Infrared-finite Observables in N=4 Super Yang-Mills Theory

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Outline

1. Introduction
   - N=4 Syper Yang Mills Theory
   - Gluon scattering amplitudes
2. Infrared Divergences
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   - Strong Coupling Case
3. Cancellation of IR Divergences
   - Toy model: electron-quark scattering
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N=4 Super Yang-Mills Theory

- $\mathcal{N} = 4$ Syper Yang-Mills theory is the most supersymmetric theory possible without gravity
- Field content: 1 massless gauge boson, 4 massless (Majorana) spin 1/2 fermions, 6 real (or 3 complex) massless spin 0 bosons
  All fields are in adjoint representation of the gauge group (Take $SU(N_c)$)
- The theory is exactly scale invariant, conformal field theory at quantum level, i.e. the β function identically vanishes at all orders of PT

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AdS/CFT Correspondence

- $N_c \rightarrow \infty$ (planar limit) is expected to be integrable and solvable
- Maldacena's conjecture: Planar Limit of N=4 SYM at strong coupling is dual to weakly coupled type II b supergravity in 10 dimensional $AdS_5 * S_5$ space.
- How might PT series be organized to produce simple strong coupling result?
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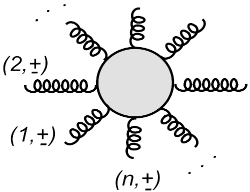

All outgoing gluons with helicity + or -
on mass shell
In the leading N_c order (planar limit)

- Colour decomposition of amplitudes in N=4 SYM theory for $N_c \to \infty$

$$\mathcal{A}_n^{(l)} = g^{n-2}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \sum_{perm} Tr(T^{a_{\sigma(1)}}, ..., T^{a_{\sigma(n)}}) A_n^{(l)}(a_{\sigma(1)}, ..., a_{\sigma(n)}),$$

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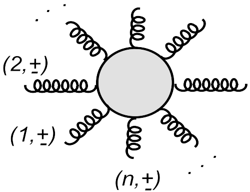

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Perturbation theory

- Bern, Dixon & Smirnov's conjecture: $M_n^{(L)}(\varepsilon) \equiv A_n^{(L)} / A_n^{(0)}$

$$\mathcal{M}_n \equiv 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\varepsilon) = exp\left[\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\varepsilon) M_n^{(1)}(l\varepsilon) + C^{(l)} + E_n^{(l)}(\varepsilon)\right)\right]$$

$$f^{(l)}(\varepsilon) = f_0^{(l)}(\varepsilon) + \varepsilon f_1^{(l)}(\varepsilon) + \varepsilon^2 f_2^{(l)}(\varepsilon)$$

- $$\mathcal{M}_n(\varepsilon) = exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_K^{(l)}}{(l\varepsilon)^2} + \frac{2G_0^{(l)}}{l\varepsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\varepsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)} F_n^{(1)}(0)\right]$$

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- Cusp anomalous dimension appears in RG eq. for the expectation value of a Wilson line with a cusp

  Loop expansion $$\gamma_K = \sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_K^{(l)}$$

  $$\gamma_K^{(1)} = 8, \quad \gamma_K^{(2)} = -16\zeta_2, \quad \gamma_K^{(3)} = 176\zeta_4, \ldots$$

- It also controls the large spin limit of anomalous dimension of leading-twist operators

  $$O_j \equiv \bar{q}(\gamma_+ \mathcal{D}^+)^j q$$

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Strong coupling expansion/AdS

- Classical solution (Alday & Maldacena) for the scattering amplitude

$$\mathcal{M}_4(\varepsilon) = exp[-S^E_{cl}]$$

- $S^E_{cl} = \frac{1}{\varepsilon^2}\frac{\sqrt{g^2N_c}}{\pi}\left[\left(\frac{\mu^2_{IR}}{-s}\right)^{\varepsilon/2} + \left(\frac{\mu^2_{IR}}{-t}\right)^{\varepsilon/2}\right]$

