BRAZIL-JINR FORUM "Frontiers in Nuclear, Elementary Particle, and Condensed Matter Physics" Dubna, June 15-19, 2015

Modeling of Intrinsic Josephson Junctions in High Temperature Superconductors

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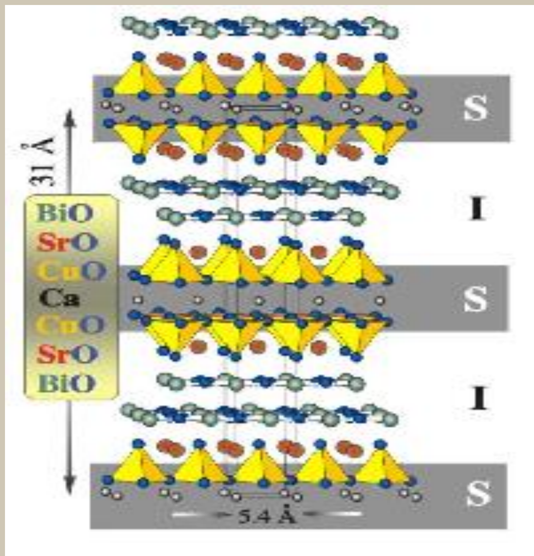
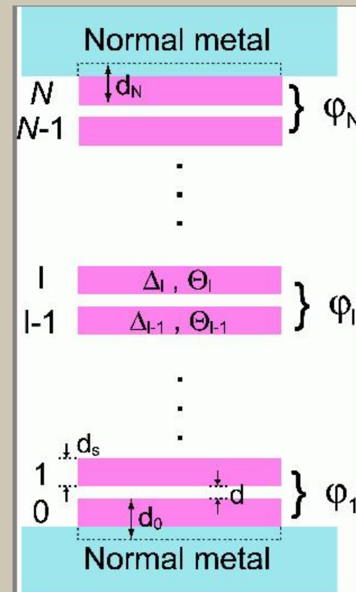
Andrej Plecenik (Slovakia)
Waldemar Nawrocki (Poland)

Outline

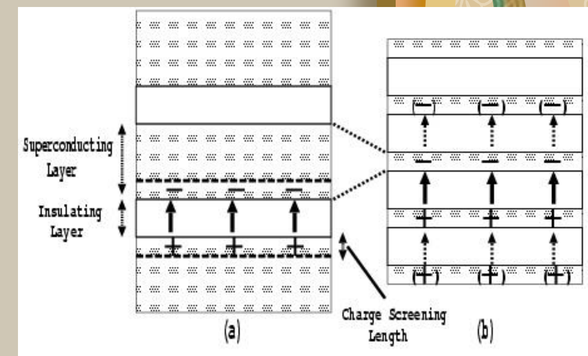
- Introduction. Models and methods
- Radiation effects
- Charging of S-layers in coupled Josephson junctions
- Chaos induced by coupling between Josephson junctions
- Shunting and radiation
- Charge density wave
- Charge imbalance effect

Layered $Bi_2Sr_2CaCu_2O_y$(Bi2212) single crystals represent natural stacks of atomic scale intrinsic Josephson junctions.





T.Koyama, M.Tachiki, 1996;
D. Ryndyk, 1997;
S. Artemenko, 1980, 1997



$$\Psi_l(t) = |\Psi_l| \exp i\theta_l(t)$$

$$\Delta_l(t) = |\Delta_l| \exp i\theta_l(t)$$

$$\rho_l = -\frac{\Phi_l}{4\pi\mu^2}; \qquad \Phi_l = \phi_l - \frac{\hbar}{2e}\frac{\partial\theta_l}{\partial t};$$

$$\frac{\hbar}{2e}\frac{\partial\varphi_{l,l+1}}{\partial t} = V_{l,l+1} + \frac{\varepsilon\mu^2}{d_s d_I}(V_{l+2,l+1} + V_{l-1,l} - 2V_{l,l+1})$$

$$\varphi_l(t) = \theta_l(t) - \theta_{l-1}(t) - \frac{2e}{\hbar}\int_{l-1}^{l} dz A_z(z,t)$$

CCJJ+DC model

$$J = C\partial V/\partial t + V/R + J_C \sin\varphi$$

$$J_D^l = -\frac{\Phi_l - \Phi_{l+1}}{R}$$

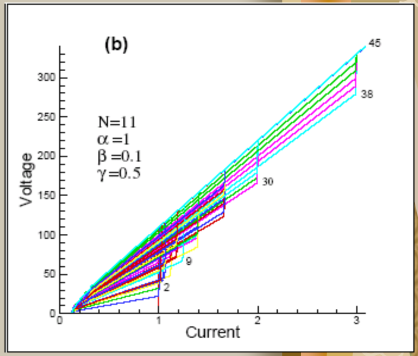
$$J = C\frac{dV_l}{dt} + J_c^l \sin(\varphi_l) + \frac{\hbar}{2eR}\dot{\varphi}_l$$

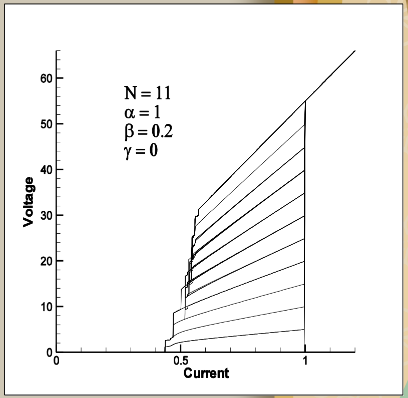
$$\ddot{\varphi}_l = \sum_{l'=1}^{n} A_{ll'}[\frac{J}{J_c} - \sin(\varphi_{l'}) - \beta\dot{\varphi}_{l'}]$$

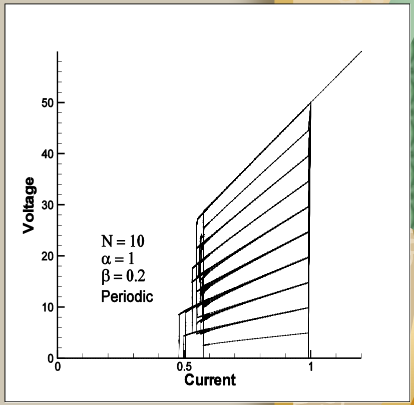
$$\frac{d^2}{dt^2}\varphi_l = (I - \sin\varphi_l - \beta\frac{d\varphi_l}{dt})$$

$$+\alpha(\sin\varphi_{l+1} + \sin\varphi_{l-1} - 2\sin\varphi_l)$$

$$+\alpha\beta(\frac{d\varphi_{l+1}}{dt} + \frac{d\varphi_{l-1}}{dt} - 2\frac{d\varphi_l}{dt})$$

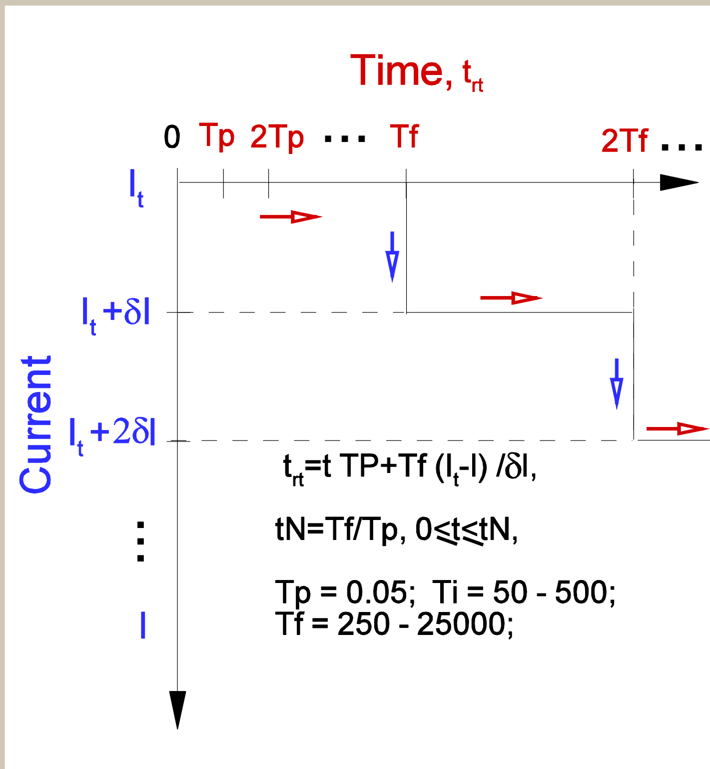






Yu.Shukrinov, F. Mahfouzi, P.Seidel, Physica C 449 (2006) 6-12.
Yu.M.Shukrinov, F.Mahfouzi. Phys.Rev.Lett, 98, 157001 (2007)

Simulation procedure for IV-characteristics



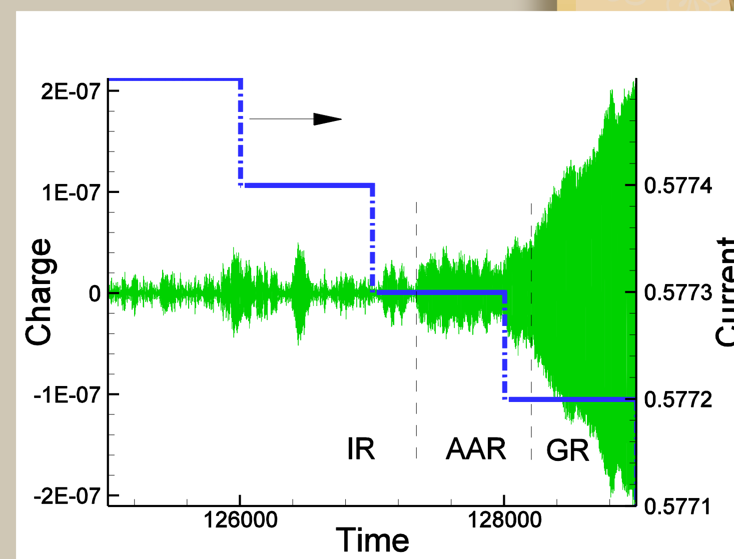
Parameters of simulation:

t_{rt} - recording time;
Tp - step in time;
t - number of steps in time;
tN - total number of time steps;
Tf - size of time domain
Ti-initial time for averaging,
δI - step in current;
I_t - initial current value for time dependence recording;

- $\mathrm{div}\,(\varepsilon\varepsilon_0 E) = Q$

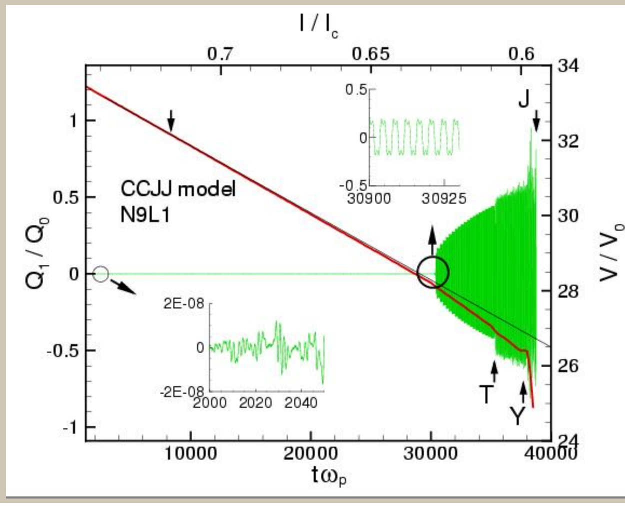
$$Q_l = Q_0\,\alpha\,(V_{l+1} - V_l)$$

$$Q_0 = \varepsilon\,\varepsilon_0\,V_0/r_D^2$$



$$\begin{cases} \dfrac{d\varphi_l}{dt} = V_l - \alpha\left(V_{l+1} + V_{l-1} - 2V_l\right) \\ \dfrac{dV_l}{dt} = I - \sin\varphi_l - \beta\varphi_l + A\sin(\omega t) \end{cases}$$

CVC and charge-time dependence in CCJJ model



The 7th International Symposium on Intrinsic Josephson Effects and Plasma Oscillations in High-Tc Superconductors (PLASMA 2010)
Hirosaki Univ., Hirosaki, Japan, April 25-28, 2010

Phase Dynamics in IJJ: Comparative Study in Different Models

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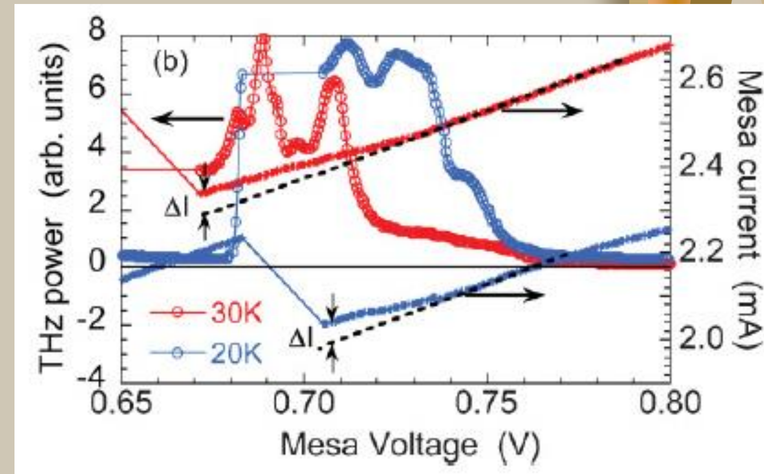

PHYSICAL REVIEW B **84**, 064523 (2011)

Tunable terahertz emission from $Bi_2Sr_2CaCu_2O_{8+\delta}$ mesa devices

T. M. Benseman,* A. E. Koshelev, K. E. Gray, W.-K. Kwok, and U. Welp
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A.Irie et al, Apl.Phys.Lett., 2008

The International Conference on Theoretical Physics 'Dubna-Nano2008' IOP Publishing
Journal of Physics: Conference Series 129 (2008) 012029 doi:10.1088/1742-6596/129/1/012029

Experimental observation of the longitudinal plasma excitation in intrinsic Josephson junctions

