Two-loop decoupling corrections to the masses of heavy SM fermions within the MSSM

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Outline

1. Introduction
   - MSSM
   - Heavy SM fermion masses
   - $\overline{\mathrm{DR}}$-scheme
2. Pole mass corrections
   - Diagrams
   - Asymptotic expansion
   - Results
3. Running b-quark mass in MSSM
4. Decoupling of heavy particles
   - Decoupling constants
   - $\overline{\mathrm{DR}} \to \overline{\mathrm{MS}}$ transition
5. Results
6. Summary

Minimal Supersymmetric Standard Model

Gauge

	Bosons		Fermions		Quantum numbers
$\hat{G}$	gluons	G^a_μ	gluinos	$\tilde{g}^a$	(8,1,0)
$\hat{V}$	$SU(2)$-bosons	A^i_μ	gauginos	$\tilde{A}^i$	(1,3,0)
$\hat{V}'$	$U(1)$-bosons	B_μ	gauginos	$\tilde{B}$	(1,1,0)

Matter

	Fermions		Bosons		
$\hat{Q}$	quarks	$Q = \binom{u}{d}_L$	squarks	$\tilde{Q} = \binom{\tilde{u}}{\tilde{d}}_L$	(3,2,1/3)
$\hat{U}$		$U = u_R$		$\tilde{U} = \tilde{u}_R$	(3,1,4/3)
$\hat{D}$		$D = d_R$		$\tilde{D} = \tilde{d}_R$	(3,1,-2/3)
$\hat{L}$	leptons	$E = \binom{\nu_l}{l}_L$	sleptons	$\tilde{E} = \binom{\tilde{\nu}_l}{\tilde{l}}_L$	(1,2,-1)
$\hat{E}$		$E = l_R$		$\tilde{E} = \tilde{l}_R$	(1,1,-2)
$\hat{H}_1$	higgsino	$(\tilde{H}^0_1, \tilde{H}^-_1)$	higgses	$H_1 = \binom{H^0_1}{H^-_1}$	(1,2,-1)
$\hat{H}_2$		$(\tilde{H}^+_2, \tilde{H}^0_2)$		$H_2 = \binom{H^+_2}{H^0_2}$	(1,2,1)

Minimal Supersymmetric Standard Model

MSSM has the following set of free parameters

1. Three gauge couplings α_i
2. Three Yukawa matrices y_u, y_d, y_l
3. Parameter $\bar{\mu}$ (mixing of higgs superfields)
4. Soft (dim < 4) supersymmetry breaking terms

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   - m_0 – a common mass for squarks and sleptons
   - A_0 – trilinear squark-squark-higgs and slepton-slepton-higgs couplings
   - B – higgs field mixing parameter

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$$y_t = \frac{\sqrt{2} m_t}{v_2}$$

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$$y_\tau = \frac{\sqrt{2} m_\tau}{v_1}$$

Heavy SM fermions

Data from Particle Data Group'08

- Top quark

$$M_t = 171.2 \pm 2.1 \text{ GeV}$$

- Bottom quark

$$m_b = 4.20 \pm 0.18 \text{ GeV}$$

- Tau lepton

$$M_\tau = 1776.84 \pm 0.17 \text{ MeV}$$

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Pole mass of a fermion. Some definitions

Fermion ressumed propagator

$$i\left(\hat{p} - m - \Sigma(\hat{p}, m_i)\right)^{-1}$$

Fermion self-energy

$$\Sigma(\hat{p}, m_i) = \hat{p}\Sigma_V(p^2, m_i^2) + \hat{p}\gamma_5\Sigma_A(p^2, m_i^2) \\ + m\Sigma_S(p^2, m_i)$$

The pole mass M satisfies the following equation:

$$\left(\left(1 - \Sigma_V(M^2, m_i^2)\right)^2 - \Sigma_A^2(M^2, m_i^2)\right) M^2 \\ - m^2\left(1 + \Sigma_S(M^2, m_i)\right)^2 = 0$$

Pole mass of a fermion. Some definitions

$$M = m\left(1 + \alpha M^{(1)} \qquad \right),$$

$$M^{(1)} = \Sigma_V^{(1)}(m^2, m_i^2) + \Sigma_S^{(1)}(m^2, m_i),$$

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\begin{aligned}
M &= m\left(1+\alpha M^{(1)}+\alpha^2 M^{(2)}\right), \\
M^{(1)} &= \Sigma_V^{(1)}(m^2, m_i^2) + \Sigma_S^{(1)}(m^2, m_i), \\
M^{(2)} &= \Sigma_V^{(2)}(m^2, m_i^2) + \Sigma_S^{(2)}(m^2, m_i) \\
&+ \frac{1}{2}{\Sigma_A^{(1)}}^2(m^2, m_i^2) + M^{(1)}\left(\Sigma_V^{(1)}(m^2, m_i^2)\right. \\
&+ \left. 2m^2 \frac{\partial}{\partial p^2}\left(\Sigma_V^{(1)}(p^2, m_i^2) + \Sigma_S^{(1)}(p^2, m_i)\right)\right)_{p^2=m^2}
\end{aligned}
$$

Quark pole mass

The pole mass of a quark is a well-defined, IR safe,, and gauge independent quantity in a finite order of perturbaation theory [Tarrach,'80].
Nevertheless, due to confinement the precision of experimental determination of the quark pole mass is limited by the ratio $\Lambda_{\rm QCD}/M_q$ [Bigi et al,'94], [Beneke&Braun,'94].
For the t-quark the uncertainty $\sim 0.05\%$, is much less then current experimental errors [PDG,'08]
For the b-quark the uncertainty is significant $\sim 10\%$.

