

# THE DECAYS $\rho^- \rightarrow \eta\pi^-$ AND $\tau^- \rightarrow \eta(\eta')\pi^-\nu$ IN THE NJL MODEL

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Studying the decays defined by the light quark mass difference can provide new information about chiral symmetry breaking mechanism.

It is possible to discover new anomalies in the weak interactions and clarify the nature of the  $a_0$  scalar meson.

Current experimental limits

$$\begin{aligned}\mathcal{B}(\rho^- \rightarrow \eta\pi^-) &< 6 \cdot 10^{-3} \\ \mathcal{B}(\tau^- \rightarrow \eta\pi^-\nu) &< 0.99 \cdot 10^{-4} \\ \mathcal{B}(\tau^- \rightarrow \eta'\pi^-\nu) &< 7.2 \cdot 10^{-7}\end{aligned}$$

# HISTORICAL OVERVIEW

- S. Tisserant and T. N. Truong, “ $\tau \rightarrow \delta\nu$  decay induces by light quark mass difference”, Phys. Lett. B **115**, 264 (1982).
- A. Bramon, S. Narison and A. Pich, “The  $\tau \rightarrow \nu_\tau \eta \pi$  Process In And Beyond Qcd”, Phys. Lett. B **196**, 543 (1987).
- H. Neufeld and H. Rupertsberger, “Isospin breaking in chiral perturbation theory and the decays  $\eta \rightarrow \pi\nu$  and  $\tau \rightarrow \eta\pi\nu$ ”, Z. Phys. C **68**, 91 (1995).
- S. Nussinov and A. Soffer, “Estimate of the branching fraction  $\tau \rightarrow \eta\pi\nu_\tau$ , the  $a_0(980)$ , and non-standard weak interactions”, Phys. Rev. D **78**, 033006 (2008)
- S. Nussinov and A. Soffer, “Estimate of the Branching Fraction of  $\tau \rightarrow \pi\eta'\nu_\tau$ ”, Phys. Rev. D **80**, 033010 (2009)
- N. Paver and Riazuddin, “On meson dominance in the ‘second class’  $\tau \rightarrow \eta\pi\nu_\tau$  decay”, Phys. Rev. D **82**, 057301 (2010)
- N. Paver and Riazuddin, “On the branching ratio of the ‘second class’  $\tau \rightarrow \eta'\pi\nu_\tau$  decay”, Phys. Rev. D **84**, 017302 (2011)

The interaction of quarks with mesons in ground state is described by the given lagrangian

$$\Delta\mathcal{L}_1 = \bar{q} \left[ i\hat{\partial} - m + \frac{g_{EW}}{\sqrt{2}} \hat{W} + g_\sigma a_0 + ig_\pi \gamma_5 \pi + \frac{g_\rho}{2} \hat{\rho} \right] q$$

$$m_u = 280 \text{ MeV}, \quad m_d - m_u \approx 3.7 \text{ MeV},$$

$$g_\pi = m_u/F_\pi, \quad F_\pi = 93 \text{ MeV},$$

$$g_\rho = 6.14, \quad g_\sigma = g_\rho/\sqrt{6},$$

# EXTENDED NJL MODEL

For the first radially excited meson states we can use the extended NJL:

$$\begin{aligned}
 \Delta\mathcal{L}_2 = & \bar{q} \left\{ i\hat{\partial} - m + \frac{g_{EW}}{\sqrt{2}} \hat{W} \right. \\
 & + \left[ g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^\perp{}^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] \pi(p) \\
 & - \left[ g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\cos(2\alpha_0)} + g_{\pi_2} f(k^\perp{}^2) \frac{\cos(\alpha - \alpha_0)}{\cos(2\alpha_0)} \right] \pi'(p) \\
 & + \left[ g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^\perp{}^2) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] \hat{\rho}(p) \\
 & \left. - \left[ g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^\perp{}^2) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right] \hat{\rho}'(p) \right\} q
 \end{aligned}$$

Angles  $\alpha_0 = 59.06^\circ$ ,  $\alpha = 59.38^\circ$ ,  $\beta_0 = 61.53^\circ$  and  $\beta = 76.78^\circ$  describe mixing of the ground and excited states for pions and vector mesons.

$$g_{\pi_1} = g_\pi, \quad g_{\rho_1} = g_\rho$$

$$g_{\pi_2} = \left[4I_2^{f^2}\right]^{-1/2}, \quad g_{\rho_2} = \left[\frac{2}{3}I_2^{f^2}\right]^{-1/2} = \sqrt{6}g_{\pi_2}$$

$$I_m^{f^n} = -iN_c \int \frac{d^4k}{(2\pi)^4} \frac{(f(k^\perp{}^2))^n}{(m^2 - k^2)^m}, \quad n = 1, 2, 3 \quad m = 1, 2, 3$$