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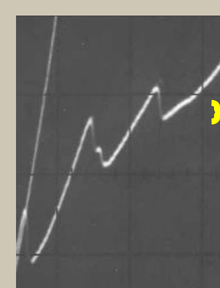
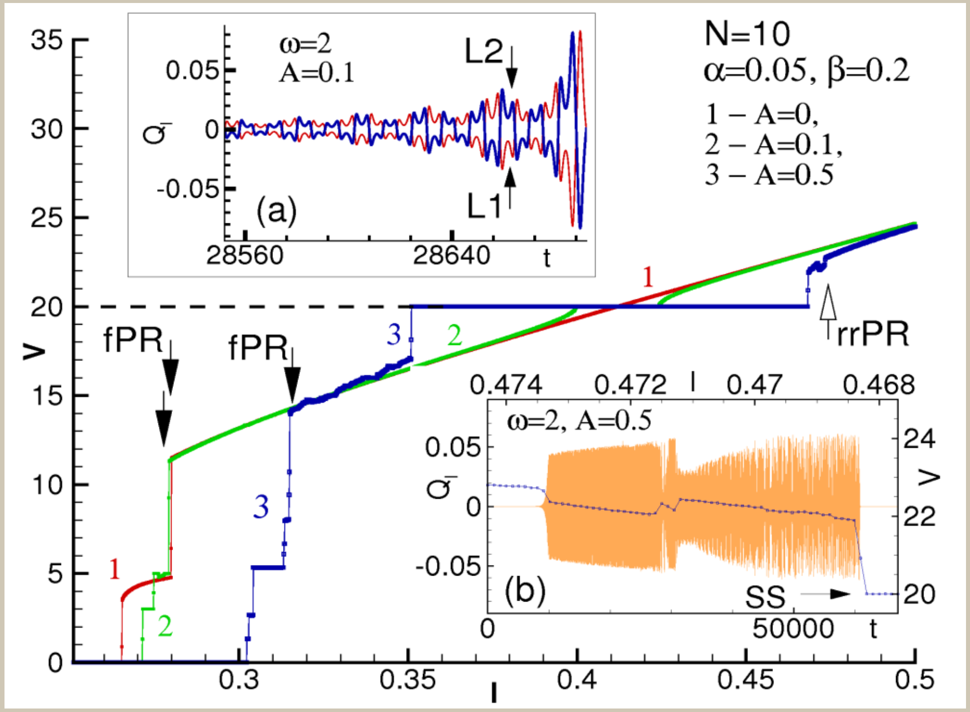


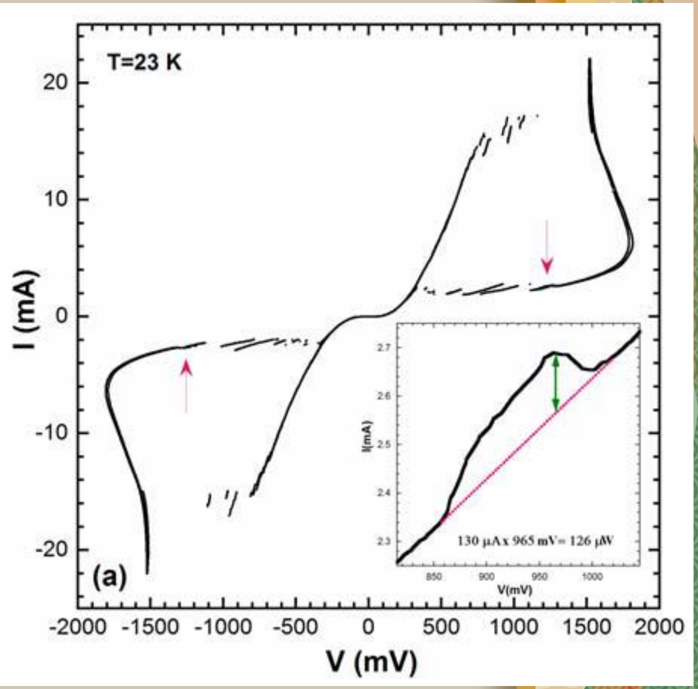
Breakpoint region

Radiation effects

IV-characteristics
without irradiation (curve 1)
under radiation with *A = 0.1 (curve 2)*
A = 0.5 (curve 3).

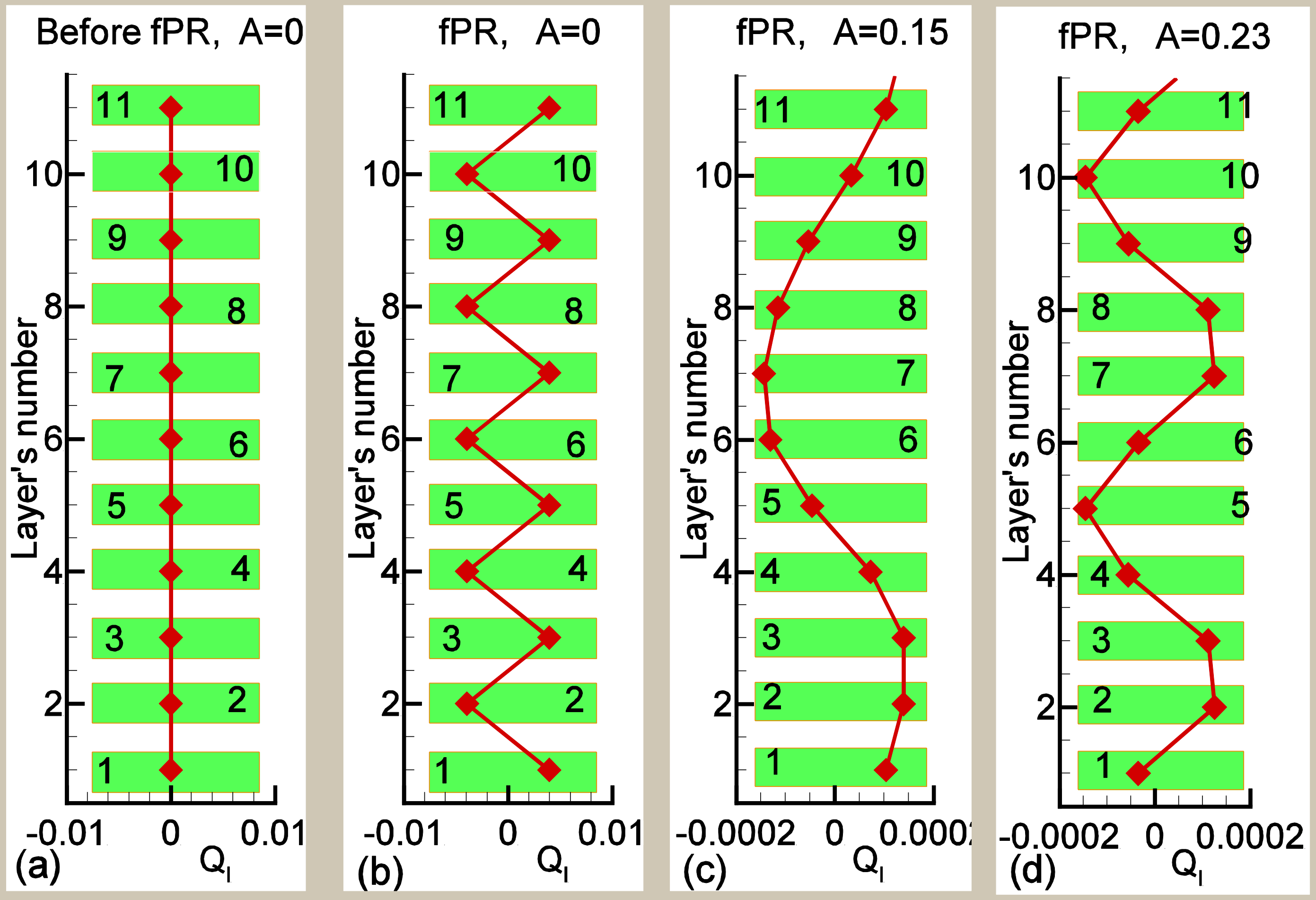


Yu.M.Shukrinov, I.Rahmonov, M. Gaafar, Phys.Rev.B, 86, 184502 (2012)

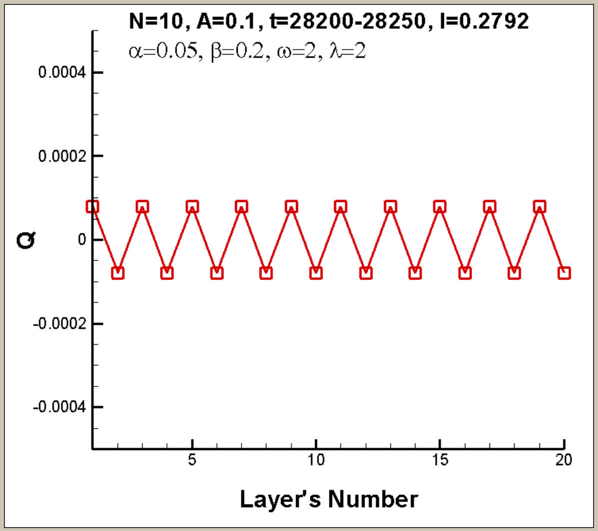


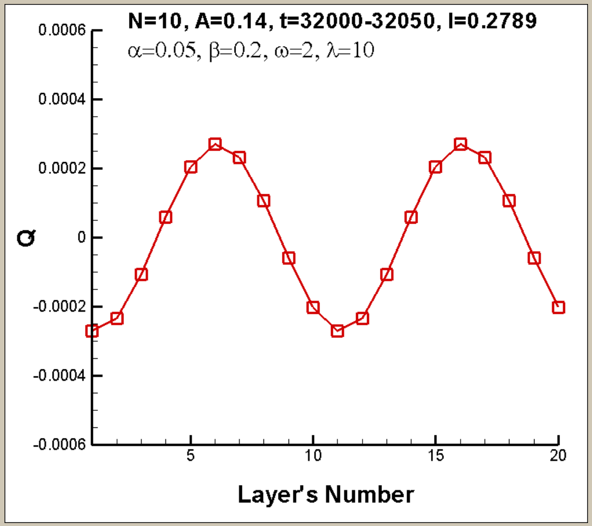
F Turkoglu, H Koseoglu, Y Demirhan, L Ozyuzer, S Preu, S Malzer, Y Simsek, P Müller, T Yamamoto and K Kadowaki 2012 *Supercond. Sci. Technol.* **25** 125004

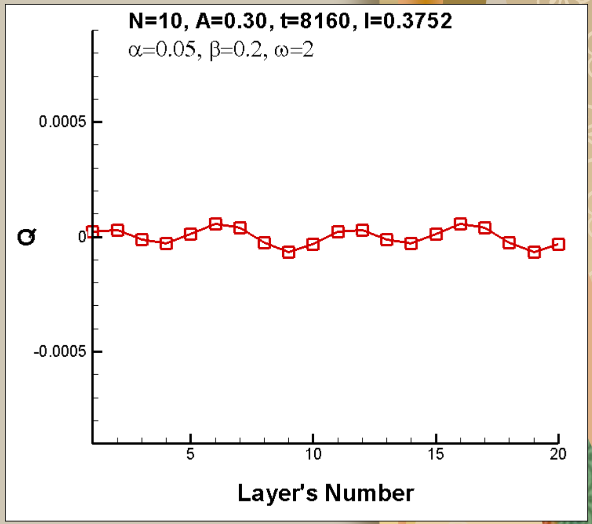
Demonstration of changing of LPW wavelength with an increase of the amplitude of radiation.

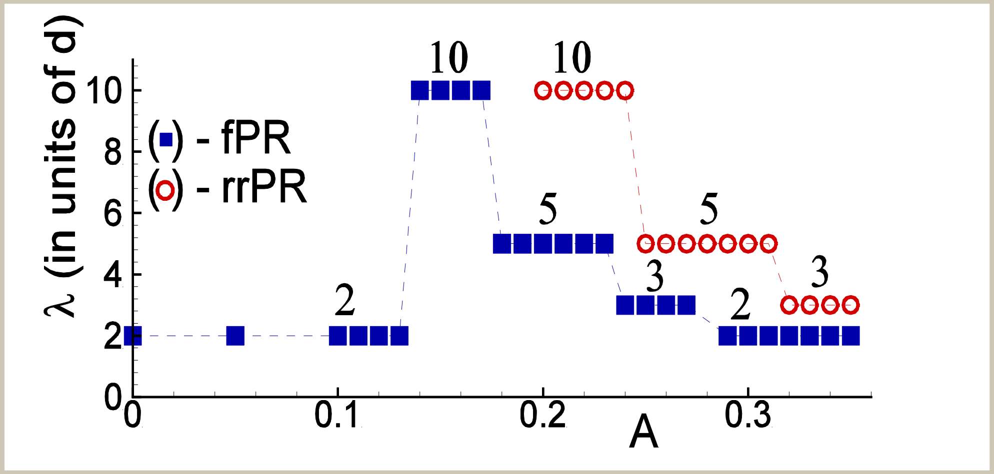


Waves in the stack of coupled JJ





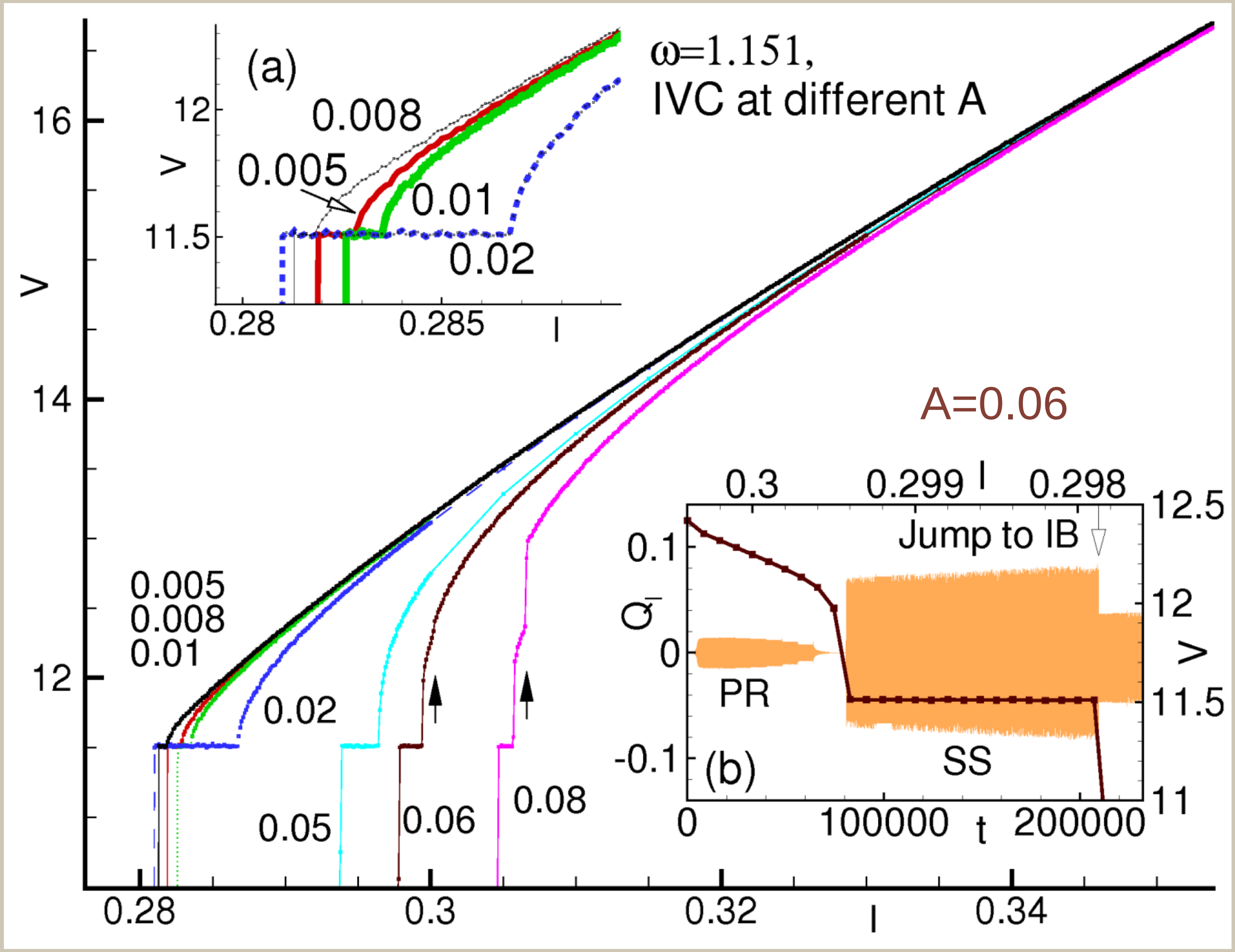




LPW wavelengths at *w = 2*
Filled squares - fundamental PR
Circles - radiation related PR