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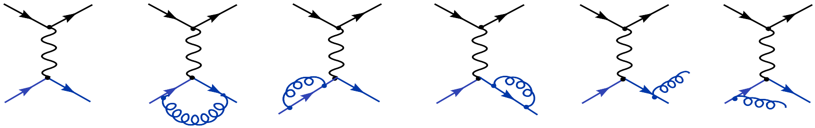

Born | Virtual | Real Emission

- Virtual Correction

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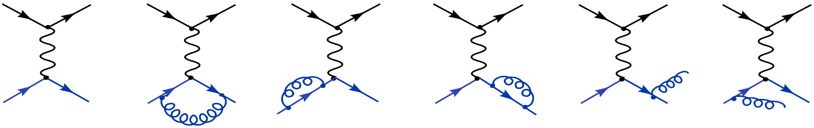

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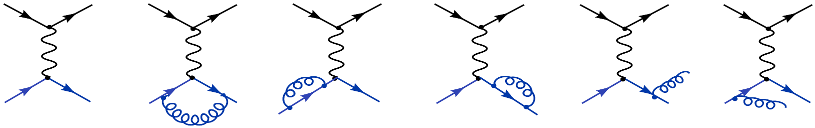

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- where the functions f_1 and f_2 in the c.m. frame are ($c = \cos\theta$)

$$f_1 = -2\frac{(c^3+5c^2-3c+5)\log(\frac{1-c}{2})+(1-c^2)(c-11)/4}{(1-c)(1+c)^2}$$

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- $$\left(\frac{d\sigma}{d\Omega}\right)_{split} = \frac{1}{\varepsilon}\frac{\alpha_s}{2\pi}\int_0^1 dz\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon} P_{qq}(z)\frac{d\sigma_0}{d\Omega}(pz)$$
- $$\left(\frac{d\sigma}{d\Omega}\right)_{split} = C_F\frac{\alpha^2}{2E^2}\frac{\alpha_s}{2\pi}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{-t}\right)^{\varepsilon}\left(-\frac{f_1}{\varepsilon}+f_3\right),$$
- where for $Q_f^2 = \hat{t}$

$$\begin{aligned} f_3 \;&=\; -\frac{1}{(1-c)^2(1+c)^2}\Big[2(1-c)(c^3+c^2-33c+7)\log(\frac{1-c}{2}) \\ &+12(9c^2+2c+5)Li_2(\frac{1+c}{2}) - (1+c)^2(c^2+5c+3)\pi^2 \\ &-\frac{1}{2}(1-c)(1+c)(11c^2-19)\Big]. \end{aligned}$$

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- $$\left(\frac{d\sigma}{d\Omega}\right)_{split} = C_F\frac{\alpha^2}{2E^2}\frac{\alpha_s}{2\pi}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{-t}\right)^{\varepsilon}\left(-\frac{f_1}{\varepsilon}+f_3\right),$$

- where for $Q_f^2 = \hat{t}$

$$f_3 = -\frac{1}{(1-c)^2(1+c)^2}\Big[2(1-c)(c^3+c^2-33c+7)\log(\frac{1-c}{2}) +12(9c^2+2c+5)Li_2(\frac{1+c}{2}) - (1+c)^2(c^2+5c+3)\pi^2 -\frac{1}{2}(1-c)(1+c)(11c^2-19)\Big].$$

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Infrared-free observable = inclusive cross-section

$$\left(\frac{d\sigma}{d\Omega}\right)_{observ} = \left(\frac{d\sigma}{d\Omega}\right)_{virt}^{2\to 2} + \left(\frac{d\sigma}{d\Omega}\right)_{real}^{2\to 3} + \left(\frac{d\sigma}{d\Omega}\right)_{split}^{2\to 2}$$
$$= \frac{\alpha^2}{2E^2}\left\{\frac{c^2+2c+5}{(1-c)^2}\right.$$
$$-\frac{\alpha_s}{2\pi}\frac{C_F}{(1-c)(1+c)^2}\left[(c^3+5c^2-3c+5)\log^2\frac{1-c}{2}\right.$$
$$\left.\left.+\frac{1}{2}(7c^3+19c^2-55c-3)\log\frac{1-c}{2}-(1+c)(3c^2+21c+2)\right]\right\}$$

