NONLINEAR TRANSPORT OF ATOMIC BOSE-EINSTEIN CONDENSATE: RELATION TO JOSEPHSON EFFECT

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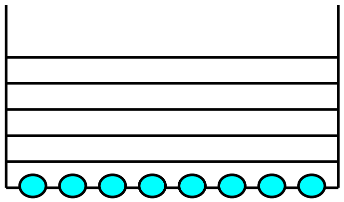


Forum Brazil-JINR, Dubna, 15-19.02.2015

Motivation

Trapped BEC:

- Unique in the precision and flexibility in control and manipulations.
  Unique laboratory for investigation of various quantum scenarios.

- Crossover of BEC with other areas
  (nano, superfluidity, **Josephson effect,** qauge theories, quantum control, quantum informatics, topological systems, quantum turbulence, nuclear physics, astrophysics,..)

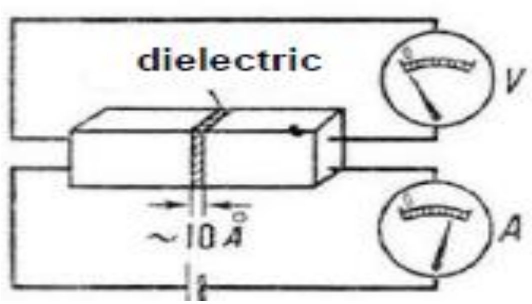
- Analogy between:
  Josephson effect in superconductors and BEC in a double-well trap

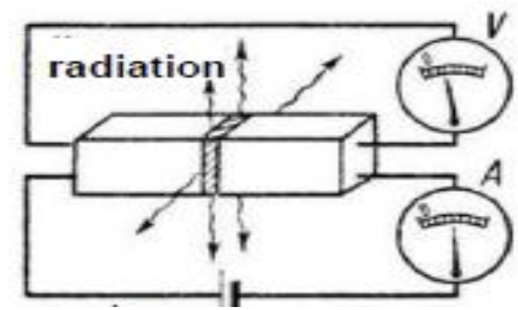
V.O.N, A.N. Novikov, and E. Suraud, Laser Phys., 24 125501 (2014)

Josephson effect

SJJ -- **S**uperconductor Josephson Junction

Tunneling transfer of Cooper pairs of electrons via a thin dielectric layer separating two superconductors





$$I_s = I_0 \sin(\theta)$$
$$\dot{\theta} = \frac{2eV}{h}$$

Josephson equations

$$\theta = \varphi_R - \varphi_L$$

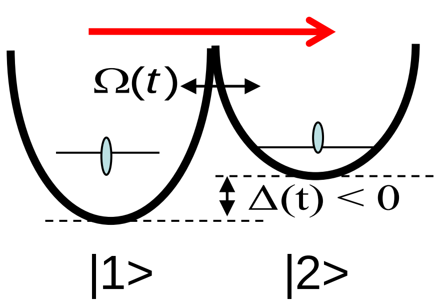
Stationary effect (dc):

- **small constant current** $I_s < I_0$
- **no voltage** V=0
- **constant phase difference** θ

Non-stationary effect (ac):

- **large current** $I > I_0$
- **non-zero voltage** V>0
- **both super and normal currents** $I = I_s + I_n$
- **emission of photons**

BJJ – Bose Josephson Junction



Analogy between SJJ and BJJ:

- 2 superconductors ⟷ 2 superfluid BECs
- thin dielectric ⟷ potential barrier
- in both cases: tunneling of bosons, <u>weak coupling</u>
- initiation by: current ⟷ barrier shift

Josephson effect in BEC:

S. Giovanazzi, A Smerzi and S. Fantoni, PRL 84, 4521 (2000)

Theoretical predictions:

- **analogy between Josephson equations for superconductors and Gross-Pitaevskii equations for BEC in a double-well trap**
- **generation of the current by the barrier shift in a double-well trap**
- **critical current**

Experiment:

S. Levy, E. Lahoud, I. Shomroni and I. Steihauer. Nature 449, 579 (2007)

- observation of dc-ac Josephson effect for BEC in a double-well trap

The aim:

- To analyze dc-ac Josephson effects in transport of BEC in a double well trap

New points:

- more generally (in terms of currents, chemical potentials and <u>phase difference θ</u>)
- important role of nonlinearity in BJJ (the critical current can be increased by 3 orders of magnitude),
- velocity time-profiles of the barrier shift
- critical velocity: similarity of ac-dc and adiabatic-nonadiabatic transitions

$$z=\frac{N_L - N_R}{N}, \quad I_s=-\frac{\dot{z}}{2}$$

$$\theta = \varphi_R - \varphi_L$$

$$\Longleftrightarrow$$

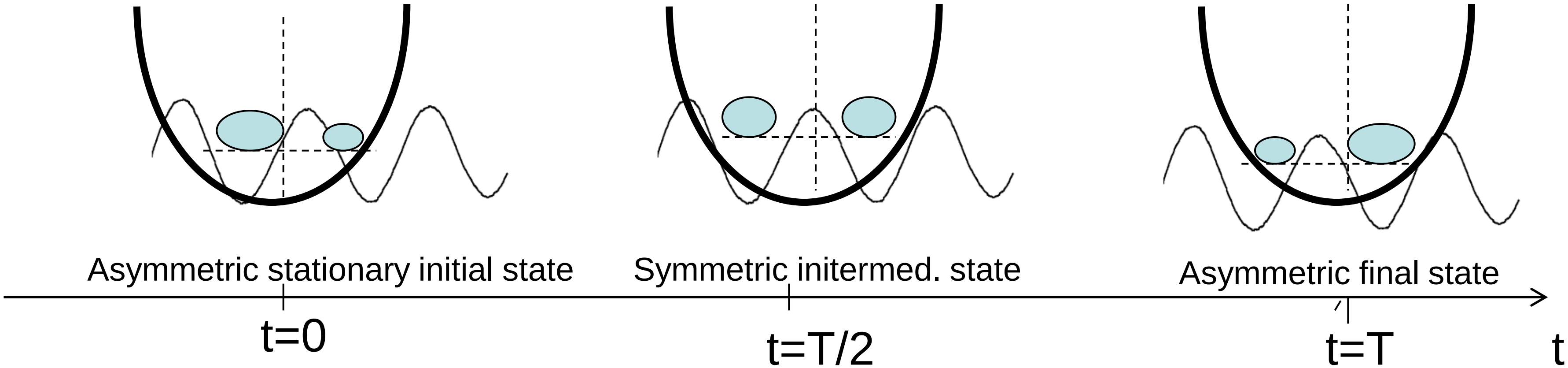
$$I_s = I_0 \sin(\theta)$$

$$\dot{\theta} = \frac{\Delta\mu}{h}, \quad \Delta\mu = 2eV$$

V.O.N, A.N. Novikov, and E. Suraud, Laser Phys., <u>24</u> 125501 (2014)

Tunneling transport induced by the barrier shift:

Potential: HO confinement + barrier (element of a periodic optical lattice)

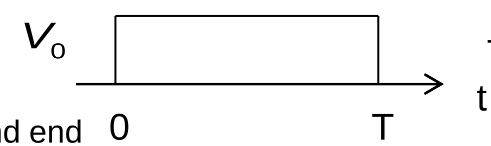


N1(0)=800, N2(0)=200

z(0) = [N1(0)-N2(0)]/N = 0.6)

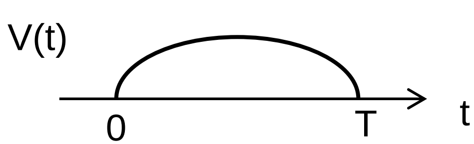
★ Fast and slow (adiabatic) transfer, velocity time profile