Yu.M.Shukrinov, I.Rahmonov, M. Gaafar, Phys.Rev.B, 86, 184502 (2012)

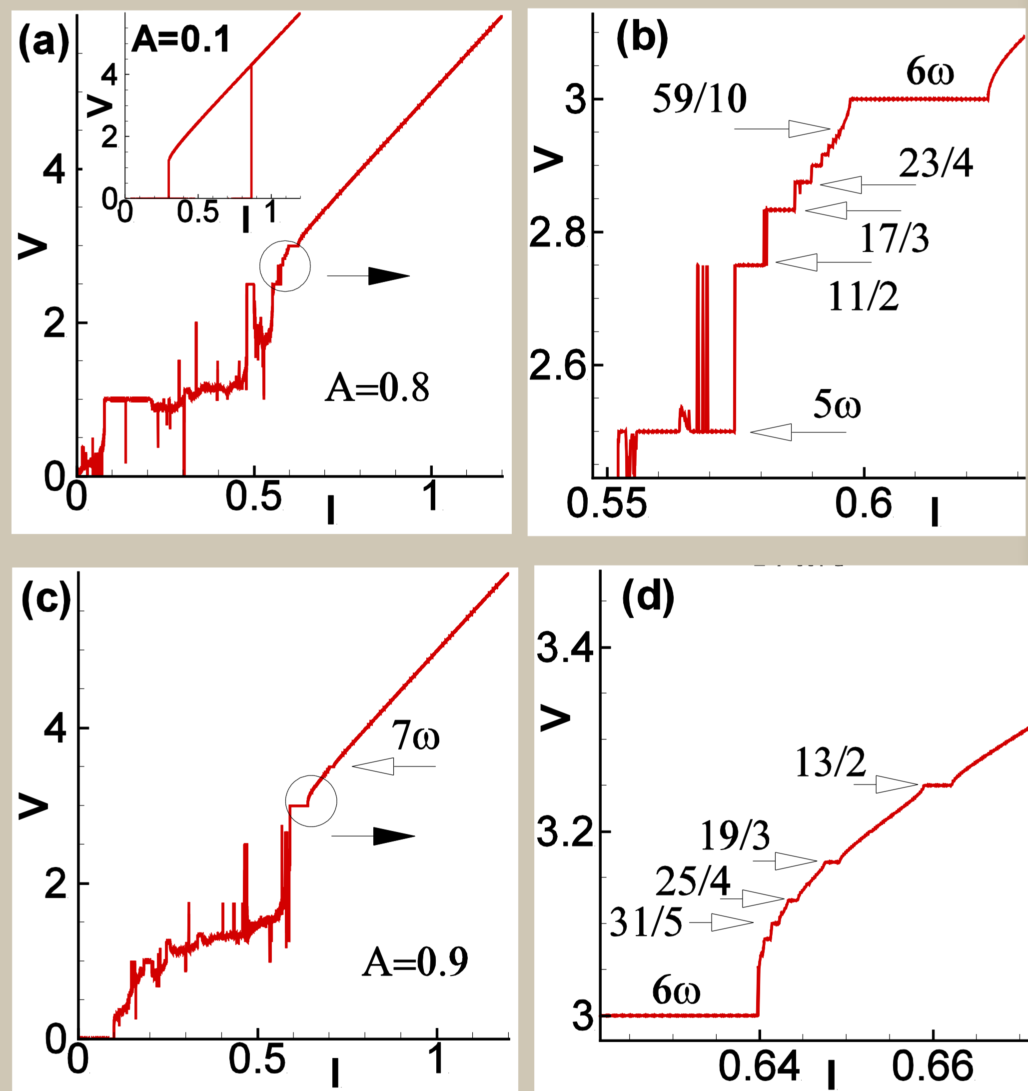
IVC of a stack with 10 JJ at *w= 1.151 and different A.*
Inset (b) shows the charge-time dependence at A = 0.06.



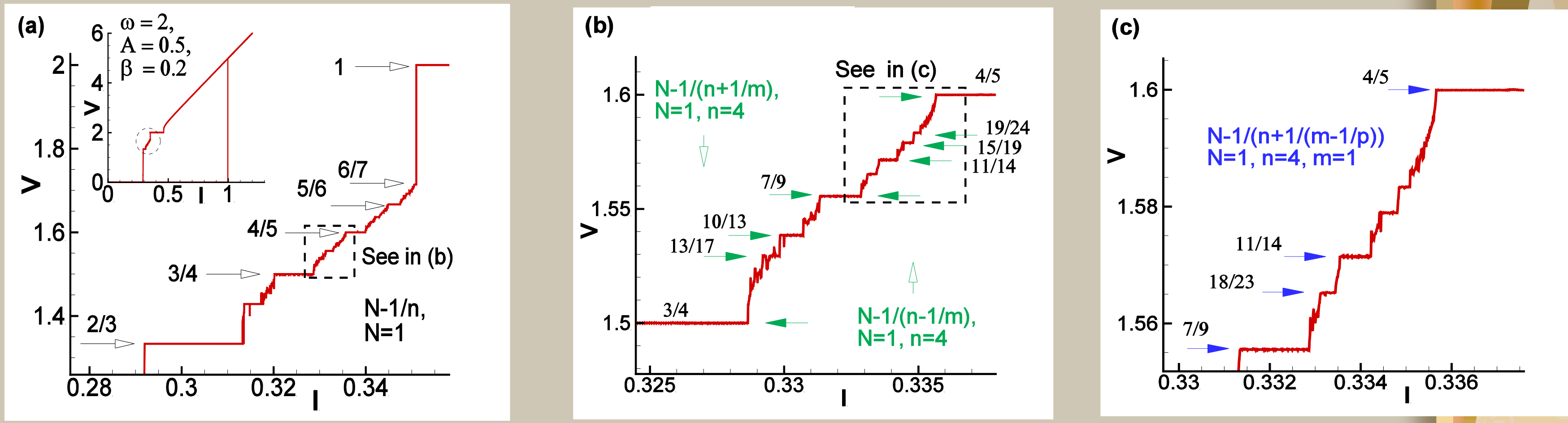
Devil's Staircases and Continued Fractions in Josephson Junctions.

DS structure, A=0.8

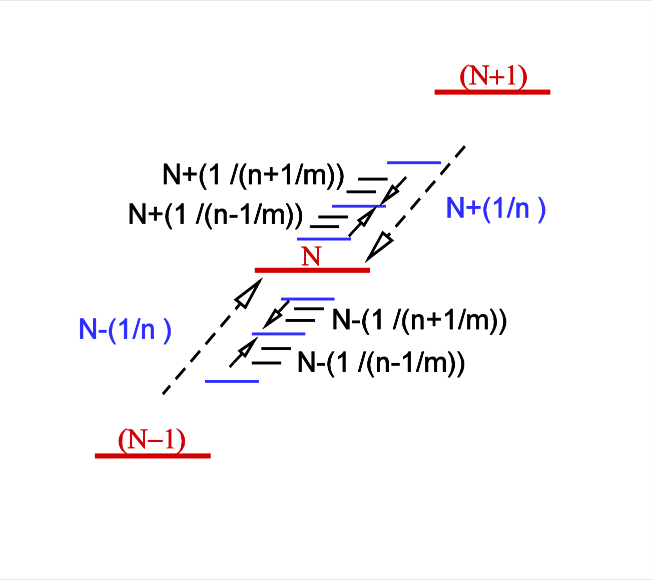
DS structure, A=0.9



Fractal structure

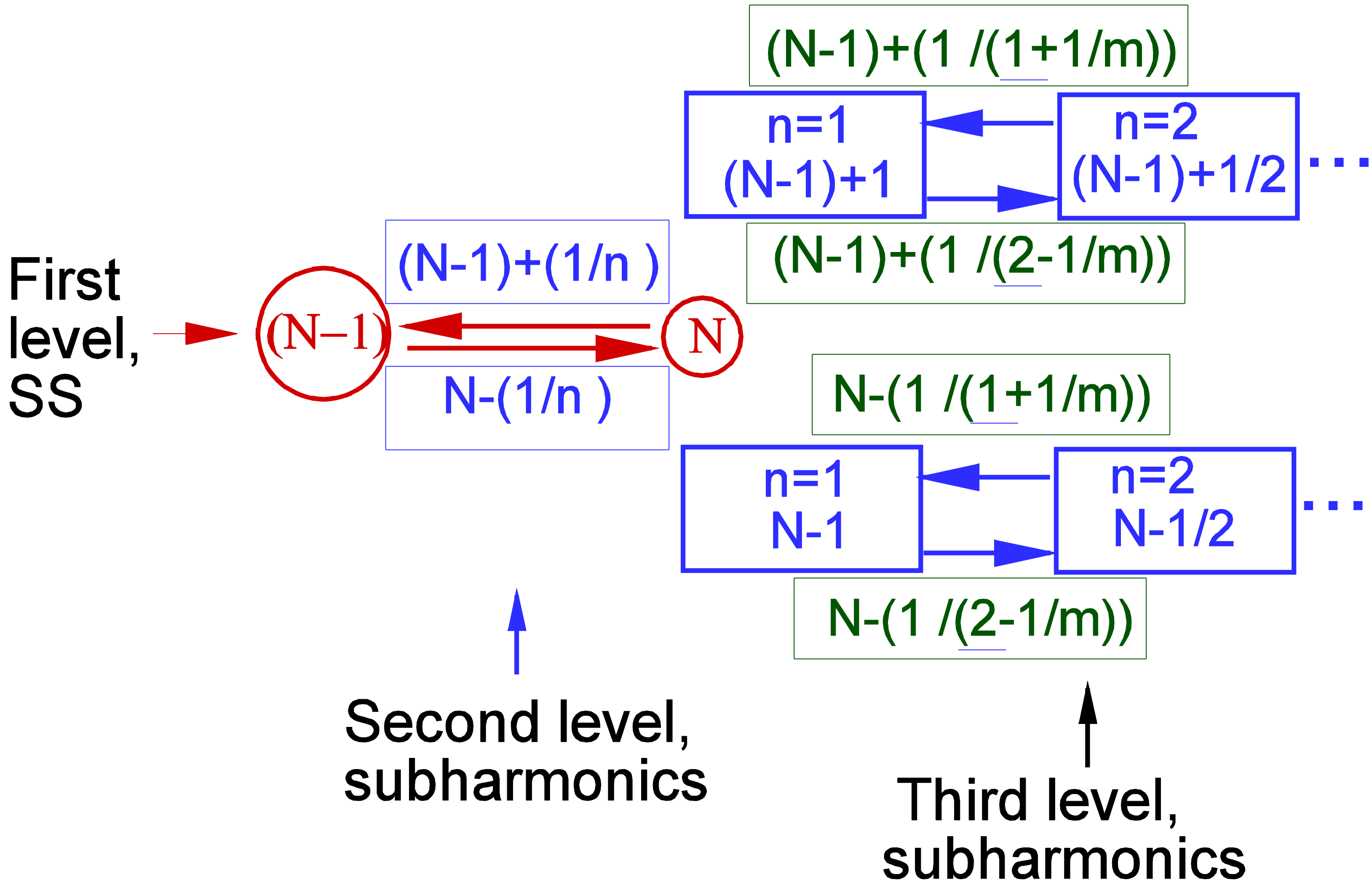


$$V = \left(N \pm \frac{1}{n \pm \frac{1}{m \pm \frac{1}{p \pm \ddots}}} \right) \omega$$