Outline

From partial amplitudes to cross-sections

To obtain the cross sections from partial amplitudes one have to compute the square of them. In the the planar limit it is just:

$$\Phi_n(p_1^{\pm},...,p_n^{\pm}) = g^{2n-4}(\frac{g^2N_c}{16\pi^2})^{2l}\sum_{colors}\mathcal{A}_n^{(l)}\mathcal{A}_n^{(l)*} =$$

$$2g^{2n-4}N_c^{n-2}(N_c^2-1)(\frac{g^2N_c}{16\pi^2})^{2l}\sum_{perm}|A_n^{(l)}(a_{\sigma(1)},...,a_{\sigma(n-1)},a_n)|^2$$

Then the cross-section is

$$d\sigma_n(p_{in}) = \Phi_n(p_1^{\pm},...,p_n^{\pm})d\phi_k,$$

where $d\phi_k$ is the phase space of the outgoing particles:

$$d\phi_k \sim \delta^D(p_{in}-p_{fin})\mathcal{S}_n\prod_k\delta^+(p_k^2)d^Dp_k,$$

where $\mathcal{S}_n$ - is so called measurement function and integration goes over $D = 4-2\varepsilon$ dimensions.

Phase space integration features

The pase space integral can be rewritten as

$$d\phi_3 \sim \delta^D(p_1+p_2-p_3-p_4-p_5)\prod_{k=3}^{5}\delta^+(p_k^2)d^Dp_k =$$
$$\delta^D(p_1+p_2-p_3-\mathbf{p}_4)\delta^+(p_3^2)d^Dp_3d^Dp_4\{\delta^+(k^2)\delta^+([\mathbf{p}_4-k]^2)d^Dk\},$$

and the typical integrant looks like

$$\int d^Dk\frac{\delta^+(k^2)\delta^+([\mathbf{p}_4-k]^2)}{(p_i,k)(p_j,\mathbf{p}_4-k)} \sim Im(I_4^{m1}).$$

After performing integration over d^Dk we have left with integrals of the typical following form

$$\int_0^1 dx\frac{x^a(1-x)^b}{(1+px)^d}F_{2,1}(1,-\varepsilon,1-\varepsilon,-qx^m(1-x)^n).$$

Phase space integration features

For $d = 0$ and "small" a, b, m, n this integral can be reduced to particular Meijer G-function and can be represented in terms of hypergeometric functions $F_{3,2}$ which can be than expanded to any order in ϵ.

$$I_{a,b,c}(\alpha, \beta, m, n, q) \quad = \quad \int_0^1 dx x^{\alpha-1}(1-x)^{\beta-1} F_{2,1}(a, b; c; -qx^m(1-x)^n)$$

Let's consider example for: $m = 1$, $n = -2$

$$I_{a,b,c}(\alpha, \beta, 1, -2, q) =$$
$$= \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}(2\pi)^{-\frac{1}{2}} 2^{\beta-\frac{1}{2}} G^{3,3}_{4,4}(\frac{4}{q}|1, 1-\frac{\beta}{2}, \frac{1}{2}-\frac{\beta}{2}, c; a, b, \alpha, 1-\alpha-\beta).$$

Virtual Correction (MHV)

- $$\left(\frac{d\sigma}{d\Omega}\right)_0^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\frac{s^4(s^2+t^2+u^2)}{s^2t^2u^2}\left(\frac{\mu^2}{s}\right)^\varepsilon = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^\varepsilon \frac{3+c^2}{(1-c^2)^2}$$

