Introduction into GLAPD- and BFKL- evolutions in perturbative QCD

Victor T. Kim

St. Petersburg Nuclear Physics Institute NRC KI, Gatchina
St. Petersburg State Polytechnic University

Outline

- Introduction & Motivation
- QCD-asymptotics at high energies: GLAPD and BFKL
- GLAPD- and BFKL- evolutions
- Collinear factorization and kT-factorization
- Search for BFKL-effects at HERA, LEP, Tevatron and LHC
- Summary

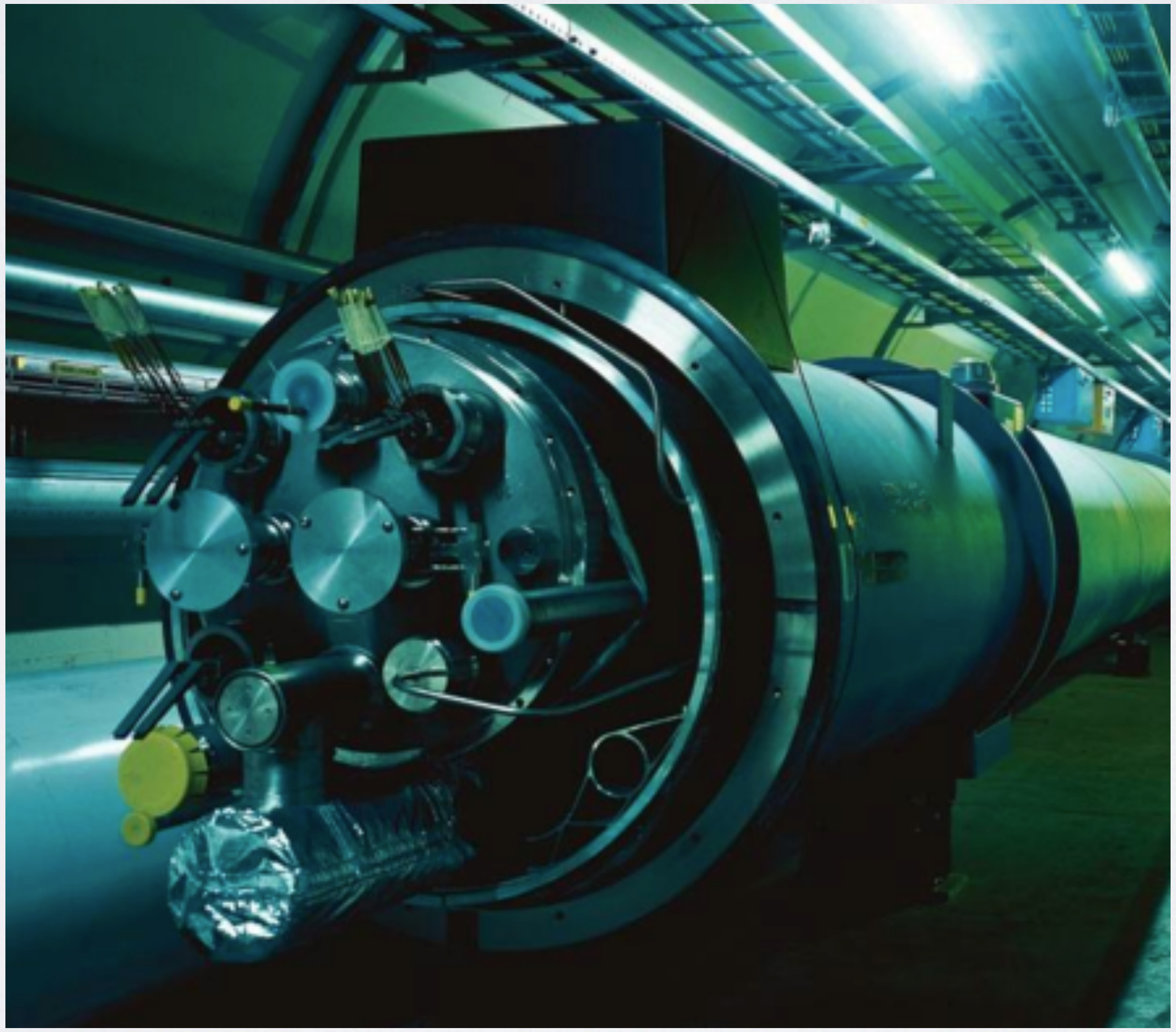
LHC physics: major goals

- Search for new physics beyond the SM

- Search for Higgs boson of the SM

- Precision measurements of SM parameters at new energy domain

- Search for new dynamics of SM at new energy domain



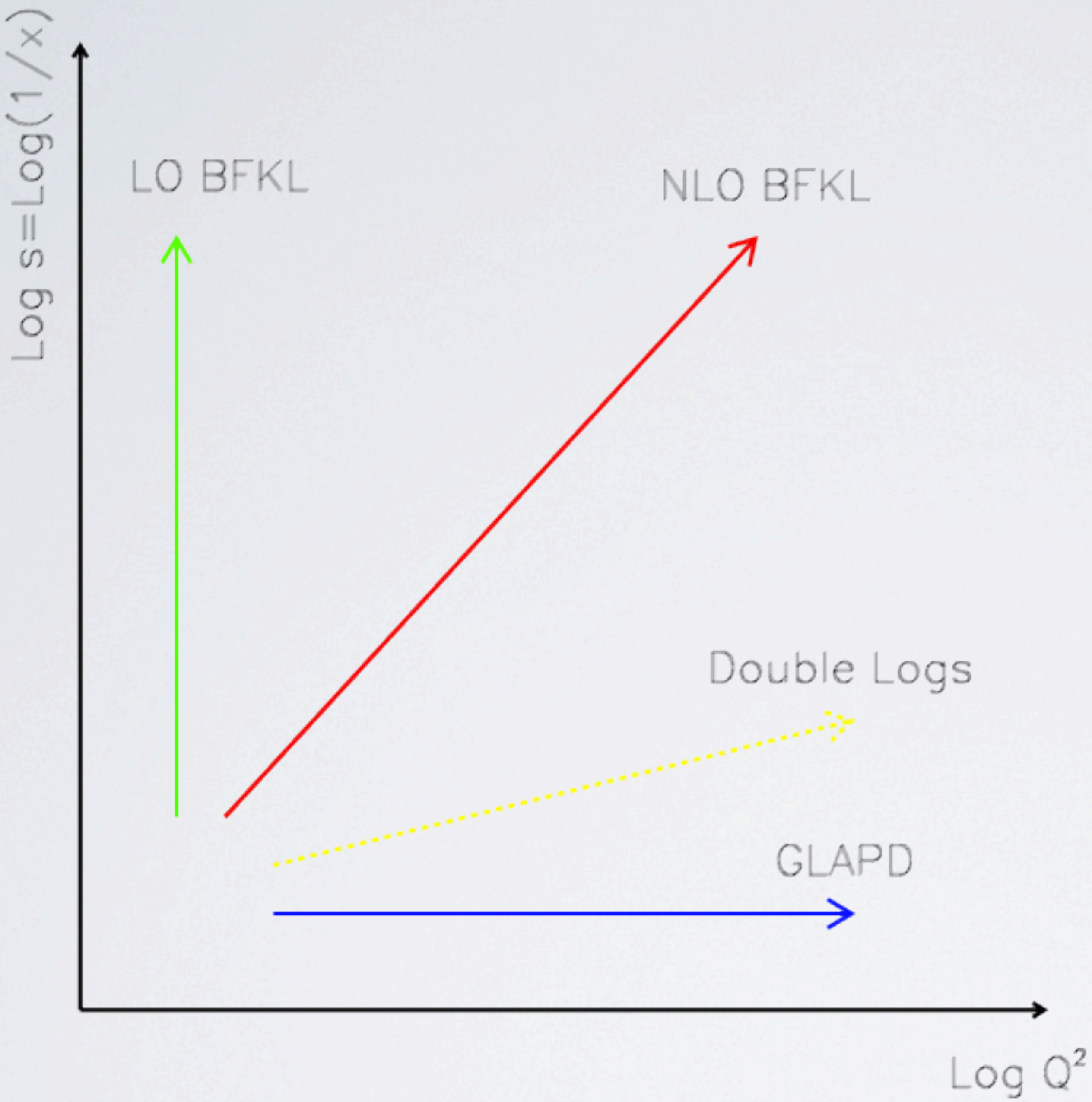
High-energy QCD asymptotics: GLAPD and BFKL

$s=(p_1+p_2)^2$
$t=(p_1-p_3)^2$ $\quad\quad$ $Q^2=-t$
Scattering in the Standard Model (QCD) at high energies:
Large logarithms: $a_s \log(s)$, $a_s \log(Q^2)$

Bjorken limit (large-angle scattering):
$s \sim Q^2 >> m^2$
$Q^2/s = x \sim 1$
Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD):
$(a_s \log(Q^2))^n$ resummation
Inclusive cross section $\sim 1/Q^4$

Regge-Gribov limit (small-angle scattering):
$s>>Q^2 >> m^2$
$Q^2/s = x \Rightarrow 0$
Balitsky-Fadin-Kuraev-Lipatov-(BFKL):
$(a_s \log(s))^n$ resummation
Total cross section $\sim s^{(a_P-1)}$
a_P – Pomeron intercept $\quad\quad$ data fit: $a_P = 1.1$

Introduction



Bjorken limit (GLAPD):
$s \sim Q^2 >> m^2$
$Q^2/s = x \sim 1$
Large-angle (large-x) scattering

Regge-Gribov limit (BFKL):
$s >> Q^2 >> m^2$
$Q^2/s = x \to 0$
Small-angle (small-x) scattering