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$\overline{\mathrm{DR}}$ renormalization scheme

- Regularization: dimensional reduction (DRED) [Siegel,'79-80],[Avdeev,Chochia&Vladimirov,'81],[Stockinger,'05]
- Renormalization: minimal (modified) subtractions, $\overline{\mathrm{DR}}$.

$$
\begin{aligned}
g^{\mu\nu} &= \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu} & \qquad g^{\mu\nu} g_{\mu\nu} &= 4 \\
g^{\mu\nu} \hat{g}^{\rho}_{\nu} &= \hat{g}^{\mu\rho} & \qquad \hat{g}^{\mu\nu} \hat{g}_{\mu\nu} &= d \\
g^{\mu\nu} \tilde{g}^{\rho}_{\nu} &= \tilde{g}^{\mu\rho} & \qquad \tilde{g}^{\mu\nu} \tilde{g}_{\mu\nu} &= 2\varepsilon = 4 - d \\
\hat{g}^{\mu\nu} \tilde{g}^{\rho}_{\nu} &= 0
\end{aligned}
$$

All fields in 4D Lagrangian depend only on $\hat{x}_{\mu} = \hat{g}_{\mu\nu} x^{\nu}$.

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$$G_\mu^{(4)} \rightarrow \left\{ G_\mu \equiv \hat{g}_{\mu\nu} G_{(4)}^\nu, W_\sigma \equiv \tilde{g}_{\sigma\rho} G_{(4)}^\rho \right\}$$

G_μ vector fields (gauge bosons),

W_σ unphysical ε-scalars.

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$$
\begin{aligned}
\mathcal{L}_B^\varepsilon &= \frac{1}{2}(D_\mu W_\sigma)_a^+ (D^\mu W_\sigma)^a \\
&- \frac{g_s^2}{4} f^{abc} f^{ade} W_\sigma^b W_\rho^c W_\sigma^d W_\rho^e - g_s \bar{q} \gamma^\sigma T^a q W_\sigma^a \\
&+ i \frac{g_s}{2} f^{abc} \overline{\tilde{g}^a} \gamma_\sigma \tilde{g}^b W_\sigma^c + g_s^2 \tilde{q}^* T^a T^b \tilde{q}\, W_\sigma^a W_\sigma^b
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$$\begin{aligned}
\mathcal{L}_B^{\varepsilon} &= \frac{1}{2}(D_\mu W_\sigma)_a^{+}(D^\mu W_\sigma)^a + \frac{1}{2} m_\varepsilon^2 W_\sigma^a W_a^\sigma \\
&- \frac{g_s^2}{4} f^{abc} f^{ade} W_\sigma^b W_\rho^c W_\sigma^d W_\rho^e - g_s \overline{q} \gamma^\sigma T^a q W_\sigma^a \\
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The problem with ε-scalar mass m_ε^2, can be solved in two equivalent ways ($\overline{\mathrm{DR}}'$).

- Minimal subtractions, but $m_\varepsilon^2 \neq 0$, with corresponding redefinition of masses of scalar superpartners [Jack et al'94].
- Non-minimal subtractions, $m_\varepsilon^2 = 0$ [Avdeev&Kalmykov,'97]

Pole mass of a fermion. Some definitions

$$
\begin{aligned}
M &= m\left(1+\alpha M^{(1)}+\alpha^2 M^{(2)}\right), \quad m \equiv m^{\overline{\mathrm{DR}}}(\mu), \alpha \equiv \alpha^{\overline{\mathrm{DR}}}(\mu) \\
M^{(1)} &= \Sigma_V^{(1)}(m^2, m_i^2) + \Sigma_S^{(1)}(m^2, m_i), \\
M^{(2)} &= \Sigma_V^{(2)}(m^2, m_i^2) + \Sigma_S^{(2)}(m^2, m_i) \\
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&+ \left.\left. 2m^2 \frac{\partial}{\partial p^2}\left(\Sigma_V^{(1)}(p^2, m_i^2) + \Sigma_S^{(1)}(p^2, m_i)\right)\right)\right|_{p^2=m^2}
\end{aligned}
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- Bottom quark

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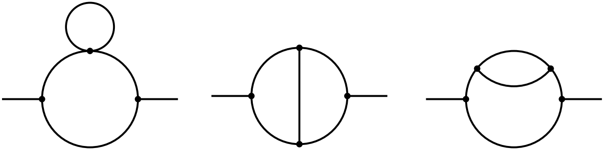
- Tau lepton

$$M_\tau = 1776.84 \pm 0.17 \text{ MeV} \qquad \Rightarrow m_\tau^{\overline{\rm DR}}(\bar{\mu})$$

Feynman diagams

We need to calculate (159, 1897, 1375) two-loop propagator-type diagrams

(Generated by FeynArts)



(63, 453, 495) (16, 304, 124) (80, 1140, 756)

Calculated by means of FORM program based on ONSHELL2.