- $$\left(\frac{d\sigma}{d\Omega}\right)_{virt}^{--++} = \frac{\alpha^2 N_c^2}{8E^2}\left(\frac{\mu^2}{s}\right)^\varepsilon \left\{\frac{\alpha N_c}{2\pi}\frac{s^4}{s^2t^2u^2}\left[-\frac{8}{\varepsilon^2}\left(((\frac{\mu^2}{-t})^\varepsilon + (\frac{\mu^2}{-u})^\varepsilon)s^2 + ((\frac{\mu^2}{s})^\varepsilon + (\frac{\mu^2}{-t})^\varepsilon)u^2 + ((\frac{\mu^2}{s})^\varepsilon + (\frac{\mu^2}{-u})^\varepsilon)t^2\right) + \frac{16}{3}\pi^2(s^2+t^2+u^2) + 4(u^2\log^2(\frac{-s}{t}) + t^2\log^2(\frac{-s}{u}) + s^2\log^2(\frac{t}{u}))\right]\right\}$$

- $$= \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\left\{\frac{\alpha N_c}{2\pi}\left[-\frac{16}{\varepsilon^2}\frac{3+c^2}{(1-c^2)^2} + \frac{4}{\varepsilon}\left(\frac{5+2c+c^2}{(1-c^2)^2}\log(\frac{1-c}{2}) + (c\leftrightarrow -c)\right) + \frac{16(3+c^2)\pi^2}{3(1-c^2)^2} - \frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

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$$\left. +((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
$$\left.\left. +\frac{16}{3}\pi^2(s^2+t^2+u^2)+4(u^2\log^2(\frac{-s}{t})+t^2\log^2(\frac{-s}{u})+s^2\log^2(\frac{t}{u}))\right]\right\}$$

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$$\left.\left.\left. +(c\leftrightarrow -c)\right)+\frac{16(3+c^2)\pi^2}{3(1-c^2)^2}-\frac{16}{(1-c^2)^2}\log(\frac{1-c}{2})\log(\frac{1+c}{2})\right]\right\}$$

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$$\left.+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-t})^{\varepsilon})u^2+((\frac{\mu^2}{s})^{\varepsilon}+(\frac{\mu^2}{-u})^{\varepsilon})t^2\right)$$
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Real Emission (MHV)

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}\right.$$
$$+\frac{1}{\varepsilon}\left[\frac{2}{(1+c)^2}\log\left(\frac{1-c}{2}\right)+\frac{2}{(1-c)^2}\log\left(\frac{1+c}{2}\right)+\frac{16\delta(2\delta-3)}{(1-c^2)^2(1-\delta)^2}\right.$$
$$\left.\left.+\frac{12(3+c^2)(\log(1-\delta)-\log(\delta))}{(1-c^2)^2}\right]+\text{Finite part}\right\}$$

- What is δ ? One has a singularity as $\delta \to 1$.
- This is the cut-off in external momenta of the scattered gluon: $|\vec{p}| \leq \frac{E}{2}(1-\delta)$.
- This allows one to distinguish identical final gluons, so that the gluon scattered at angle θ has non-zero momentum

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Real Emission (Anti MHV)

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{Born} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\Bigg\{\frac{1}{\varepsilon^2}\frac{8(3+c^2)}{(1-c^2)^2}$$

$$+\frac{2}{\varepsilon}\Bigg[\frac{64}{3(1-c^2)^3}+\delta\frac{(6\delta^2-3\delta+30)c^2+(10\delta^2-57\delta+66)}{3(1-c^2)^2}-\frac{6(c^2+3)\log\delta}{(1-c^2)^2}+$$

$$\Bigg(\frac{3c^2-24c+85}{(1-c)(1+c)^3}\log\frac{1-c}{2}-\frac{4(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-(1-\delta)c}{2}$$