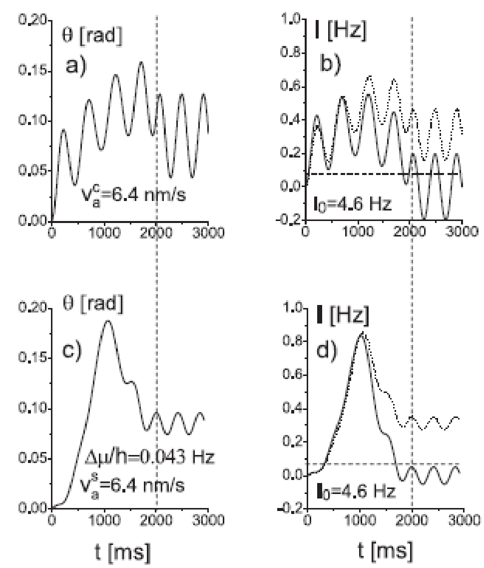
1) $V_0 = const$



Sharp changes at the beginning and end of the evolution result in strong undesirable dipole oscillations

2) $V(t) = V_m \cos^2(\frac{\pi t}{T} + \frac{\pi}{2})$





Model

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=[-\frac{\hbar^2}{2m}\nabla^2+V_{ext}(\vec{r})+g\,|\Psi(\vec{r},t)|^2]\Psi(\vec{r},t)$$

Time-dependent 3D Gross-Pitaevskii equation for order parameter

confinement + barrier

BEC interaction, nonlinearity

$$V_{ext}(\vec{r})=\frac{m}{2}(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)+V_0\cos^2(\frac{\pi(x-x_0(t))}{q_0})$$

HO confinement $j=L,R$ barrier

$$\int_{-\infty}^{+\infty}dr^3|\Psi(\vec{r},t)|^2=N$$

$$\varphi_j=\arctan\frac{\int_{-\infty}^{+\infty}dr^3\mathrm{Re}\Psi_j(\vec{r},t)|\Psi_j(\vec{r},t)|^2}{\int_{-\infty}^{+\infty}dr^3\mathrm{Re}\Psi_j(\vec{r},t)|\Psi_j(\vec{r},t)|^2},\quad j=L,R$$

$$N_L=\int_{-\infty}^{+\infty}dydz\int_{-\infty}^{x_0(t)}dx|\Psi(\vec{r},t)|^2$$

$$N_R=\int_{-\infty}^{+\infty}dydz\int_{x_0(t)}^{+\infty}dx|\Psi(\vec{r},t)|^2$$

$$\rho(x,t)=\int_{-\infty}^{+\infty}dydz|\Psi(x,y,z,t)|^2$$

$z=\frac{N_L-N_R}{N}$ population imbalance

$I_s=-\frac{\dot{z}}{2}$ current

$\theta=\varphi_R-\varphi_L$ phase difference

$\Delta\mu=h\dot{\theta}$ chemical potential difference

GPE

$$i\hbar \frac{\partial}{\partial t}\Psi(\vec{r},t) = [-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\vec{r}) + g\,|\Psi(\vec{r},t)|^2]\Psi(\vec{r},t)$$

⇩

In two-mode approximation (TMA), the GPE is reduced to the system of equations similar to Josephson equations:

$$\dot{z} = -2I = -2K\sqrt{1-z^2}\sin\theta$$

$$\dot{\theta} = \frac{\Delta\mu}{2} + K\frac{z}{\sqrt{1-z^2}}\cos\theta + \frac{NU}{2}z$$