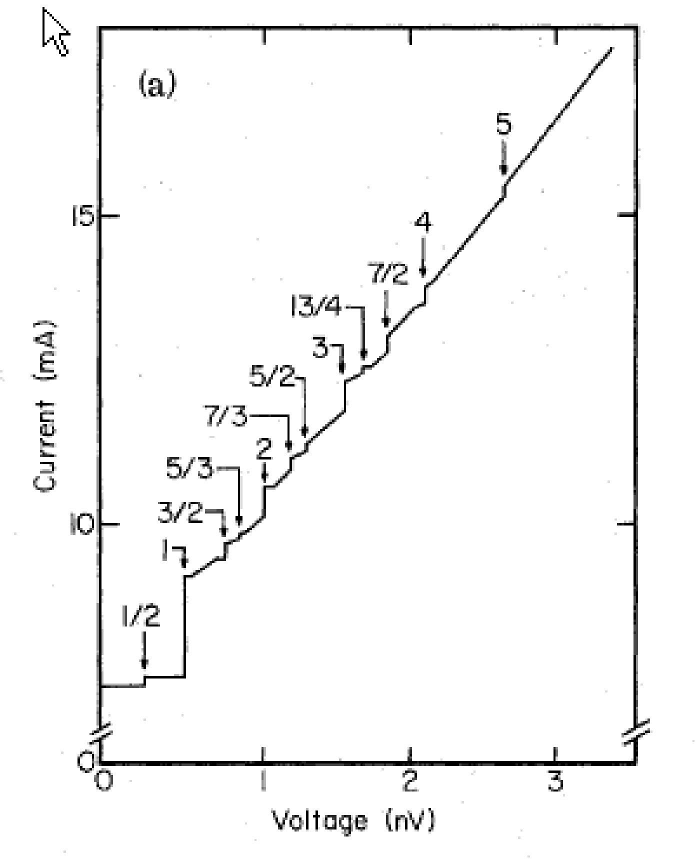


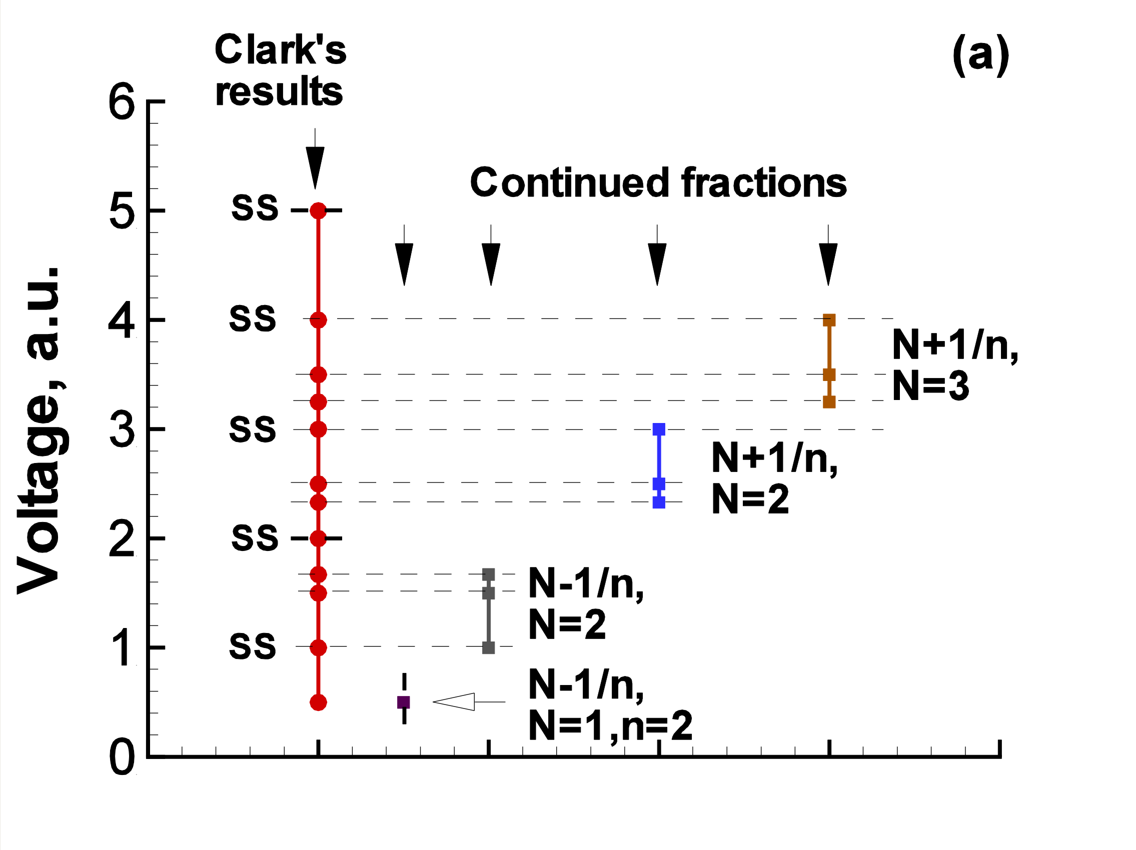
Yu. M. S., S. Yu. Medvedeva, A. E. Botha, M. R. Kolahchi, and A. Irie.,
- Phys. Rev. B, 88, 214515 (2013)

Continued fraction algorithm for SS subharmonics (underlined is value of n)

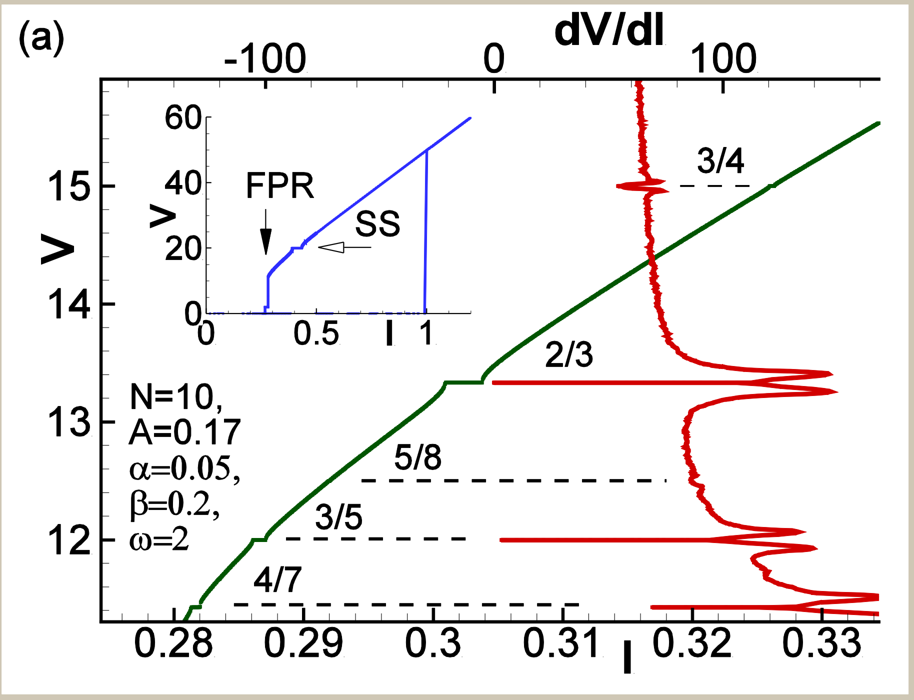


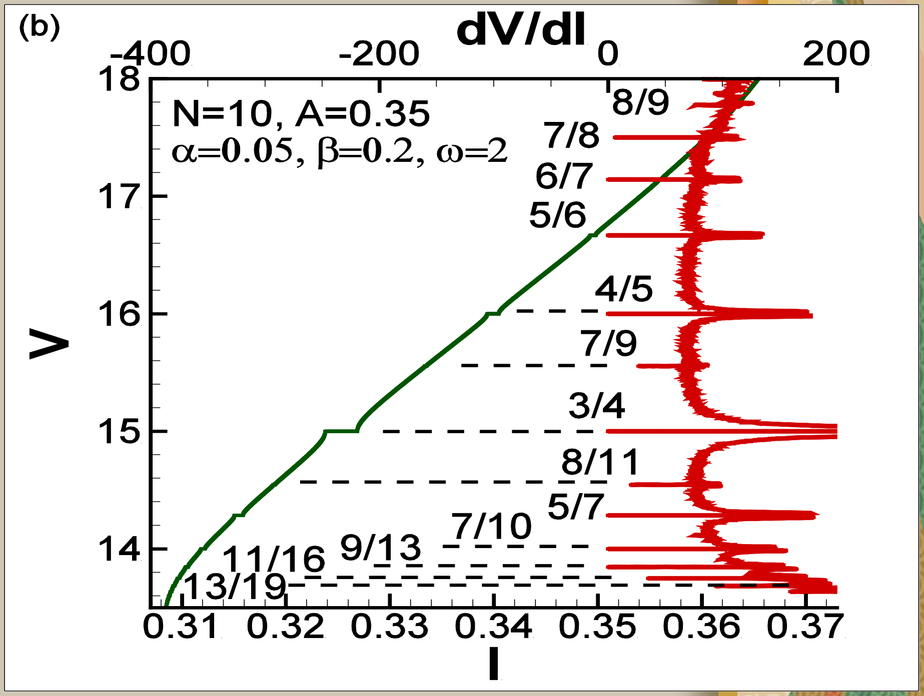
Comparison of the J. Clarke's experimental results [Phys. Rev 155, 419 (1967)] with continued fractions.





Analysis of subharmonic appearance with an increase in A
(a) A=0.17; (b) A=0.35



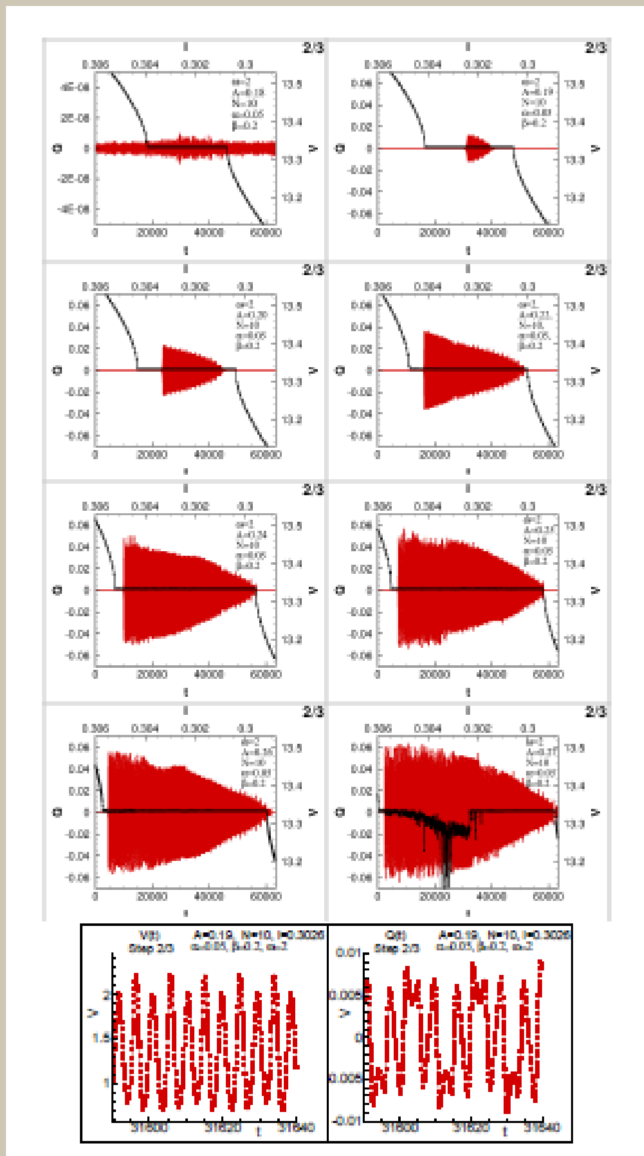


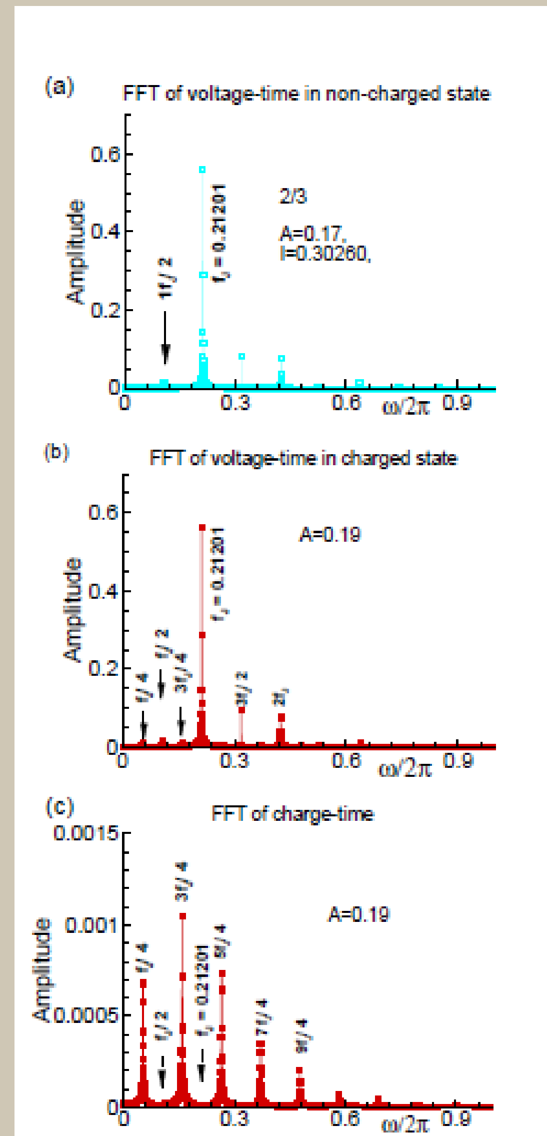
2/3, 3/5, 4/7…→1/2
N-1/(n+(1/m)) with N=1, n=2

3/4, 4/5, 5/6, 6/7…→ 1
N-1/n with N=1
3/4, 5/7, 7/10… → 2/3
N-1/(n+(1/m)) with N=1, n=3

Step 2/3

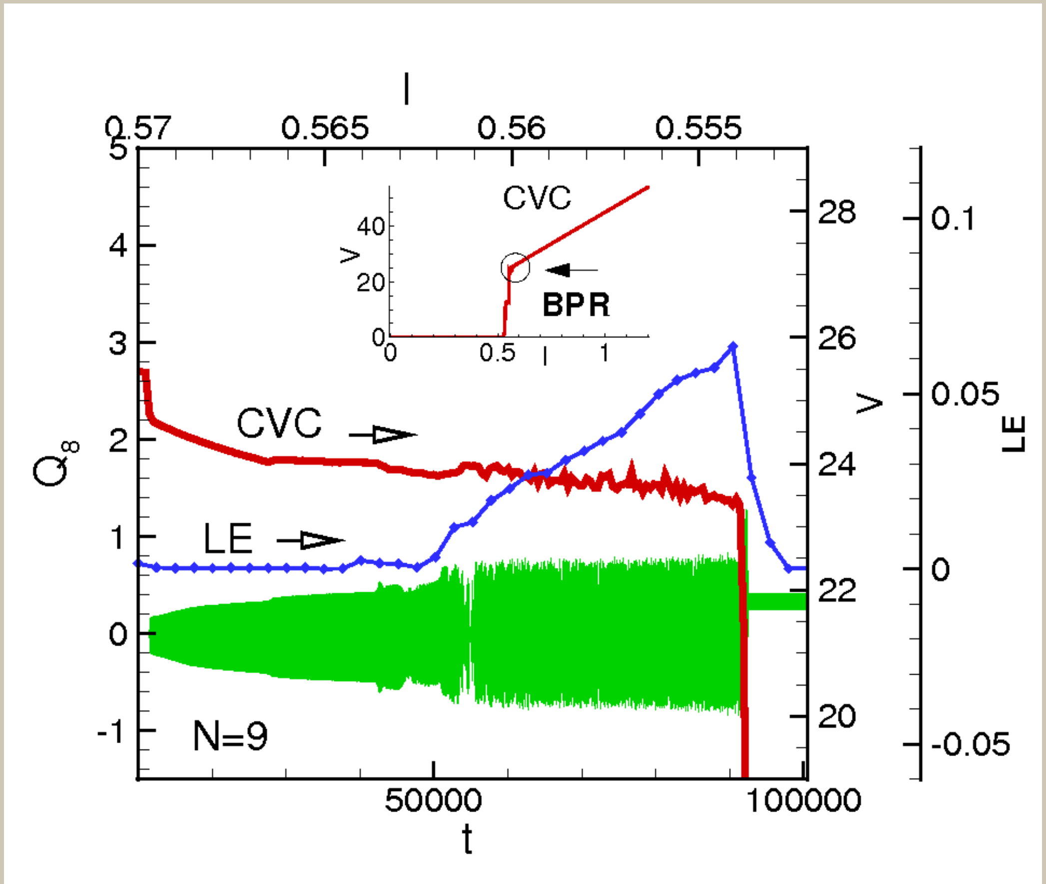
Parametric resonance and double resonance conditions in case of SSS