Minimal Supersymmetric Standard Model

Gauge

	Bosons		Fermions		Quantum numbers
$\hat{G}$	gluons	G^a_μ	gluinos	$\tilde{g}^a$	(8,1,0)
$\hat{V}$	$SU(2)$-bosons	A^i_μ	gauginos	$\tilde{A}^i$	(1,3,0)
$\hat{V}'$	$U(1)$-bosons	B_μ	gauginos	$\tilde{B}$	(1,1,0)

Matter

	Fermions		Bosons		
$\hat{Q}$	quarks	$Q = \binom{u}{d}_L$	squarks	$\tilde{Q} = \binom{\tilde{u}}{\tilde{d}}_L$	(3,2,1/3)
$\hat{U}$		$U = u_R$		$\tilde{U} = \tilde{u}_R$	(3,1,4/3)
$\hat{D}$		$D = d_R$		$\tilde{D} = \tilde{d}_R$	(3,1,-2/3)
$\hat{L}$	leptons	$E = \binom{\nu_l}{l}_L$	sleptons	$\tilde{E} = \binom{\tilde{\nu}_l}{\tilde{l}}_L$	(1,2,-1)
$\hat{E}$		$E = l_R$		$\tilde{E} = \tilde{l}_R$	(1,1,-2)
$\hat{H}_1$	higgsino	$(\tilde{H}^0_1, \tilde{H}^-_1)$	higgses	$H_1 = \binom{H^0_1}{H^-_1}$	(1,2,-1)
$\hat{H}_2$		$(\tilde{H}^+_2, \tilde{H}^0_2)$		$H_2 = \binom{H^+_2}{H^0_2}$	(1,2,1)

Large Mass Expansion

Mass hierachy:

- For t quark: $m_t \ll m_{\tilde{g}}, m_{\tilde{q}}$
- For b quark: $m_b \ll m_t, m_{\tilde{g}}, m_{\mathcal{H}}, m_{\tilde{q}}, m_{\tilde{\chi}}$
- For τ lepton: $m_\tau \ll m_t, m_{\mathcal{H}}, m_{\tilde{q}}, m_{\tilde{l}}, m_{\tilde{\chi}}$

Asymptotic expansion has the form (see e.g.,[Tkachov,'97], [Smirnov,'02]):

$$F_\Gamma(p_1, \ldots, M_1, \ldots, m_1, \ldots) \simeq \sum_\gamma F_{\Gamma/\gamma}(p, m) \bullet \mathcal{M}_\gamma(p_\gamma, m) F_\gamma(M, m, p_\gamma).$$

The operator $\mathcal{M}_\gamma$ perform Taylor expansion of F_γ in small (with respect to subgraph γ) momenta and masses. The summation is over all *asymptotically* irreducible subgraphs of Γ.

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- For b quark: $m_b \ll m_t, m_{\tilde{g}}, m_{\mathcal{H}}, m_{\tilde{q}}, m_{\tilde{\chi}}$
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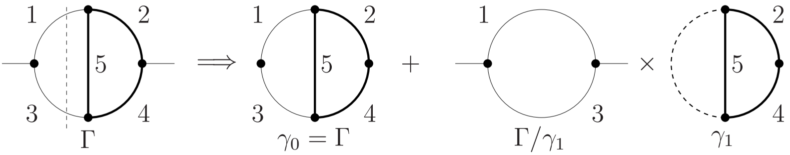

Two contributions to an asymptotic expansion:

1. Taylor expansion of the integrand ($\log M_{\text{hard}}$, but no $\log m_f$)
2. subgraph contribution that
   - cancel spurious IR poles (in dimensional regularization)
   - restore non-analytical dependence of the result on "small" parameters ($\log m_f$ and/or $\log M_{\text{hard}}$).

Pole masses of heavy SM fermions in MSSM

$$\frac{M - m_f^{\overline{\mathrm{DR}}}(M_Z)}{m_q^{\overline{\mathrm{DR}}}(M_Z)} \equiv \frac{\Delta m_f}{m_f} =$$

$$\ldots \alpha_s + \ldots \alpha_f + \ldots g^2 + \ldots g'^2$$ [Pierce et al,'97]

$$+ \ldots \alpha_s^2$$ [Bednyakov et al,'02]

[Bednyakov et al,05]

$$+ \ldots \alpha_s \alpha_f + \ldots \alpha_f^2$$ [Bednyakov&Sheplyakov,'04]

[Bednyakov'09]

For t-quark we calculated $\mathcal{O}(\alpha_s^2)$ corrections, for b-quark and τ-lepton two-loop corrections include ones proportional to

$$\alpha_f \equiv \frac{y_f^2}{4\pi}$$

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$$\ldots \alpha_s + \ldots \alpha_f + \ldots g^2 + \ldots g'^2 \quad 5\text{–}8\ \%,\ 20\text{–}25\ \%,\ \text{-}(2\text{-}10)\ \%$$

$$+ \ldots \alpha_s^2 \quad 2\text{–}3\ \%,\ 10\text{–}20\ \%$$

$$+ \ldots \alpha_s \alpha_f + \ldots \alpha_f^2 \quad -(5\text{–}15)\ \%,\ -(0.02\text{–}2)\ \%$$

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Running mass of b quark

MSSM, $\overline{\mathrm{DR}}$-scheme

$m_b^{\overline{\mathrm{DR}}}(\mu)$ running mass $m_b^{\overline{\mathrm{DR}}} = y_b^{\overline{\mathrm{DR}}} v_1^{\overline{\mathrm{DR}}} / \sqrt{2}$