Chronicles of GLAPD

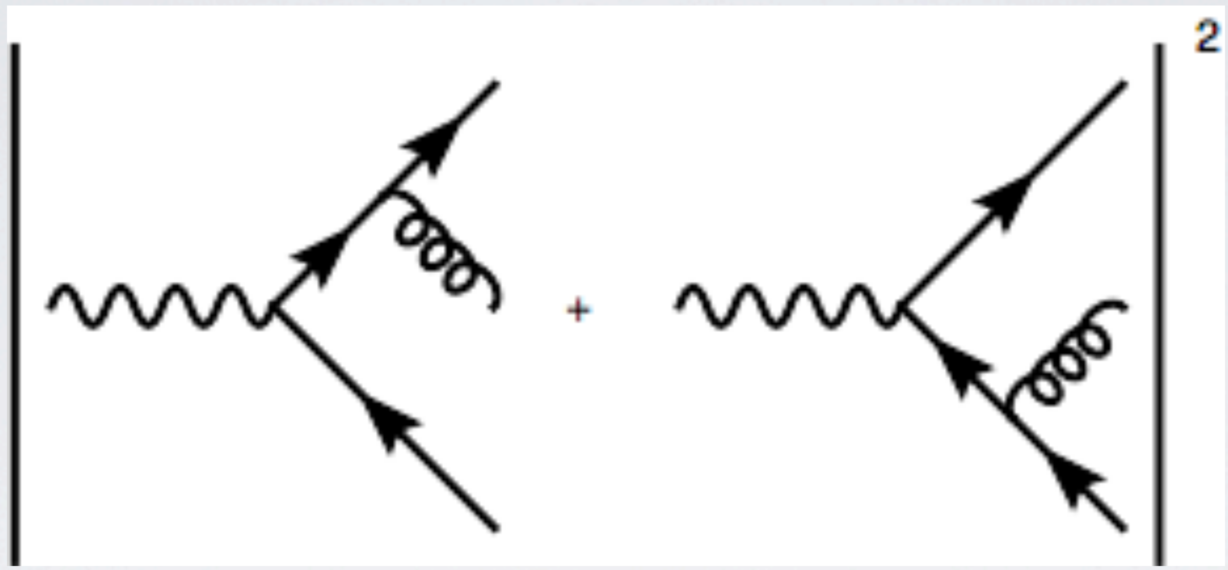
V.N. Gribov & L.N. Lipatov (1971-72) parton model in QED
L.N. Lipatov (1974) evolution equation for parton model
G. Altarelli & G. Parisi (1976-77) evolution for QCD
Yu.L. Dokshitzer (1977) evolution equation for QCD
Leading order approximation (LO) GLAPD

GLAPD equation <-> RG equation

Furmanski, Petronzio, Curci
Bardeen, Buras, Muta et el.
Indurain, Lopez, et al.
(1977 - 1980s)
NLO:
anomalous dimensions
coefficient functions

GLAPD at glance

Example: e+e- -> q qbar g in perturbative QCD



$$\frac{d\sigma}{d\cos\theta\, dz_g} \sim \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta} \frac{1+(1-z_g)^2}{z_g}$$

$E_g/E_{g,max}$

$e^+e^- \to$ q qbar

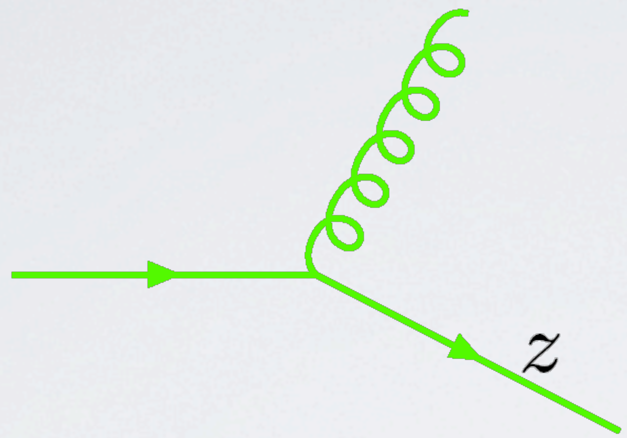
"quark charge squared"

GLAPD at glance: collinear limit

Uniiversal: $$d\sigma = \sigma_0 \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz\, P(z,\phi) d\phi$$

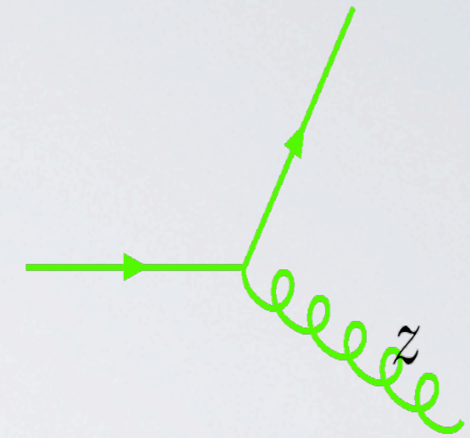
GLAPD kernels
Splitting functions:

$$P(z,\phi) =$$



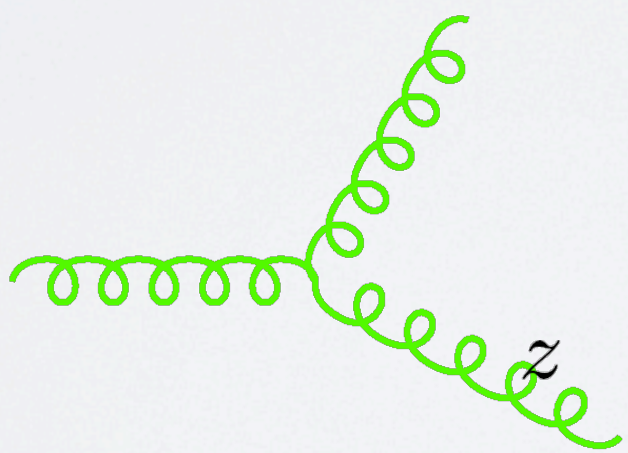
$q \to qg$

$$C_F \frac{1+z^2}{1-z}$$



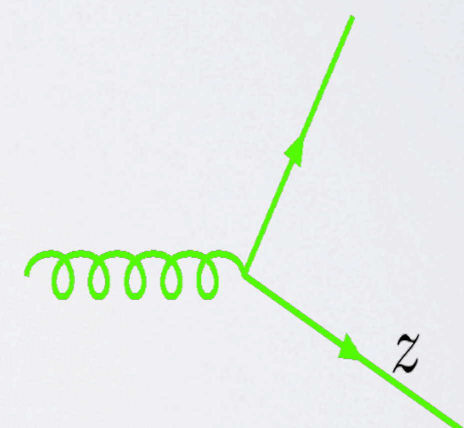
$q \to gq$

$$C_F \frac{1+(1-z)^2}{z}$$



$g \to gg$

$$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$$



$g \to q\bar{q}$

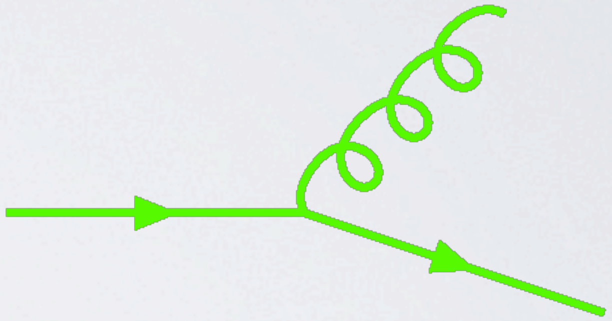
$$T_R \left(z^2 + (1-z)^2\right)$$

GLAPD at glance: resolvable partons

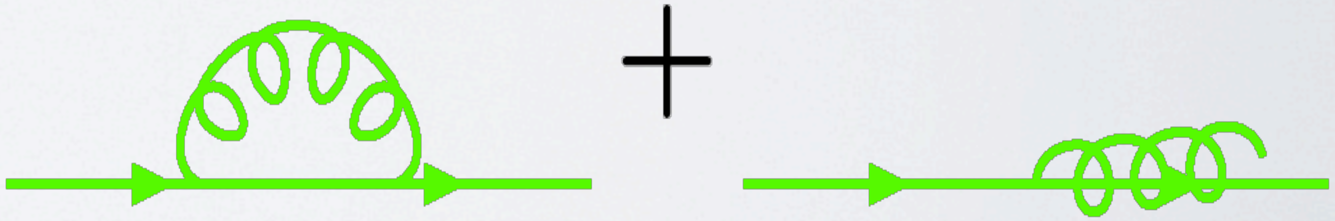
Resolvable partons at higher scales:
collinear parton pair <-> parton

$$k_\perp > Q_0.$$

Resolvable emission:



Virtual + unresolvable emmssion:



GLAPD at glance: evolution equation

Probability of emission between q^2 and $q^2 + dq^2$:

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Chronicles of BFKL

V.S. Fadin, E.A. Kuraev & L.N. Lipatov, Phys. Lett. (1975) intercept: a_P=1.5
L.N. Lipatov, ЯФ (1976) vector boson reggezation
E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETP (1976-77) BFKL equation SU(2)
I.I. Balitsky, L.N. Lipatov, Yad. Fiz. (1978) BFKL-Pomeron in QCD
Leading order approximation (LO) BFKL
Cross section: $\sigma_0 (S/S_0)^{(a_P-1)}$ $a_P = 1 + C\, a_S \approx 1.5$

L.V. Gribov, E.M. Levin & M. G. Ryskin, Phys. Rep. (1983) small-x physics:
Rise of parton distribution functions and their saturation (unitarization)

L.N. Lipatov (1989) graviton reggezation

L.N. Lipatov (1986) Pomeron at $t < 0$ and 2D-symmetries

L.N. Lipatov (93), L.D. Faddeev & G.P. Korchemsky (94)
QCD at high energies and large Nc: 2D-integrable system

L. McLerran, R. Venugopalan,
A. Kovner, A. Leonidov, J. Maria, H. Weigenert (1996-99)
Strong color charge: nonperturbative version of BFKL ($A \gg 1$: $x \ll 1$)
color glass condensate

Chronicles of BFKL: our time

V.S. Fadin & L.N. Lipatov (89-98)
C.Camici & M. Ciafaloni (96-98)
next-to-leading order approximation (NLO) BFKL
MS-renormalization scheme: large corrections

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99)
D. Colferai, M. Ciafaloni, & G. Salam (99)
NLO BFKL: resummation of running coupling a_S
Pomeron intercept: a_P=1.2 - 1.3
Cross section: $\sigma_0\ (S/S_0)^{(aP-1)}$ $a_P = 1 + C\ a_S$