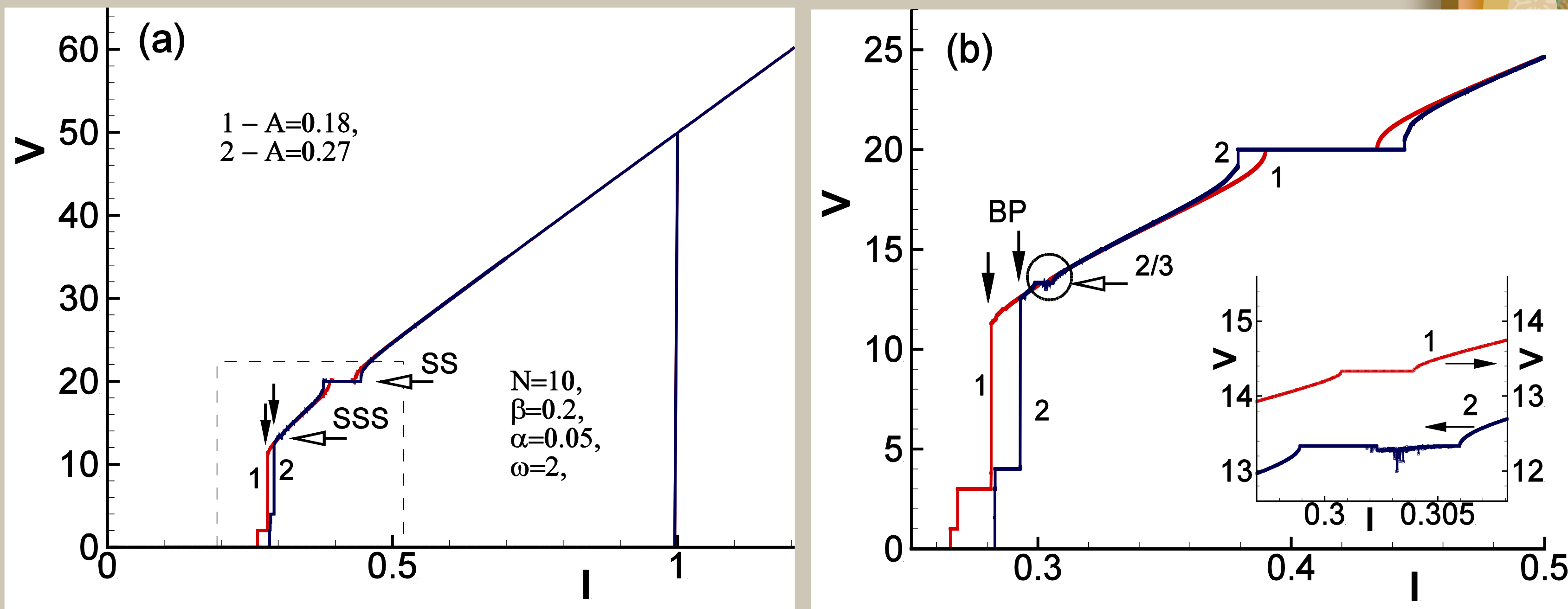
Chaos induced by coupling between Josephson junctions

CVC and Lyapunov exponent as function of current with the charge in the S-layer as a function of time.



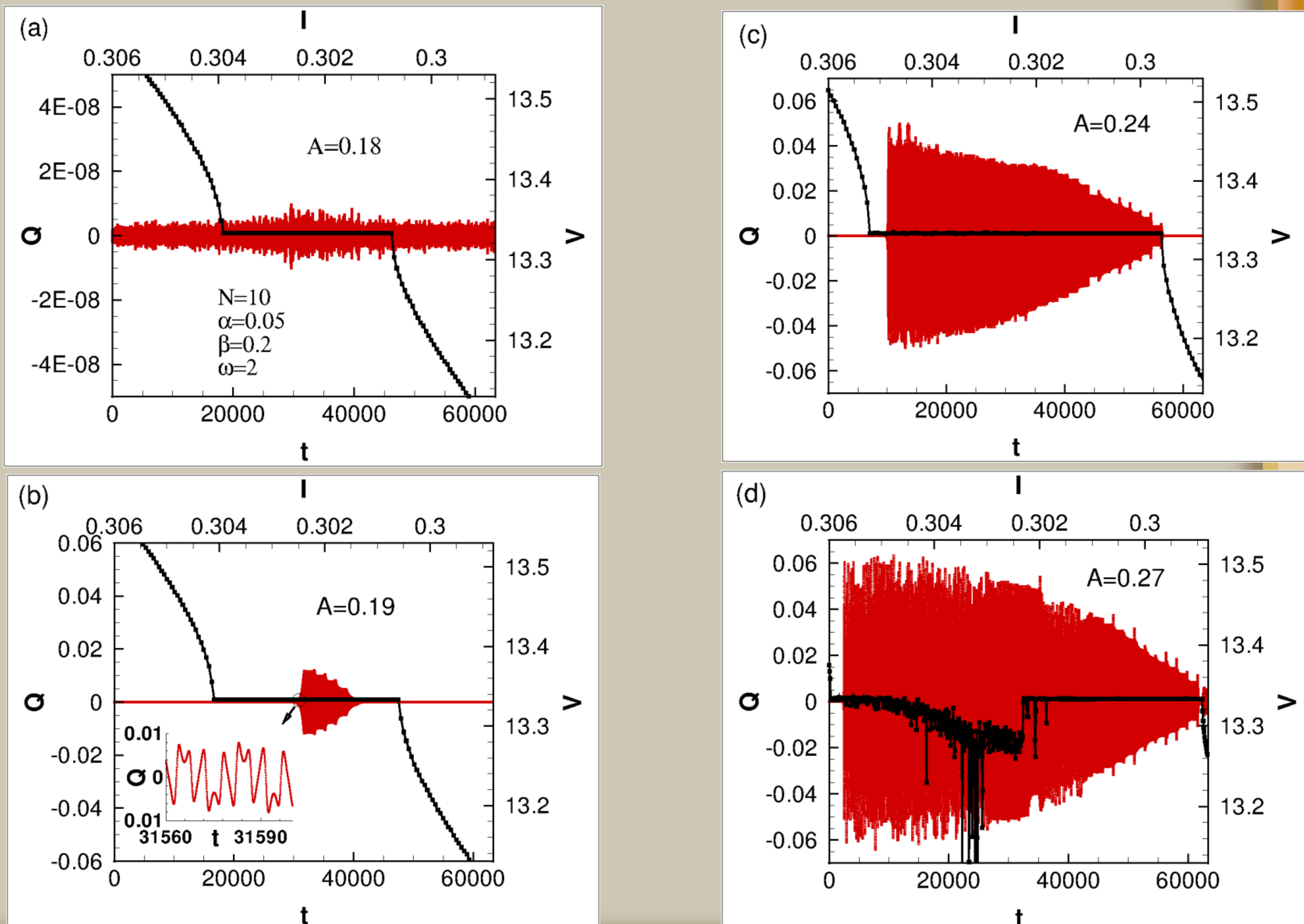
Yu. M. S., M. Hamdipour, M. R. Kolahchi, A. E. Botha, and M. Suzuki,
Physics Letters A 376 (2012) 3609–3619

IV-characteristics at two amplitudes of radiation

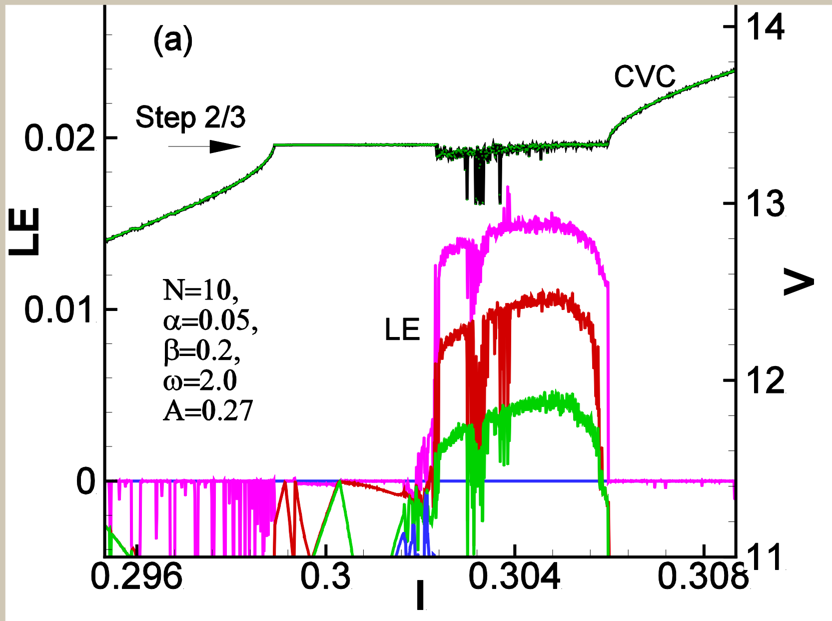


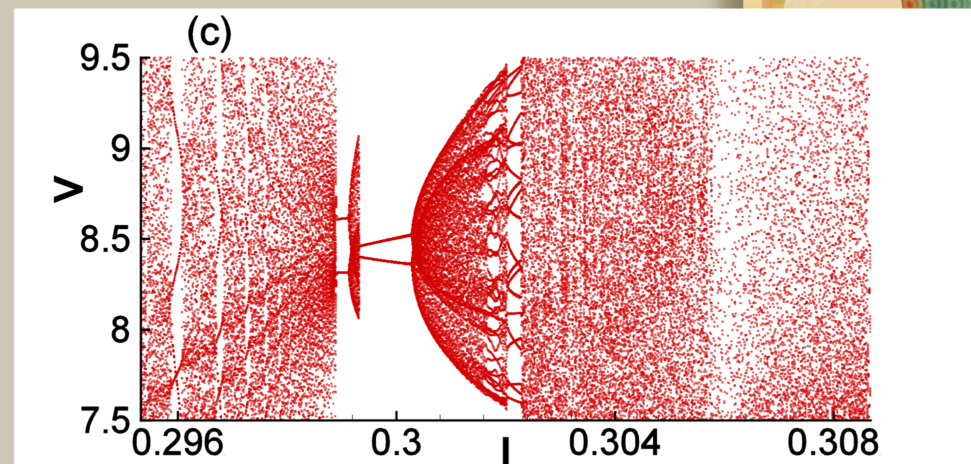
Yu.M. Shukrinov, H.Azemtsa-Donfack, À.E.Botha.
JETP Letters, 101, 251--257 (2015)

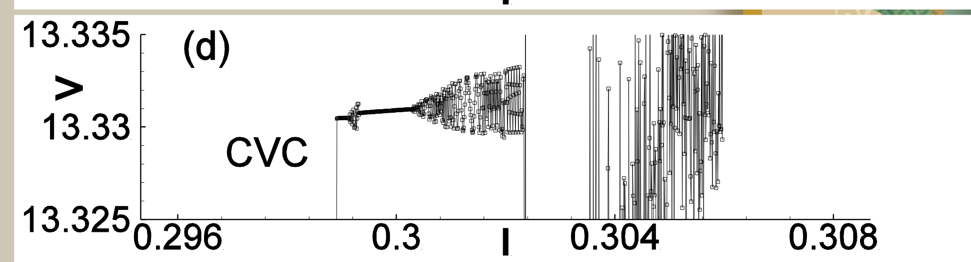
Charging of superconducting layers and chaos