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Pole mass M_b

$$M_b = 4.19^{+0.79}_{-0.91} \text{ GeV [DELPHI,'06]}$$

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Pole mass M_b

M_b depends both on $\log m_b/\mu$ and $\log M_{\mathrm{hard}}/\mu$
decoupling theorem [Appelquist&Carazzone,'74]
"does not" hold

Running mass of b quark

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$m_b^{\overline{\mathrm{DR}}}(\mu)$ running mass $m_b^{\overline{\mathrm{DR}}} = y_b^{\overline{\mathrm{DR}}} v_1^{\overline{\mathrm{DR}}}/\sqrt{2}$

↓ Effective theory, $E \ll M_{\mathrm{hard}}$

QCD, $\overline{\mathrm{MS}}$-scheme

$m_b^{\overline{\mathrm{MS}}}(\mu)$ running mass, $m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}}) = 4.20 \pm 0.18$ GeV [PDG,'08].

Decoupling of heavy particles

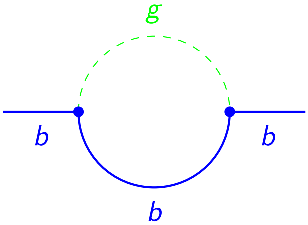
MSSM, $\overline{\mathrm{DR}}$-scheme. up to $\mathcal{O}(m_b^2/M_{\text{hard}}^2) \sim 0.1\ \%$

$$M_b = m_b \left(1 \right)$$

Decoupling of heavy particles

MSSM, $\overline{\mathrm{DR}}$-scheme. up to $\mathcal{O}(m_b^2/M^2_{\text{hard}}) \sim 0.1\ \%$

$$M_b = m_b \left(1 + \frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right) \right)$$



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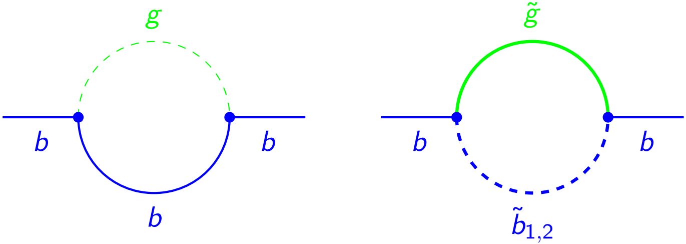
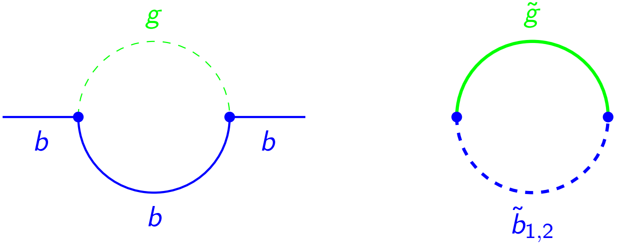


Decoupling of heavy particles

MSSM, $\overline{\mathrm{DR}}$-scheme. up to $\mathcal{O}(m_b^2/M_{\text{hard}}^2) \sim 0.1$ %

$$M_b = m_b \left(1 + \frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right) + \frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right)$$



Decoupling of heavy particles

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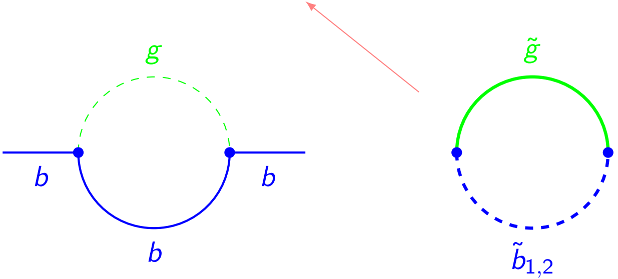
$$M_b = m_b\left(1+\frac{\alpha_s}{3\pi}\left(4-3\log\frac{m_b^2}{\mu^2}\right)+\frac{\alpha}{4\pi}\left(c_1+c_2\log\frac{M_{\mathrm{hard}}^2}{\mu^2}\right)\right)$$

$$= m_b\left[1+\frac{\alpha}{4\pi}\left(c_1+c_2\log\frac{M_{\mathrm{hard}}^2}{\mu^2}\right)\right]\left[1+\frac{\alpha_s}{3\pi}\left(4-3\log\frac{m_b^2}{\mu^2}\right)\right]$$



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$$
\begin{aligned}
M_b &= m_b \left(1+\frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right)+\frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right) \\
&= m_b \left[1 + \frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right]\left[1 + \frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right)\right]
\end{aligned}
$$

QCD, $\overline{\mathrm{MS}}$-scheme

$$
M_b = m_b \left(1 + \frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right)\right)
$$

Decoupling of heavy particles

MSSM, $\overline{\mathrm{DR}}$-scheme. up to $\mathcal{O}(m_b^2/M_{\text{hard}}^2) \sim 0.1$ %

$$
\begin{aligned}
M_b &= m_b \left(1+\frac{\alpha_s}{3\pi}\left(4-3\log\frac{m_b^2}{\mu^2}\right)+\frac{\alpha}{4\pi}\left(c_1+c_2\log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right) \\
&= m_b \left[1+\frac{\alpha}{4\pi}\left(c_1+c_2\log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right]\left[1+\frac{\alpha_s}{3\pi}\left(4-3\log\frac{m_b^2}{\mu^2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
m_b^{\overline{\mathrm{MS}}}(\mu) &= m_b^{\overline{\mathrm{DR}}}(\mu)\left(1+\frac{\alpha^{\overline{\mathrm{DR}}}(\mu)}{4\pi}\left(c_1+c_2\log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right) \\
\alpha_s^{\overline{\mathrm{MS}}}(\mu) &= \alpha_s^{\overline{\mathrm{DR}}}(\mu)
\end{aligned}
$$

