One-Loop Integrals at vanishing external momenta and applications for extended Higgs potentials reconstructions

CALC'2012

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1. Basic examples and an idea for extended Higgs potentials analysis by catastrophe theory methods
[Dolgopolov, Dubinin, Rykova, Journal of Modern Physics, 2011]
2. Basic examples for one-loop integrals need for finite-temperature and zero-temperature effective Higgs potential reconstruction
3. Higgs potential bifurcation sets in MSSM and NMSSM (Elena Petrova talk)

Effective potential

General form

$$U_{eff} = -\mu_1^2 (\Phi_1^+ \Phi_1) - \mu_2^2 (\Phi_2^+ \Phi_2) -$$
$$\mu_{12}^2 (\Phi_1^+ \Phi_2) - \mu_{12}^{2*} (\Phi_2^+ \Phi_1) +$$
$$\lambda_1 (T) (\Phi_1^+ \Phi_1)^2 + \lambda_2 (T) (\Phi_2^+ \Phi_2)^2 +$$
$$\lambda_3 (T) (\Phi_1^+ \Phi_1) (\Phi_2^+ \Phi_2) +$$
$$\lambda_4 (T) (\Phi_1^+ \Phi_2) (\Phi_2^+ \Phi_1) +$$
$$\frac{1}{2} \lambda_5 (T) (\Phi_1^+ \Phi_2)^2 + \frac{1}{2} \lambda_5^* (T) (\Phi_2^+ \Phi_1)^2 +$$
$$\lambda_6 (T) (\Phi_1^+ \Phi_1) (\Phi_1^+ \Phi_2) +$$
$$\lambda_6^* (T) (\Phi_1^+ \Phi_1) (\Phi_2^+ \Phi_1) +$$
$$\lambda_7 (T) (\Phi_2^+ \Phi_2) (\Phi_1^+ \Phi_2) + \lambda_7^* (T) (\Phi_2^+ \Phi_2) (\Phi_2^+ \Phi_1)$$

Vacume averages

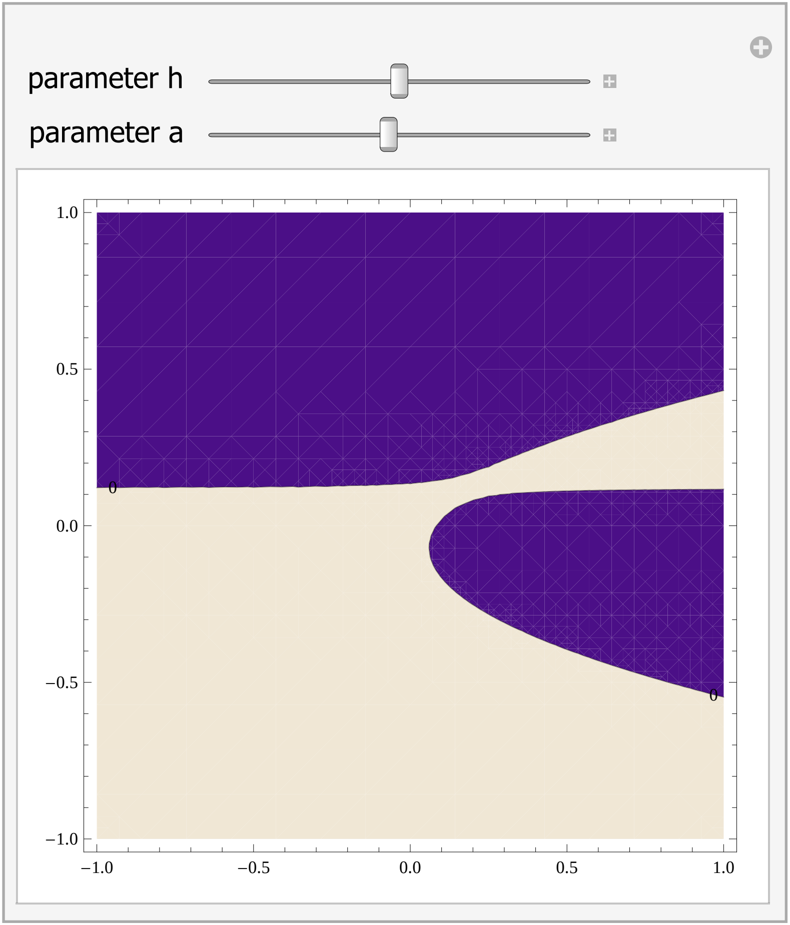
$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

