Atomic Bose-Einstein Condensates, with optical lattices, Field-induced dipole moments and spin-orbit coupling

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1. Introduction
2. 3D BEC with linear and nonlinear optical lattices
3. Bright solitons in BEC with field-induced dipole moments
4. Bright solitons: existence and stability
5. Dynamics of bright solitons
6. Spin-Orbit Coupling in Ultracold-atom systems
7. General conclusions

Outline

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4 Bright solitons: existence and stability

5 Dynamics of bright solitons

6 Spin-Orbit Coupling in Ultracold-atom systems

7 General conclusions

BEC and universal aspects of few-body systems

- Our investigations are mainly concerned with two aspects of atomic systems:
  – Stability and dynamics of Bose-Einstein condensates (BEC) under different trap conditions.
  – Universal aspects of quantum few-body systems, which can emerge from such studies.
  In the second case, a larger group is involved. I will mention the leading ones, which can be contacted for those interested in our pos-docs, visitors or graduate programs: Tobias Frederico (ITA, São José dos Campos), Marcelo Yamashita (IFT-UNESP, São Paulo), Antonio Delfino (UFF, Niterói).
- Few-body aspects can be studied in BEC systems, through the effects due to non-linear terms, from the main two-body term to higher order nonlinear terms, represented by three- and four-body interactions.

BEC Stability - History of a group collaboration

- A. Gammal, LT, T. Frederico, *Improved Numerical Approach For Time-Independent Gross-Pitaevskii Nonlinear Schrödinger Equation*, Physical Review E 60, 2421 (1999).
- A. Gammal, T. Frederico, LT, P. Chomaz, Liquid-Gas phase transition in BEC with time evolution, PRA 61, 051602 (2000); Atomic BEC with 3B Interactions and Collective Excitations, JPB 33, 4053 (2000), where we study effects of a repulsive 3B interaction on a trapped BEC with attractive 2B interaction.
- V. S. Filho, A. Gammal, T. Frederico, LT, Chaos in collapsing Bose-condensed gas, PRA 62, 033605 (2000).
- A. Gammal, T. Frederico, LT, F.K. Abdullaev, Stability analysis of the D-dimensional nonlinear Schrödinger equation with trap and two- and three-body interactions, PLA 267, 305 (2000).
- A. Gammal, T. Frederico, LT, Critical Number of Atoms for Attractive BEC with Cylindrically Symmetrical Traps, PRA 64, 055602 (2001).
- V. S. Filho, F.K. Abdullaev, A. Gammal, LT, Autosolitons in Bose-Einstein condensates, PRA 63, 053603 (2001).

Collaborators in recent works I am reporting

from Brazilian institutions:
- **Arnaldo Gammal (IF-USP, São Paulo)**
- **Hedhio L. F. da Luz (UFABC, Santo André) - pos-doc fellow**

Long-term visiting professor from Uzbek Science Academy:
- **Fatkhulla Abdullaev (IFT-UNESP and UFABC)**

Other short-time visitors:
- **Mario Salerno (Salerno University, Italy)**
- **Boris Malomed (Tel Aviv University, Israel)**

Collaborations going on in UFABC:

- Valery Shchesnovich
- Marijana Brtka
- Marcelo Pires

Introduction - BEC and soliton dynamics

First, within a full 3D approach, I present some results obtained on the existence of matter-wave solitons when considering cross-combined optical lattices (OL) in the three perpendicular directions, with a nonlinear OL in one of the directions.

These results can also be useful in practical applications, opening the possibility to manage stable 3D solitons through spatial modulations of the scattering length in one of the optical lattice directions.

Introduction - BEC and soliton dynamics

Next, we consider a bosonic gas of particles carrying collinear dipole moments, induced by an external polarizing field with the strength periodically modulated along the coordinate. In this case, one can obtain an effective nonlocal nonlinear lattice in a condensate, where bright solitons can be manifested. The existence, stability and mobility of such solitons appearing in the condensate are investigated by effective 1D model.

The interest of such theoretical studies relies on recent experimental investigations on Bose-Einstein condensation (BEC) in gases made of atoms carrying permanent magnetic moments, such as chromium, dysprosium, and erbium.

The long-range and anisotropic character of the dipole-dipole interactions leads to new physical phenomena, which are not expected in BEC with contact interactions.

Finally, I will report some recent investigations on a condensate with spin-orbit coupling.

Outline

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3D BEC with linear and nonlinear optical lattices

Let us consider a cross-combined linear and nonlinear OLs, with no harmonic trap. In this case, we can have

$$\mathrm{i}\frac{\partial u}{\partial t} = -\nabla^2 u - Vu - \Gamma|u|^2 u$$

where

$$\begin{aligned} V &\equiv V(x,z) = \varepsilon_x \cos(2x) + \varepsilon_z \cos(2z), \\ \Gamma &\equiv \Gamma(y) = \chi + \gamma \cos(\lambda y), \end{aligned}$$

denote the linear and nonlinear OLs, respectively,

parametrized by $\epsilon_{x,y}$, χ, γ and λ.