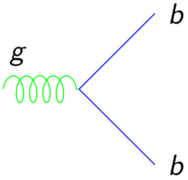
Decoupling constants

$$
\begin{aligned}
m_b^{\overline{\mathrm{MS}}}(\mu) &= m_b^{\overline{\mathrm{DR}}}(\mu)\left(1+\frac{\alpha^{\overline{\mathrm{DR}}}}{4\pi}\left(c_1+c_2\log\frac{M_{\mathrm{hard}}^2}{\mu^2}\right)\right)\\
\alpha_s^{\overline{\mathrm{MS}}}(\mu) &= \alpha_s^{\overline{\mathrm{DR}}}(\mu)
\end{aligned}
$$

Decoupling constants

$$m_b^{\overline{\mathrm{MS}}}(\mu) = m_b^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathsf{hard}}, \mu)$$

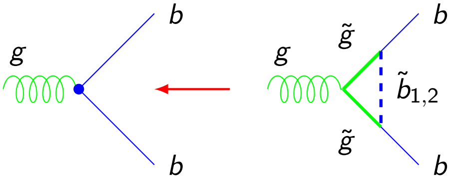
$$\alpha_s^{\overline{\mathrm{MS}}}(\mu) = \alpha_s^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathsf{hard}}, \mu)$$



Decoupling constants

$$m_b^{\overline{\mathrm{MS}}}(\mu) = m_b^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathrm{hard}}, \mu)$$

$$\alpha_s^{\overline{\mathrm{MS}}}(\mu) = \alpha_s^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathrm{hard}}, \mu)$$



Decoupling constants

$$m_b^{\overline{\mathrm{MS}}}(\mu) = m_b^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathrm{hard}}, \mu)$$

$$\alpha_s^{\overline{\mathrm{MS}}}(\mu) = \alpha_s^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathrm{hard}}, \mu)$$

We calculate two-loop contribution to ζ_{m_b} proportional to

$$\alpha^{\overline{\mathrm{DR}}} = \left\{ \alpha_s, \alpha_b \equiv \frac{y_b^2}{4\pi}, \alpha_t \equiv \frac{y_t^2}{4\pi} \right\}^{\overline{\mathrm{DR}}}.$$

We have

$$M_{\mathrm{hard}} = \{m_t, m_{\tilde{q}}, m_{\tilde{g}}, m_{\tilde{\chi}}, m_{\mathcal{H}}\}.$$

Decoupling constants

$$m_b^{\overline{\rm MS}}(\mu) = m_b^{\overline{\rm DR}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\rm DR}}(\mu), M_{\rm hard}, \mu)$$

$$\alpha_s^{\overline{\rm MS}}(\mu) = \alpha_s^{\overline{\rm DR}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\rm DR}}(\mu), M_{\rm hard}, \mu)$$

Decoupling constants have the following perturbative expansion

$$\zeta_{m_b} = 1 + \alpha\, \delta\zeta_{m_b}^{(1)} + \alpha^2\, \delta\zeta_{m_b}^{(2)} + \cdots$$

$$\zeta_{\alpha_s} = 1 + \alpha\, \delta\zeta_{\alpha_s}^{(1)} + \alpha^2\, \delta\zeta_{\alpha_s}^{(2)} + \cdots$$

$$M_b^{\rm MSSM}(m_b^{\overline{\rm DR}}, \alpha_s^{\overline{\rm DR}}, \cdots) = M_b^{\rm QCD}(m_b^{\overline{\rm MS}}, \alpha_s^{\overline{\rm MS}}) + \mathcal{O}\left(\frac{m_b^2}{M_{\rm hard}^2}\right)$$

Decoupling constants

$$m_b^{\overline{\rm MS}}(\mu) = m_b^{\overline{\rm DR}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\rm DR}}(\mu), M_{\rm hard}, \mu)$$

$$\alpha_s^{\overline{\rm MS}}(\mu) = \alpha_s^{\overline{\rm DR}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\rm DR}}(\mu), M_{\rm hard}, \mu)$$

$$\begin{aligned} m_b^{\overline{\rm DR}} \delta\zeta_{m_b}^{(n)} &= \left[M_b^{\rm MSSM}(m_b^{\overline{\rm DR}}, \alpha_s^{\overline{\rm DR}}, \cdots) \right]^{(n)} \\ &- \left[M_b^{\rm QCD}(m_b^{\overline{\rm DR}} \zeta_{m_b}^{(n-1)}, \alpha_s^{\overline{\rm DR}} \zeta_{\alpha_s}^{(n-1)}) \right]^{(n)}, \end{aligned}$$

where

$$\zeta^{(n)} = 1 + \sum_{i=1}^{n} \alpha^i \delta\zeta^{(i)}$$

Decoupling constants

$$m_b^{\overline{\rm MS}}(\mu) = m_b^{\overline{\rm DR}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\rm DR}}(\mu), M_{\rm hard}, \mu)$$