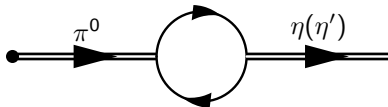
The form factor is taken in a simple polynomial form:

$$f(k^\perp{}^2) = (1 - d|k^\perp{}^2|)\Theta(\Lambda^2 - |k^\perp{}^2|),$$

$$k^\perp = k - \frac{(kp)p}{p^2}, \quad d = 1.78 \text{ GeV}^{-2},$$

$k$  and  $p$  are the quark and meson momenta,  $\Lambda = 1.03 \text{ GeV}$

The  $\pi^0 - \eta(\eta')$  transitions are described by the given diagram



The amplitude of the transition  $\pi^0 - \eta(\eta')$  has the form

$$\epsilon_{\pi\eta(\eta')} = 2g_\pi^2 \left( (2I_1(m_d) + m_{\eta(\eta')}^2 I_2(m_d)) - (2I_1(m_u) + m_{\eta(\eta')}^2 I_2(m_u)) \right) \frac{\epsilon_{\eta(\eta')}}{m_\pi^2 - m_{\eta(\eta')}^2}$$

$$\begin{aligned}\epsilon_\eta &= \sin \bar{\theta} \text{ for } \eta \text{ meson} \\ \epsilon_{\eta'} &= \cos \bar{\theta} \text{ for } \eta' \text{ meson} \\ &\text{where } \bar{\theta} = -54^\circ\end{aligned}$$



The  $I_1$  and  $I_2$  are divergent integrals describing quark loops

$$I_1(m) = -i \frac{N_c}{(2\pi)^4} \int^{\Lambda_4} \frac{d^4 k}{(m^2 - k^2)} = \frac{N_c}{(4\pi)^2} \left[ \Lambda_4^2 - m^2 \log \left( \frac{\Lambda_4^2}{m^2} + 1 \right) \right]$$
$$I_2(m) = -i \frac{N_c}{(2\pi)^4} \int^{\Lambda_4} \frac{d^4 k}{(m^2 - k^2)^2} = \frac{N_c}{(4\pi)^2} \left[ \log \left( \frac{\Lambda_4^2}{m^2} + 1 \right) - \left( 1 + \frac{m^2}{\Lambda_4^2} \right)^{-1} \right]$$

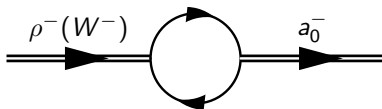
$$N_c = 3, \quad \Lambda_4 = 1250 \text{ MeV.}$$

Obtained estimates coincide with estimates given by Paver & Riazuddin

$$\left| \begin{array}{l} \epsilon_{\pi\eta}^{PR} \\ 1.34 \cdot 10^{-2} \end{array} \right| \left| \begin{array}{l} \epsilon_{\pi\eta}^{NJL} \\ 1.55 \cdot 10^{-2} \end{array} \right| \left| \begin{array}{l} \epsilon_{\pi\eta'}^{PR} \\ (3 \pm 1) \cdot 10^{-3} \end{array} \right| \left| \begin{array}{l} \epsilon_{\pi\eta'}^{NJL} \\ 6.79 \cdot 10^{-3} \end{array} \right|$$

# $\rho^-(W^-) - a_0^-$ TRANSITIONS

The  $\pi^0 - \eta(\eta')$  transitions are described by the given diagram



The transition  $\rho^- - a_0^-$  takes the form

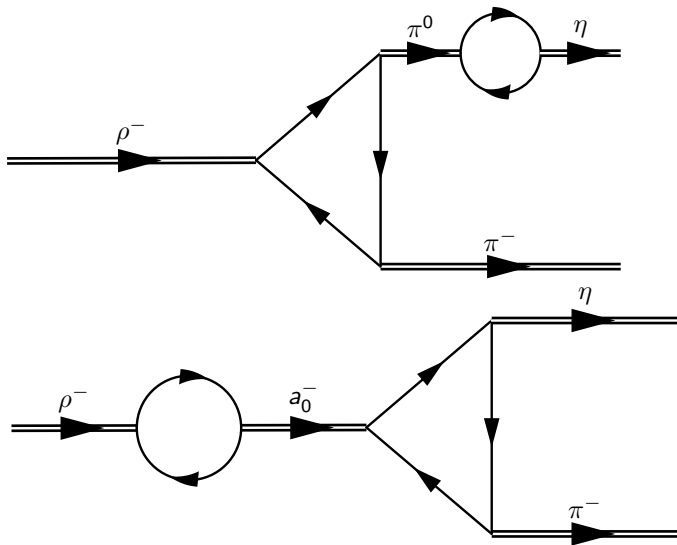
$$\frac{\sqrt{6}}{2}(m_d - m_u)p^\mu \rho_\mu^- a_0^-$$