3D BEC with linear and nonlinear optical lattices

We consider variational and full numerical approach.
The variational *ansatz*, with parameters A, a, b and c, is given by

$$U(x,y,z) = Ae^{-(ax^2+by^2+cz^2)/2}.$$

Given μ as the chemical potential, we look for solutions of the form

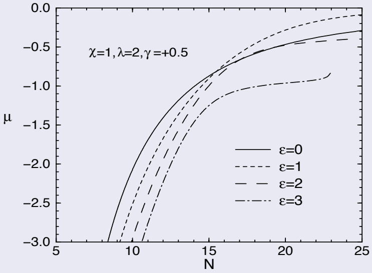
$$u(x,y,z,t) = U(x,y,z)\exp(-\mathrm{i}\mu t).$$

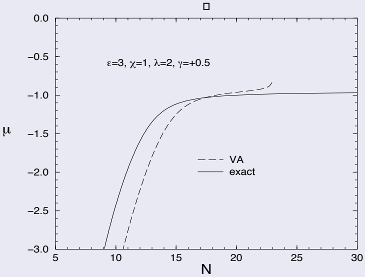
3D Solitons with quasi-2D OLs

First, we consider a quasi-2D OLs, were $\varepsilon_z = 0$ (no constraint in the z direction) and $\varepsilon_x = \varepsilon$. Sample results are given in the following, for the chemical potential μ in terms of the number of atoms N, for attractive 2-body interactions ($\chi = 1$). No stable 3D solitons are predicted to exist, in the attractive case, according to the Vakhitov-Kolokolov (VK) criterion for stability $d\mu/dN < 0$.

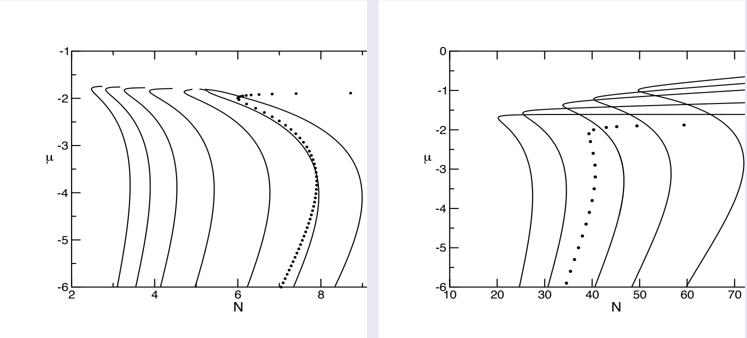
3D Solitons with quasi-2D OLs

In the right panel we have compared one of the VA curve (for $\varepsilon_x = 3$) with the corresponding one obtained from numerical simulations of the GP equation, showing a good quantitative agreement between numerical and analytical results. PDE simulations also confirm the prediction of the VK criterion about the instability of 3D solitons in this case.





Next, we have shown that one can stabilize solitons, even in the repulsive case, by adding a linear OL in the $z-$direction. We consider $\varepsilon = \varepsilon_x = \varepsilon_z = 3$ in the next figure. VA are given by solid lines and PDE by dotted lines. In both cases we observe branches for which stable 3D solitons are possible. In the left, $\chi = 1$, varying the NOL strength $\gamma =$ 3.0, 2.5, 2.0, 1.5, 1.0, 0.7, 0.5 (left to right). A PDE result is shown for $\gamma =$ 0.5. In the right panel, $\gamma =$ 0.5, with several repulsive mean nonlinearity: $\chi =$ 0.0, -0.1, -0.2, -0.25, -0.3 (left to right). The PDE result is for $\chi = -$ 0.2 (other parameters are the same).



For more details, see Abdullaev et al. JPB, 45, 115302 (2012).

Outline

1. Introduction
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7. General conclusions

BEC with field-induced dipole moments

We consider a condensate elongated along axis x, with dipole moments of polarizable molecules or atoms induced by an external field directed along x too. The local strength of the polarizing field varies along x, which gives rise to an effective *nonlocal nonlinear lattice* in the condensate.

One can use an effectively 1D GPE, with the DDI term derived from the underlying 3D GPE. The necessary spatially modulated dc electric and/or magnetic field can be imposed by ferroelectric or ferromagnetic lattices.

In our approach, we have considered local dipole moment induced by a polarizing electric field. (One could also consider magnetic dipole moments induced by a solenoid.)

From 3D to 1D GPE

The GPE for the 3D mean-field wave function $\Psi(\mathbf{r}, t)$ is given by

$$
\begin{aligned}
i\hbar\frac{\partial\Psi}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\Psi + \frac{m}{2}\left[\omega_{\parallel}^2 x^2 + d(x)\mathcal{E}(x) + \omega_{\perp}^2(y^2+z^2)\right]\Psi + \\
&+ g_{3\mathrm{D}}|\Psi|^2\Psi + \left[\int |\Psi(\mathbf{r}', t)|^2 W_{\mathrm{DD}}(\mathbf{r}-\mathbf{r}')d^3\mathbf{r}'\right]\Psi,
\end{aligned}
$$

where $d(x) = \gamma\mathcal{E}(x)$ is the local dipole moment, induced by external field $\mathcal{E}(x)$, which is directed and modulated along x, γ is the molecular or atomic polarizability, $g_{3\mathrm{D}} \equiv 4\pi\hbar^2 a_s/m$. Further, the DDI kernel is given by

$$
W_{\mathrm{DD}}(\mathbf{r}-\mathbf{r}') = \frac{d(x)d(x')}{|\mathbf{r}-\mathbf{r}'|^3}\left[1 - \frac{3\,(x-x')^2}{|\mathbf{r}-\mathbf{r}'|^2}\right]
$$