L.N. Lipatov, A.V. Kotikov et al. (2001-06)
SUSY N=4 BFKL-Pomeron
Anamalous dimensions: test of AdS/CFT-conjecture

Interesting BFKL features

- BFKL equation: quantization of renormalization group

Euler-Lagrange equation ⇔ GLAPD (RG) equation

Schroedinger equation ⇔ BFKL equation

L. Lipatov (86)

Effective action with Reggeons: L. Lipatov (94-97)

Effective Feynman Rules:

L. Lipatov, E. Kuraev, I. Cherednikov, E.Antonov (2004)

Effective Feynman rules for x-sections(!):

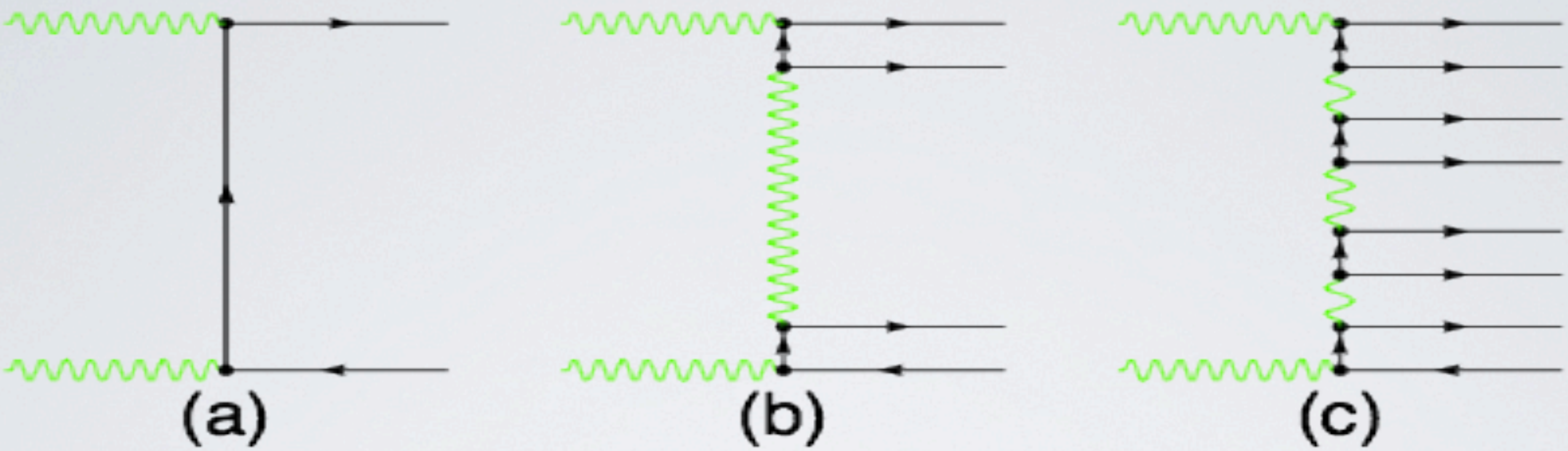
VK & G.Pivovarov (96)

- Duality: BFKL Pomeron ⇔ gravition

- All Standard Model particles:

BFKL QCD asymptotics for high-energy cross sections!

Asymptotics of QED cross sections



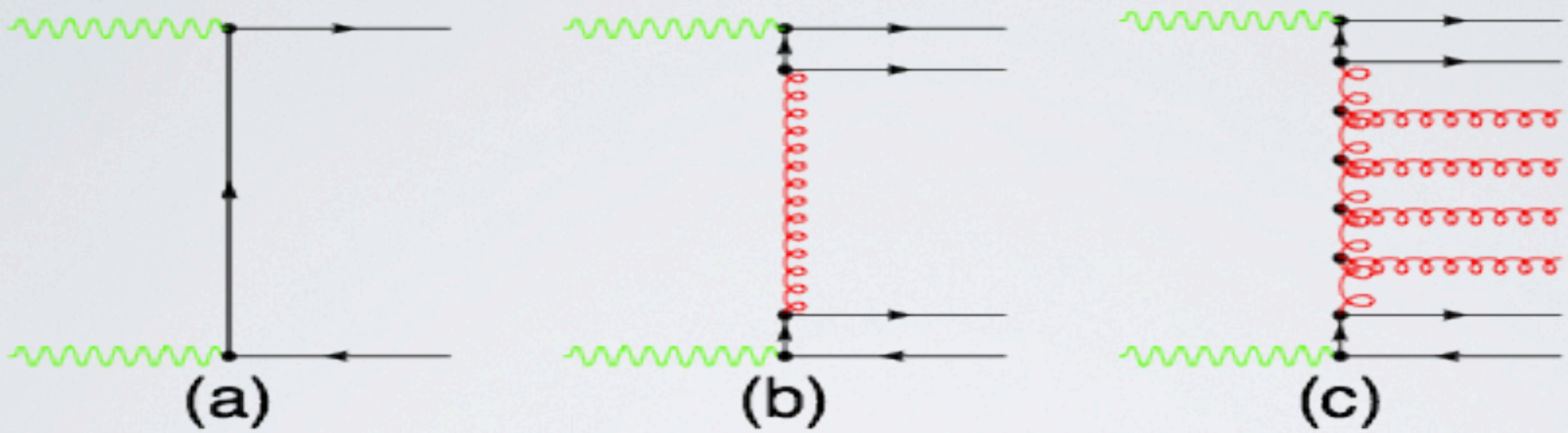
$\sigma \sim (a_{QED})^2 \log(s)/s$ $\sigma \sim (a_{QED})^4 \log^2(s)$

V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
H. Cheng & T.T. Wu (69-70)

Cross section (at s -> ∞): $\sim (a_{QED})^4 (S/S_0)^{(aP-1)}$

$a_P = 1 + C\,(a_{QED})^2 \approx 1.002$

Asymptotics of QCD cross sections: γγ



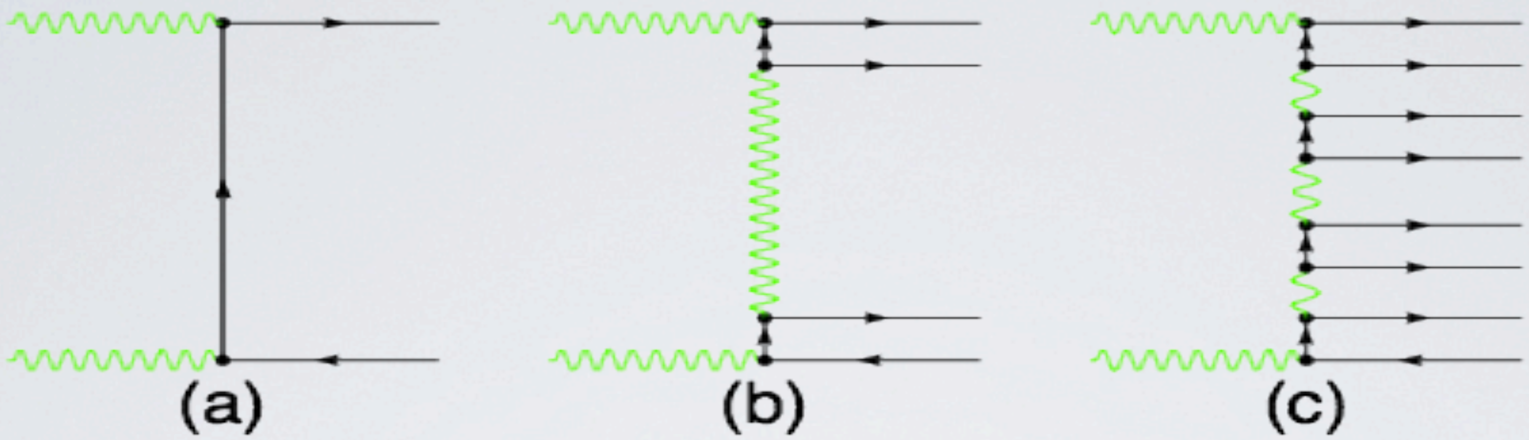
$\sigma \sim (a_{QED})^2 \log(s)/s$ $\sigma \sim (a_{QED})^2 (a_S)^2 \log^2(s)$

LO BFKL

Cross section (at s -> ∞): $\sim (a_{QED})^2 (a_S)^2 (S/S_0)^{(aP-1)}$

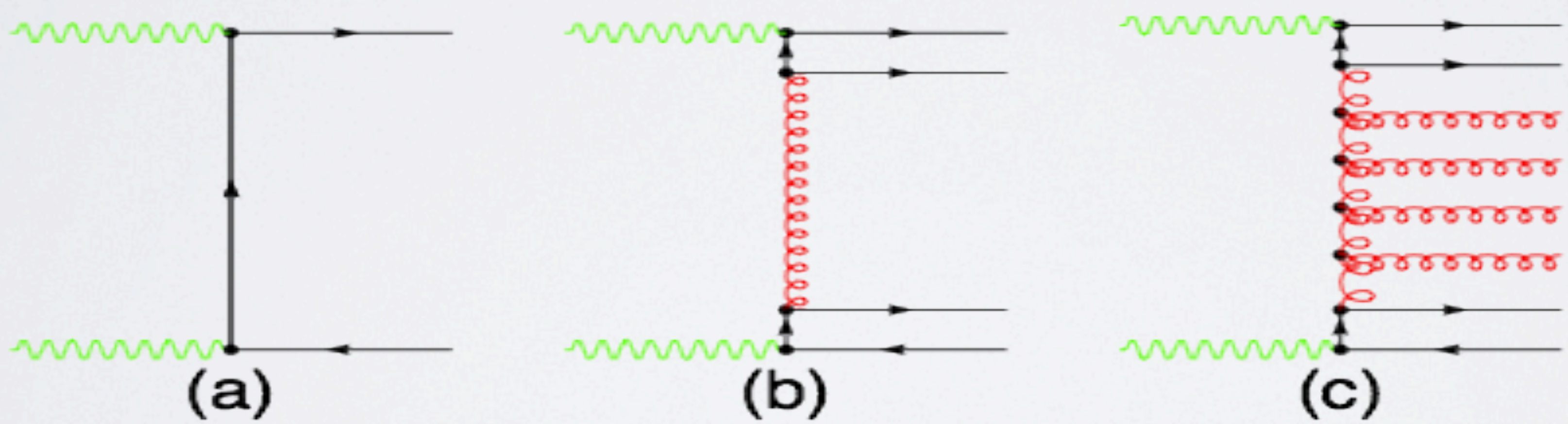
$a_P = 1 + C\,(a_S) \approx 1.5$ **LO BFKL S. Brodsky & F. Hautmann (96)**
$a_P = 1 + C\,(a_S) \approx 1.2$ **NLO BFKL S.Brodsky, V Fadin, VK, L. Lipatov, G. Pivovarov (2001-02)**