Effective potential in terms of vevs:

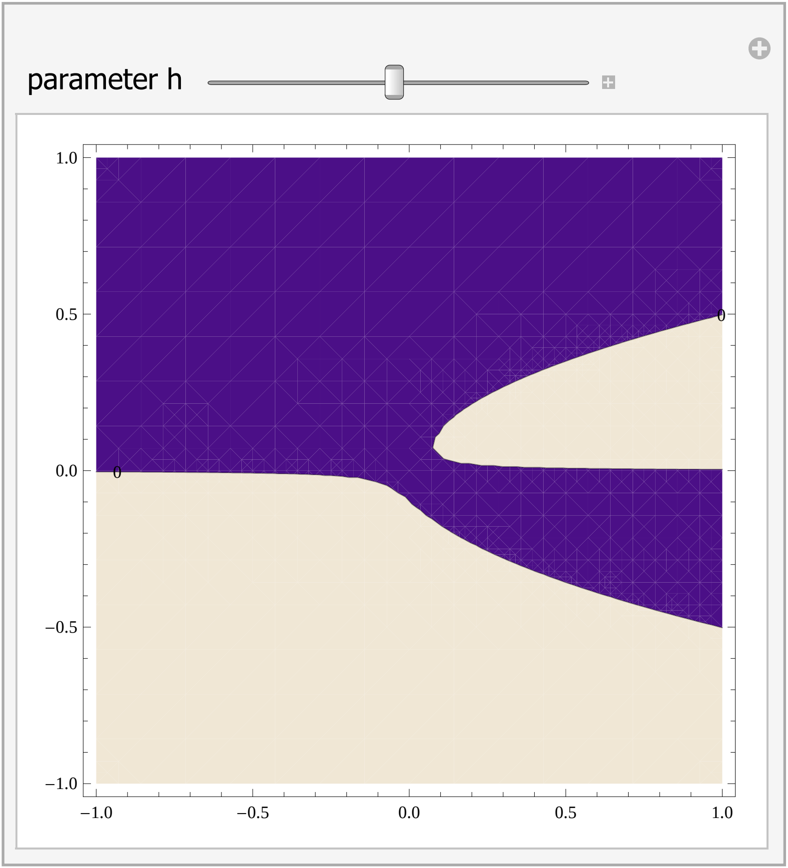
$$U_{eff} = \frac{1}{2} \mu_1^2 v_1^2 - \frac{1}{2} \mu_2^2 v_2^2 - \mathrm{Re}\mu_{12}^2 v_1 v_2 + \frac{\lambda_1}{4} v_1^4 +$$
$$\frac{\lambda_2}{4} v_2^4 + \frac{\lambda_{345}}{4} v_1^2 v_2^2 + \frac{\mathrm{Re}\lambda_6}{2} v_1^3 v_2 + \frac{\mathrm{Re}\lambda_7}{2} v_2^3 v_1,$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5$

Example 1



Example 2



Minimization conditions

Equivilibrium

$$\nabla U_{eff} = 0$$

Stationary conditions

$$\mu_1^2 = -\mathrm{Re}\mu_{12}^2 \frac{v_2}{v_1} + \lambda_1 v_1{}^2 + \frac{1}{2} \lambda_{345} v_2^2 + \frac{3}{2} \mathrm{Re}\lambda_6 v_1 v_2 + \frac{1}{2} \mathrm{Re}\lambda_7 \frac{v_2^3}{v_1}$$

$$\mu_2^2 = -\mathrm{Re}\mu_{12}^2 \frac{v_1}{v_2} + \lambda_2 v_2{}^2 + \frac{1}{2} \lambda_{345} v_1^2 + \frac{3}{2} \mathrm{Re}\lambda_7 v_1 v_2 + \frac{1}{2} \mathrm{Re}\lambda_6 \frac{v_1^3}{v_2}$$

Type of equilibrium is defined by Hessin

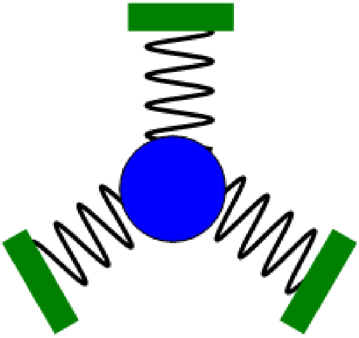
$$\frac{\partial^2 U_{eff}}{\partial v_i \, \partial v_j} > 0$$