and the wave function is normalized to the number of atoms,

$$
N = \int |\Psi(\mathbf{r}, t)|^2\, d^3\mathbf{r}.
$$

From 3D to 1D GPE

The field-induced dipole moment is essential in the range of

$$d \cdot \mathcal{E} \sim B,$$

where B is the rotational constant, determined by to the equilibrium internuclear distance r and reduced mass m_r of the polarizable molecule: $B = \hbar^2/(2m_r r^2)$. Typical values of the parameters are:
$d \sim 1$ Debye, $B \sim h \times$ 10 GHz, which yields an estimate for the necessary electric-field strength, $\mathcal{E} \sim 10^4$ V·cm^{-1}.
Such fields are accessible to experiments with BEC in atomic gases [see Appendix B of Lahaye et al, *Rep. Prog. Phys.* **72** 126401(2009)].
Thus, the spatial variation of the polarizing dc electric field,

$$\mathcal{E}(x) = \mathcal{E}_0 f(x),$$

leads to the respective spatial modulation of the DDI, with $d(x) = d_0 f(x)$.
This induces the mentioned effective nonlocal nonlinear lattice in the GPE.

From 3D to 1D GPE

For the quasi-1D case, we follow Sinha and Santos [*Phys. Rev. Lett.* **99** 140406 (2007)]. If the ground state in the transverse plane, (y, z), is imposed by the trapping potential, the 3D wave function may be factorized as

$$\Psi(\mathbf{r}, t) = \psi(x, t)\left(\sqrt{\pi}a_\perp\right)^{-1}\exp\left(-\rho^2/2a_\perp^2\right),$$

with $\rho^2 \equiv y^2 + z^2$, and $a_\perp^2 \equiv \hbar/m\omega_\perp$. Substituting into the 3D expression and integrating over (y, z), the effective 1D DDI is derived with kernel

$$W_{\mathrm{1DD}} = \frac{2d^2}{a_\perp^3}\left[\frac{2|x|}{a_\perp} - \sqrt{\pi}\left(1 + \frac{2x^2}{a_\perp^2}\right)\exp\left(\frac{x^2}{a_\perp^2}\right)\mathrm{erfc}\left(\frac{|x|}{a_\perp}\right)\right].$$

Let us go to dimensionless variables:

$x \to a_\perp x$, $t \to \frac{t}{\omega_\perp}$, $\psi(x,t) \to \sqrt{\frac{5}{\pi^{3/2}a_d}}\phi(x,t)$, $\alpha = \frac{\omega_\parallel^2}{2\omega_\perp^2}$, $g = \frac{10a_s}{\pi^{3/2}a_d}$,
where $a_d = md^2/\hbar^2$ is the characteristic DDI length

From 3D to 1D GPE

With $\phi \equiv \phi(x,t)$ and $\phi' \equiv \phi(x',t)$, the reduced 1D equation is:

$$\begin{aligned} \mathrm{i}\frac{\partial \phi}{\partial t} &= -\frac{1}{2}\frac{\partial^2 \phi}{\partial x^2} + \alpha x^2 \phi + \beta f^2(x)\phi + g|\phi|^2\phi \\ &- f(x)\phi \int_{-\infty}^{+\infty} f(x')|\phi'|^2 R(x-x')dx'. \end{aligned}$$

The effective 1D kernel is

$$R(x) = \sigma\frac{10}{\pi}\left[\left(1+2x^2\right)\exp\left(x^2\right)\mathrm{erfc}(|x|) - \frac{2}{\sqrt{\pi}}|x|\right].$$

($\sigma = +1$ for the attractive DDI, and $\sigma = -1/2$ for the repulsive DDI between dipoles oriented perpendicular to x).

Actually, this rather complex kernel $R(x)$ can be replaced by

$$R(x) = \frac{10\sigma}{\pi\sqrt{(\pi\, x^2+1)^3}},$$

(See Cuevas et al [*Phys. Rev. A* **79** 053608 (2009)]).

From 3D to 1D GPE

The DDI can be represented by a pseudopotential which includes a contact-interaction term. Then, the spatially modulated $d(x)$ may induce a position-depending part of the contact interactions too.
However, in the present setting, the regularization scale $a_\perp$ eliminates the singular part of the DDI at scales $|x| \lesssim a_\perp$. Therefore, we restrict ourselves to the consideration of the pure nonlinear nonlocal lattice.

We assume that the dynamics of the system in the perpendicular directions is completely frozen, i.e., the transverse trapping frequency, $\omega_\perp$, is much larger then the longitudinal one, $\omega_\perp \gg \omega_{||}$.
On the other hand, if $\omega_\perp$ is not too large, interesting transverse effects may occur, which is beyond the scope of our work.

From 3D to 1D GPE

In the equation for the potential we can identify the usual harmonic-trap potential, αx^2, the nonlinear term $g|\phi|^2\phi$, and an effective DDI potential, composed of linear and a nonlinear terms:

$$V_{\mathrm{eff}}^{(\mathrm{DDI})}(x;|\phi|^2) = f(x)\left[\beta f(x) - \int_{-\infty}^{+\infty} f(x')|\phi'|^2 R(x-x')dx'\right],$$

where the modulation function is

$$f(x) = f_0 + f_1\cos(kx).$$

The parameters are f_0, f_1, and $k \equiv 2\pi a_\perp/\lambda = 2\pi/\Lambda$. β can vary from 1 to 10, under typical physical conditions, with the constant part fixed as $f_0 \equiv 1$.

