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Conformal symmetry and scheme (in)dependence of high order corrections to the generalized Crewther relation in QED

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Work in progress

Plan of Presentation

Reminding of the concept of Conformal Symmetry and applications for the 3-point AVV function

Generalized Crewther Relations (GCR) as consequences of CS and of its violation in massless PT in $\overline{MS}$-scheme-derivation

Q1: Why $\overline{MS}$-scheme ?

A1: It respects gauge invariance

Q2: Does GKRs "factorized" form , discovered in the $\overline{MS}$-scheme at 3-rd order of PT in Broadhurst-Kataev, 1993 and proved by Crewther, 1997 and Braun, Korchemsky and Mueller, 2003 in all orders of PT in the same scheme, also holds in other renormalization schemes, which respect gauge-invariance?

Consequences of CS in the expressions for the Green functions under study and scheme **(in)dependence** of the concrete contributions to GKRs in QED

$SU(N_c)$ $\overline{MS}$-scheme analytical 4-th order results for Adler D-function, Bjorken and Gross-Llewellyn Smith sum rules Baikov, Chetykin, Kuhn, 2010 and BChK+Rittinger, 2012, are studied in QED ($U(1)$) in $\overline{MS}$, MOM and OS schemes.

For D-function analogy with results presented by J. Hehn, CALC-2012, Part II, July 25, 2012 are observed

$$Bjp(Q^2) = \int_0^1 \left(g_1^{lp}(x,Q^2) - g_1^{ln}(x,Q^2)\right)dx = \frac{1}{6} g_a C_{Bjp}^{NS}(a_s)$$

Where $a_s(Q^2) = \alpha_s(Q^2)/\pi$, while in QED $a = \alpha(Q^2)/\pi$ GLS-sum rule series contains SI (lbl-type) and Adler PT series $D_A = Q^2 \int_0^\infty \frac{R(s)ds}{(s+Q^2)^2} = 3\sum_F \left(Q_F^2\right) C_D^V(a_s)$ as well:

$$\begin{aligned} GLS(Q^2) &= \frac{1}{2}\int_0^1 F_3^{\nu p+\overline{\nu}p}(x,Q^2)dx = 3C_{GLS}(a_s) = 3\left(C_{Bjp}^{NS}(a_s) + C_{GLS}^{SI}(a_s)\right) \\ C_{GLS}(a) &= 1 + c_0 a + \sum_{l=1}^{3} c_0 c_l a^{l+1} + c_0^{SI} a^3 + c_0^{SI} {}_1^{SI} a^4 + O(a^5) \\ C_D^V(a) &= 1 + d_0 a + \sum_{l=1}^{3} d_0 d_l a^{l+1} + d_0^{SI} a^3 + d_0^{SI} d_1^{SI} a^4 + O(a^5) \end{aligned}$$

What are the consequences of the CS and of its breaking on the relations of c_i and d_i?

Conformal Invariance is the symmetry under the following transformations of coordinates

1. Translations $x'^{\mu} = x^{\mu} + \alpha^{\mu}$ with 4 parameters α^{μ},
2. Scale (or dilaton) transformation $x'^{\mu} = \rho x^{\mu}$ with 1 parameter $\rho > 0$,
3. Special conformal transformations $x'^{\mu} = \frac{x^{\mu} + \beta^{\mu} x^2}{1 + 2\beta x + \beta^2 x^2}$ with 4 parameters β^{μ} and
4. Homogeneous Lorentz transformations $x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$, which also contain 4 parameters.
5. Consequences are widely studied at present, though in renormalized QFT models the CI is violated- appearance of β-function and the effects of running of the coupling constants- QCD, QED

For the quantities considered the CS and the effects of its violation are manifesting as expressions with **factorized** RG $\beta(a)$ in the $\overline{MS}$ for sure Broadhurst, Kataev. 1993 a^3, all orders indictions Gabadadze, Kataev, 1995 Crewther 1997 all orders proof supported by BChK+R 2010, 2012 a^4. Below 1-consequence of Conformal Symmetry Crewther, 1972))

$$C_D^{NS} C_{Bjp} = 1 + \left(\frac{\beta(a)}{a}\right)\left(K_1 a + K + 2a^2 + K_3 a^3 + O(a^4)\right) \quad (1)$$

$$C_D^{V} C_{GLS} = 1 + \left(\frac{\beta(a)}{a}\right)\left(K_1 a + K + 2a^2 + \tilde{K}_3 a^3 + O(a^4)\right) \quad (2)$$

where $\tilde{K}_3 = K_3 + K_3^{SI}$ (BChK+Rittinger 2012). This is valid in $\overline{MS}$-scheme. The CI parts, which corresponds to **unity** are derivable from the following 3-point function

$$T^{abc}_{\mu\alpha\beta}(p,q) = i\int <0|TA^a_\mu(y)V^b_\alpha(x)V^c_\beta(0)|0> e^{ipx+iqy}dxdy = d^{abc}\Delta^{(1-loop)}_{\mu\alpha\beta}(p,q) \quad (3)$$

where $A^a_\mu(y) = \overline{\psi}\gamma_g(\lambda^a/2)\psi$, $V^b_\beta = \overline{\psi}(\lambda^b/2)\psi$. CS presumes that this 3-point function is proportional to 1-loop result $\Delta^{(1-loop)}_{\mu\alpha\beta}(p,q)$ (Schrier 1971). This was confirmed at 2-loops by explicit calculation Jegerlehner and Tarasov (2006). In QED we have the same result.