Classic analogy

$$E = \frac{1}{2}(m_{ij}\dot{x}_i\dot{x}_j + k_{ij}x_ix_j) + l_{ijk}x_ix_jx_k + \ldots$$

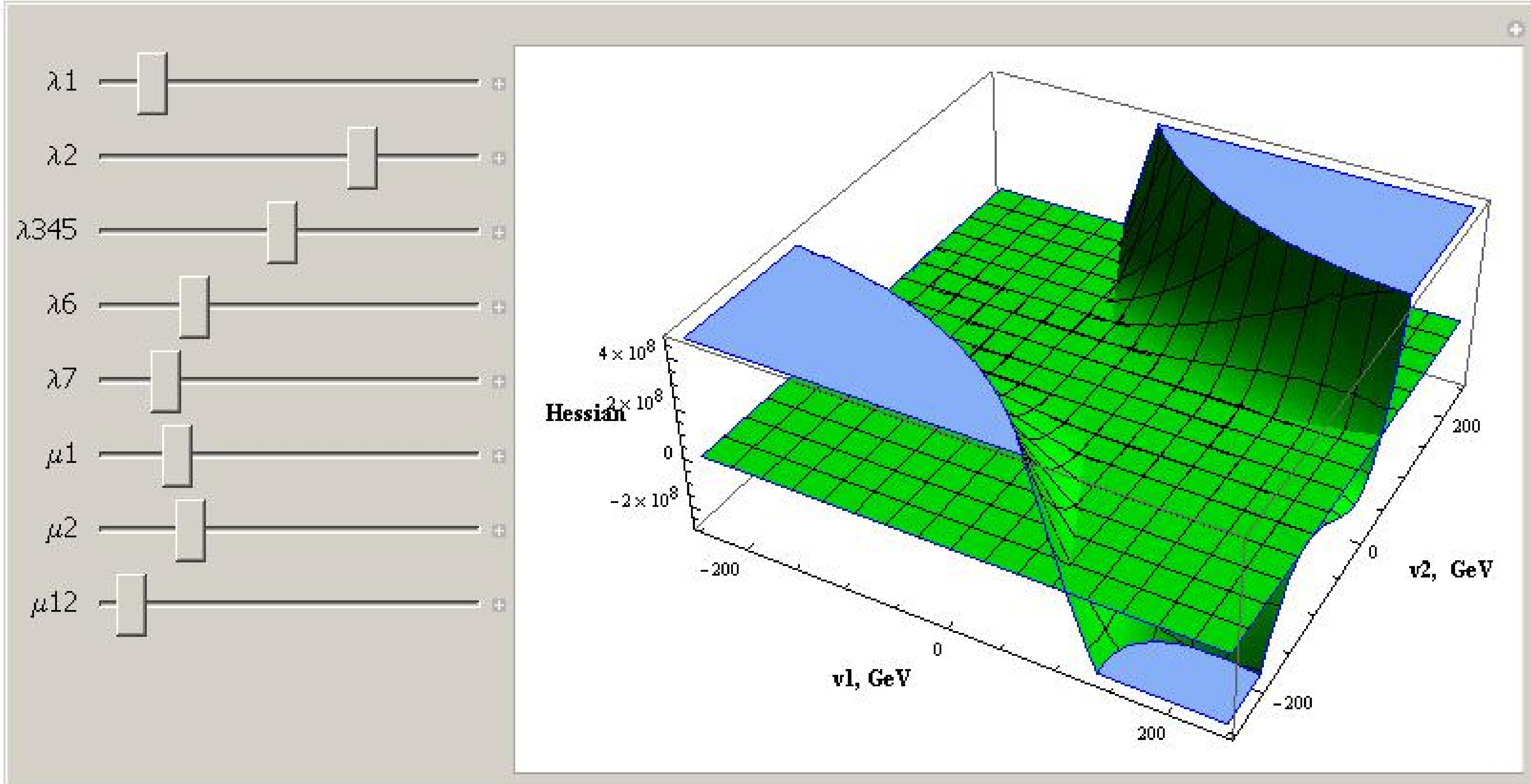
$$\Theta = \|A\|x$$

$$E = \frac{1}{2}(M_i\dot{\Theta}_i^2 + K_i\Theta_i^2) + L_{ijk}\Theta_i\Theta_j\Theta_k + \ldots$$



Hessian

H (μ , λ)



$v^2 = v_1^2 + v_2^2 = 246$ GeV, $\quad tg\beta = \frac{v_2}{v_1}$

Morse lemma

Two-doublet potential evolution depends on two fields ν_1 & ν_2 evolution and 7 parameters λ_i $(i = 1..\ 7)$.