For the  $W^-$  boson we get

$$\frac{\sqrt{3}g_{EW}|V_{ud}|}{4g_\rho}(m_d - m_u)p^\mu W_\mu^- a_0^-$$

where  $p$  is a vector boson momentum

# THE DECAY $\rho^- \rightarrow \eta\pi^-$



# THE DECAY $\rho^- \rightarrow \eta\pi^-$

The first diagram describes the amplitude which contains the  $\pi^0 - \eta$  transitions in the final state

$$T_1 = g_\rho \epsilon_{\pi\eta} (p_-^\mu - p_0^\mu) \rho_\mu^- \eta \pi^- ,$$

The second diagram describes the amplitude containing the intermediate  $a_0^-$  meson

$$T_2 = 2Zg_\rho \frac{m_u(m_d - m_u)}{m_{a_0}^2 - m_\rho^2} \epsilon_\eta p^\mu \rho_\mu^- \eta \pi^- ,$$

This amplitude contains the amplitude of the  $a_0^- \rightarrow \eta\pi^-$  decay

$$\frac{4}{\sqrt{6}} Zg_\rho m_u \epsilon_\eta a_0^- \eta \pi^-$$

where  $Z$  is the renormalizing factor after taking into account  $\pi - a_1$  transitions in the final state

# THE DECAY $\rho^- \rightarrow \eta\pi^-$

For branching fractions we get

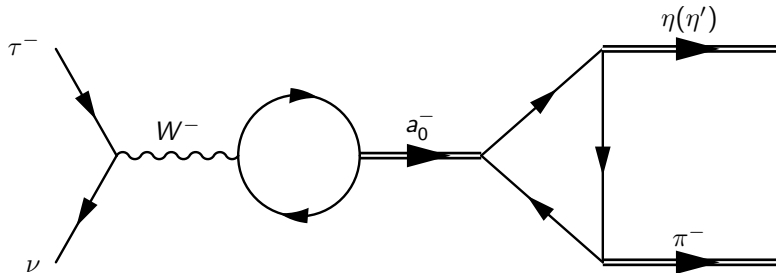
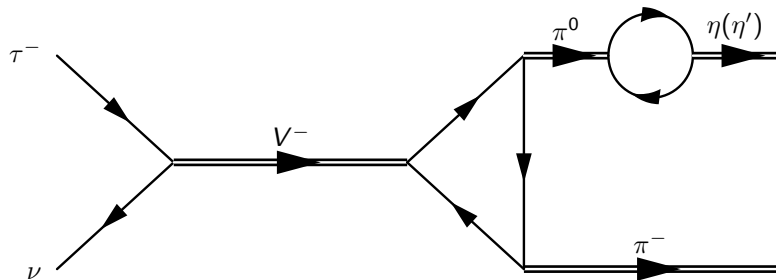
$$\mathcal{B}_1 = \epsilon_{\pi\eta}^2 \frac{\lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} = 1.78 \cdot 10^{-5}$$

$$\mathcal{B}_2 = 4Z^2 \sin^2 \bar{\theta} \left( \frac{m_u(m_d - m_u)}{m_{a_0}^2 - m_\rho^2} \right)^2 \frac{\lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} = 0.33 \cdot 10^{-5}$$

$$\mathcal{B}_{NS} = 1.4 \cdot 10^{-5} \simeq \mathcal{B}_1 \quad (\text{Nussinov \& Soffer})$$

$$\text{where } \lambda(s, m_{\eta(\eta')}^2, m_\pi^2) = (s - m_{\eta(\eta')}^2 - m_\pi^2)^2 - 4m_{\eta(\eta')}^2 m_\pi^2$$

# THE DECAYS $\tau^- \rightarrow \eta(\eta')\pi^-$



# THE DECAYS $\tau^- \rightarrow \eta(\eta')\pi^-$

The amplitude with  $\pi^0 - \eta(\eta')$  transitions in final state

$$\epsilon_{\pi\eta(\eta')} m_\rho^2 \left( \left( 1 - \frac{i\sqrt{q^2}\Gamma_\rho(p^2)}{m_\rho^2} \right) BW_\rho(p^2) + \beta \frac{p^2}{m_\rho^2} BW_{\rho'}(p^2) \right) (p_{\pi^-}^\mu - p_{\eta(\eta')}^\mu) l_\mu \pi^- \eta(\eta')$$