↔

Josephson equations

$$I_s = I_0 \sin(\theta)$$

$$\dot{\theta} = \frac{\Delta\mu}{h}$$

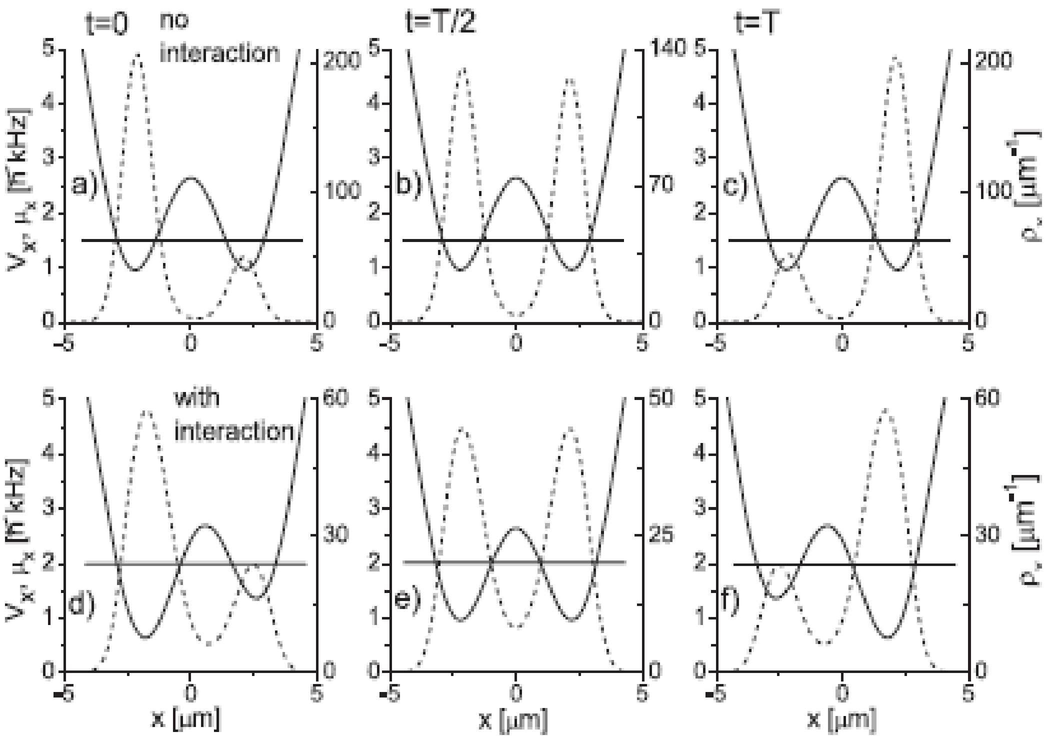
Exact current: $I = -\dfrac{\dot{z}}{2}$

Critical current : $I_0 = \dfrac{I(t)}{\sin\theta(t)}$ at $t = \dfrac{T}{2},\ z = 0$

Approx. current: $\tilde{I} = I_0\sqrt{1-z^2}\sin\theta$

Trap-density configuration

M. Albiez, et al,
PRL 95 010402 (2005)



Initial barrier shifts for z(0)=0.6

d= 3 nm - no inter.

d=500 nm - with inter.

- trap parameters and initialization of the process like in Heisenberg experiment ↑

-initial equilibrium asymmetric state

- barrier position as a control parameter

- transfer during the time T to get the inverse population

- weak coupling

-- the chemical potential is always below the barrier top → tunneling

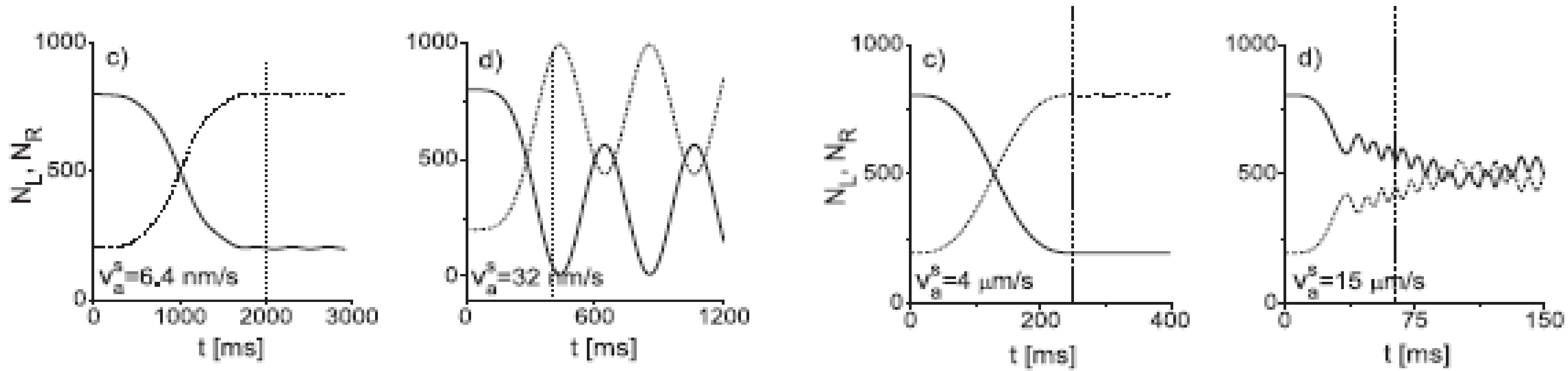
-interaction:
  - more coupling
  - larger initial barrier shift

