Domain wall network as QCD vacuum: confinement, chiral symmetry, hadronization

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An overall task pursued by most of the approaches to QCD vacuum structure is an identification of the properties of nonperturbative gauge field configurations able to provide a coherent resolution of the confinement, the chiral symmetry breaking, the $U_{\mathrm{A}}(1)$ symmetry realization and the strong CP problems, both in terms of color-charged fields and colorless hadrons.

The other side of this task is identification of the conditions for deconfinement and chiral symmetry restoration, if any.

- **Confinement of both static and dynamical quarks** $\longrightarrow W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu} \rangle$
  $S(x,y) = \langle \psi(y)\bar{\psi}(x) \rangle$
- **Dynamical Breaking of chiral $SU_L(N_f) \times SU_R(N_f)$ symmetry** $\longrightarrow \langle \bar{\psi}(x)\psi(x) \rangle$
- **$U_A(1)$ Problem** $\longrightarrow \eta'$ (χ, Axial Anomaly)
- **Strong CP Problem** $\longrightarrow Z(\theta)$
- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:
  hadron masses, formfactors, scattering

Light mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**

What would be a formalism for coherent simultaneous description of all these nonperturbative features of QCD?

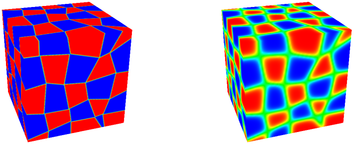
QCD vacuum as a medium characterized by certain condensates,
quarks and gluons - elementary coloured excitations (confined),
mesons and baryons - collective colourless excitations (masses, form factors, etc)

Quantum effective action of QCD!

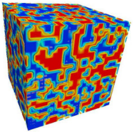
An ensemble of almost everywhere (in R^4) homogeneous Abelian (anti-)self-dual gluon fields

P. Minkowski, Phys. Lett. **B 76** (1978) 439.
H. Pagels, and E. Tomboulis, Nucl. Phys. B **143** (1978) 485.
P. Minkowski, Nucl. Phys. B**177** (1981) 203.
H. Leutwyler, Nucl. Phys. B **179** (1981) 129.

$$\langle : g^2 F^2 : \rangle \neq 0, \quad \chi = \int d^4x \langle Q(x) Q(0) \rangle \neq 0, \quad \langle Q(x) \rangle = 0$$



Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$



Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$

P.J. Moran,Derek B. Leinweber, arXiv:0805.4246v1 [hep-lat] 2008

m_u (MeV)	m_d (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)	Λ (MeV)	g
198.3	198.3	413	1650	4840	319.5	9.96

Meson	π	ρ	K	K^*	ω	ϕ
M	140	770	496	890	770	1034
$M^{\exp}$	140	770	496	890	786	1020
f_P	126	-	145	-	-	-
$f_P^{\exp}$	132	-	157	-	-	-
h	6.51	4.16	7.25	4.48	4.16	4.94
M^*	630	864	743	970	864	1087

Meson	D	D^*	D_s	D_s^*	B	B^*	B_s	B_s^*
M	1766	1991	1910	2142	4965	5143	5092	5292
$M^{\exp}$	1869	2010	1969	2110	5278	5324	5375	5422
f_P	149	-	177	-	123	-	150	-

Meson	η_c	J/ψ	χ_{c0}	χ_{c1}	χ_{c2}	ψ'	ψ''
n	0	0	0	0	0	1	2
ℓ	0	0	1	1	1	0	0
j	0	1	0	1	2	1	1
M (MeV)	3000	3161	3452	3529	3531	3817	4120
$M^{\exp}$ (MeV)	2980	3096	3415	3510	3556	3770	4040

Meson	ℓ	j	M	$M^{\exp}$
π	0	0	140	140
b_1	1	1	1252	1235
K	0	0	496	496
$K_1(1270)$	1	1	1263	1270
ρ	0	1	770	770
	1	0	1238	
a_1	1	1	1311	1260
a_2	1	2	1364	1320
K^*	0	1	890	890
	1	0	1274	
K_1(1400)	1	1	1342	1400
K_2^*	1	2	1388	1430