Asymptotics of QED cross sections



V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
Cheng & T.T. Wu (69-71)

Asymptotics of QCD cross sections



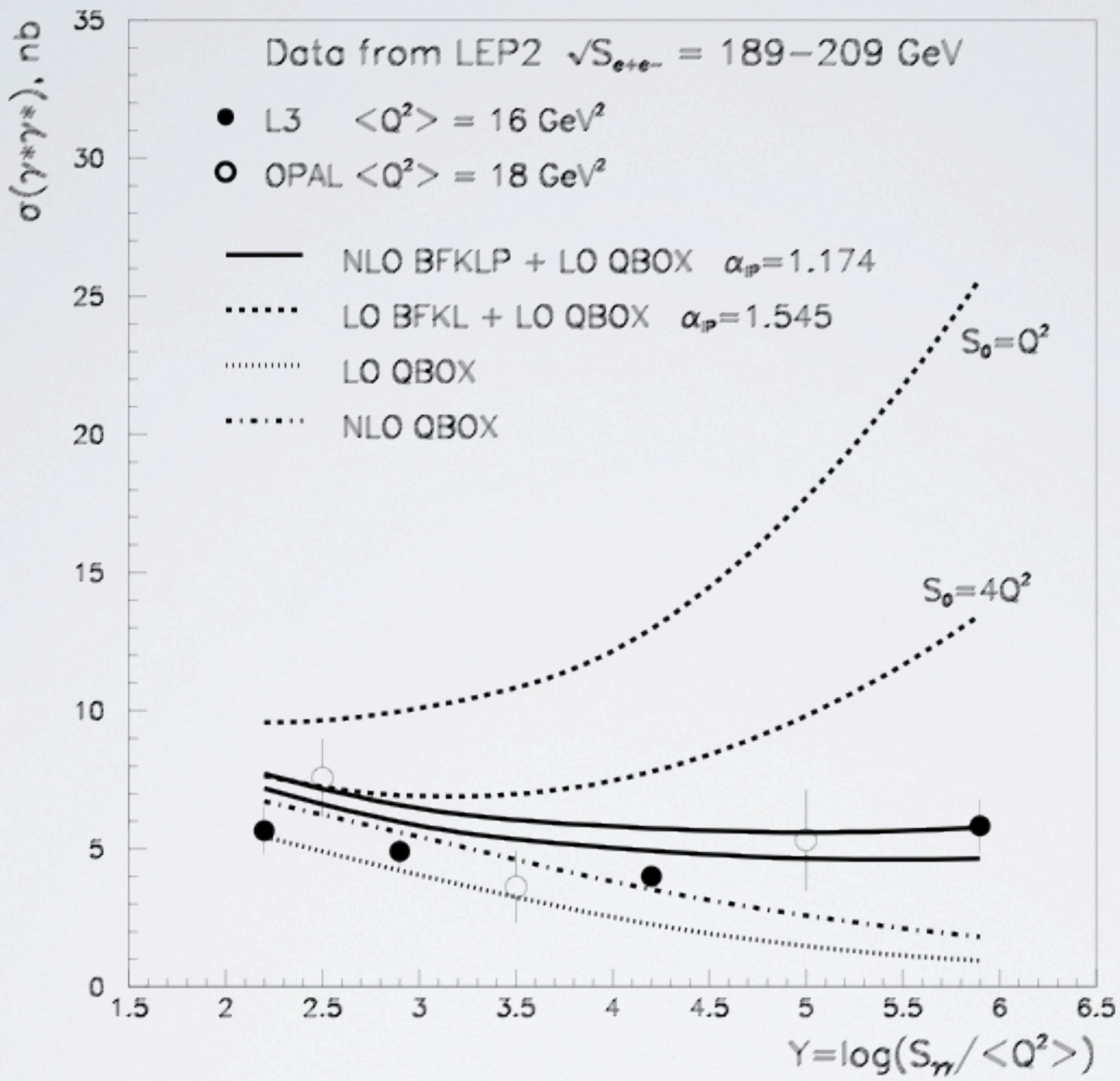
NLO BFKL

S.J. Brodsky, VK, L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2001-02)

full NLO BFKL:

I. Balitsky, J.Chirolli, J. Bartels et al.

Highly virtual photon scattering at LEP-2



S.J Brodsky, VK, L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2002) NLO BFKL

Search for manisfestions of BFKL

- Rise of PDFs at small x: $\sim 1/x^{(a_P-1)}$
ep HERA: $a_P = 1.2 - 1.3$

- Highly virtual photon cross sections
LEP2: $a_P = 1.2$

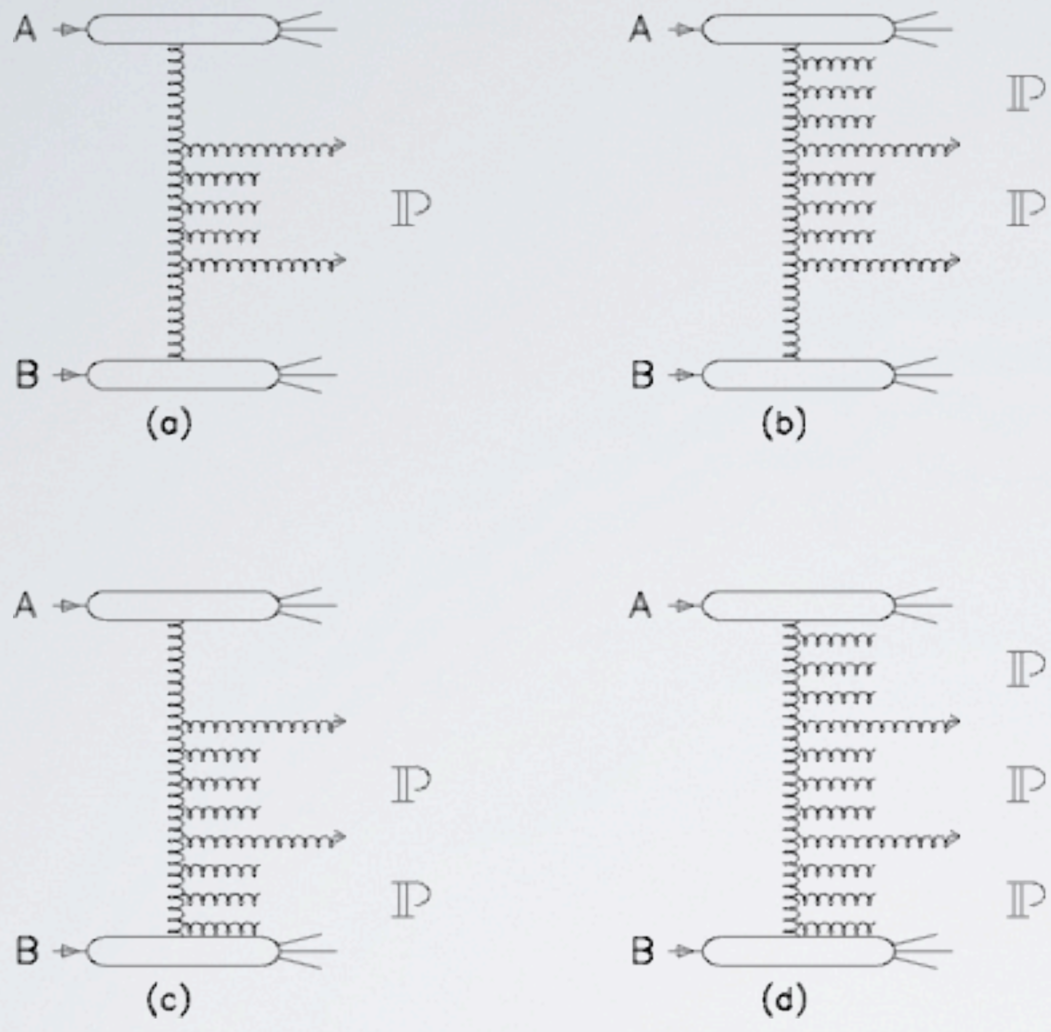
- Heavy quark cross sections
HERA, Tevatron, LHC (7 TeV) $a_P = 1.2 - 1.3$

- Dijet azimuthal angle decorrelations
Tevatron & LHC:

- Dijet "K-factor"

- Standard Model particle cross section
- BFKL asymptotics!