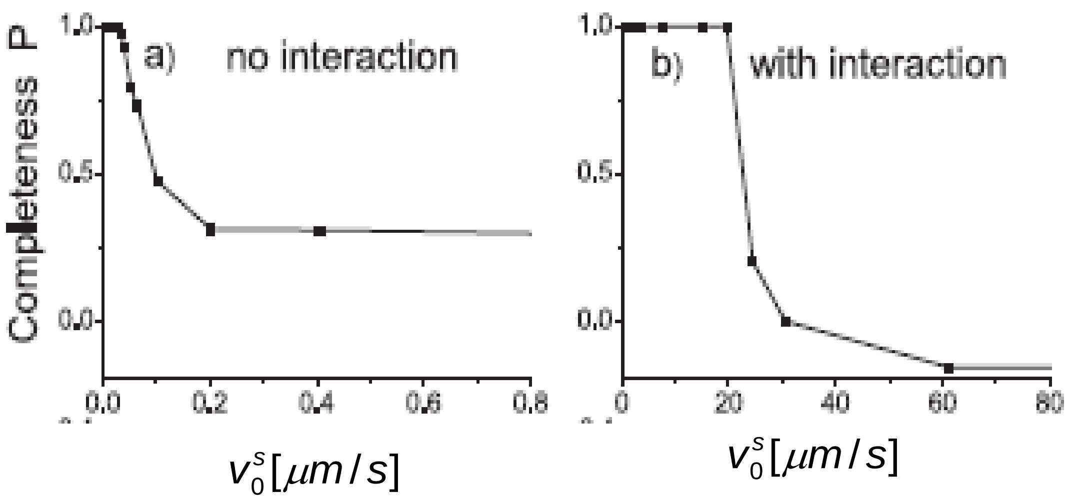
- the similar technique was used in

V.O.N., A.N. Novikov, and E. Suraud,
J. Phys. B 45 225303 (2012)