Local propities of potential could be described by methods of catastroph theory. We use Morse lemma for effective potential to rewrite one in canonical form.

$$(\nabla U_{eff} = 0) \wedge$$
$$\left(\det \frac{\partial^2 U_{eff}}{\partial \nu_i \, \partial \nu_j} \neq 0\right) \Longrightarrow U_{eff} \longrightarrow U_{canon} = \tilde{\mu}_1 \, \tilde{\nu}_1{}^2 + \tilde{\mu}_2 \, \tilde{\nu}_2{}^2$$

$\tilde{\mu}_1, \tilde{\mu}_2$ – **Hessian eigenvalues**

$\tilde{\nu}_1, \tilde{\nu}_2$ – new variables

U_{eff} and U_{canon} are quality equivalent

Transform potential due to Morse lemma:

- Linear transformation, diagonalization

$$\nu_1 = \mathrm{Cos}[\theta]\, \nabla_1 - \mathrm{Sin}[\theta]\, \nabla_2$$

$$\nu_2 = \mathrm{Sin}[\theta]\, \nabla_1 + \mathrm{Cos}[\theta]\, \nabla_2$$

$$\frac{1}{2}\mu_1{}^2\, \nabla_1{}^2 - \frac{1}{2}\mu_2{}^2\, \nabla_1{}^2 - \mathrm{Re}\,\mu_{12}^2\, \nabla_1\, \nabla_2 + \,..\longrightarrow$$

$$\overline{\mu}_1{}^2\, \nabla_1{}^2 - \frac{1}{2}\overline{\mu}_2{}^2\, \nabla_1{}^2 + \,..$$

where $\overline{\mu}_{1,2} = \frac{1}{2}\left(\mu_1^2 + \mu_2^2 \pm \sqrt{\left(\mu_1^2 - \mu_2^2\right)^2 + 4\,\mu_{12}^4}\right)$

- Nonlinear transformation (axis-conserve)

$$V1 = \nabla_1 + \left(A_{20}\, \nabla_1^2 + A_{11}\, \nabla_1\, \nabla_2 + A_{02}\, \nabla_2^2\right)$$

$$+ \left(A_{30}\, \nabla_1^3 + A_{21}\, \nabla_1^2\, \nabla_2 + A_{12}\, \nabla_1\, \nabla_2^2 + A_{03}\, \nabla_1^3\right)$$

$$V\,2 = \nabla_2 + \left(B_{20}\, \nabla_1^2 + B_{11}\, \nabla_1\, \nabla_2 + B_{02}\, \nabla_2^2\right) +$$

$$\left(B_{30}\, \nabla_1^3 + B_{21}\, \nabla_1^2\, \nabla_2 + B_{12}\, \nabla_1\, \nabla_2^2 + B_{03}\, \nabla_1^3\right)$$