$$\alpha_s^{\overline{\rm MS}}(\mu) = \alpha_s^{\overline{\rm DR}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\rm DR}}(\mu), M_{\rm hard}, \mu)$$

$$\begin{aligned} m_b^{\overline{\rm DR}} \delta\zeta_{m_b}^{(2)} &= \left[M_b^{\rm MSSM}(m_b^{\overline{\rm DR}}, \alpha_s^{\overline{\rm DR}}, \cdots) \right]^{(2)} \\ &- \left[M_b^{\rm QCD}(m_b^{\overline{\rm DR}} \zeta_{m_b}^{(1)}, \alpha_s^{\overline{\rm DR}} \zeta_{\alpha_s}^{(1)}) \right]^{(2)}, \end{aligned}$$

where

$$\zeta^{(n)} = 1 + \sum_{i=1}^{n} \alpha^i \delta\zeta^{(i)}$$

Decoupling constants

$$m_b^{\overline{\mathrm{MS}}}(\mu) = m_b^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{m_b}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathrm{hard}}, \mu)$$

$$\alpha_s^{\overline{\mathrm{MS}}}(\mu) = \alpha_s^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{\alpha_s}(\alpha^{\overline{\mathrm{DR}}}(\mu), M_{\mathrm{hard}}, \mu)$$

Decoupling constants have the following perturbative expansion

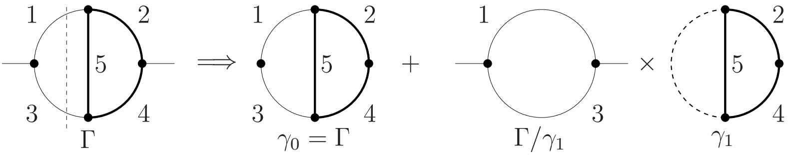
$$\zeta_{m_b} = 1 + \alpha\, \delta\zeta_{m_b}^{(1)} + \alpha^2\, \delta\zeta_{m_b}^{(2)} + \cdots$$
$$\zeta_{\alpha_s} = 1 + \alpha\, \delta\zeta_{\alpha_s}^{(1)} + \alpha^2\, \delta\zeta_{\alpha_s}^{(2)} + \cdots$$

$$M_b^{\mathrm{MSSM}}(m_b^{\overline{\mathrm{DR}}}, \alpha_s^{\overline{\mathrm{DR}}}, \cdots) = M_b^{\mathrm{QCD}}(m_b^{\overline{\mathrm{MS}}}, \alpha_s^{\overline{\mathrm{MS}}}) + \mathcal{O}\left(\frac{m_b^2}{M_{\mathrm{hard}}^2}\right)$$

Large Mass Expansion

Mass hierachy:

- For t quark: $m_t \ll m_{\tilde{g}}, m_{\tilde{q}}$
- For b quark: $m_b \ll m_t, m_{\tilde{g}}, m_{\mathcal{H}}, m_{\tilde{q}}, m_{\tilde{\chi}}$
- For τ lepton: $m_\tau \ll m_t, m_{\mathcal{H}}, m_{\tilde{q}}, m_{\tilde{l}}, m_{\tilde{\chi}}$



Two contributions to an asymptotic expansion:

1. Taylor expansion of the integrand ($\log M_{\text{hard}}$, but no $\log m_f$)
2. subgraph contribution that
   - cancel spurious IR poles (in dimensional regularization)
   - restore non-analytical dependence of the result on "small" parameters ($\log m_f$ and/or $\log M_{\text{hard}}$).

Optimal strategy

$$m_b^{\overline{\rm MS}}(\mu = m_b^{\overline{\rm MS}})$$

(4.20 GeV)

No large logs in the relation between $m_b^{\overline{\rm MS}}$ and $m_b^{\overline{\rm DR}}$!

Optimal strategy

$$m_b^{\overline{\rm MS}}(\mu = m_b^{\overline{\rm MS}}) \xrightarrow[\text{QCD}]{\text{RGE}} m_b^{\overline{\rm MS}}(\mu = M_{\rm hard})$$

(4.20 GeV) (2.87 GeV при $\mu = 91$ GeV)

No large logs in the relation between $m_b^{\overline{\rm MS}}$ and $m_b^{\overline{\rm DR}}$!

Optimal strategy

$$m_b^{\overline{\rm DR}}(\mu = M_{\sf hard})$$

$$m_b^{\overline{\rm MS}}(\mu) = m_b^{\overline{\rm DR}}(\mu)\left(1 + \frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\sf hard}}{\mu}\right)\right)$$

$$m_b^{\overline{\rm MS}}(\mu = m_b^{\overline{\rm MS}}) \xrightarrow[\text{QCD}]{\text{RGE}} m_b^{\overline{\rm MS}}(\mu = M_{\sf hard})$$

(4.20 GeV) (2.87 GeV при $\mu = 91$ GeV)

No large logs in the relation between $m_b^{\overline{\rm MS}}$ and $m_b^{\overline{\rm DR}}$!

Optimal strategy

$$m_b^{\overline{\rm DR}}(\mu = M_{\rm hard})$$

$$m_b^{\overline{\rm MS}}(M_{\rm hard}) = m_b^{\overline{\rm DR}}(M_{\rm hard})\left(1 + \frac{\alpha}{4\pi} c_1\right)$$

$$m_b^{\overline{\rm MS}}(\mu = m_b^{\overline{\rm MS}}) \xrightarrow[\text{QCD}]{\text{RGE}} m_b^{\overline{\rm MS}}(\mu = M_{\rm hard})$$

(4.20 GeV) (2.87 GeV при $\mu = 91$ GeV)

No large logs in the relation between $m_b^{\overline{\rm MS}}$ and $m_b^{\overline{\rm DR}}$!