Impact of interaction between BEC atoms

no interaction   with interaction



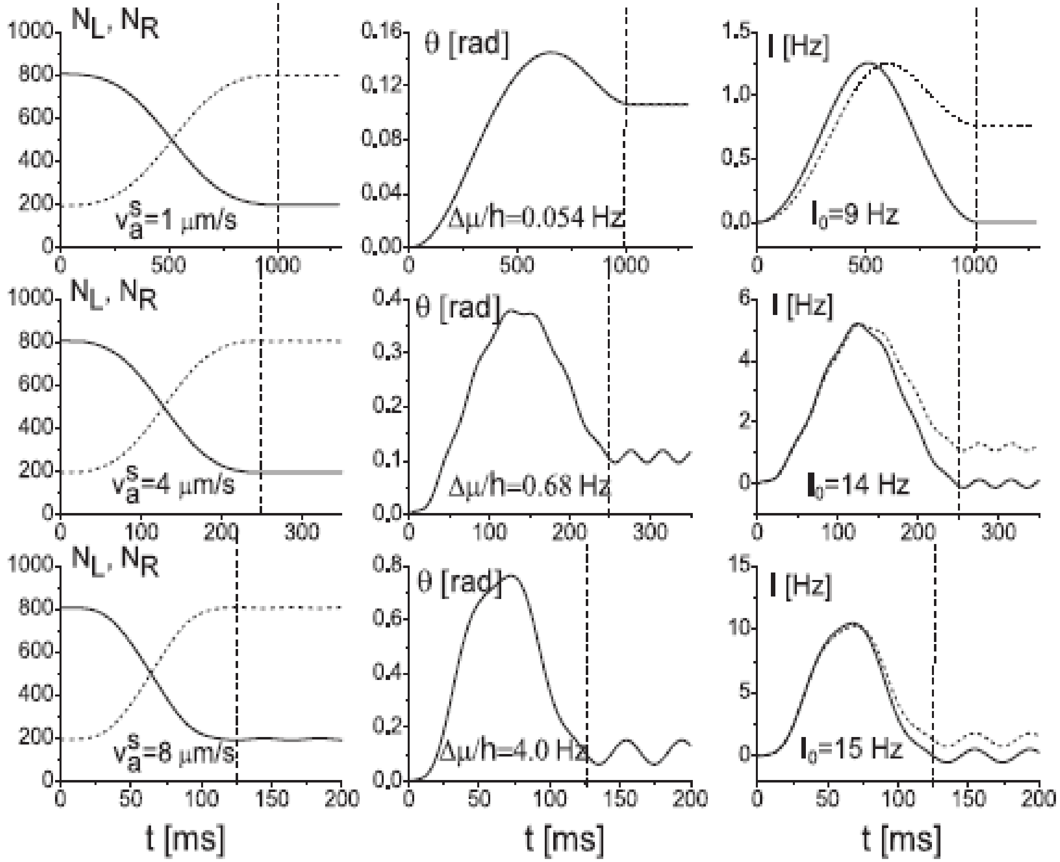


Interaction:
- allows robust transfer three orders of magnitude faster,
- provides a wide plateau for velocities with a complete population transfer
- At some critical velocity ~20, we get the adiabatic limit and the transfer fails.

Does this process correspond to Josephson effects?
Does the transport critical velocity correspond to Josephson critical current?

Stationary Josephson effect (dc) →

- $I_s < I_0$
- $\Delta\mu = 0$
- θ =const



- exact current I(t) from GPE

.. approximate current i(t) from TMA-GPE

The transport:

- $I < I_0$
- $\frac{\Delta\mu}{h} \sim$ 1-4 Hz   not zero but small
- $\theta < 1$ rad   changes a bit

The difference is caused by the change of the current generated by the barrier shift with a time-dependent velocity

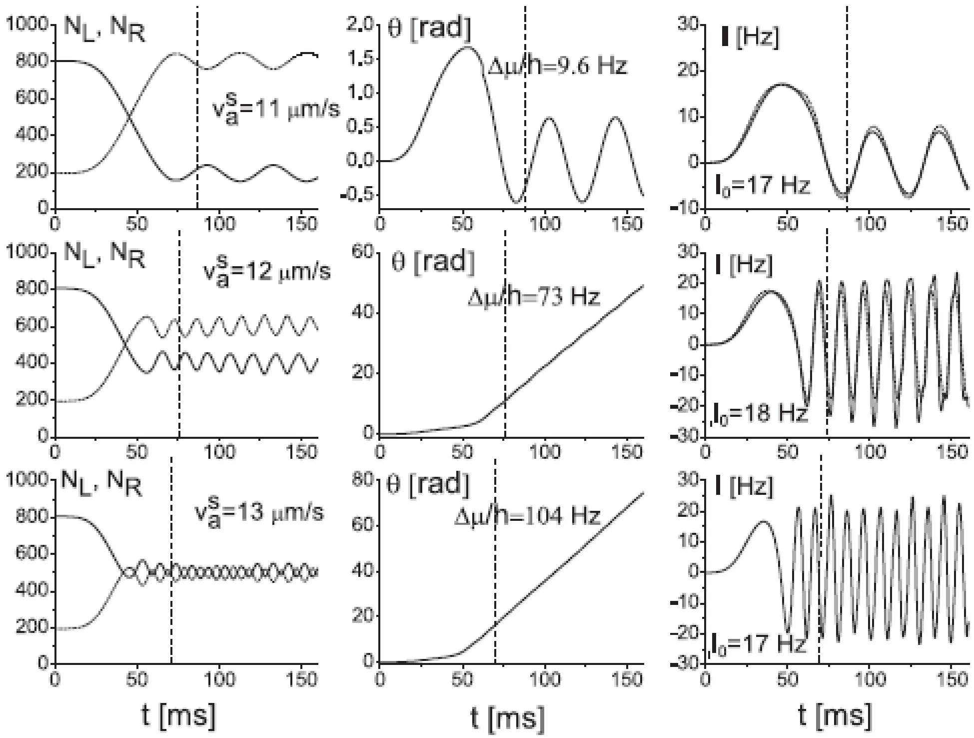
Up to this small difference, BEC transport can be associated with dc (stationary) Josephson effect.