БФКЛ: dijet processes



Jet production

GLAPD: ordering on κT
y – no ordering

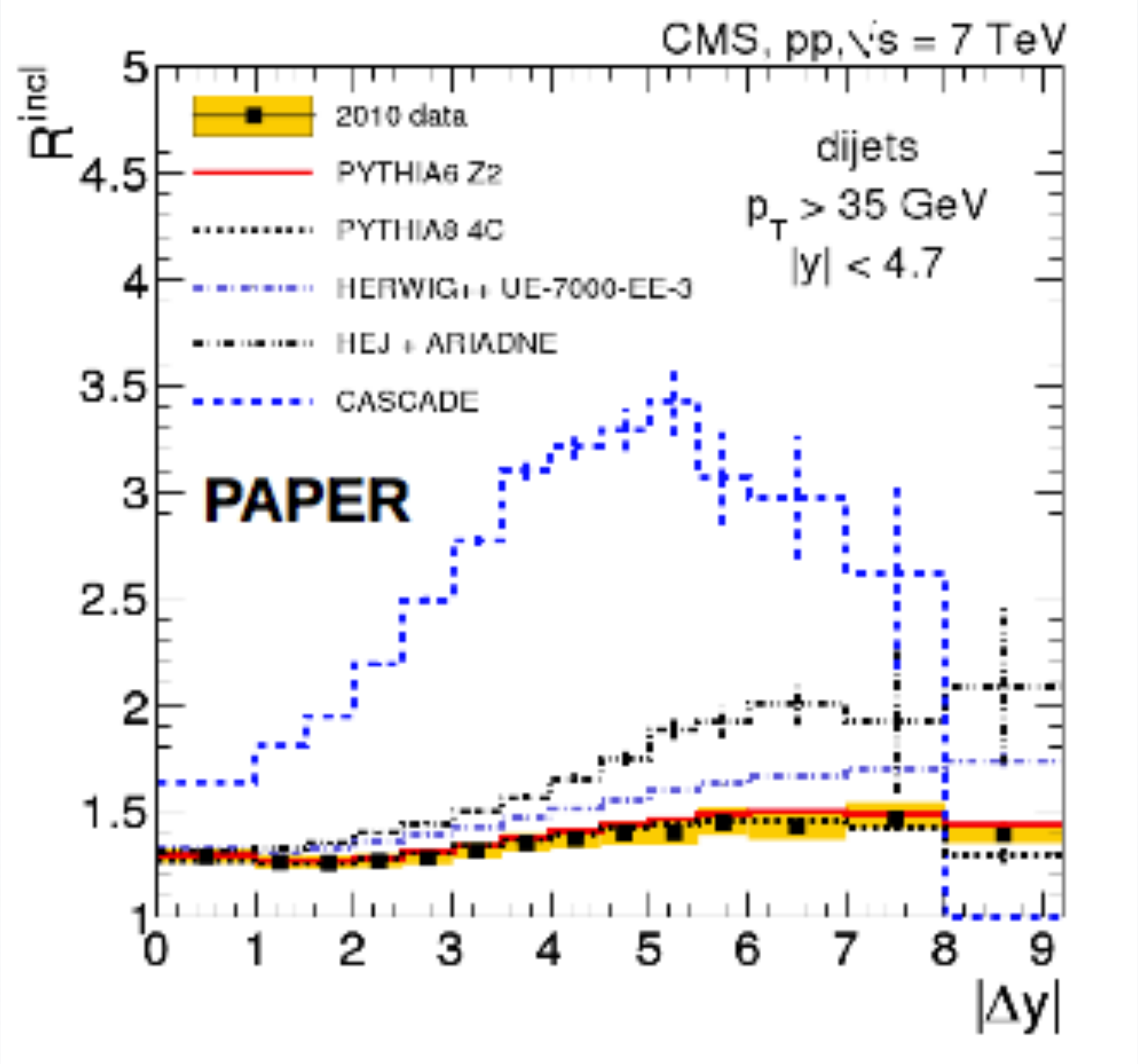
BFKL: ordering on y
κT – no ordering

A. Mueller & H. Navelet, Nucl. Phys. (87)
Most forward/backward (Mueller-Navelet) dijets: x-section ~ exp(y)

V.T. Kim & G.B. Pivovarov, Phys. Rev. (96)
Inclusive dijets

E.M.Levin, M.G.Ryskin, Yu.M.Shabelsky, A.G.Shuvaev (91)
J.C. Collins, R.K. Ellis (91), S. Catani et al (91)
kT-factorization

CMS @LHC 7 TeV



Dijet "K-factor" = inclusive dijets/ "exclusive" dijets

CMS Coll. (2012) "**Measurement of inclusive to exclusive dijet production ratios at large rapidity intervals at √s = 7 TeV**"

BFKL at the LHC

Search for asymptotic QCD effects at high-energies

First stage of LHC: first stage (parameter tuning):

- tuned LO GLAPD MC generators describe dijet "K-factor"

- Available BFKL generators require NLO-corrections

- **2012-13: additional observables:**
- **Dijet azimuthal decorrelations, K-factor with veto, etc.**

Second stage of LHC at larger energy (6.5 x 6.5 TeV): Observation of BFKL-effects (or stringent limit?)

Introduction into GLAPD- and BFKL-evolutions in perturbative QCD - II

GLAPD-evolution: selected topics

GLAPD: factorization of hard procees

Two milestones of perturbative QCD for hard processes:

- **Asymptotic freedom**
- **Factorization of hard processes**

Factorization theorem:
Inclusive cross section factorizes into
parton subprocess and parton distribution functions
Amati, Petronzio & Veneziano (77)
Efremov & Radyushkin (78-80)
Collins, Soper & Sterman (86)

Independence on separation boundary
between hard subrpocess and soft part
governs by RG (GLAPD) equation

Sudakov resummation

Probability of emission between q^2 **and** $q^2 + dq^2$ **:**

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\, P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability of NO emission between Q^2 **and** q^2 **:** $\Delta(Q^2, q^2)$

$$\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2, q^2) = \exp - \int_{q^2}^{Q^2} \frac{dk^2}{k^2}\bar{P}(k^2).$$

$$\Delta_q(Q^2) \sim \exp - C_F \frac{\alpha_s}{2\pi} \log^2 \frac{Q^2}{Q_0^2}$$

Sudakov resummation

Probability of no gluon emission by quark (Sudakov form factor):

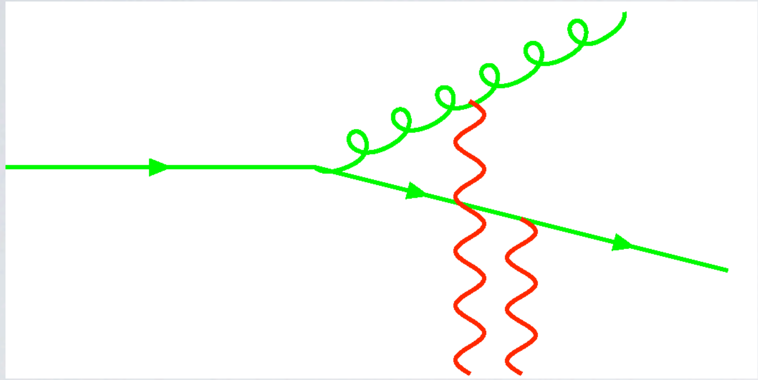
$$\Delta_q(Q^2) \sim \exp -C_F \frac{\alpha_s}{2\pi} \log^2 \frac{Q^2}{Q_0^2}$$

Unitarity (probability):

[resolvable emission] + [virtual + unresolvable emmssion] = 1

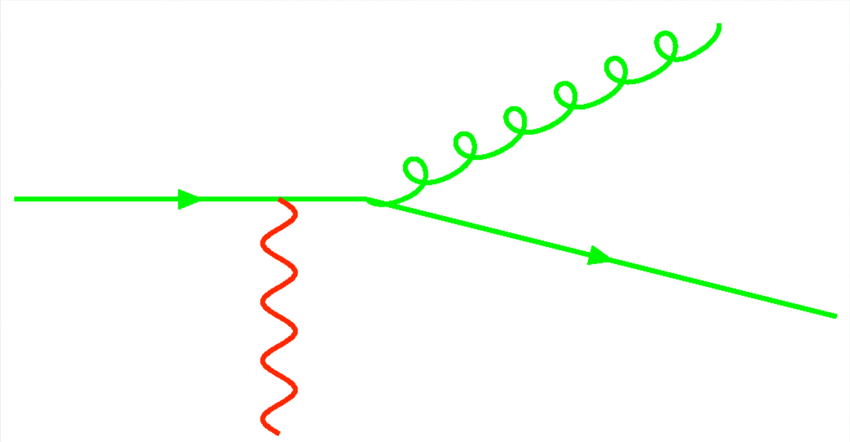
-> MC event generators

MC event generators: parton shower



Soft limit: universal in amplitude level
-> spoils independent evolution?

NO: angular ordering!
Outside hard angle ordered gluons: soft gluons sum coherently
One can see the color charge of whole jet only



GLAPD MC event generators: parton shower

Exact matrix elements vs
independent branching in MC parton shower:

where is quantum interference?

kT (or/and angle) ordering!

Introduction into GLAPD- and BFKL-evolutions in perturbative QCD - III

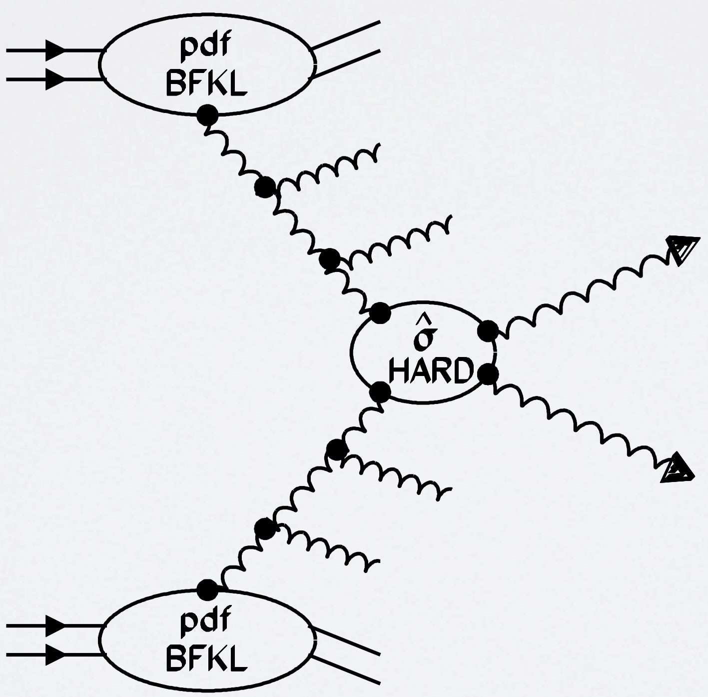
BFKL: selected topics

- kT-factorization
- NLO BFKL: Pomeron intercept

BFKL: kT-factorization?