From 3D to 1D GPE

So, the Hamiltonian is

$$
\begin{aligned}
H &= \int_{-\infty}^{+\infty} dx \left[\frac{1}{2} \left| \frac{\partial \phi}{\partial x} \right|^2 + \frac{g}{2} |\phi|^4 + \alpha x^2 |\phi|^2 + \beta f^2(x) |\phi|^2 \right] \\
&- \frac{1}{2} \int_{-\infty}^{+\infty} dx f(x) |\phi|^2 \int_{-\infty}^{+\infty} dx' f(x') |\phi'|^2 R(x - x').
\end{aligned}
$$

It also contains the additional linear potential, $\beta f^2(x)$, which is induced by the interaction of the locally-induced dipole moment with the polarizing field.

Outline

1 Introduction

2 3D BEC with linear and nonlinear optical lattices

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4 **Bright solitons: existence and stability**

5 Dynamics of bright solitons

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7 General conclusions

Bright soliton solutions

The existence of bright-soliton solutions can be investigated by solving the eigenvalue problem, with $\phi = |\phi| e^{-\mathrm{i}\mu t}$:

$$-\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + \alpha x^2\phi + g|\phi|^2\phi + V_{eff}^{DDI}(x;|\phi|^2)\phi = \mu\phi.$$

We consider full numerical solutions of this equation, as well as corresponding variational approaches (VA), for two characteristic cases: $\alpha = 0$, $\beta \neq 0$ and $\alpha = \beta = 0$.

Variational approach

In the corresponding Lagrangean formalism We assume the following Gaussian ansatz with center set at $x = \zeta$:

$$\phi = A \exp\left(-\frac{(x-\zeta)^2}{2a^2}\right). \tag{1}$$

Variational approach

The corresponding averaged Lagrangian L is given by

$$\begin{aligned}\frac{L}{N} &= \mu_r - \frac{1}{4a^2} - \beta\left(2f_1 f_0 e^{-a^2k^2/4}\cos(k\zeta) + \frac{f_1^2}{2}e^{-a^2k^2}\cos(2k\zeta)\right)\\ &- \frac{gN}{2\sqrt{2\pi}a} + \frac{N}{2\pi a^2}F(a,\zeta,f_0,f_1),\end{aligned}$$

where $N = \sqrt{\pi}A^2 a$, $\mu_r \equiv \mu - \beta\left[f_0^2 + (1/2)f_1^2\right]$, and

$$F(a,\zeta,f_0,f_1) \equiv \int\int_{-\infty}^{+\infty} dydx f(x)f(y)e^{-[(x-\zeta)/a]^2}R(x-y)e^{-[(y-\zeta)/a]^2}.$$

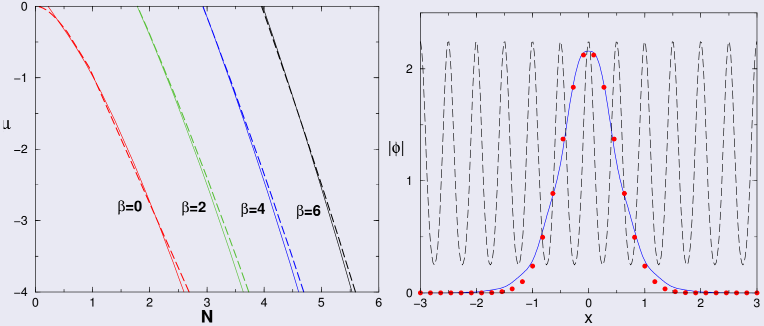
$\alpha = 0$, $\beta \neq 0$

The VA solutions are compared with their numerical counterparts in the next two figures, for $g = \alpha = 0$, $f_0 = 1$, $f_1 = 0.5$, and $\Lambda = 0.5$ ($k = 4\pi$).

The numerical solutions were obtained by a relaxation technique [See Brtka et al. *Phys. Lett. A* **359** 339 (2006)].

The effect of β is illustrated by plots of chemical potential μ versus the number of atoms, N, in the left panel. As shown, VA provides good agreement with the numerical results. The corresponding profiles are compared in the right panel, for $\mu = 0$ and $\beta = 6$. In the right panel, we also show the oscillatory function, $f^2(x)$, by the dashed line.

$\alpha = 0, \beta \neq 0$



Left panel: μ vs N, from full numerical solutions (solid lines), and from VA (dashed lines). Other parameters are $\alpha = g = 0$, $\sigma = 1$, $f_0 = 1$, $f_1 = 0.5$ and $\Lambda = 0.5$ $k = 4\pi$).

Right panel: the numerical solution (the solid line) for the profile of the wave function, centered in $x = 0$, is compared with VA results (dotted curve), for $\beta = 6$ and $\mu = 0$. In this panel, we also plot $f^2(x)$ by dashed line.

$\alpha = 0$, $\beta \neq 0$

The VA produces more accurate results for large β because, in this case, the contribution of the linear lattice grows, and it is known that the Gaussian ansatz, that we use here, works well with linear lattice potentials [see Kartashov et al. *Rev. Mod. Phys.* **83** 247(2011)]. Further, the steady increase of μ with β is also explained by the fact that the linear potential is multiplied by β.