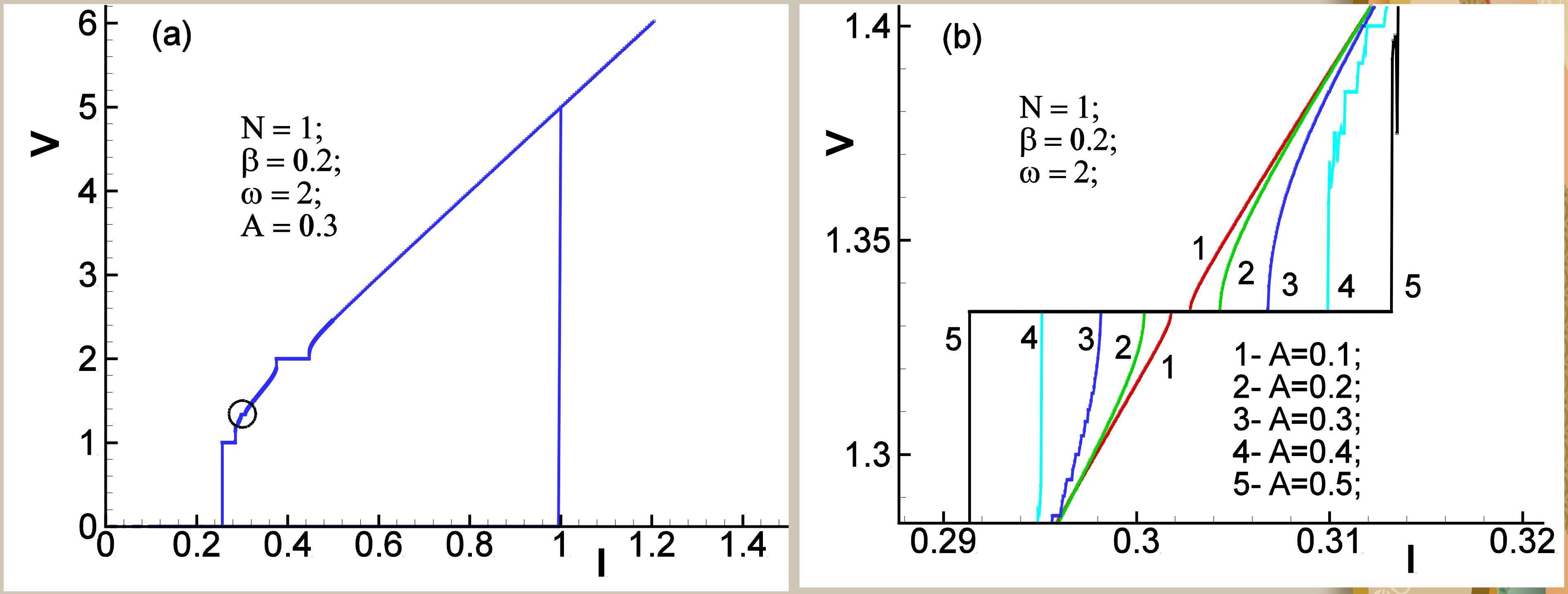
Lyapunov exponents and Poincare section





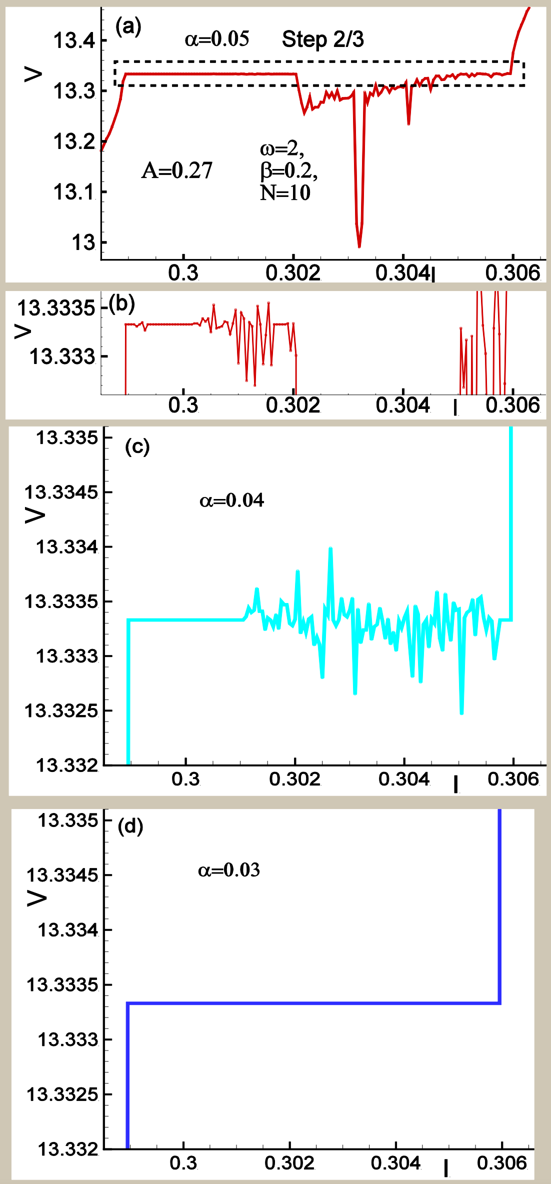


Comparison with a case of single Josephson junction

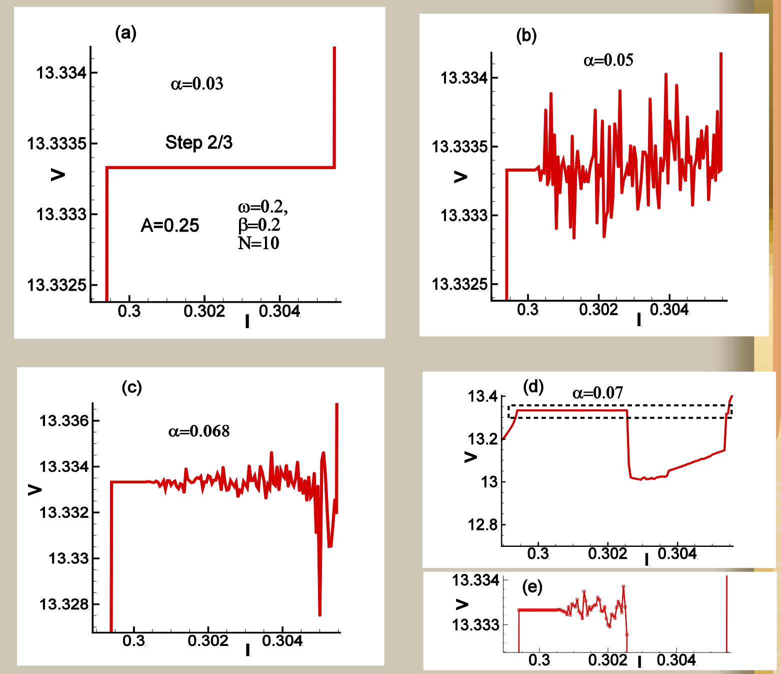


Effect of coupling between junctions

A=0.27

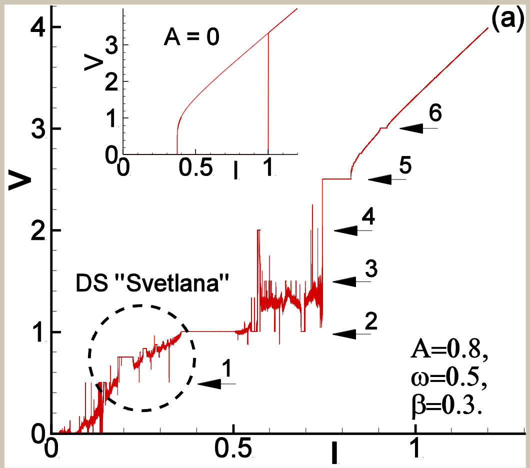


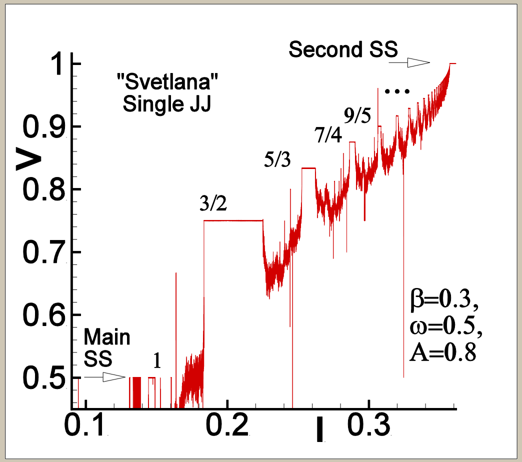
A=0.25

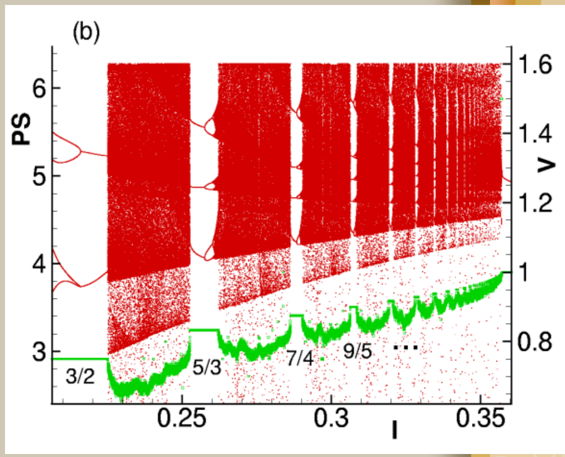


Yu.M. Shukrinov, H.Azemtsa-Donfack, À.E.Botha.
JETP Letters, 101, 251--257 (2015)

DS structure “Svetlana”



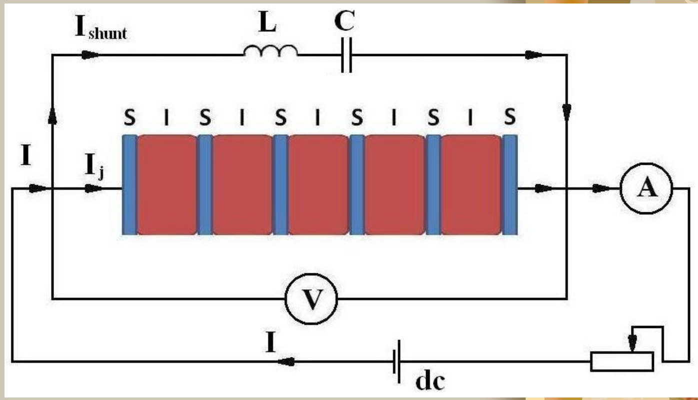




Yu. M. Shukrinov, S. Yu. Medvedeva, A. E. Botha, M. R. Kolahchi, A. Irie,
Chaos 24, 033115, 2014

Shunting of Josephson junctions

Shunting



$$
\begin{cases}
\dfrac{\partial \varphi_l}{\partial \tau} = V_l - \alpha (V_{l+1} + V_{l-1} - 2V_l) \\
\dfrac{\partial V_l}{\partial \tau} = I - \sin \varphi_l - \beta \dfrac{\partial \varphi_l}{\partial \tau} - CU \\
\dfrac{\partial U}{\partial \tau} = \dfrac{1}{LC} \left(\sum_{l=1}^{N} V_l - u_c \right) \\
\dfrac{\partial u_c}{\partial \tau} = U
\end{cases}
$$

$I \to I_c$;

time $\to \tau = \omega_p t$, $\omega_p = \sqrt{\dfrac{2eI_c}{C_j \hbar}}$;

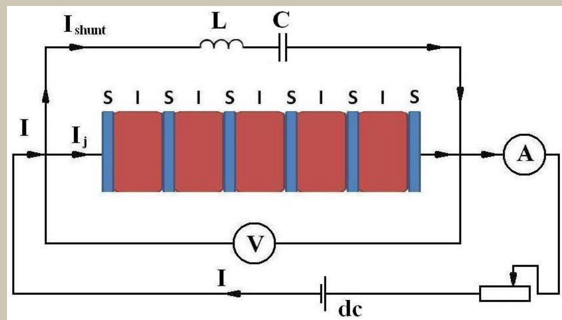
voltages V_l и $u_c \to V_0 = \dfrac{\hbar \omega_p}{2e}$;

$C \to C_j$;

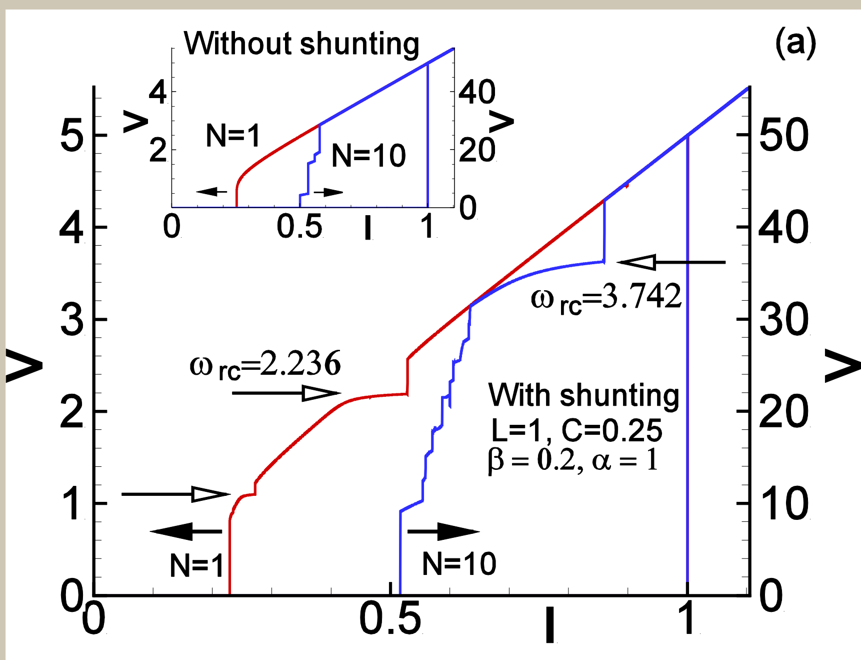
$L \to (C_j \omega_p^2)^{-1}$.

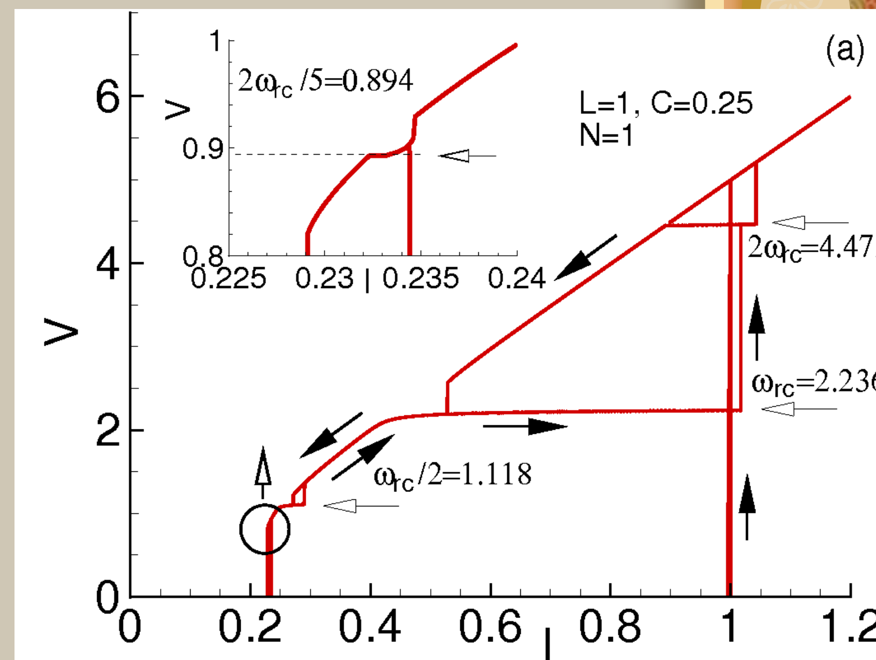
$$\beta = \frac{1}{R_j} \sqrt{\frac{\hbar}{2eI_c C_j}} = \frac{1}{\sqrt{\beta_c}}.$$

Resonance circuit branches



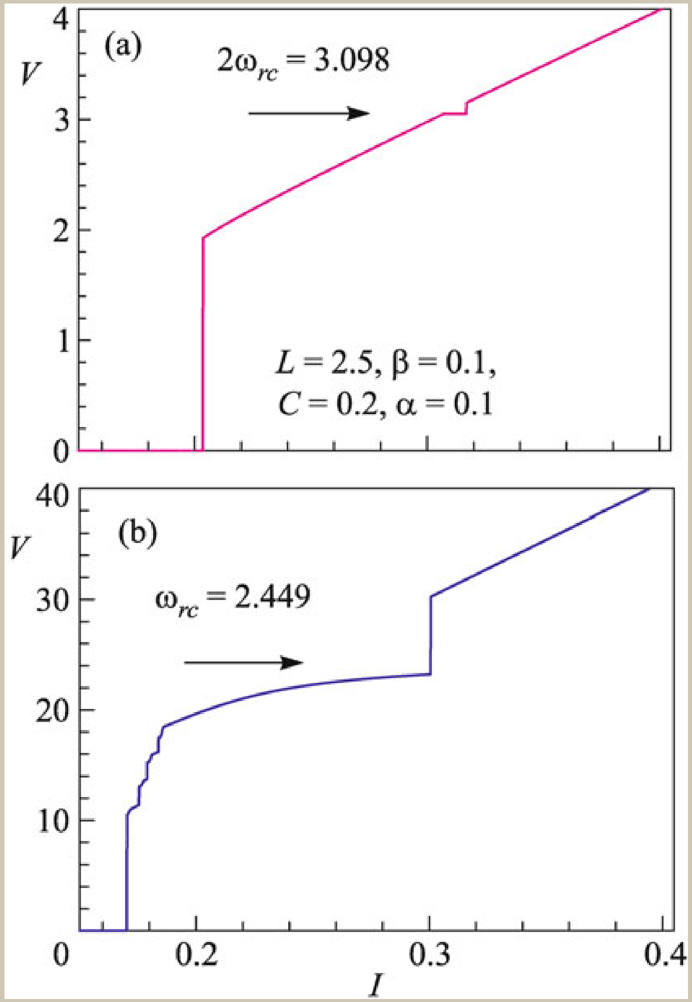
$$\omega_{rc} = \sqrt{\frac{1+NC}{LC}}$$





Yu. M. Shukrinov, I. R. Rahmonov, K. Kulikov - JETP Letters, 96, 657 (2012).