J. Collins & R.K. Ellis (91)
M. Ciafaloni, S. Catani, F. Hautmann, G. Marchesini (91)
E.M. Levin, M.G. Ryskin, Yu.M. Shabelsky & A.G. Shuvaev (91)

Unintegrated on kT parton distribution function (uPDF)



(a)

NLO BFKL: Pomeron intercept

NLO BFKL in MSbar-scheme: negative eigen value
-> a huge problem: falling down cross sections

Solution:
NLO BFKL in physical renormalization schemes with resummed large running coupling terms

LO BFKL

LO BFKL:
Pomeron intercept is too large
multi-Regge kinematics
non-running coupling

BFKL:
effective field theory for LO (and partly NLO)

Effective action

L.N. Lipatov (1994-97)

Effective Feynman rules:

I. Cherdnikov E. Kuraev & E. Antonov, L. Lipatov (2004)

Regge-behaviour for MHV amplitudes:

L.N. Lipatov, J. Bartels, A. Prygarin (2010-2011)

LO BFKL: effective theory for x-sections

VK & G.B. Pivovarov (1996)

Conformal symmetries:
2 gluon -> n-gluon x-section for multi-Regge kinematics

LO BFKL: equation solution

V.S. Fadin, E.A. Kuraev & L.N. Lipatov (1975-77)

**LO BFKL equation solution:
2 gluon - -> 2 gluon in all orders of perturbative theory in multi-Regge kinematics**

$$f^{\mathrm{BFKL}}(k_{1\perp}, k_{2\perp}, y)$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \, \chi_{n,\nu}(k_{1\perp}) e^{y\omega(n,\nu)} \chi^{*}_{n,\nu}(k_{2\perp})$$

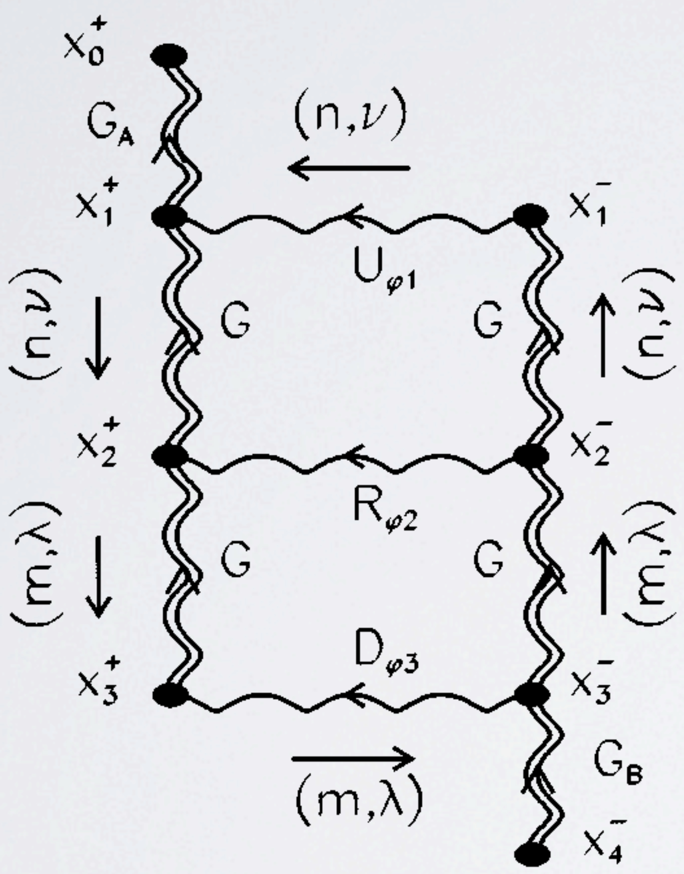
$$\chi_{n,\nu}(k_{\perp}) = (k_{\perp}^{2})^{-1/2+i\nu} e^{in\varphi} / 2\pi$$ **LO BFKL eigen functions**

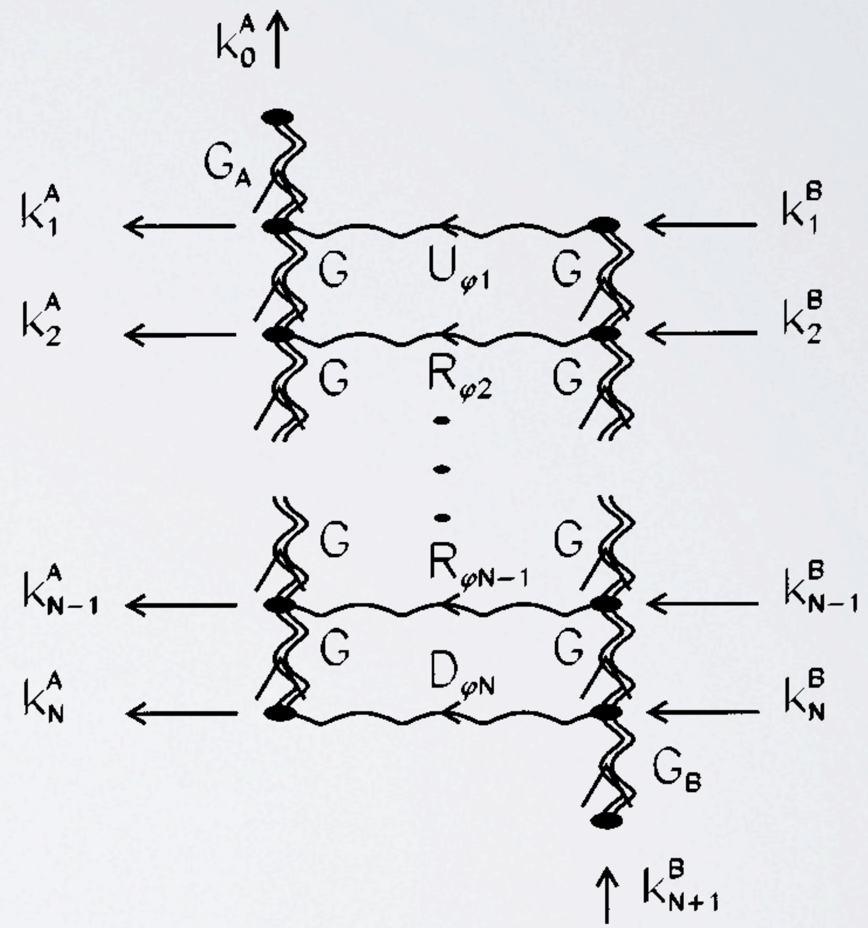
$$\omega(n,\nu) = \frac{2\alpha_S C_A}{\pi}\left[\psi(1) - \mathrm{Re}\,\psi\left(\frac{|n|+1}{2} + i\nu\right)\right]$$ **LO BFKL eigen value**

LO BFKL: effective theory for x-sections

VK & G.B. Pivovarov (1996)

2 gluon -> n-gluon x-section for multi-Regge kinematics





2 gluon -> 3-gluon

2 gluon -> n-gluon

NLO BFKL in MSbar-scheme

Eigen value

V.S. Fadin & L.Lipatov (98) C.Camici & M.Ciafaloni (98)

$$\omega_{\overline{MS}}(Q_1^2,\nu) = \int d^2Q_2 \; K_{\overline{MS}}(\vec{Q}_1,\vec{Q}_2)\left(\frac{Q_2^2}{Q_1^2}\right)^{-\frac{1}{2}+i\nu} =$$

$$= N_C\chi_L(\nu)\frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi}\left[1 + r_{\overline{MS}}(\nu)\frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi}\right]$$

where

$(Q_2^2/Q_1^2)^{-1/2+i\nu}$ **LO eigen functions**

$$\chi_L(\nu) = 2\psi(1) - \psi(1/2+i\nu) - \psi(1/2-i\nu)$$

ν - conformal weight parameter

Q_1, Q_2 – virtualities of Reggeized gluons

NLO BFKL: conformal and beta-dependent part

Eigen value in MSbar-scheme

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov [BFKLP] (1998)

$$r_{\overline{MS}}(\nu) = r^{\beta}_{\overline{MS}}(\nu) + r^{conf}_{\overline{MS}}(\nu)$$

where

$$r^{\beta}_{\overline{MS}}(\nu) = -\frac{\beta_0}{4}\left[\frac{1}{2}\chi_L(\nu) - \frac{5}{3}\right]$$

$$r^{conf}_{\overline{MS}}(\nu) = -\frac{N_C}{4\chi_L(\nu)}\left[\frac{\pi^2 \sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3 + \left(1 + \frac{N_F}{N_C^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) - \chi_L''(\nu) + \frac{\pi^2-4}{3}\chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu)\right] \quad (4)$$

$$\varphi(\nu) = 2\int_0^1 dx \frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right], \; \mathrm{Li}_2(x) = -\int_0^x dt \frac{\ln(1-t)}{t}$$

NLO BFKL with resummed running coupling

Eigen value in MSbar-scheme

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov
[BFKLP] (1998)

$$\alpha_{I\!P}^{\overline{MS}} - 1 = \omega_{\overline{MS}}(Q^2, 0) = 12 \, \ln 2 \, \frac{\alpha_{\overline{MS}}(Q^2)}{\pi} \left[1 + r_{\overline{MS}}(0) \frac{\alpha_{\overline{MS}}(Q^2)}{\pi} \right]$$

$$r_{\overline{MS}} \, (0) \; \simeq \; -20.12 - 0.1020 N_F + 0.06692 \beta_0 \; ,$$

$$r_{\overline{MS}}(0)_{|N_F=4} \; \simeq \; -19.99 \; .$$

Non-physical negative value of NLO BFKL eigen value in MSbar-scheme

What about NLO BFKL in physical renormalization schemes?

