4\alpha^2 e_q^2}{9Q^2}(Q^2 + t_1 + t_2)$$

Loop K-factor, see refs. 22, 23 in G. Watt, A.D. Martin, M.G. Ryskin, Phys. Rev. D **70**, 014012 (2004):

$$K = exp\left[C_F \frac{\alpha_s(Q^{2/3} Q_T^{4/3})}{2\pi} \pi^2\right]$$

KMR unintegrated PDFs.

Kimber M. A. , Martin A. D. , Ryskin M. G., Phys. Rev. D **63**, 114027, (2001), [arXiv:hep-ph/0101348]

KMR prescription to obtain unintegrated PDF from collinear one is based on the mechanism of last step parton k_T-dependent radiation and the assumption of strong angular ordering:

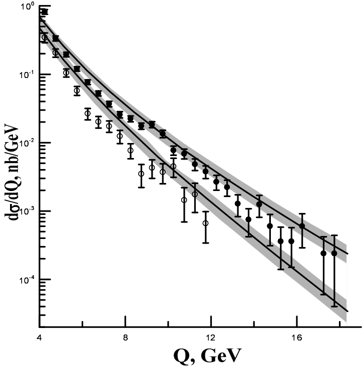
$$\Phi_q(x,k_T^2,\mu^2) = T_q(k_T,\mu)\frac{\alpha_s(k_T^2)}{2\pi}\int\limits_x^{\mu/(\mu+k_T)} dz \int^{k_T^2} \frac{dq_T^2}{q_T^2} \times$$

$$\times \left[P_{qg}(z) f_g\left(\frac{x}{z},q_T^2\right) + P_{qq}(z) f_q\left(\frac{x}{z},q_T^2\right)\right].$$

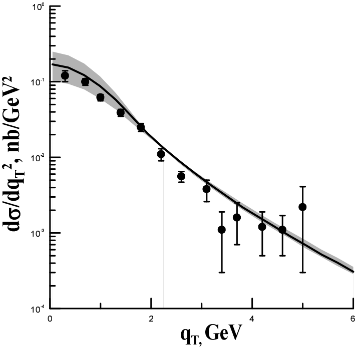
Where $P_{qg}(z)$, $P_{qq}(z)$- DGLAP splitting functions, $T_q(k_T,\mu)$- Sudakov formfactor:

$$T_q(k_T,\mu) = exp\left\{-\int\limits_{k_T^2}^{\mu^2}\frac{dq_T^2}{q_T^2}\frac{\alpha_s(q_T^2)}{2\pi}\sum_{a'}\int\limits_0^{1-\Delta} P_{qa'}(z')dz'\right\}$$

DY-pair spectra, CERN R-209.



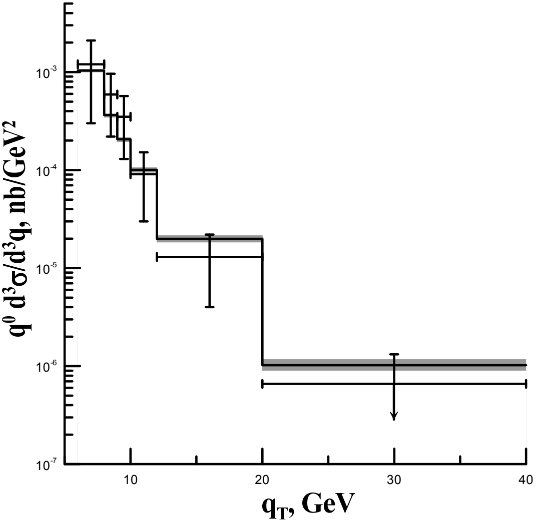
Open circles - $\sqrt{S} = 44$ GeV.
Closed circles - $\sqrt{S} = 62$ GeV



$\sqrt{S} = 62$ GeV, $5 < Q < 8$ GeV.