in different variables

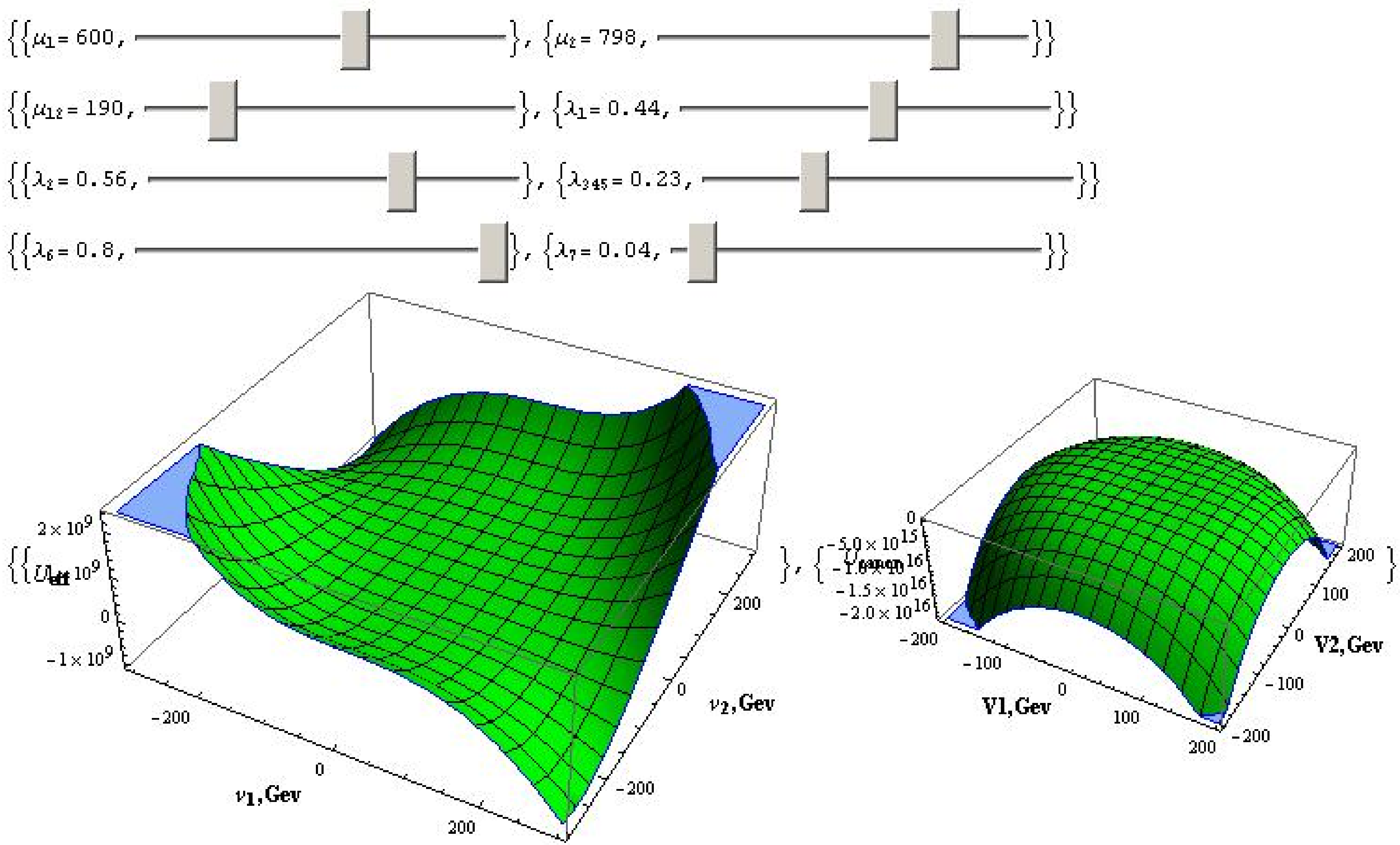
Equal coefficients for same powers of variables.

$$U_{eff}(\overline{V}_1, \overline{V}_2) = U_{canon}(V1, V2)$$

$$\overline{\mu}_1 \overline{V}_1^{\,2} + \overline{\mu}_2 \overline{V}_2^{\,2} + \ldots = \overline{\mu}_1 V1^2 + \overline{\mu}_2 V2^2$$

In system of nonphysical fields potential is presented in unique form, which defines stable equivilibrium state.

Minimum surfaces in terms of physical and unphysical fileds



Physical

Unphysical

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The smallness of external state masses relative to the masses of loop particles allows us to take the limit where the external particles are massless.
These integrals are particularly easy to solve using algebraic recursion relations or residue method
Remark. But it is necessary to keep the external momentum p_μ general (i. e. nonzero) until it can be converted into an external mass in the amplitude, after which point one may take accurate limit $p \to 0$, especially in the cases with equivalent internal masses in loop.

In d dimensions such two-point loop integral B_0 is defined as:

$$\frac{1}{(4\pi)^2} B_0(m^2, M^2) \quad = \quad \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{i}{(k^2 - m^2)\,[k^2 - M^2]}. \quad (1)$$

In order to derive asymptotic results we expand $B_0(p^2, m_1^2, m_2^2)$ in powers of the external momentum p^2:

$$B_0(p^2, m_1^2, m_2^2) = -\Delta_\mu - \frac{m_1^2 \log m_1^2 - m_2^2 \log m_2^2}{m_2^2 - m_1^2} + \mathcal{O}\left(\frac{p^2}{m_1^2 + m_2^2}\right).$$

The 3- and 4-point loop integrals at vanishing external momenta are defined as:

$$\frac{1}{(4\pi)^2} C_{2n}(m_1^2, m_2^2, m_3^2) = \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{ik^{2n}}{\prod_i^3 (k^2 - m_i^2)}, \quad (2)$$

$$\frac{1}{(4\pi)^2} D_{2n}(m_1^2, m_2^2, m_3^2, m_4^2) = \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{ik^{2n}}{\prod_i^4 (k^2 - m_i^2)}. \quad (3)$$

Note that the above equations are manifestly equivalent under interchange of the arguments and explicit calculations show that one may reshuffle the order of the arguments without changing the result (for functions with unequal arguments it is obvious, and for functions with the equivalent arguments it demands accuracy!).