NLO BFKL with resummed running coupling

Eigen value in MSbar-scheme

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov [BFKLP] (1998)

physical schemes: MOM (ggg-vertex), Υ-scheme

transition to an another scheme
-> finite renormalization

$$\alpha_S \;\rightarrow\; \alpha_S\left[1+T\frac{\alpha_S}{\pi}\right]$$

$$\omega_{MOM}(Q^2,\nu) \;=\; N_C\chi_L(\nu)\frac{\alpha_{MOM}(Q^2)}{\pi}\left[1+r_{MOM}(\nu)\;\frac{\alpha_{MOM}(Q^2)}{\pi}\right]$$

$$r_{MOM}(\nu) \;=\; r_{\overline{MS}}(\nu)+T_{MOM}.$$

NLO BFKL in MOM-scheme

Eigen value in MOM-scheme

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov
[BFKLP] (1998)

Scheme		$T = T^{conf} + T^{\beta}$	$r(0) = r^{conf}(0) + r^{\beta}(0)$	$r(0)$ $(N_F = 4)$
M	$\xi = 0$	$7.471 - 1.281\beta_0$	$-12.64 - 0.1020 N_F - 1.214\beta_0$	-22.76
O	$\xi = 1$	$8.247 - 1.281\beta_0$	$-11.87 - 0.1020 N_F - 1.214\beta_0$	-21.99
M	$\xi = 3$	$8.790 - 1.281\beta_0$	$-11.33 - 0.1020 N_F - 1.214\beta_0$	-21.44
V		$2 - 0.4167\beta_0$	$-18.12 - 0.1020 N_F - 0.3497\beta_0$	-21.44
Υ		$6.47 - 0.923\beta_0$	$-13.6 - 0.102 N_F - 0.856\beta_0$	-21.7

No scheme dependence:
values of r(0) is similar to MSbar-scheme

Conformal part of r(0) is small for
non-Abelian physical renormalization schemes

NLO BFKL with resummed coupling

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov [BFKLP] (1998)

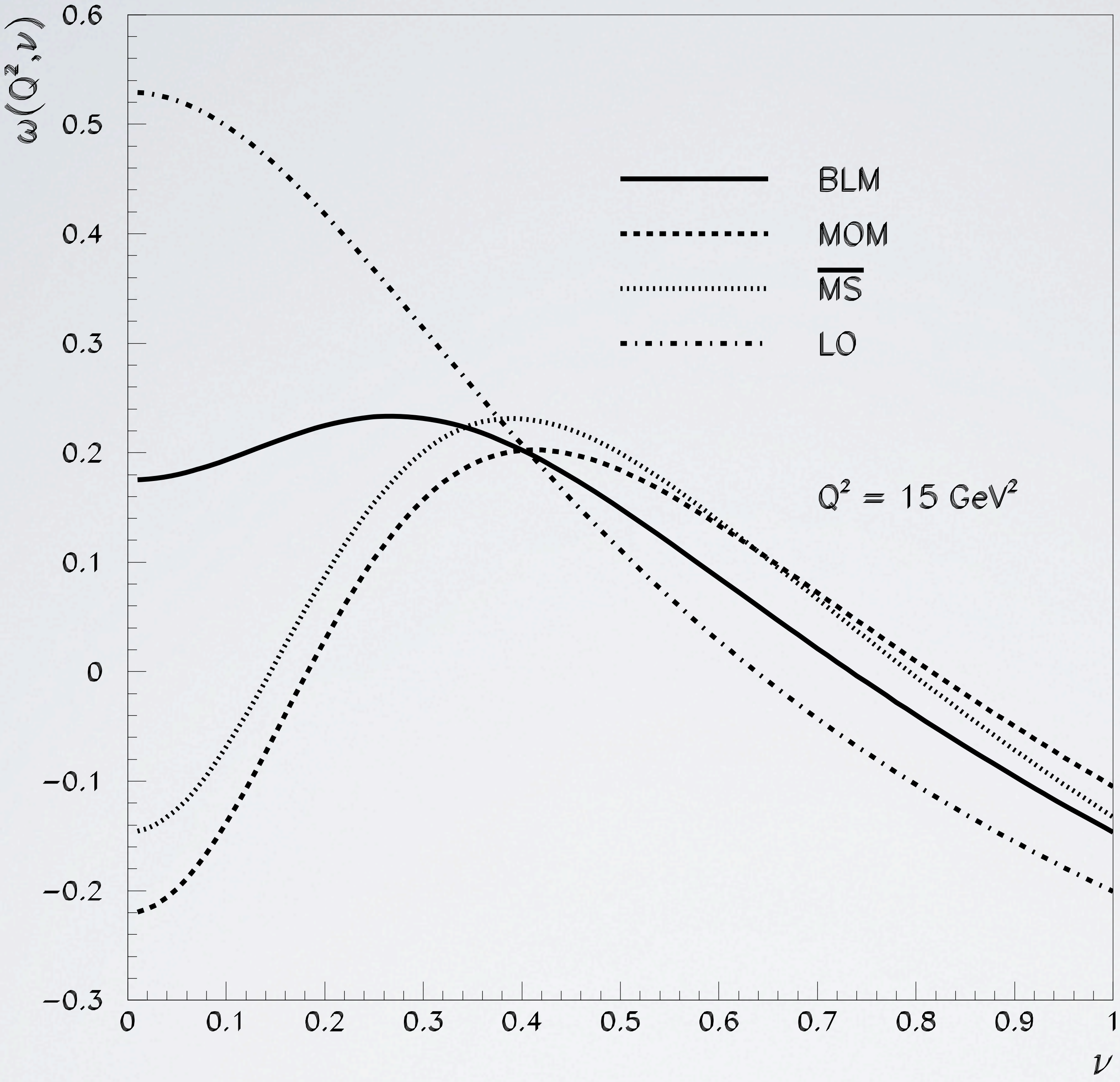
$$\omega_{BLM}^{MOM}(Q^2,\nu) = N_C \chi_L(\nu) \frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi} \left[1 + r_{BLM}^{MOM}(\nu) \frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi}\right] \quad (1$$

$$r_{BLM}^{MOM}(\nu) = r_{MOM}^{conf}(\nu)\,. \quad (1$$

Beta-dependent part of r(ν) defines BLM scale:

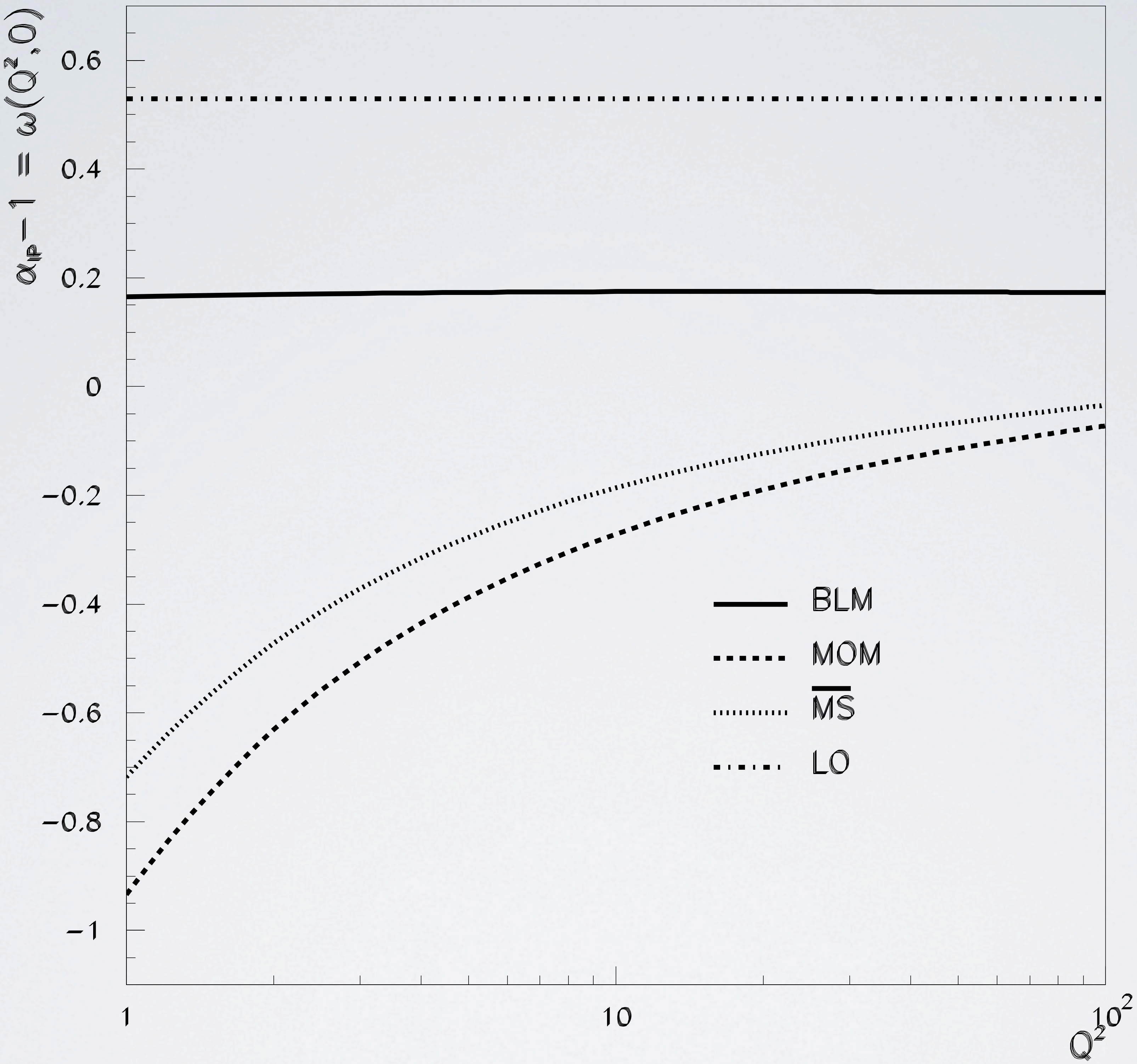
$$Q_{BLM}^{MOM\,2}(\nu) = Q^2 \exp\left[-\frac{4 r_{MOM}^{\beta}(\nu)}{\beta_0}\right] = Q^2 \exp\left[\frac{1}{2}\chi_L(\nu) - \frac{5}{3} + 2\left(1 + \frac{2}{3} I\right)\right]$$