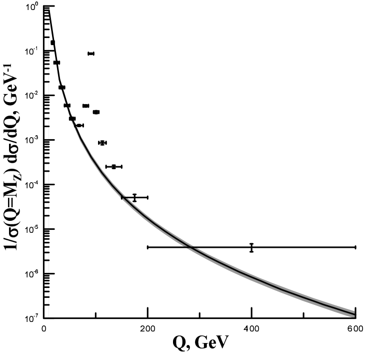
Comparation with UA1 data.

$Q < 2.5$ GeV, $|y| < 1.7$, $Q \ll q_T$

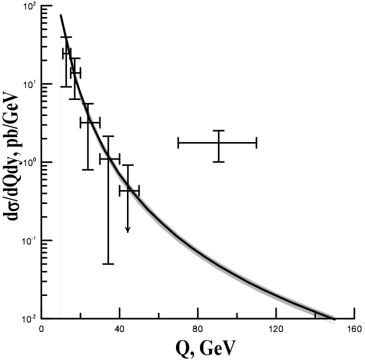


DY-pair spectrum, CDF (Tevatron) and CMS (LHC).

$p + p \to l^+ l^- + X$,
$\sqrt{S} = 7$ TeV



$p + \bar{p} \to l^+ l^- + X$, $|y| < 1$,
$\sqrt{S} = 1.8$ TeV



Angular distribution of DY dileptons.

We are working in the Collins-Soper reference frame:

$$\frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega} = \frac{\alpha^2}{64\pi^3 S Q^2}\Big[W_T(1+\cos^2\theta) + W_L(1-\cos^2\theta) + \\ + W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta}\sin^2\theta\cos 2\phi\Big].$$

$$\frac{dN}{d\Omega} = (1+\cos^2\theta) + A_0\left(\frac{1}{2} - \frac{3}{2}\cos^2\theta\right) + \\ + A_1 \sin 2\theta\cos\phi + \frac{A_2}{2}\sin^2\theta\cos 2\phi$$

or

$$\frac{dN}{d\Omega} = \frac{4}{\lambda+3}\left(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right),$$

Spin structure functions and angular coefficients.

$$\epsilon^\mu_{\pm 1} = \frac{1}{\sqrt{2}}(\mp X^\mu - iY^\mu),\ \epsilon^\mu_0 = Z^\mu$$

Spin structure functions:

$$W_T = W_{\mu\nu}\epsilon^{\mu\star}_{+1}\epsilon^\nu_{+1},$$
$$W_L = W_{\mu\nu}\epsilon^\mu_0\epsilon^\nu_0,$$
$$W_\Delta = W_{\mu\nu}(\epsilon^{\mu\star}_{+1}\epsilon^\nu_0 + \epsilon^{\mu\star}_0\epsilon^\nu_{+1})/\sqrt{2},$$
$$W_{\Delta\Delta} = W_{\mu\nu}\epsilon^{\mu\star}_{+1}\epsilon^\nu_{-1}$$

Unit vectors of CS-frame in the CM frame:

$$Z^\mu = \frac{2}{Q_T}\left[\frac{(qP_2)}{\sqrt{S}}\tilde{P}^\mu_1 - \frac{(qP_1)}{\sqrt{S}}\tilde{P}^\mu_2\right],$$
$$X^\mu = -\frac{2Q}{q_T Q_T}\left[\frac{(qP_2)}{\sqrt{S}}\tilde{P}^\mu_1 + \frac{(qP_1)}{\sqrt{S}}\tilde{P}^\mu_2\right],$$
$$Y^\mu = \varepsilon^{\mu\nu\alpha\beta}T_\nu Z_\alpha X_\beta$$
$$T^\nu = \frac{q^\nu}{Q},\ \tilde{P}^\mu_i = \frac{1}{\sqrt{S}}\left(P^\mu_i - \frac{qP_i}{Q^2}q^\mu\right)$$

Angular coefficients.

$$A_0 = \frac{W_L}{W_{TL}}, \quad A_1 = \frac{W_\Delta}{W_{TL}}, \quad A_2 = \frac{2W_{\Delta\Delta}}{W_{TL}},$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0}, \quad \mu = \frac{2A_1}{2 + A_0}, \quad \nu = \frac{2A_2}{2 + A_0}.$$

Spin structure functions in PRA.