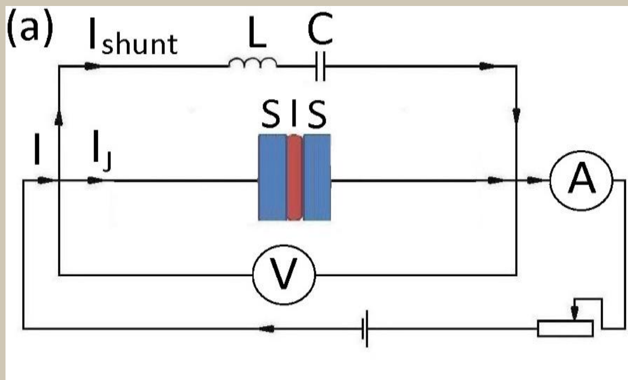
Current–voltage characteristics for the cases *N = (a) 1 and (b) 10 calculated at the parameters** corresponding to BSCCO

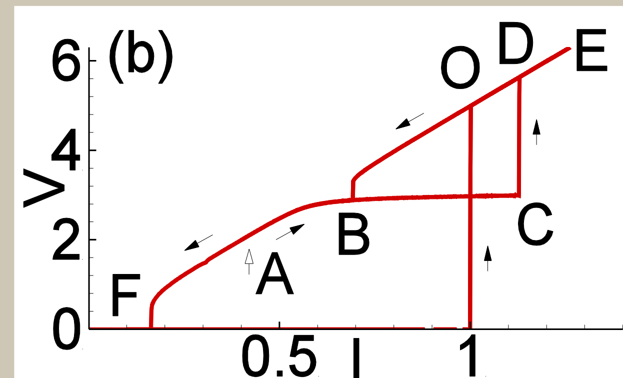


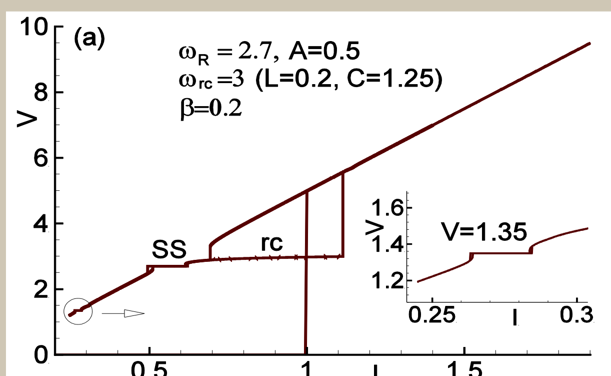
- We calculate the necessary capacitance of theshunt at a given inductance *L = 50 pH using the typi*cal parameters for BSCCO
- *S = 1 μm^2,*
- *d_I = 12 × 10^(–10)m,*
- ε = 25,
- β = 0.1,
- α = 0.1,
- ω_p = 0.5 THz.
- At these parameters, the capacitance of the Josephson junction is *C_J = 0.2 pF.*
- At real inductance *L = 50 pH, the* dimensionless inductance is *L = 2.5. Consequently, the shunting capacitance Csh* = 0.04 pF is sufficient for the observation of the *rc* branch at *LC* =0.5,

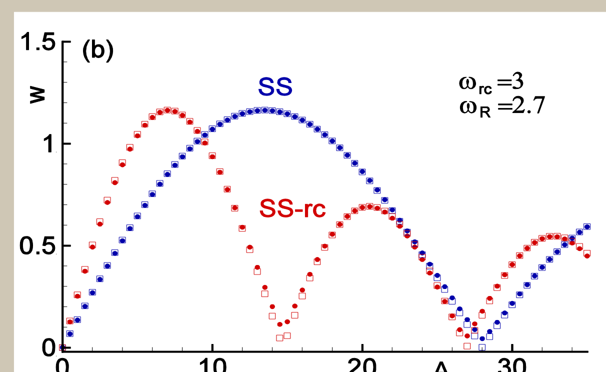
* Presented by A. Ustinov and E. Ilichev

Variation of amplitude dependence of SS width in resonance region





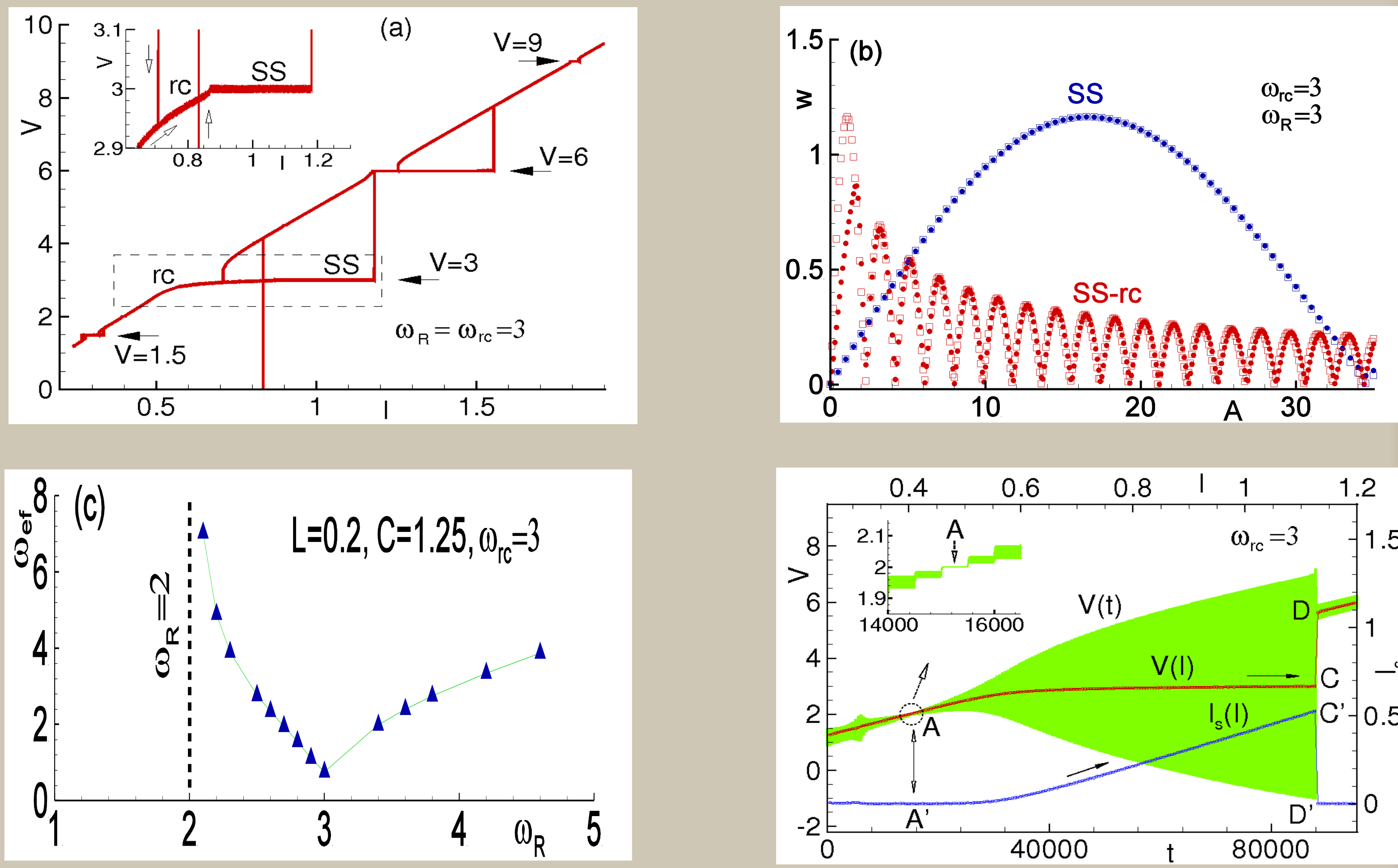




$$\Delta I = 2|J_n(Z)|, \qquad Z = \frac{A}{\omega_R}\frac{1}{\sqrt{\beta^2+\omega_R^2}}$$

Yu. M. Shukrinov, I. R. Rahmonov, K. V. Kulikov and P. Seidel. - EPL, 110, 47001 (2015)

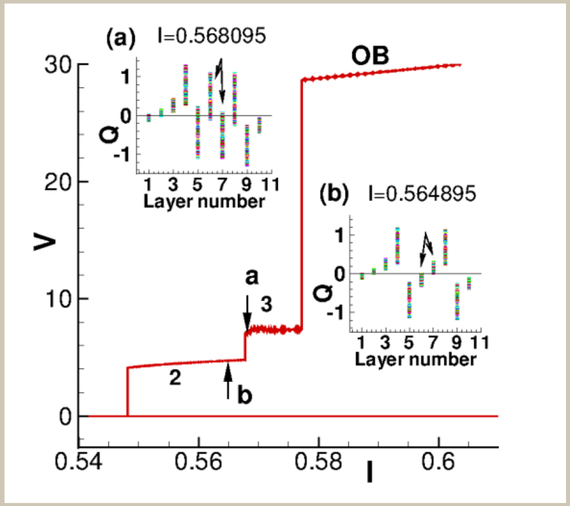
Resonance conditions

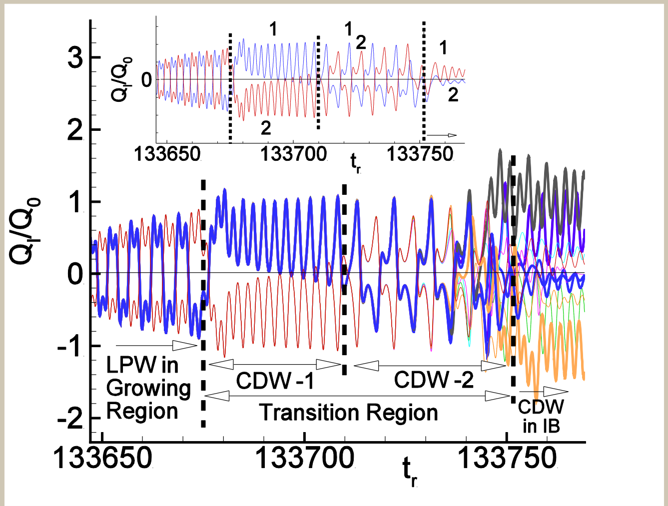


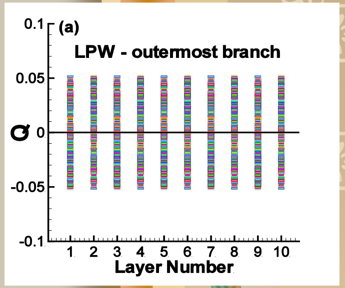
Yu. M. Shukrinov, I. R. Rahmonov, K. V. Kulikov and P. Seidel. - EPL, 110, 47001 (2015)

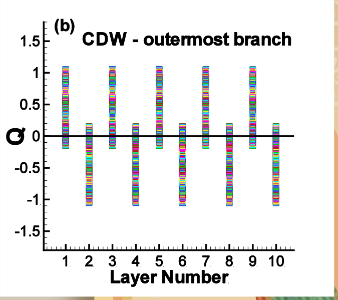
- Charge density wave

Charge Density Waves

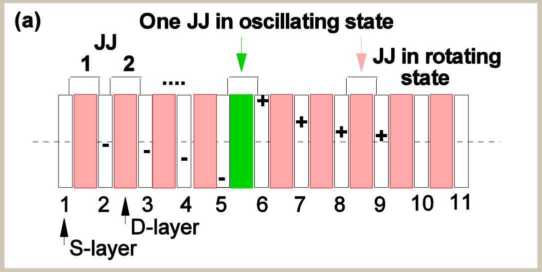


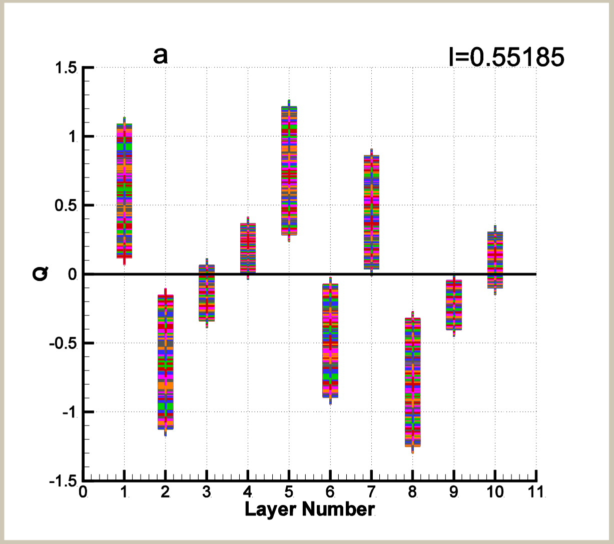


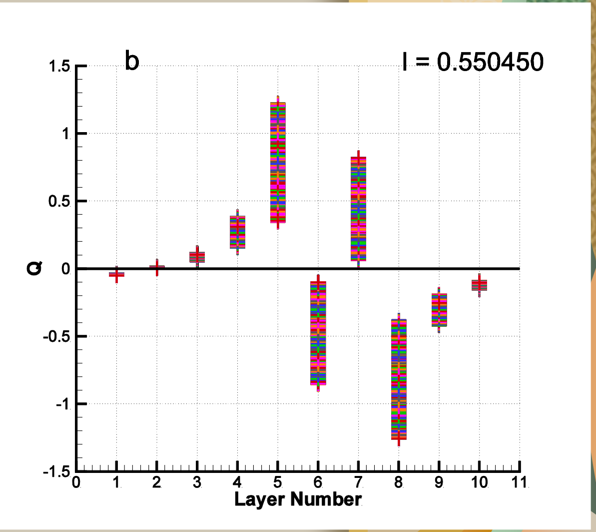




LPW→ CDW, CDW → CDW



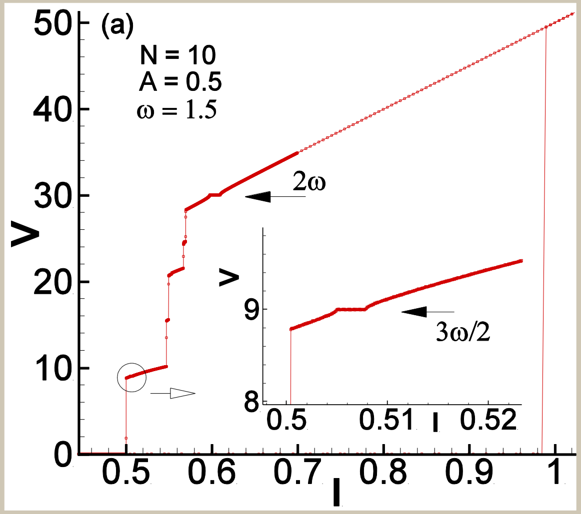


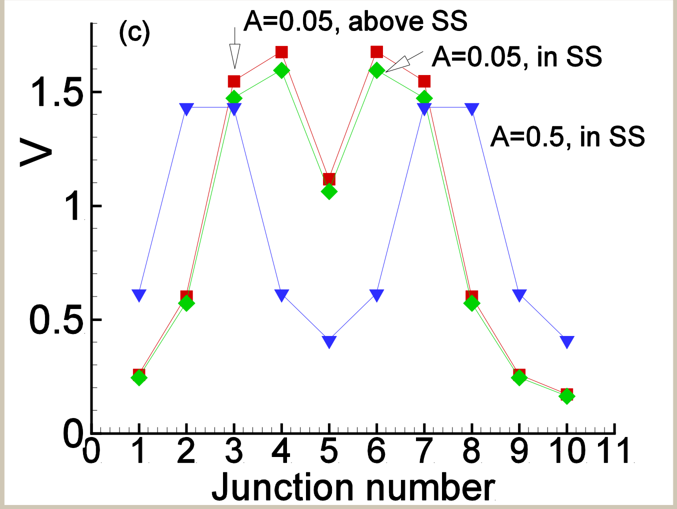


Yu. M. S., and H. Abdelhafiz., JETP Letters, 2013, Vol. 98, No. 9, 551

Breathing Charge Density Waves in Intrinsic Josephson Junctions

The effect of external electromagnetic radiation on the system of coupled Josephson junctions in the CDW state is completely different from the case of single JJ. It causes the appearance of the set of the Shapiro steps in the IV-characteristics of JJ of the stack related to the voltage distribution among JJs. However, usual harmonics and subharmonics of radiation frequency are observed in the total IV-characteristics of the stack.