NLO BFKL Eigen Value



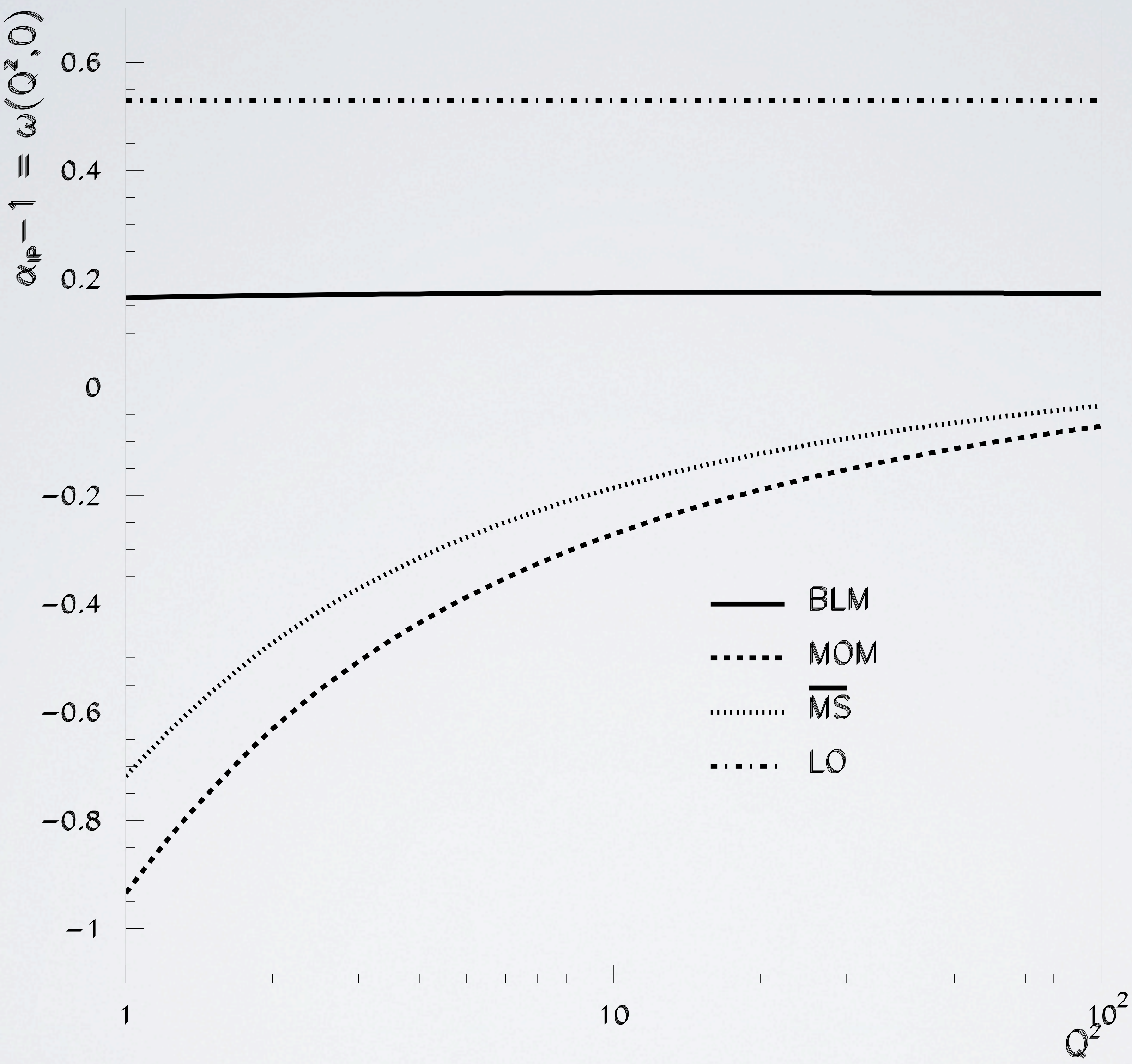
Eigen value vs conformal weight parameter ν

NLO BFKL Eigen Value



Eigen value vs Q^2

NLO BFKL Pomeron intercept



NLO BFKL Pomeron intercept vs Q^2

NLO BFKL Pomeron intercept with resummed running coupling terms

Scheme		$r_{BLM}(0)$ $(N_F = 4)$	$\alpha_{IP}^{BLM} - 1 = \omega_{BLM}(Q^2, 0)$		
			$Q^2 = 1$ GeV^2	$Q^2 = 15$ GeV^2	$Q^2 = 100$ GeV^2
M	$\xi = 0$	-13.05	0.134	0.155	0.157
O	$\xi = 1$	-12.28	0.152	0.167	0.166
M	$\xi = 3$	-11.74	0.165	0.175	0.173
Υ		-14.01	0.133	0.146	0.146

NLO BFKL Pomeron intercept
in non-Abelian schemes with the BLM scale setting

NLO BFKL:
scattering of highly virtual photons

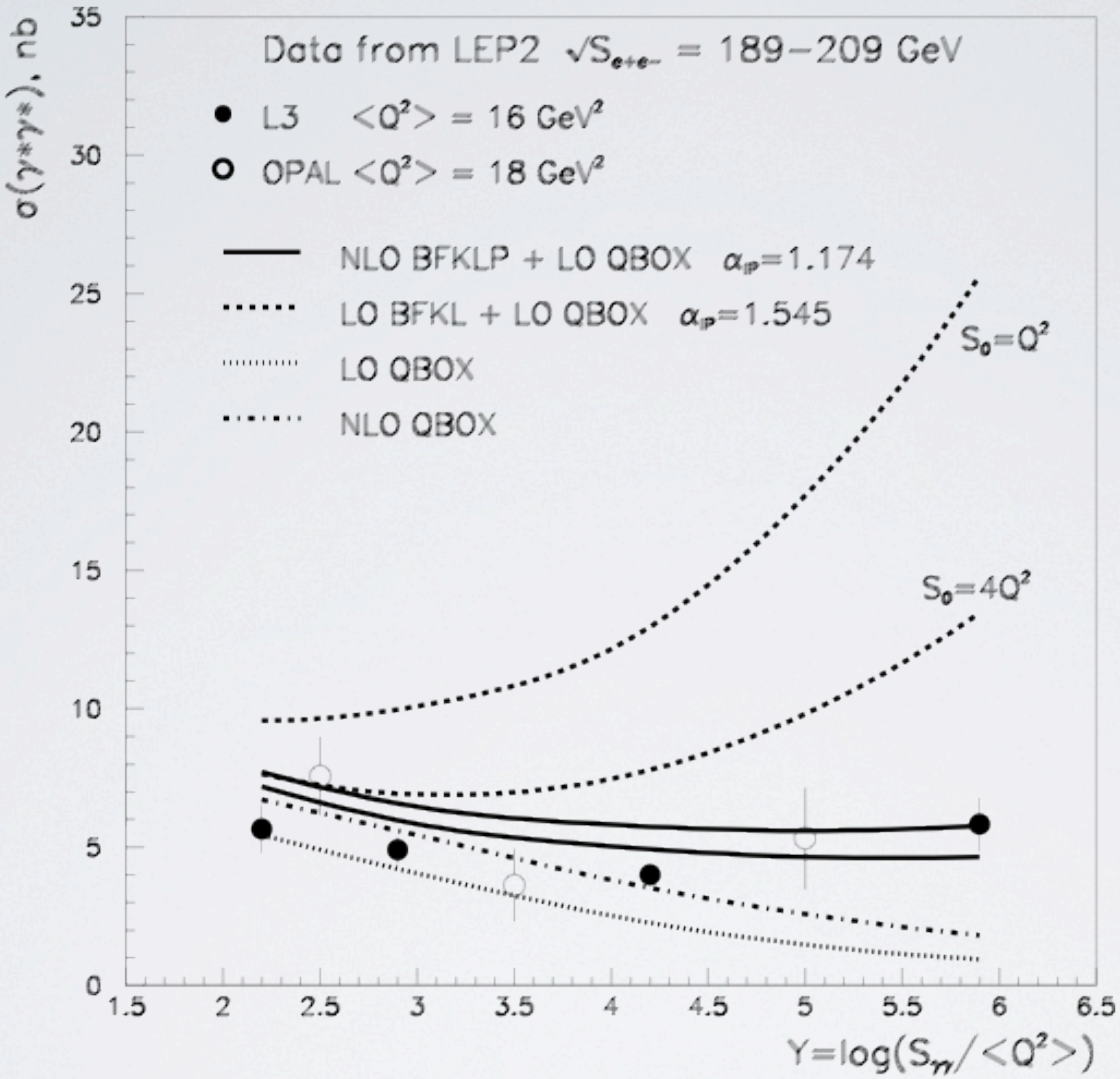
$$\sigma(s, Q_A^2, Q_B^2) = \sum_{i,k=T,L} \frac{1}{\pi Q_A Q_B} \int_0^\infty \frac{d\nu}{2\pi} \cos\left(\nu \ln\left(\frac{Q_A^2}{Q_B^2}\right)\right) F_i(\nu) F_k(-\nu) \left(\frac{s}{s_0}\right)^{\omega(Q^2,\nu)}$$

$$F_T(\nu) = \alpha_{QED}\,\alpha_S \left(\sum_q e_q^2\right) \frac{\pi}{2} \frac{\left[\frac{3}{2} - i\nu\right]\left[\frac{3}{2} + i\nu\right]\Gamma\left(\frac{1}{2} - i\nu\right)^2 \Gamma\left(\frac{1}{2} + i\nu\right)^2}{\Gamma(2 - i\nu)\Gamma(2 + i\nu)}$$

$$F_L(\nu) = \alpha_{QED}\,\alpha_S \left(\sum_q e_q^2\right) \pi \frac{\Gamma\left(\frac{3}{2} - i\nu\right)\Gamma\left(\frac{3}{2} + i\nu\right)\Gamma\left(\frac{1}{2} - i\nu\right)\Gamma\left(\frac{1}{2} + i\nu\right)}{\Gamma(2 - i\nu)\Gamma(2 + i\nu)}$$

Photon LO impact factors
NLO impact factors I. Balitsky & J. Chirilli (2010)

Highly virtual photon scattering at LEP-2



S.J Brodsky, V.S. Fadin, VK, L.N. Lipatov & G.B. Pivovarov (2002)

Full NLO BFKL will be soon

Summary: GLAPD- and BFKL- evolutions

- GLAPD-evolution is a main ingredient of modern high-energy physics phenomenology for precision measurements

- BFKL is an important theoretical tool for high-energy limit

- NLO BFKL phenomenology is developing

- NLO BFKL MC generators

- BFKL searches at the LHC