The amplitude with intermediate  $a_0^-$  meson

$$2Zm_u(m_d - m_u)\epsilon_{\eta(\eta')}(BW_{a_0}(p^2) + \beta_{a_0\eta(\eta')\pi} BW_{a_0'}(p^2)) p^\mu l_\mu \pi^- \eta(\eta')$$

For branching fractions we get

$$\mathcal{B}_V^{\pi\eta} = 4.35 \cdot 10^{-6}$$

$$\mathcal{B}_S^{\pi\eta} = 0.37 \cdot 10^{-6}$$

$$\mathcal{B}_V^{\pi\eta'} = 1.11 \cdot 10^{-8}$$

$$\mathcal{B}_S^{\pi\eta'} = 2.63 \cdot 10^{-8}$$



# CONCLUSIONS

We can compare our results with ones obtained in previous works

	$\mathcal{B}_V^{\pi\eta} \times 10^{-6}$	$\mathcal{B}_S^{\pi\eta} \times 10^{-6}$	$\mathcal{B}_{tot}^{\pi\eta} \times 10^{-6}$
NJL	4.35	0.37	4.72
PR	$1.58 \div 5.70$	$10.7 \div 65.9$	$\simeq 26$
NS	3.6	$\sim 10$	$3 \div 10$
EXP	-	-	$< 99$

	$\mathcal{B}_V^{\pi\eta'} \times 10^{-8}$	$\mathcal{B}_S^{\pi\eta'} \times 10^{-8}$	$\mathcal{B}_{tot}^{\pi\eta'} \times 10^{-8}$
NJL	1.11	2.63	3.74
PR	$0.14 \div 3.4$	$6 \div 18$	-
NS	$< 2 + 8$	$< 10 + (20 \div 120)$	$< 140$
EXP	-	-	$< 72$

We can get decay width and branching fraction for  $\tau \rightarrow a_0 \nu$

$$\Gamma = \frac{G_F^2 |V_{ud}|^2 m_\tau^3}{16\pi} \left( \frac{\sqrt{6} m_d - m_u}{2 g_\rho} \right)^2 \left( 1 - \frac{m_{a_0}^2}{m_\tau^2} \right)^2$$

$$\mathcal{B}^{NJL} = 3.28 \cdot 10^{-6}$$

$$\mathcal{B}^{NS} = 8 \cdot 10^{-6} \quad (\text{Nussinov \& Soffer})$$

$$\mathcal{B}^{TT} = 16 \cdot 10^{-6} \quad (\text{Tisserant \& Truong})$$

# CONCLUSIONS

The branching fraction for  $\tau^- \rightarrow a_0^- \nu$  confirms relevancy of our expression for the vertex  $\tau a_0 \nu$ . For the vertex  $a_0^- \rightarrow \eta \pi^-$  also was used well-known expression. It contradicts with the ansatz used by Bramon, Narison, Pich (1987) and Paver & Riazuddin (2010)

$$\frac{\epsilon_{\pi\eta} M_R^2}{M_R^2 - p^2 - iM_R \Gamma_R(p^2)}$$

On the other side, if we use this ansatz for vector to scalar transition and calculate  $\rho^- \rightarrow \eta \pi^-$  with this ansatz then we get

$$\mathcal{B} \sim \epsilon_{\pi\eta}^2 \left( \frac{m_{a_0}^2}{m_{a_0}^2 - m_\rho^2} \right)^2 \frac{\Lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\Lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} \sim 10^{-3}$$

This limit can be reached in the experiment in the near future.

Thanks for your attention

$$C_{W\rho'} = - \left( \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right)$$

$$C_{\rho'\pi\pi} = - \left( \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} g_{\rho_1} + \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \frac{l_2^f}{l_2} g_{\rho_2} \right)$$

$$\beta = e^{i\pi} C_{W\rho'} C_{\rho'\pi\pi} / g_\rho$$

$$C_{Wa'_0} = \frac{\sqrt{3}}{4g_\rho} g_{EW} |V_{ud}| (m_d - m_u) p^\mu W_\mu^- a_0^- \left( \frac{\cos(\phi + \phi_0)}{\sin(2\phi_0)} + \Gamma \frac{\cos(\phi - \phi_0)}{\sin(2\phi_0)} \right)$$

$$\beta_{a_0\eta(\eta')\pi} = e^{i\pi} C_{Wa'_0} \frac{\sqrt{6}}{4Z} \frac{A_{a'_0 \rightarrow \eta(\eta')\pi}}{m_u}$$

$$\Gamma = \frac{l_2^f}{\sqrt{l_2 l_2^{f^2}}} = 0.54$$