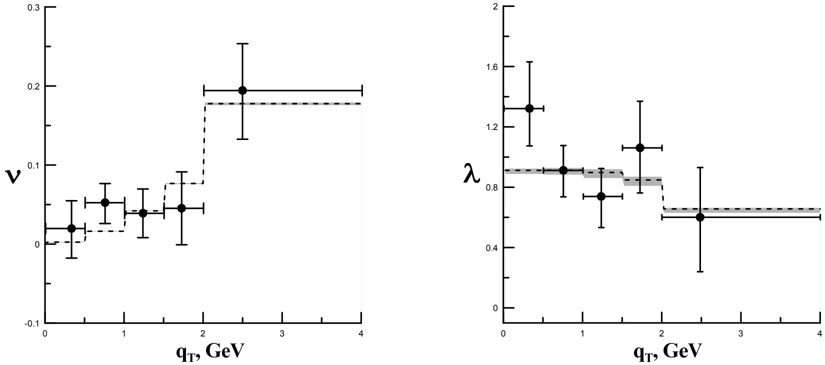
Relation between hadronic and partonic structure functions:

$$W_{T,L,...} = \frac{8\pi^2 S}{3Q_T^4} \int dt_1 \int d\phi_1 \sum_f e_f^2 \Phi_f^{(1)}(x_1, t_1, \mu^2) \Phi_{\bar{f}}^{(2)}(x_2, t_2, \mu^2) w_{T,L,...}$$

$$w_T^{PM} = Q^2, w_L^{PM} = w_\Delta^{PM} = w_{\Delta\Delta}^{PM} = 0$$

$$w_T^{PRA} = Q^2 + \frac{\mathbf{q}_T^2}{2},\ w_L^{PRA} = (\mathbf{q}_{1T} - \mathbf{q}_{2T})^2,\ w_\Delta^{PRA} = 0,\ w_{\Delta\Delta}^{PRA} = \frac{\mathbf{q}_T^2}{2}$$

Angular coefficients from NuSea (FNAL E866) data.



$p + p \to \mu^+\mu^- + X$, $E_p = 800$ GeV, $\sqrt{S} = 39$ GeV

$4.5 < Q < 15$ GeV, $0 < x_F < 0.8$, $x_F = \frac{2Q_T}{\sqrt{S}} sinh(y)$

Angular coefficients in the Z-boson region.

The $Q\bar{Q}Z$ effective vertex in PRA:

$$\gamma_{\mu}^{Q\bar{Q}Z}(q_1,q_2)=\Gamma_{\mu}(q_1,q_2)(c_V+c_A\gamma^5)$$

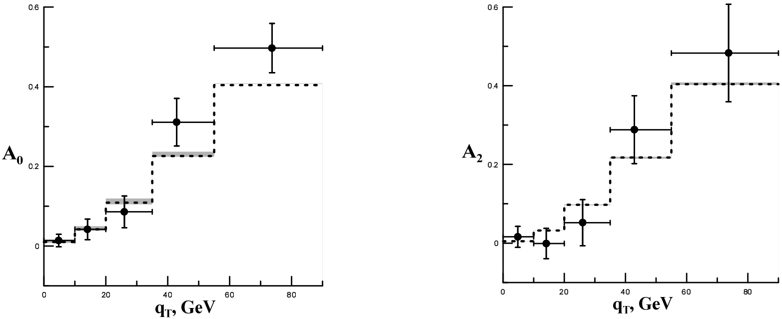
Effect of the Z-boson exchange inclusion on the spin SF-s:

$$w_{T,L,\Delta\Delta}^{Z,\ PRA}(t_1,t_2,Q^2)=w_{T,L,\Delta\Delta}^{PRA}(t_1,t_2,Q^2)f(Q^2)$$

Where $f(Q^2)\sim\frac{1}{(Q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}$, $\Gamma_Z\ll M_Z$, so:

$$A_0^Z=\frac{W_L^{Z,PRA}}{W_{TL}^{Z,PRA}}\approx\frac{W_L^{PRA}}{W_{TL}^{PRA}}=A_0,\ \ A_2^Z\approx A_2$$

Comparation with CDF data.