$\alpha = 0$, $\beta \neq 0$

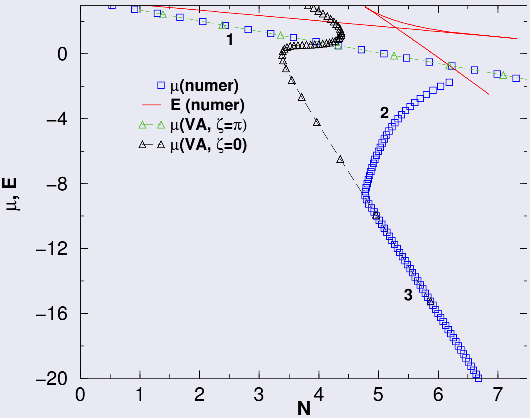


Figure: The chemical potential (VA and full numerical) and total energy (full numerical), are shown for $\Lambda = \pi$ (i.e., $k = 2$). Other parameters are $\alpha = g = 0$, $\sigma = 1$, $f_0 = 1$, $f_1 = 0.5$ and $\beta = 6$. We indicate three regions (1, 2 and 3) for the numerical solutions, following the variation of μ (μ_1, μ_2 and μ_3), to identify the corresponding profiles shown in the next two panels.

$\alpha = 0$, $\beta \neq 0$

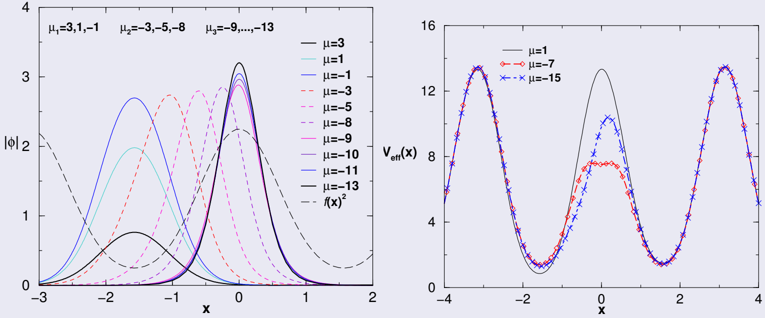


Figure: Profiles corresponding to regions identified in the previous figure. The modulation function, $f^2(x)$, is also shown by the black-dashed line in the left panel. The numerical results for the corresponding effective DDI potential are shown in the right panel for three different values of the chemical potential.

$\alpha = 0$, $\beta \neq 0$ - repulsive DDI

For the perpendicular orientation of the dipoles (repulsive DDI, $\sigma = -1/2$) (while other parameters are the same as in previous figures, we demonstrate, by means of numerical results for different values of μ, that the wave-function profiles are delocalized, i.e., they do not build bright solitons.

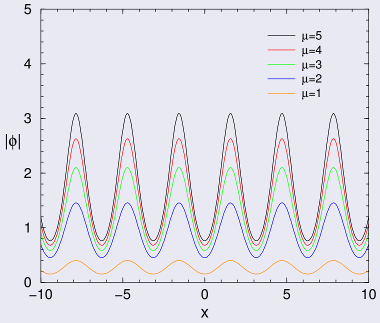


Figure: (Color online) Solution profiles for $\sigma = -1/2$, with other parameters the same as shown in previous two figures (in particular, $\Lambda = \pi$ and $\beta = 6$).

$\alpha = \beta = 0$

Following the case with $g = 0$, we present μ vs N (left panels), compared with VA for $\beta = 0$, in the absence of the linear trap ($\alpha = 0$). We consider different values of f_1, with $\Lambda = 1$ (left top) and 0.5 (left bottom). According to the Vakhitov-Kolokolov (VK) criterion for stability, $\partial\mu/\partial N < 0$, this assumption is well verified by our numerical results, for the whole range of parameters that we have analyzed. Simulations of the corresponding temporal evolution (not shown) validate the VK criterion.

We note that VA results cannot follow the results to the full extension, besides the fact that present very good agreement for large negative μ.

As seen in the left top panel, for $\Lambda = 1$ the VA correctly predicts the stability and converges to numerical results for $\mu < -1.5$ at all values of f_1. In the case of $\Lambda = 0.5$ (the left bottom panel), the VA results are equally accurate at $\mu < -6$. On the other hand, in the case of $f_1 = 2.0$ and $\Lambda = 0.5$, the VA results represent a set of two solutions, one being nearly insensitive to variations of f_1.