In practice this means, that CS gives the following relations

$$\begin{aligned} c_0 + d_0 &= 0 \quad c_0 c_1 + d_0 d_1 \ \textbf{scheme independent} \\ c_0^{SI} + d_0^{SI} &= 0 \quad c_0^{SI} c_1^{SI} + d_0^{SI} d_1^{SI} \ \textbf{scheme independent} \end{aligned} \quad (4)$$

Does factorization of factor $(\beta(a)/a)$- related to violation of CS in PT, holds in other schemes- QED case study (Garkusha, Kataev 2012)

The order $O(a^4)$ results for $C_{Bjp}(a)$, $C_{GLS}(a)$ and $C_D^{NS}(a)$ and $C_D^V(a)$ were transformed from $\overline{MS}$ to MOM and OS- schemes.
$\overline{a}_{MOM} = \overline{\alpha}_{MOM}/\pi$ coincides with INVARIANT CHARGE and $\beta_{MOM}(a_{MOM}) \equiv \Psi(a)$ known as Gell-Mann-Low function; in OS $\overline{a}_{OS} = \overline{\alpha}_{OS}/\pi$ while $\overline{\alpha}_{OS}$- running QED coupling in the OS-scheme; $\overline{\alpha}_{OS}(0) = 1/(137, \ldots)$ Jegerlehner, CALC-2012

$$\beta(a) = \mu^2 \frac{\partial a}{\partial \mu^2} = \sum_{i \geq 0} \beta_i a^{i+2} \tag{5}$$

For study of scheme-dependence of generalized Crewther relations the 4-loop $\overline{MS}$ and MOM results Gorishny, Kataev, Larin and Surguladze, 1991 and OS result by Broadhurst, Kataev, Tarasov, 1993 is needed The comments on existing 5-loop analytical results for QED β-function is beyond the scope of this talk

Factorization is true in QED is MS-like MOM and OS-schemes- reason : all of them in QED respect gauge invariance

$$C_D^{NS} C_{Bjp} = 1 + \left(\frac{\beta(a)}{a}\right)\left(K_1 a + K + 2a^2 + K_3 a^3 + O(a^4)\right) \quad (6)$$

$$C_D^{V} C_{GLS} = 1 + \left(\frac{\beta(a)}{a}\right)\left(K_1 a + K + 2a^2 + \tilde{K}_3 a^3 + O(a^4)\right) \quad (7)$$

$$K_1^{\overline{MS}} = K_1^{MOM} = K_1^{OS} \quad (8)$$

$$K_2^{\overline{MS}} = K_2^{MOM} = K_2^{OS} \quad (9)$$

$$K_3^{AS} - K_3^{\overline{MS}} = 3K_1 \frac{\beta_2^{\overline{MS}} - \beta_2^{AS}}{\beta_0} \quad ; \quad K_3^{SI} \quad scheme \quad invariant \quad (10)$$

The unknown coefficient K_4 depends from scheme strongly:

$$K_4^{AS} - K_4^{\overline{MS}} = 4K_2 \frac{\beta_2^{\overline{MS}} - \beta_2^{AS}}{\beta_0} + 2K_1 \frac{\beta_3^{\overline{MS}} - \beta_3^{AS}}{\beta_0} \quad (11)$$

Conclusion

There are the number of useful features, which can be revealed in PT using notion of Conformal Symmetry and perturbative Conformal Symmetry Breaking- including understanding of scheme-independence of definite analytical PT contributions Work in Progress and FEATURES

The similar holds in the case of consideration of for polynomial GKR's by Kataev, Mikhailov, 2010-2012

$$C_D^{NS}C_{Bjp} = 1+\sum_{n=1}^{n=3}\left(\frac{\beta(a)}{a}\right)^n P_n(a)$$

$$C_D^{V}C_{GLS} = 1+\sum_{k=1}^{k=3}\left(\frac{\beta(a)}{a}\right)^k \tilde{P}_k(a)$$

The most striking feature is IDENTITY of the definite a^4 PT contributions to $C_D^{NS}(a)$ (in $SU(N_c)$ coefficient $(d^{abcd}d^{abcd}/d_R)N_F$) and C_D^{SI} (in $SU(N_c)$ coefficient of $(d^{abc}d^{abc}/d_R)C_F$ In the case of QED limit this coefficient is the same: $-\frac{13}{16}-\zeta_3+\frac{5}{2}\zeta_5$

Why: can be understood on the comparison of the related diagrams with the ones presented in the lecture ofJ. Hehn, CALC-2012, Part II, July 25, 2012

General Conclusion: better understanding of the property of the Conformal Invariance and of its PT breaking should be useful in future to simplify complicated analytical PT calculations and study their theoretical and phenomenological outcomes

As they say: STAY TUNED