Non-stationary Josephson effect (ac)

$$I_s > I_0$$

$$\Delta\mu \neq 0$$

$$\theta = \frac{\Delta\mu}{h} t$$



So we obviously get the transfer from dc to ac.

The transport:

- at a critical velocity ~ 12-13 $\mu m/s$ the transport is transformed into high-frequency modulated oscillations

  $\frac{\Delta\mu}{h}$ ~ 10-100 Hz becomes large

- θ changes linearly with time

- a slight modulation is caused by dipole oscillations caused by the rapid barrier velocity

Conclusions

- Analysis of SJJ-BJJ analogy in terms of currents, chemical potentials and phase differences.

- Approximate analogy between SJJ and BJJ is confirmed

- BJJ demonstrates both dc and ac

- Further:
  - important role of nonlinearity in BJJ: the critical current is increased by 3 orders of magnitude,
  - need in soft velocity time-profile,
  - critical velocity: similarity of ac-dc and adiabatic-nonadiabatic transitions
- Perspectives:
  - various Josephson scenarios (Shapiro steps, ….)
  - role of nonlinearity in BJJ $\longleftrightarrow$ capacity in SJJ
  - toroidal traps and atomic SQUIDs

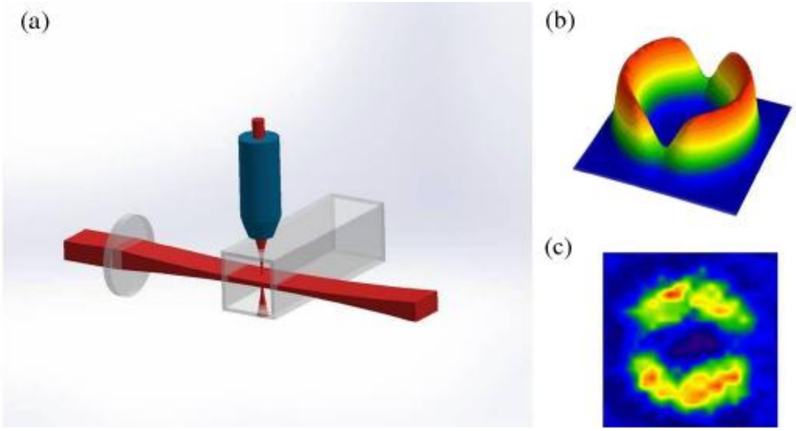
(Superconductor QUantum Interference Devices)

**Superconductor SQUIDS are widely used for precise measurements of weak magnetic fields

Atomic SQUIDs

C. Ryu, P.W. Blackburn, A. A. Blinova, and M. G. Boshier (Los Alamos),
“Experimental Realization of Josephson Junctions for an Atom SQUID”
PRL, 111, 205301 (2013)

C.A. SACKETT,
“An atomic SQUID”
Nature, 505, 166 (2014)



- two moving barriers (weak couplings) in the toroidal trap

-slow and rapid shift of the barrier

- measurements of BEC density to see if the system is yet able to adapt the barrier shifts

-rotation sensing

(instead of measurement of magnetic fields in superconductor SQUIDs)

Thank you for the attention