$\alpha = \beta = 0$

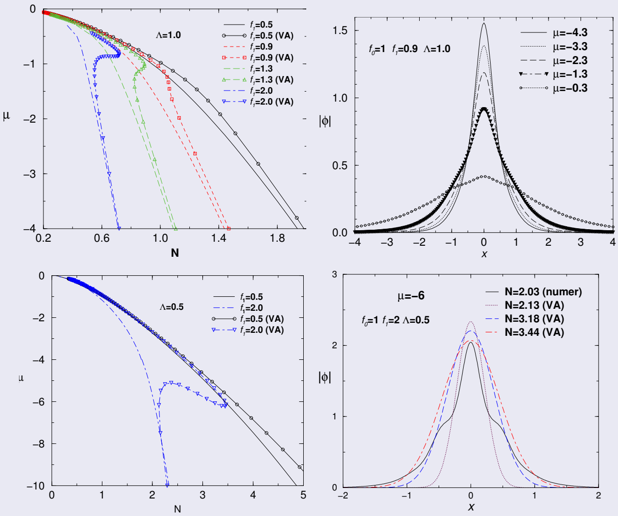


Figure: Parameters indicated inside the frames, for the case of $\beta = \alpha = g = 0$, $f_0 = 1$, and $\Lambda = 1$ or 0.5

$\alpha = \beta = 0$ (Pure nonlocal nonlinear lattice), with $g > 0$

By fixing $\alpha = 0$ and $\beta = 0$, we have analyzed the model with repulsive contact interactions ($g > 0$). The corresponding term in the formalism tends to expand the wave function, on the contrary to the attractive nonlocal nonlinear interaction. In next panels, for the given parameters, with $\mu = -2$, we show stationary solutions for different magnitudes of g (left panel). As g increases, the wave function indeed gets broader and the number of atoms trapped in the soliton increases. Beyond a critical value, $g_c = 3.45$, no solution can be found. In the right panel, we consider different values of f_1 (keeping the other parameters as in the left panel), to verify how the behavior of critical value g_c and the corresponding values of N. It is observed that N decreases as f_1 and g_c increase.

This dependence can be explained considering the broad soliton case, when the nonlocal term can be approximated as the local one with an effective nonlinearity coefficient, $\gamma_{\mathrm{eff}} = f^2(x) \int_{-\infty}^{+\infty} dyR(y)$ and the bright solitons should exist, provided that $g > f^2 \int_{-\infty}^{+\infty} dyR(y)$. This arguments explain the growth of g_c with the increase of f_1.

$\alpha = \beta = 0$ (Pure nonlocal nonlinear lattice), with $g > 0$

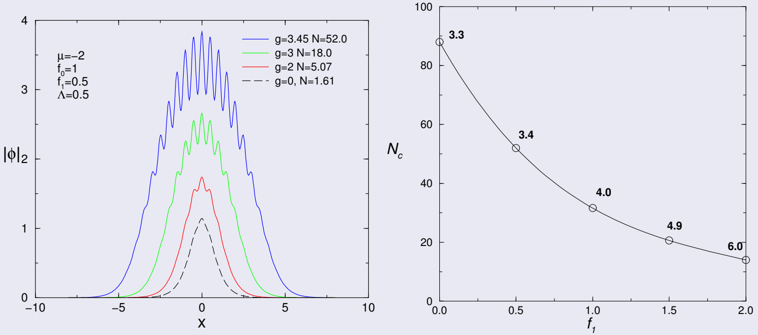


Figure: The left panel: wave-function profiles with fixed $\mu = -2$ and a few values of g. The parameters are indicated inside the frame ($f_0 = 1$, $f_1 = 0.5$ and $\Lambda = 0.5$). These results for $f_1 = 0.5$ define critical maximum values $g_c = 3.45$ and $N_c = 52$, above which no bright soliton were found. In the right panel, by varying the modulation amplitude f_1, with other parameters as in the left panel), we display a curve corresponding to the critical values of N and g.

Outline

1. Introduction
2. 3D BEC with linear and nonlinear optical lattices
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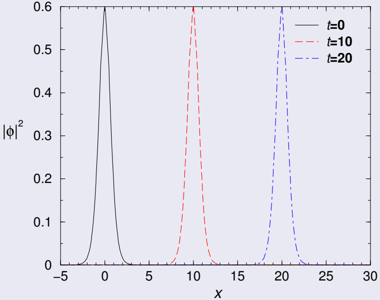
7. General conclusions

Dynamics of bright solitons

The mobility of solitons and collisons are shown by considering full numerical solutions for the 1D GPE, exploring a parameter region to obtain stable bright-soliton solutions.

Next figure with two panels, we show the soliton propagation by considering, in the dimensionless units, a time interval from 0 to 20, with velocity set to 1. In the left panel, we have $\mu = -1$ and $N \approx 1.02$; and, in the right panel, $\mu = -10$ and $N \approx 4.86$, with profiles separated by time intervals $\Delta t = 2$. In the latter case, the soliton ends up getting trapped at a fixed position. In both the cases, we have $\alpha = \beta = g = 0$, and $f_0 = 1$, $f_1 = 0.5$, $\Lambda = 0.5$.

Dynamics of bright solitons



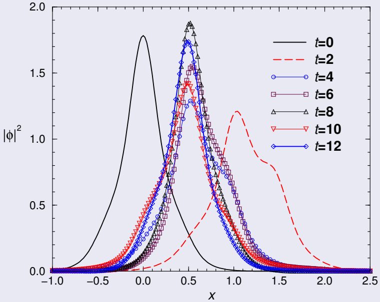


Figure: Left panel: time intervals $\Delta t = 10$ (the velocity is 1), with $\mu = -1$ and $N \approx 1.02$. Right panel: $\mu = -10$, $N \approx 4.86$, with intervals $\Delta t = 2$. In both cases, $f_0 = 1$, $f_1 = 0.5$ and $\Lambda = 0.5$ (with $\alpha = \beta = 0$).