Optimal strategy

$$m_b^{\overline{\mathrm{DR}}}(\mu) \xleftarrow[\text{MSSM}]{\text{RGE}} m_b^{\overline{\mathrm{DR}}}(\mu = M_{\mathrm{hard}})$$

$$m_b^{\overline{\mathrm{MS}}}(M_{\mathrm{hard}}) = m_b^{\overline{\mathrm{DR}}}(M_{\mathrm{hard}}) \left(1 + \frac{\alpha}{4\pi} c_1\right)$$

$$m_b^{\overline{\mathrm{MS}}}(\mu = m_b^{\overline{\mathrm{MS}}}) \xrightarrow[\text{QCD}]{\text{RGE}} m_b^{\overline{\mathrm{MS}}}(\mu = M_{\mathrm{hard}})$$

(4.20 GeV) — (2.87 GeV при $\mu = 91$ GeV)

No large logs in the relation between $m_b^{\overline{\mathrm{MS}}}$ and $m_b^{\overline{\mathrm{DR}}}$!

Optimal strategy

$$m_b^{\overline{\rm DR}}(\mu) \xleftarrow[\text{MSSM}]{\text{RGE}} m_b^{\overline{\rm DR}}(\mu = M_{\text{hard}})$$

$$\updownarrow$$

$$m_b^{\overline{\rm MS}}(M_{\text{hard}}) = m_b^{\overline{\rm DR}}(M_{\text{hard}}) \left(1 + \frac{\alpha}{4\pi} c_1\right)$$

$$m_b^{\overline{\rm MS}}(\mu = m_b^{\overline{\rm MS}}) \xrightarrow[\text{QCD}]{\text{RGE}} m_b^{\overline{\rm MS}}(\mu = M_{\text{hard}})$$

(4.20 GeV) (2.87 GeV при $\mu = 91$ GeV)

No large logs in the relation between $m_b^{\overline{\rm MS}}$ and $m_b^{\overline{\rm DR}}$!

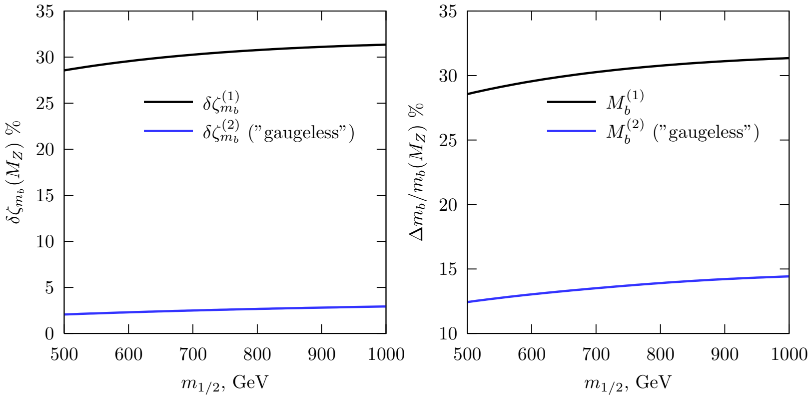
$\overline{\mathrm{DR}} \to \overline{\mathrm{MS}}$ transition

- Since ε-scalar mass m_ε is an unphysical parameter one can take it to be infinitely large, $m_\varepsilon \gg m_b, M_{\text{hard}}$. One can formally decouple ε-scalars together with superpartners. This trick allows one to make a simultaneous transition from MSSM to QCD and from $\overline{\mathrm{DR}}$-scheme to $\overline{\mathrm{MS}}$ [Bednyakov,'07].
- There is another ("standard") way [Harlander et al,'06]. The transition from $\overline{\mathrm{DR}}$ to $\overline{\mathrm{MS}}$ is realized by two (independent?) steps:

$$\begin{array}{ccccc} (\text{MSSM}, \ \overline{\mathrm{DR}}) & \xrightarrow{(1)} & (\text{QCD}, \ \overline{\mathrm{DR}}) & \xrightarrow{(2)} & (\text{QCD}, \ \overline{\mathrm{MS}}). \\ (\alpha_s, m_b, M_{\text{SUSY}}, ...) & \longrightarrow & (\alpha_s, m_b, \alpha_y, \lambda) & \longrightarrow & (\alpha_s, m_b) \end{array}$$