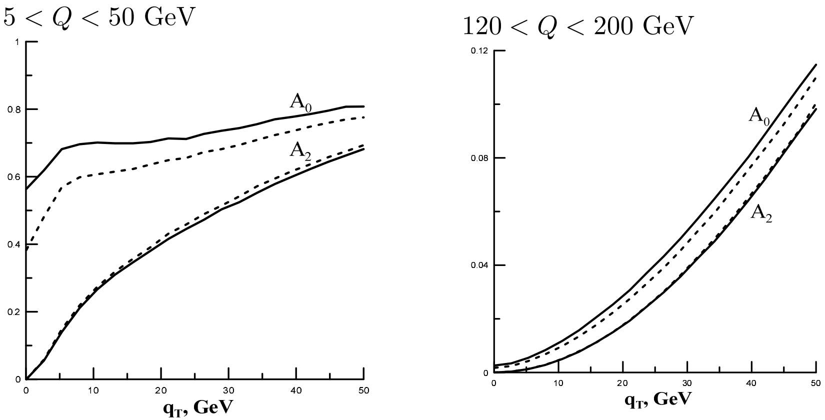
$p + \bar{p} \to e^+e^- + X,$
$\sqrt{S} = 1.96$ TeV, $66 < Q < 116$ GeV, $|\eta| < 3.6.$

Lam-Tung relation

- On the partonic level and in NLO CPM one has $A_0 = A_2$.
- In the PRA we predict large difference at more large energy, or at small $x \sim \frac{2Q_T}{\sqrt{S}}$, independently from unintegrtated PDF.

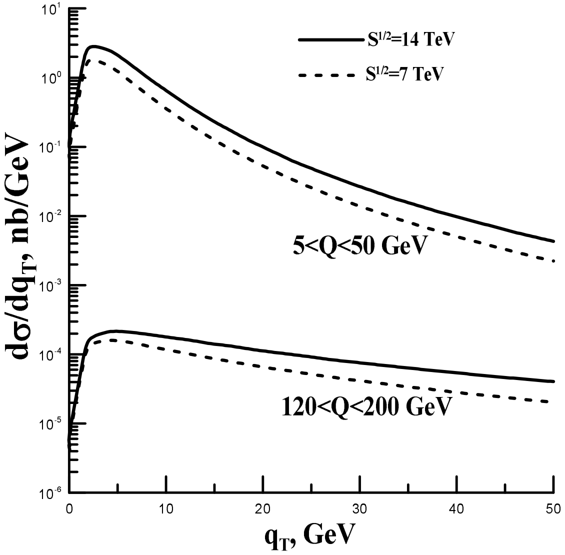
$$w_L^{PRA} = (\mathbf{q}_{1T} - \mathbf{q}_{2T})^2 \sim A_0, \; w_{\Delta\Delta}^{PRA} = \frac{\mathbf{q}_T^2}{2} \sim A_2$$

Lam-Tung relation at the LHC energies



Dashed line - $\sqrt{S} = 7$ TeV, solid line - $\sqrt{S} = 14$ TeV.

DY-dilepton spectra at the LHC



Summary of results.

- We describe q_T and Q spectra of Drell-Yan dileptons at the region of large $\sqrt{S} \gg q_T, Q$.
- We describe q_T-dependence of angular coefficients A_0, A_2, λ, ν.
- We predict strong breaking of the Lam-Tung relations when $q_T, Q \ll \sqrt{S}$ (in small-x region).
- We predict q_T and Q spectra for LHC, outside the Z-boson mass region.
- We predict q_T-dependence of angular coefficients for the LHC

Conclusions.

- LO PRA describes existing data well
- We have agreement between LO PRA and NLO Collinear PM
- To describe q_T-spectra of DY-dileptons in PM special resummation procedure is needed. In PRA it can be done by the simple way already in LO.

Thank you for your attention!