Two solitons interacting

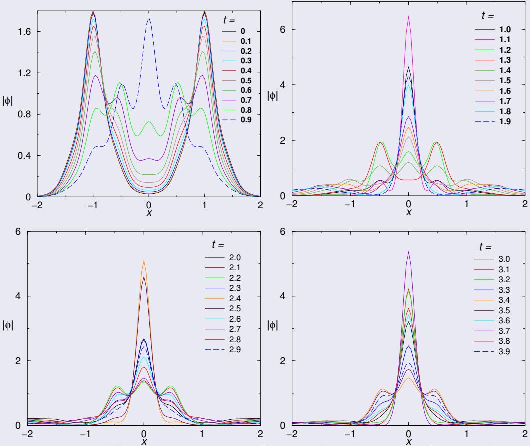
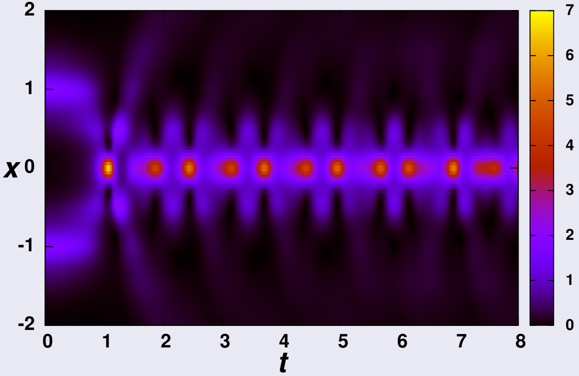


Figure: With $\mu = -10$, $N \approx 4.86$, we show the interaction of two solitons in four panels at different moments of time, with $\Delta t = 0.1$ (the average velocity is zero). In all the cases, the parameters are $f_0 = 1$, $f_1 = 0.5$ and $\Lambda = 0.5$ ($a_\perp = 2, \lambda = 1$). The phase difference between the solitons is zero, and they attract each other. One can see a transition from a bound state to a breather.

Density plot - two solitons interacting

Next, we show the corresponding density plot.



For the same parameters, we observe almost no interaction when the phase between them is π.

Outline

1. Introduction
2. 3D BEC with linear and nonlinear optical lattices
3. Bright solitons in BEC with field-induced dipole moments
4. Bright solitons: existence and stability
5. Dynamics of bright solitons
6. **Spin-Orbit Coupling in Ultracold-atom systems**
7. General conclusions

Work in progress: Spin-Orbit Coupling in ultracold-atom systems

Motivated by some recent experiments with ultracold-atoms, we are considering GPE with spin-orbit coupling.
Spin-orbit coupling links a particle's velocity to its quantum-mechanical spin, and is essential in numerous condensed matter phenomena. As pointed out by Galitski and Spielman [Nature 494, 49(2013)], for ultracold atomic systems, the engineered material parameters are tunable and a variety of synthetic spin-orbit couplings can be engineered by using laser fields.
As also shown by Jimínez-garcia et al [PRL 114, 125301 (2015)], in ultra cold BEC system, we have an opportunity to investigate and control spin-orbit coupling using amplitude-modulated Raman coupling.

Work in progress: Spin-Orbit Coupling

Let us follow a model with periodic variation in space of the two-body scattering length, which can be given by

$$\begin{aligned} \mathrm{i}\frac{\partial u}{\partial t} &= -\frac{1}{2}\frac{\partial^2 u}{\partial x^2} - g(x)(|u|^2+|v|^2)u - \mathrm{i}\alpha\frac{\partial u}{\partial x} - Kv \\ \mathrm{i}\frac{\partial v}{\partial t} &= -\frac{1}{2}\frac{\partial^2 v}{\partial x^2} - g(x)(|u|^2+|v|^2)v + \mathrm{i}\alpha\frac{\partial v}{\partial x} - Ku \end{aligned}$$

where $u \equiv u(x,t)$, $v \equiv v(x,t)$, with $g(x)$ being a periodic function in space, $g(x+L) = g(x)$, with period L. One particular choice for $g(x)$, which we consider is given by

$$g(x) = g_0 + g_1 \cos(kx).$$

In the following we will fix k to $\pi/2$, as this corresponds to adjust our length scale to the lattice parameter such that $L = 4$.

Spin-Orbit Coupling

The physical model is a Bose-Einstein condensate (BEC) with spin-orbit (SO) coupling and variable in space atomic scattering length.
Two limits we consider:
1) Large spin-orbit coupling α, leading the system to the massive Thirring model with periodic non-linearity.
2) Small spin-orbit coupling α - two component BEC with periodic scattering length.

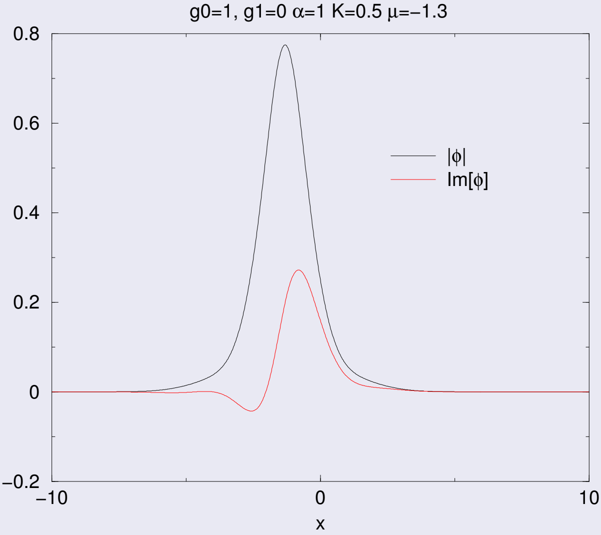
...