The two-point loop integral in tensor reduction B_1 can be defined as:

$$\frac{1}{(4\pi)^2} p_\mu B_1(p, m^2, M^2) = \tag{4}$$

$$\mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{ik_\mu}{(k^2 - m^2)\left[(k+p)^2 - M^2\right]}. \tag{5}$$

Such integrals appear in the self-energy contributions.

It turns out that this is very simple to evaluate integrals (1), (2), (3) with the following algebraic relations:

$$\frac{1}{(k^2-m_1^2)(k^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left(\frac{1}{k^2-m_1^2}-\frac{1}{k^2-m_2^2}\right), \quad (6)$$

$$\frac{k^2}{k^2-m^2} = 1 + \frac{m^2}{k^2-m^2}. \quad (7)$$

Note that denominators of integrals under consideration are spherically symmetric.

And for the case of all equivalent masses the particularly well-known general integral can be performed using Γ function:

$$\int \frac{d^n k}{(2\pi)^n} \frac{(k^2)^b}{(k^2 - A^2)^a} = \tag{8}$$

$$\frac{i}{(4\pi)^{n/2}} (-1)^{a+b} (A^2)^{b-a+n/2} \frac{\Gamma(b+n/2)\Gamma(-b+a-n/2)}{\Gamma(n/2)\Gamma(a)}, \tag{9}$$

where the left-hand side is an integral in n-dimensional Minkowski space. In the private case at $b = 0$

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - A^2)^a} = \frac{i}{(4\pi)^{n/2}} (-1)^a (A^2)^{-a+n/2} \frac{\Gamma(a-n/2)}{\Gamma(a)}. \tag{10}$$

The explicit formulae at $d \to 4$ are listed below (we give also expressions for some 3-point functions proportional to higher momenta powers, may be useful in contributions for Higgs potential calculations) and may be represented only in terms of B_0 integral, or A_0 (10):

$$\frac{1}{(4\pi)^2}A_0(m^2) = \mu^{4-d}\int \frac{d^d k}{(2\pi)^d}\frac{i}{k^2-m^2} = \tag{11}$$

$$= i\,\frac{\mu^{4-d} i}{(4\pi)^{d/2}}(-1)(m^2)^{-1+d/2}\Gamma(1-d/2) = \tag{12}$$

$$= \frac{-i\,i}{16\pi^2}\left(1+\frac{\varepsilon}{2}\log(4\pi\mu^2)+\mathcal{O}(\varepsilon^2)\right)m^2 \tag{13}$$

$$\times \left(1-\frac{\varepsilon}{2}\log(m^2)+\mathcal{O}(\varepsilon^2)\right)\times$$

$$\times (-1)\left(\frac{2}{\varepsilon}+1-\gamma_E+\mathcal{O}(\varepsilon)\right) \Rightarrow$$

$$\Rightarrow A_0(x) = x(-\Delta_\mu + \log x), \tag{14}$$

$$
\begin{aligned}
B_0(x,y) &= \frac{1}{x-y}\left[A_0(x) - A_0(y)\right] = && (15)\\
&= \frac{-1}{x-y}\left[x\Delta_\mu - y\Delta_\mu - x\log x + y\log y\right] = &&\\
&= -\Delta_\mu - \frac{-x\log x + y\log x - y\log x + y\log y}{x-y} = &&\\
&= -\Delta + \log\frac{x}{\mu^2} - \frac{y}{x-y}\log\frac{y}{x} = && (16)\\
= B_0(y,x) &= -\Delta + \log\frac{y}{\mu^2} - \frac{x}{y-x}\log\frac{x}{y}, && (17)
\end{aligned}
$$

$$C_0(x,y,z) = \frac{1}{y-z}\left[B_0(x,y) - B_0(x,z)\right] \tag{18}$$

$$= \frac{y\log\frac{y}{x}}{(x-y)(z-y)} + \frac{z\log\frac{z}{x}}{(x-z)(y-z)}, \tag{19}$$