Meson	Υ	χ_{b0}	χ_{b1}	χ_{b2}	Υ'	χ'_{b0}	χ'_{b1}	χ'_{b2}	Υ''
n	0	0	0	0	1	1	1	1	2
ℓ	0	1	1	1	0	1	1	1	0
j	1	0	1	2	1	0	1	2	1
M (MeV)	9490	9767	9780	9780	10052	10212	10215	10215	10292
$M^{\exp}$ (MeV)	9460	9860	9892	9913	10230	10235	10255	10269	10355

$$M_\eta = 640 \text{ MeV}, \; M_{\eta'} = 950 \text{ MeV}, \; h_\eta = 4.72, \; h_{\eta'} = 2.55, \; \sqrt{B}R = 1.56.$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)

Features of **the spectrum of light vector and pseudoscalar mesons** are driven by the chiral symmetries and are correctly reproduced by the model quantitatively.

$$B^a_\mu = n^a B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, \quad B_{\mu\alpha} B_{\alpha\nu} = \delta_{\mu\nu} B^2, \quad B^2 = \mathrm{const}$$

$$D^2(x) G(x,y) = -\delta(x-y) \quad G(x,y) = e^{ixBy} H(x-y) \quad \tilde{H}(p^2) = \frac{1 - e^{-p^2/B}}{p^2}$$

$$\tilde{H}_f(p \mid B) \to O\left(\exp\left\{\frac{p^2}{\Lambda^2}\right\}\right), \; F_{n\ell}\left(p^2\right) \to O\left(\exp\left\{\frac{p^2}{\Lambda^2}\right\}\right),$$

Regge behaviour of the spectrum is due to nonlocality of the vertices and propagators.

- $M^2_{aJ\ell n} = \frac{8}{3} \ln\left(\frac{5}{2}\right) \cdot \Lambda^2 \cdot n + O(\ln n)$, for $n \gg \ell$, $\qquad M^2_{aJ\ell n} = \frac{4}{3} \ln 5 \cdot \Lambda^2 \cdot \ell + O(\ln \ell)$, for $\ell \gg n$.

Heavy-light mesons and heavy quarkonia

- $m_Q \gg \Lambda, m_Q \gg m_q, \qquad M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q)$
- $m_Q \gg \Lambda, \qquad M_{Q\bar{Q}} = 2m_Q - \Delta_{Q\bar{Q}}, \qquad \Delta^{(P)}_{Q\bar{Q}} = 2\Delta^{(V)}_{Q\bar{Q}}$

- QCD effective action and vacuum gluon configurations
- Gluon condensates and domain wall network as QCD vacuum
- Domain bulk - confinement
- Domain wall junctions - deconfinement
- The domain model of QCD vacuum
- Testing the domain model - static characteristics of QCD vacuum
- Hadronization: spectrum, decay constants
- Summary

QCD effective action and vacuum gluon configurations

In Euclidean functional integral for YM theory one has to allow the gluon condensates to be nonzero:

$$Z = N \int_{\mathcal{F}_B} DA \int_{\Psi} D\psi D\bar{\psi} \exp\{-S[A,\psi,\bar{\psi}]\}$$

$$\mathcal{F}_B = \left\{ A : \lim_{V\to\infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2 \right\}.$$

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017

L. D. Faddeev, [arXiv:0911.1013 [math-ph]]

H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$A^a_\mu = B^a_\mu + Q^a_\mu$, background gauge fixing condition $D(B)Q = 0$:

$$1 = \int_{\mathcal{B}} DB\Phi[A,B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega]\delta[D(B^\omega)Q^\omega]$$

Q^a_μ – local (perturbative) fluctuations of gluon field with zero gluon condensate: $Q \in \mathcal{Q}$;
B^a_μ are long range field configurations with nonzero condensate: $B \in \mathcal{B}$.