After some manipulation, the system can be written as a single equation, for $\phi \equiv \phi_1$:

$$\mu\phi + \frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + \frac{\alpha^2}{2}\phi + 2g(x)|\phi|^2\phi \pm K\phi^* e^{2i\alpha x} = 0, \tag{2}$$

which is still a coupled equation for the real and imaginary parts of ϕ.

...



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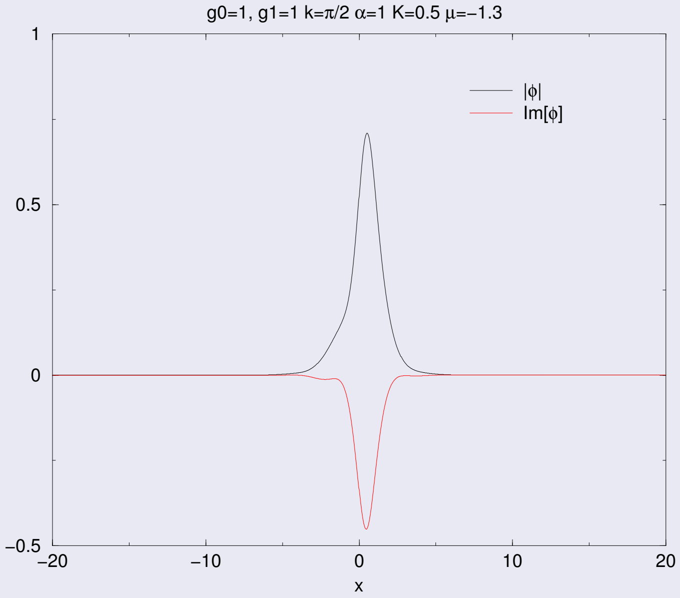

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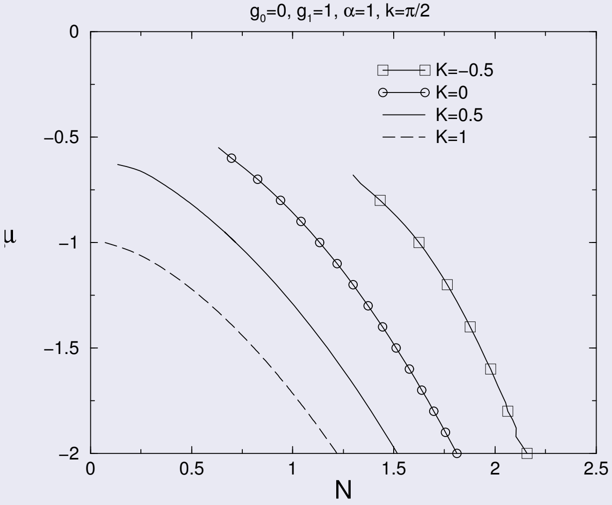

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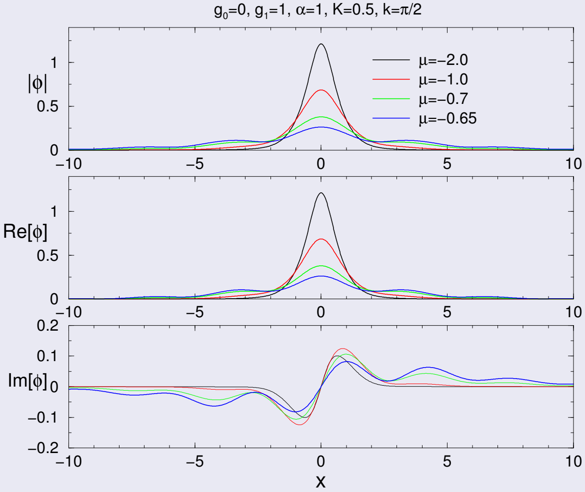

Figure: Chemical potential μ as a function of the number of particles N, by considering the Eq. (2) with $g_0 = 0$, $g_1 = 1$, $\alpha = 1$ and $k = \pi/2$, for a few values of the coupling parameter K.

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General conclusions

First, we report some results on the existence of solitons in the case of cross-combined linear and nonlinear optical lattices, within a full three-dymensional model. As shown, we can obtain families of 3D solitons, which can be stable for both attractive and repulsive interactions.

These results can be useful in practical applications, opening the possibility to manage stable 3D solitons through spatial modulations of the scattering length in one of the optical lattice directions.

Next, we report investigations within a reduced 1D model, where we study the parameter conditions for the existence of bright solitons considering induced dipole-dipole interactions. The results were verified by comparison with numerical solutions of the respective one-dimensional GPE. The stability of the soliton families exactly obeys by the VK criterion. The dynamics of solitons and interactions between them, including merger into breathers, were also investigated. It was also found that the dynamical version of the VA provides for a good prediction for frequencies of small oscillations of perturbed solitons.
Finally, it was also presented preliminar results on spin-orbit coupling in BEC.

Main references

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THANK YOU!