Results

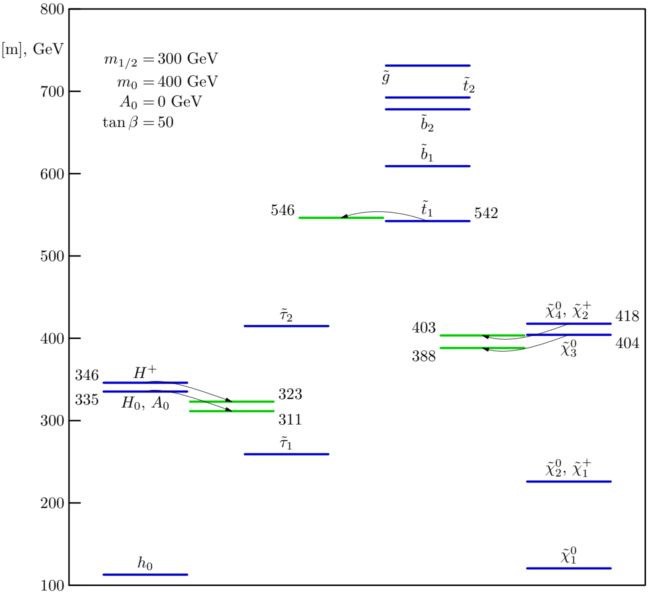
$$m_b^{\overline{\rm MS}} = m_b^{\overline{\rm DR}}\left(1 + \delta\zeta_{m_b}^{(1)} + \delta\zeta_{m_b}^{(2)}\right), \quad M_b = m_b^{\overline{\rm DR}}\left(1 + \tilde{M}_b^{(1)} + \tilde{M}_b^{(2)}\right)$$



CMSSM parameters: $\tan\beta = 50$, $A_0 = 0$, $\bar{\mu} > 0$, $m_0 = 1000$ GeV
Masses and couplings at M_Z are obtained with the help of [SoftSusy].

Results

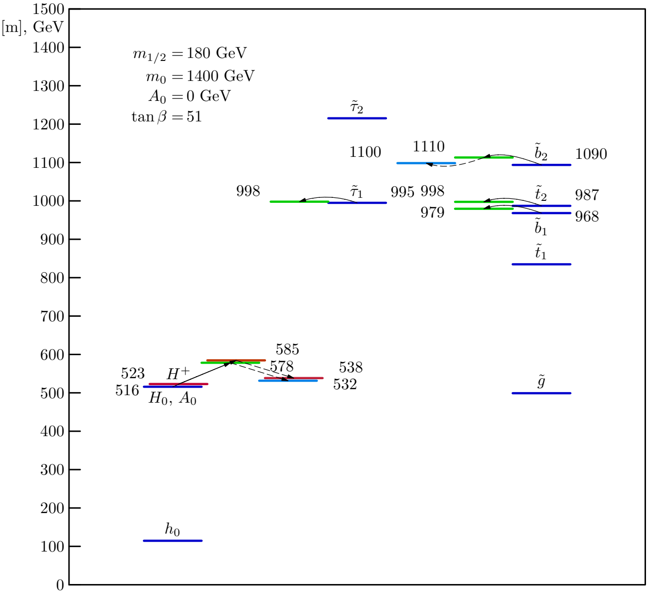
t-quark



The task is to incorporate the correction into computer codes, e.g. in [SoftSusy], [ffmssmsc].

Results

b-quark



The task is to incorporate the correction into computer codes, e.g. in [SoftSusy], [ffmssmsc].

Conclusions

Main results include

1. Application of the effective theory approach both to the problem of decoupling of heavy particles and to the problem of $\overline{\mathrm{DR}} \to \overline{\mathrm{MS}}$ transition (decoupling of ε-scalars).
2. Calculation of $\mathcal{O}(\alpha_s^2)$ contribution to the relation between the running mass of the t quark in MSSM and the pole mass M_t
3. Calculation of $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s\alpha_f + \alpha_f^2)$ contribution to the relation between the running mass of the b quark in MSSM and QCD $m_b^{\overline{\mathrm{MS}}}(\mu) = m_b^{\overline{\mathrm{DR}}}(\mu) \times \zeta_{m_b}(\mu)$.
4. Calculation of $\mathcal{O}(\alpha_f^2)$ contribution to the relation between the running mass of the τ lepton in MSSM and its pole mass M_τ

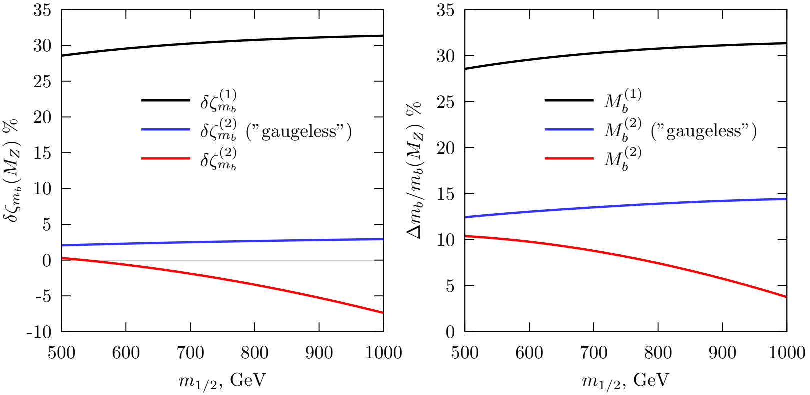
Conclusions

Issues:

1. We neglect electroweak gauge couplings. Masses of the would-be Goldstone bosons G are gauge-parameter dependent in a linear R_ξ gauge. Numerical analysis shows that different prescriptions for the Goldstone boson masses $M_G = M_{W,Z}$ and $M_G = 0$ gives different results in a region of large $m_{1/2}$
2. Optimization of the computer code is needed

Gauge dependence of the two-loop yukawa corrections

Blue curve corresponds to $M_G = 0$ (Landau gauge),
Red one — to $M_G = M_{W,Z}$ (Feynman gauge)



CMSSM parameters: $\tan\beta = 50$, $A_0 = 0$, $\bar{\mu} > 0$, $m_0 = 1000$ GeV

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