$$+\frac{16\delta(5-c)}{(1-c^2)^2(1+\delta-(1-\delta)c)}-\frac{32(1-c)}{3(1+c)^3(1+\delta-(1-\delta)c)^3}+(c\leftrightarrow -c)\Bigg)\Bigg]$$

$$+\text{Finite part}\Bigg\}$$

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$$+\frac{2}{\varepsilon}\left[\frac{64}{3(1-c^2)^3}+\delta\frac{(6\delta^2-3\delta+30)c^2+(10\delta^2-57\delta+66)}{3(1-c^2)^2}-\frac{6(c^2+3)\log\delta}{(1-c^2)^2}+\right.$$
$$\left(\frac{3c^2-24c+85}{(1-c)(1+c)^3}\log\frac{1-c}{2}-\frac{4(c^2-6c+21)}{(1-c)(1+c)^3}\log\frac{1+\delta-(1-\delta)c}{2}\right.$$
$$\left.\left.+\frac{16\delta(5-c)}{(1-c^2)^2(1+\delta-(1-\delta)c)}-\frac{32(1-c)}{3(1+c)^3(1+\delta-(1-\delta)c)^3}+(c\leftrightarrow -c)\right)\right]$$
$$\left.+\text{Finite part}\right\}$$

Real Emission (Matter)($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q)}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{4}{\varepsilon}\left[\frac{32(79-25c^2)}{3(1-c^2)^2)} + \frac{64(3-c)^2}{(1-c)(1+c)^3})\log(\frac{1-c}{2}) + \frac{64(3+c)^2}{(1-c)^3(1+c)}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

- Sfermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\tilde{\bar{q}}\tilde{q})}_{Real} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{2\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{128(10+7c^2)}{(1-c^2)^2} - \frac{192(5-c)}{(1+c)^3}\log(\frac{1-c}{2}) - \frac{192(5+c)}{(1-c)^3}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

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$$\left.\left.+\frac{64(3-c)^2}{(1-c)(1+c)^3})\log(\frac{1-c}{2}) + \frac{64(3+c)^2}{(1-c)^3(1+c)}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

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$$\left.\left.-\frac{192(5-c)}{(1+c)^3}\log(\frac{1-c}{2}) - \frac{192(5+c)}{(1-c)^3}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

Real Emission (Matter)($\delta = 1$)

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Initial and final state splitting (MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

$$\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[-\frac{4(c^2+3)}{(1-c^2)^2}\left(\log\frac{1-c}{2}+\log\frac{1+c}{2}\right)-\frac{8(c^2+3)}{(1-c^2)^2}\log\frac{1-\delta}{\delta}\right.\right.$$
$$\left.\left.-\frac{16\delta(2\delta-3)}{(1-\delta^2)(1-c^2)^2}\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}$$

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Initial and final state splitting (MHV)

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Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.\right.\right.$$
$$-\frac{16(c^2-3c+3)}{(1-c)^2(1+c)^3}\log\frac{1+\delta-c(1-\delta)}{2}+\frac{8(c^2+3)}{(1-c^2)^2}\log\delta+(c\leftrightarrow -c)\Bigg)$$
$$-\frac{4\delta}{3(1-c^2)^2((1+\delta)^2-c^2(1-\delta)^2)^3}\Big(c^8(1-\delta)^6(2\delta^2+3\delta+6)-4c^6(1-\delta)^4(\delta^4+10\delta^3$$
$$-23\delta^2+114\delta-33)+2c^4(1-\delta)^2(39\delta^5-102\delta^4+86\delta^3+658\delta^2+183\delta-312)$$
$$+4c^2(\delta^8-12\delta^7+39\delta^6+216\delta^5-42\delta^4-720\delta^3-421\delta^2+300\delta+208)$$
$$\left.\left.-(1+\delta)^3(2\delta^5-9\delta^4+63\delta^3+455\delta^2+579\delta+198)\Big)\right]+\text{Finite part}\right\}$$

- Final

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\left[\log\delta-\frac{\delta}{3}(2\delta^2-9\delta+18)\right]+\text{F.p.}\right\}$$

Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.\right.\right.$$
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$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{FnSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\frac{4(c^2+3)}{(1-c^2)^2}\left[\log\delta-\frac{\delta}{3}(2\delta^2-9\delta+18)\right]+\text{F.p.}\right\}$$

Initial and final state splitting (Anti MHV)

- Initial

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++-)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{1}{\varepsilon}\left[\left(\frac{4(c^3-15c^2+51c-45)}{(1-c)^2(1+c)^3}\log\frac{1-c}{2}\right.\right.\right.$$
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Initial state splitting (Matter) ($\delta = 1$)

- Fermions

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+\bar{q}q)}_{InSplit} = \frac{\alpha^2 N_c^2}{E^2}\left(\frac{\mu^2}{s}\right)^{\varepsilon}\left(\frac{\mu^2}{Q_f^2}\right)^{\varepsilon}\frac{\alpha N_c}{2\pi}\left\{\frac{4}{\varepsilon}\left[\frac{32(79-25c^2)}{3(1-c^2)^2)}\right.\right.$$
$$\left.\left.+\frac{64(3-c)^2}{(1-c)(1+c)^3})\log(\frac{1-c}{2}) + \frac{64(3+c)^2}{(1-c)^3(1+c)}\log(\frac{1+c}{2})\right] + \text{Finite part}\right\}$$

- Sfermions

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Infrared-free sets (for any arbitrary δ)

- $$A^{MHV} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--++)}_{Virt} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{Real} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{InSplit} + \left(\frac{d\sigma}{d\Omega_{14}}\right)^{(--+++)}_{FnSplit}$$

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- Registration of two fastest gluons of positive chirality

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- Registration of one fastest gluon of positive chirality

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$$B^{AntiMHV}\Big|_{\delta=1} + C^{Matter}\Big|_{\delta=1} \quad \Rightarrow \text{Finite Part}$$

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The simplest IR finite answer so far ($Q_f = E$): N=4 SYM Anti MHV

$$\left(\frac{d\sigma}{d\Omega_{14}}\right)_{AntiMHV} = \frac{\alpha^2 N_c^2}{E^2}\left\{\frac{3+c^2}{(1-c^2)^2}\right.$$
$$-\frac{\alpha N_c}{2\pi}\left[2\frac{(c^4+2c^3+4c^2+6c+19)\log^2(\frac{1-c}{2})}{(1-c)^2(1+c)^4} + 2\frac{(c^4-2c^3+4c^2-6c+19)\log^2(\frac{1+c}{2})}{(1-c)^4(1+c)^2}\right.$$
$$-8\frac{(c^2+1)\log(\frac{1+c}{2})\log(\frac{1-c}{2})}{(1-c^2)^2} - \frac{6\pi^2(c^2-1)+5(61c^2+99)}{9(1-c^2)^2}$$
$$\left.\left.+2\frac{(11c^3+31c^2-47c+133)\log(\frac{1+c}{2})}{3(1-c)^3(1+c)^2} - 2\frac{(11c^3-31c^2-47c-133)\log(\frac{1-c}{2})}{3(1+c)^3(1-c)^2}\right]\right\}$$

Summary

- In observable cross-sections the IR divergences do cancel in accordance with Kinoshita-Lee-Nauenberg theorem

$$d\sigma_{obs}^{incl} = \sum_{n=2}^{\infty} \int_0^1 dz_1 q_1(z_1, \frac{Q_f^2}{\mu^2}) \int_0^1 dz_2 q_2(z_2, \frac{Q_f^2}{\mu^2}) \prod_{i=1}^{n} \int_0^1 dx_i q_i(x_i, \frac{Q_f^2}{\mu^2}) \times \times d\sigma^{2\to n}(z_1 p_1, z_2 p_2)$$

- The simple structure of the MHV amplitude DOES NOT reveal at the level of IR finite cross-sections;
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