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \int_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$

The character of long range fields has yet to be identified by the dynamics of fluctuations:

$$Z = N' \int\limits_{\mathcal{B}} DB \int\limits_{\Psi} D\psi D\bar{\psi} \int\limits_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S_{\rm QCD}[B+Q,\psi,\bar{\psi}]\}$$
$$= \int\limits_{\mathcal{B}} DB \exp\{-S_{\rm eff}[B]\}$$

Global minima of $S_{\rm eff}[B]$ – field configurations that are dominant in the thermodynamic limit $V \to \infty$. Homogeneous Abelian (anti-)self-dual fields are of particular interest.

$$\langle F^2 \rangle : \quad A_\mu = -\frac{1}{2} n F_{\mu\nu} x_\nu, \; \tilde{F}_{\mu\nu} = \pm F_{\mu\nu}$$
$$n = T^3 \; \cos\xi + T^8 \; \sin\xi.$$

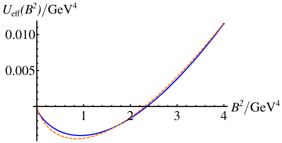
P. Minkowski, Nucl. Phys. B177 (1981) 203
H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$$G\left(z^2\right) \sim \frac{e^{-B_{\rm vac} z^2}}{z^2}, \quad \tilde{G}\left(p^2\right) \sim \frac{1}{p^2}\left(1 - e^{-p^2/B_{\rm vac}}\right)$$

H. Leutwyler, Phys. Lett. B 96 (1980) 154

Gluon propagator ⇒ Regge trajectories

G.V. Efimov, and S.N. Nedelko, Phys. Rev. D 51 (1995)



A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D 83, 045014 (2011)

Gluon condensates and domain wall network

$$\mathcal{L}_{\rm eff} = -\frac{1}{4B^2}\left(D^{ab}_{\nu}F^{b}_{\rho\mu}D^{ac}_{\nu}F^{c}_{\rho\mu} + D^{ab}_{\mu}F^{b}_{\mu\nu}D^{ac}_{\rho}F^{c}_{\rho\nu}\right) - U_{\rm eff}$$
$$U_{\rm eff} = \frac{B^4}{12}{\rm Tr}\left(C_1F^2 + \frac{4}{3}C_2F^4 - \frac{16}{9}C_3F^6\right),$$

B.V. Galilo, S.N. Nedelko, Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

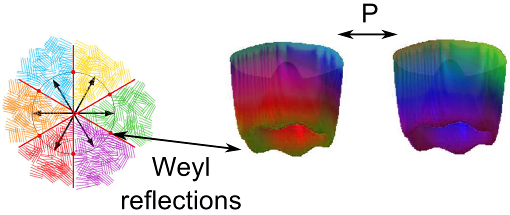
$$D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - iA^{ab}_{\mu} = \partial_{\mu} - iA^{c}_{\mu}(T^{c})^{ab},$$
$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - if^{abc}A^{b}_{\mu}A^{c}_{\nu},$$
$$F_{\mu\nu} = F^{a}_{\mu\nu}T^{a}, \quad T^{a}_{bc} = -if^{abc}$$
$$C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

U_{eff} possesses 12 degenerate discrete minima:

$$A_\mu = -\frac{1}{2} n_k F_{\mu\nu} x_\nu,\ \tilde{F}_{\mu\nu} = \pm F_{\mu\nu},$$

where the matrix n_k belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3 \ \cos(\xi_k) + T^8 \ \sin(\xi_k)\,,$$
$$\xi_k = \frac{2k+1}{6}\pi,\ k = 0, 1, \ldots, 5.$$



Domain wall network

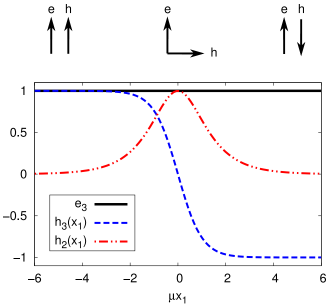
$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\Lambda^2 b_{\rm vac}^2 \partial_\mu\omega\partial_\mu\omega - b_{\rm vac}^4\Lambda^4\left(C_2 + 3C_3 b_{\rm vac}^2\right)\sin^2\omega,$$