$$C_0(x,y,y) = \frac{\partial}{\partial y}B_0(x,y) = \frac{1}{y-x}\left[1 + \frac{x\log\frac{x}{y}}{y-x}\right], \tag{20}$$

$$C_2(x,y,z) = B_0(x,y) + zC_0(x,y,z) =$$

$$= -\Delta - \log\frac{\mu^2}{x} + \frac{y^2\log\frac{y}{x}}{(x-y)(z-y)} + \frac{z^2\log\frac{z}{x}}{(x-z)(y-z)},$$

$$
\begin{aligned}
C_2(x,y,y) &= B_0(y,x) + yC_0(x,y,y) = \\
&= -\Delta_\mu + \log y - \frac{x}{y-x}\log\frac{x}{y} + \frac{y}{y-x}\left[1 + \frac{x\log\frac{x}{y}}{y-x}\right] = \\
&= -\Delta - \log\frac{\mu^2}{y} - \frac{x}{x-y}\left[\frac{y}{x} - \frac{x\log\frac{x}{y}}{x-y}\right] \qquad (21)
\end{aligned}
$$

$$B_0(x,y) + yC_0(x,y,y) =$$
$$= -\Delta - \log\frac{\mu^2}{x} + \frac{y}{y-x}\left[1 - \frac{(y-2x)\log\frac{x}{y}}{y-x}\right], \qquad (22)$$
$$= \frac{\partial}{\partial y}A_0(y) + xC_0(x,y,y) =$$
$$= -\Delta - \log\frac{\mu^2}{y} - \frac{x}{x-y}\left[\frac{y}{x} - \frac{x\log\frac{x}{y}}{x-y}\right]. \qquad (23)$$

The answers (21), (23), and (22) are identical indeed, check by Mathematica is done. Check for last equations:

$$\frac{1}{(4\pi)^2}\frac{\partial}{\partial m^2}A_0(m^2) = \mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{i}{(k^2-m^2)^2} = \quad (24)$$

$$= i\,\frac{\mu^{4-d}i}{(4\pi)^{d/2}}(-1)^2(m^2)^{-2+d/2}\Gamma(2-d/2) =$$

$$= \frac{i\,i}{16\pi^2}\left(1+\frac{\varepsilon}{2}\log(4\pi\mu^2)+\mathcal{O}(\varepsilon^2)\right)\times \quad (25)$$

$$\times \left(1-\frac{\varepsilon}{2}\log(m^2)+\mathcal{O}(\varepsilon^2)\right)\times$$

$$\times \left(\frac{2}{\varepsilon}-\gamma_E+\mathcal{O}(\varepsilon)\right) = (-\Delta_\mu+1)+\log x$$

$$yes \quad \frac{\partial}{\partial x}A_0(x) = -\Delta_\mu+\log x+1. \quad (26)$$

Equal arguments correspond to equivalent masses. The divergent part is

$$\Delta_{\{\mu;1\}} = \frac{2}{4-d} + \log(4\pi\{\mu^2;1\}) - \gamma_{\mathrm{E}} + 1$$

where μ is the renormalization scale.
And

$$C_{11}(x,y) = -\frac{x-3y}{4(x-y)^2} + \frac{y^2}{2(x-y)^3}\log\frac{y}{x}, \qquad (27)$$

$$C_{12}(x,y) = -\frac{x+y}{2(x-y)^2} - \frac{xy}{3(x-y)^3}\log\frac{y}{x}. \qquad (28)$$

The explicit formulae for the four-point integrals at vanishing external momenta are (possible cross-checks are provided):

$$D_0(x,y,z,t) = \frac{1}{z-t}\left(C_0(x,y,z) - C_0(x,y,t)\right) =$$

$$= \frac{y\log\frac{y}{x}}{(y-x)(y-z)(y-t)} + \frac{z\log\frac{z}{x}}{(z-x)(z-y)(z-t)} + \quad (29)$$

$$+ \frac{t\log\frac{t}{x}}{(t-x)(t-y)(t-z)}, \quad (30)$$

$$
\begin{aligned}
D_0(x,y,z,z) &= \frac{\partial}{\partial z} C_0(z,y,x) \\
&= \frac{\partial}{\partial z}\left(\frac{y\log\frac{y}{z}}{(z-y)(x-y)} + \frac{x\log\frac{x}{z}}{(z-x)(y-x)}\right) = \\
&= \frac{1}{(x-z)(y-z)} + \frac{x\log\frac{x}{z}}{(x-y)(x-z)^2} + \frac{y\log\frac{y}{z}}{(y-x)(y-z)^2},
\end{aligned} \tag{31}
$$