Yu. M. Shukrinov and H. Abdelhafiz. - Pis'ma v ZhETF, 98, (2013) 619--624; JETP Letters, 2013, Vol. 98, No. 9, pp. 551–556.

Generalized Josephson Relation with Charge Imbalance:

$$\frac{\hbar}{2e}\dot{\varphi}_l(t) = (1+2\alpha)V_l - \alpha(V_{l-1} + V_{l+1}) + \Psi_{l+1} - \Psi_l$$

Where $\alpha = r_D^2 \epsilon_o / d d_s^l$

CCJJ+DC Model

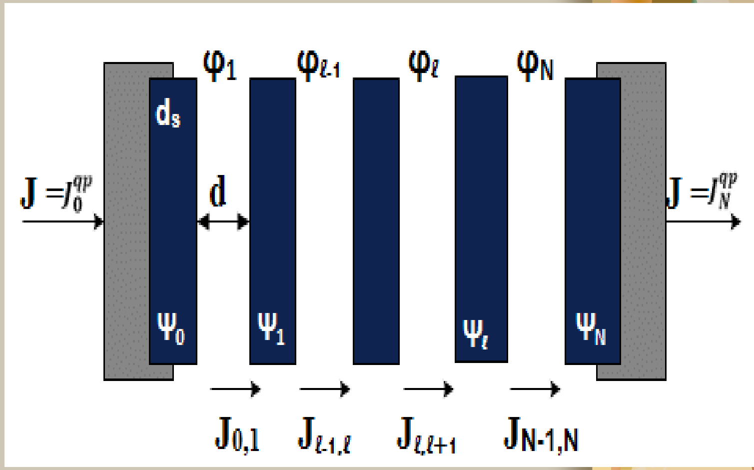
Total current :

$$J = C\frac{dV_l}{dt} + J_c^l \sin\varphi_l(t) + \frac{\hbar}{2eR}\dot{\varphi}_l(t) + \frac{\Psi_{l-1}(t) - \Psi_l(t)}{R}$$

Dynamics for the Quasi-particle Potential for Nonstationary Case

$$\dot{\Psi}_l = \frac{4\pi r_D^2}{d_s^l}\left(J_l^{qp} - J_{l+1}^{qp}\right) - \frac{\Psi_l}{\tau_{qp}^l}$$

$$J_{qp}^l = \frac{\hbar}{2eR}\dot{\varphi}_l + \frac{\Psi_{l-1} - \Psi_l}{R}$$



System of kinetic equ's:

$$\dot{\Psi}_0 = \frac{4\pi r_D^2}{d_s^0}\left(J - \frac{\hbar}{2eR}\dot{\varphi}_{0,1} + \frac{\Psi_1 - \Psi_0}{R}\right) - \frac{\Psi_0}{\tau_{qp}^0}$$

$$\dot{\Psi}_l = \frac{4\pi r_D^2}{d_s^l}\left(\frac{\hbar}{2eR}\dot{\varphi}_{l-1,l} - \frac{\hbar}{2eR}\dot{\varphi}_{l,l+1} + \frac{\Psi_{1-1} + \Psi_{l+1} - 2\Psi_l}{R}\right) - \frac{\Psi_l}{\tau_{qp}^l}$$

$$\dot{\Psi}_N = \frac{4\pi r_D^2}{d_s^N}\left(-J + \frac{\hbar}{2eR}\dot{\varphi}_{N-1,N} + \frac{\Psi_{N-1} - \Psi_N}{R}\right) - \frac{\Psi_N}{\tau_{qp}^N}$$

For Numerical Calculations:

$$\eta_l = 4\pi r_D^2 \tau_{qp}^l / R d_s^l$$

$$\zeta_l = \omega_p \tau_{qp}^l$$

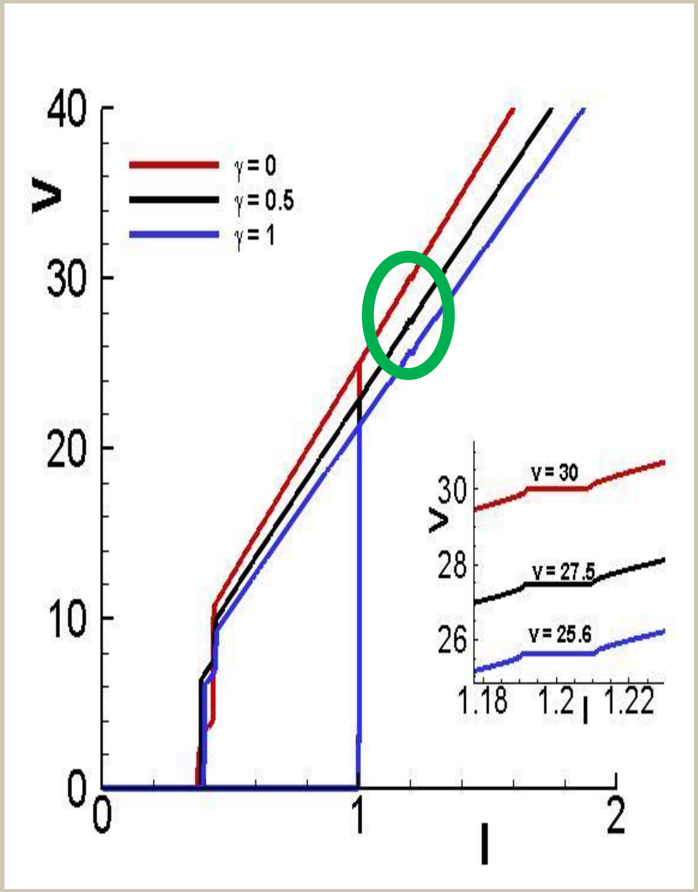
Where :

η is the disequilibrium parameter

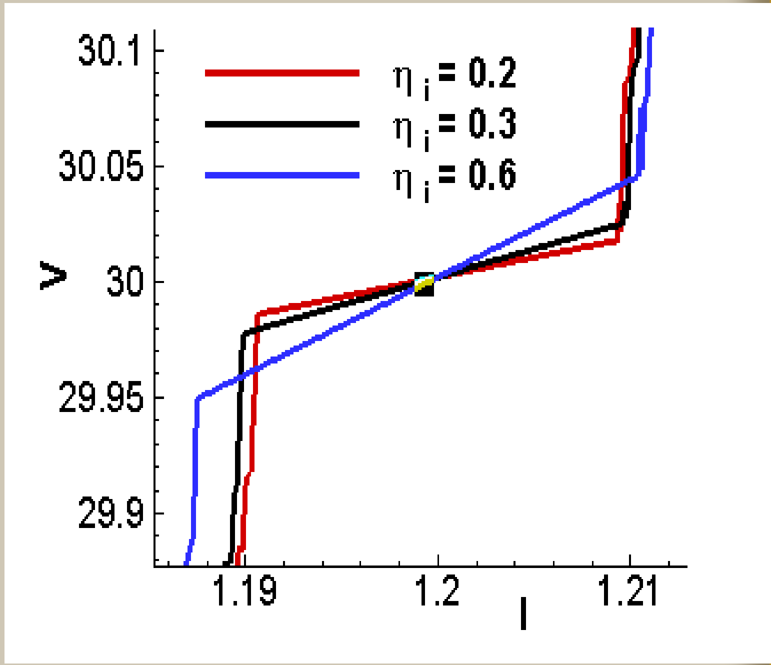
ζ is the normalized QP relaxation time

ω_p is the plasma freq.

Effect of the boundary conditions on the Shapiro Step



Variation of the Shapiro step slope with η



Yu. M. Shukrinov , *M. Nashaat , K. V. Kulikov , R. Dawood*
Effect of Charge Imbalance on Shapiro Step in Intrinsic Josephson Junctions, in preparation , 2015

Conclusions

We predict a series of effects in intrinsic Josephson junction in HTSC, particularly:

- Variation of longitudinal plasma wavelength with an increase of the amplitude of radiation.
- Breathing charge density waves
- Variation of amplitude dependence of SS width in resonance region
- Chaos induced by coupling between Josephson junctions
- Slope and shift of the Shapiro step in IV-characteristics

Thank you

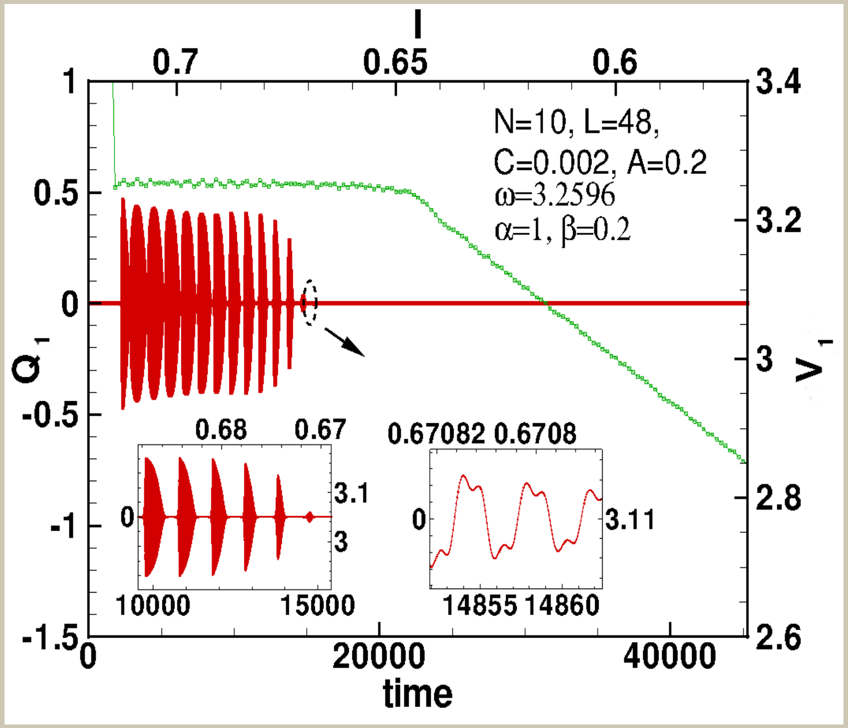
for your

attention!

Estimation of charge

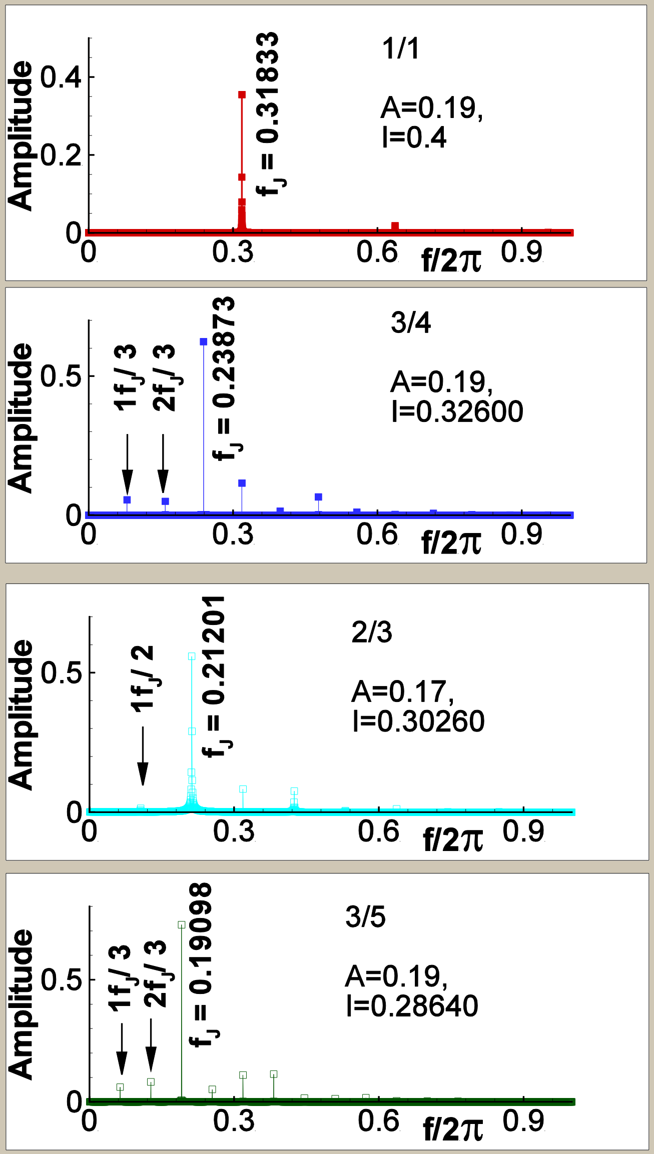
Using Maxwell equation, we express the charge density in the S-layer by the voltages in the neighbor insulating layers $Q_l = Q_0\alpha(V_l - V_{l-1})$, where $Q_0 = \varepsilon\varepsilon_0 V_0 / r_D^2$, $V_0 = \hbar\omega_p / 2e$, ω_p is plasma frequency and r_D is Debye screening length. We can estimate the value of electric charge in the superconducting layer. Using $\varepsilon_0 = 0.0885 * 10^{-11}$ F/m, $e/\hbar = 2.4 * 10^{14}$ 1/V $*$ s and consider $r_D = 3 * 10^{-10}$ m, $\varepsilon = 25$, $\omega_p = 10^{12}$ 1/s, we get $V_0 = 2 * 10^{-3}$ V and $Q_0 = 5 * 10^6 K/m^3$. So, at $Q = Q_0$ for the superconducting layer with area $S = 1\mu m^2$ and thickness $d = 3 * 10^{-10} m$, the charge in the superconducting layer is equal to $1.5 * 10^{-15} K$. This value is not high enough, but the charge dynamics on the superconducting layers determines the features of current voltage characteristics of the coupled Josephson junctions.

Shunted stack with N=10 under external radiation



- Yu.Shukrinov, I. Rahmonov, K. Kulikov, P. Seidel, E. Il'ichev, ISEC'13, 219-221, 2013.

Results of FFT analysis of V(t)



Charging of S-layers