And some additional private checks:

$$B_0(m^2) = \frac{\partial}{\partial m^2} A_0(m^2) = -\Delta_\mu + \log m^2 + 1, \qquad (32)$$

$$C_0(m^2) = \frac{1}{2} \frac{\partial}{\partial m^2} B_0(m^2) = \frac{1}{2m^2}, \qquad (33)$$

$$D_0(m^2) = \frac{1}{3} \frac{\partial}{\partial m^2} C_0(m^2) = -\frac{1}{6m^4}. \qquad (34)$$

[CALC'2009] In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies $\omega_n = 2\pi n T$ $(n = 0, \pm 1, \pm 2, ...)$, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{i=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)}, \quad (35)$$

Here $\mathbf{k}$ is the three-dimensional momentum in a system with the temperature T. In the following calculations first we perform integration with respect to $\mathbf{k}$ and then take the sum, using the reduction to three-dimensional theory in the high-temperature limit for zero frequencies.

$$I[m_1, m_2, ..., m_b] = 2T\,(2\pi T)^{3-2b}\ \frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\ \frac{\Gamma(b-3/2)}{\Gamma(b)}\ S(M, b{-}3/2), \quad (36)$$

where

$$S(M, b-3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2 + M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2. \tag{37}$$

For $b > 1$ the parameter m^2 is a linear function dependent on m_i^2 and the variables $\{dx\}$ of Feynman parametrization, which are the integration variables in (37). At the integer values of b the integrand in (3) is a generalized Hurwitz zeta-function Note that for the leading threshold corrections to effective parameters of the two-doublet potential $b > 2$, so the wave-function renormalization appears in connection with the divergence at $b = 2$

$M^2(M_a, M_b, x) = (M_a^2 - M_b^2)x + M_b^2$. Then we get

$$\frac{1}{[\mathbf{k}^2 + m_a^2][\mathbf{k}^2 + m_b^2]} = \frac{1}{(2\pi T)^4} \int_0^1 \frac{dx}{(\mathbf{p}^2 + n^2 + M^2)^2}. \quad (54)$$

and divergent series for (45) ($d\mathbf{k} = (2\pi T)^3 d\mathbf{p}$)

$$I_0[M_a, M_b] = \frac{1}{2\pi T} \int_0^1 dx \sum_{n=-\infty, n\neq 0}^{\infty} \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{(\mathbf{p}^2 + n^2 + M^2)^2}, \quad (55)$$

With the help of dimensional regularization or differentiating the integral

$$\int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{(\mathbf{p}^2 + M^2)} = -\frac{M}{4\pi} + \mathcal{O}(\frac{M^2}{T^2}) \quad (56)$$

over the parameter M, the equation (55) can be reduced to

$$I_0[M_a, M_b] = \frac{1}{16\pi^2 T} \int_0^1 dx\, \zeta(2, \frac{1}{2}, M^2), \quad (57)$$

where $\zeta(u,s,t)$ is the generalized Hurwitz zeta-function [1]

$$\zeta(u,s,t) = \sum_{n=1}^{\infty} \frac{1}{(n^u + t)^s}. \tag{58}$$

So in the case under consideration the sums of integrals (51) and (52) can be calculated by differentiation of (57) with respect to mass parameters participating in $M = M(M_a, M_b, x)$.
Differentiation increases the power s in the denominator of (57) giving convergent integrals

$$I_1[M_a, M_b] = \frac{T}{2M_a}\frac{\partial}{\partial M_a} I_0 = -\frac{1}{64\pi^4 T^2}\int_0^1 \, dx \; x \; \zeta[2, \frac{3}{2}, M^2(x)], \tag{59}$$

$$I_2[M_a, M_b] = -\frac{1}{2M_b}\frac{\partial}{\partial M_b}(-I_1) = \frac{3}{256\pi^6 T^4}\int_0^1 dx \; x \; (1-x) \; \zeta[2, \frac{5}{2}, M^2(x) \tag{60}$$

Summary

1. Method for stable state research

2. Constrains on parameters.

3. Temperature evolution.