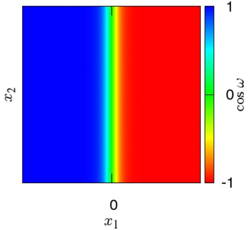
leads to sine-Gordon equation

$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\rm vac}^2\Lambda^2\left(C_2 + 3C_3 b_{\rm vac}^2\right),$$

and the standard kink solution

$$\omega(x_\nu) = 2\ \mathrm{arctg}\left(\exp(\mu x_\nu)\right)$$





The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta^i_\nu x_\nu - q^i)).$$

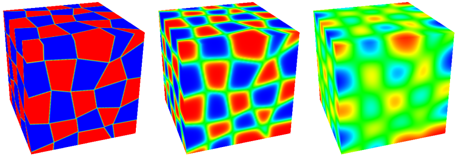
A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i).$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta^{ij}_\nu x_\nu - q^{ij})$$

S.N. Nedelko, V.E. Voronin, Eur.Phys.J. A51 (2015) 4



$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle = B^2$

$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle \ll B^2$

H. Pagels, and E. Tomboulis, Nucl. Phys. B 143 (1978) 485

P. Minkowski, Phys. Lett. B 76 (1978) 439

H. Leutwyler, Nucl. Phys. B 179 (1981);

G.V. Efimov, and S.N. Nedelko, Phys. Rev. D 51 (1995) 176

Domain bulk - confinement

The case of finite size spherical domains was considered in

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001)

Elementary color charged excitations - fluctuations decaying in all four directions.

Eigenvalue problem for scalar field in $\mathbb{R}^4$:

$$B_\mu = B_{\mu\nu}x_\nu, \widetilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu}.$$

$$-\left(\partial_\mu - iB_\mu\right)^2 G = \delta \quad \longrightarrow \quad G\left(x-y\right) \sim \frac{e^{-B(x-y)^2/4}}{(x-y)^2}$$

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = \lambda\Phi \quad \longrightarrow \quad \left[\beta^+_\pm\beta_\pm + \gamma^+_+\gamma_+ + 1\right]\Phi = \frac{\lambda}{4B}\Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}}x_\mu + \partial_\mu,$$

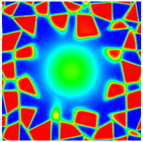
$$\beta^+_\pm = \frac{1}{2}(\alpha^+_1 \pm i\alpha^+_2), \quad \gamma^+_\pm = \frac{1}{2}(\alpha^+_3 \pm i\alpha^+_4), \quad \alpha^+_\mu = \frac{1}{\sqrt{B}}x_\mu - \partial_\mu.$$

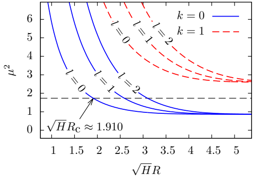
The eigenfunctions and eigenvalues - 4-dim harmonic oscillator

$$\Phi_{nmkl}(x) = \frac{1}{\pi^2\sqrt{n!m!k!l!}}\left(\beta^+_+\right)^k\left(\beta^+_-\right)^l\left(\gamma^+_+\right)^n\left(\gamma^+_-\right)^m\Phi_{0000}, \; \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$\lambda_r = 4B(r+1), \; r = k+n$ self-dual field, $r = l+n$ anti-self-dual field

Domain wall junctions - deconfinement





The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = 0,$$

$$\Phi(x) = 0, \quad x \in \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2,\ (x_3, x_4) \in \mathrm{R}^2\right\}$$

The solutions are quasi-particle excitations

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta} \phi_{alk}(r),$$

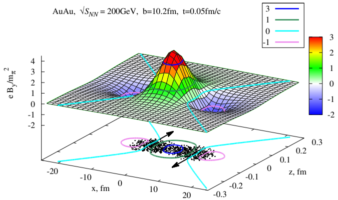
$$p_0^2 = p_3^2 + \mu_{akl}^2, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu_{akl}^2},$$

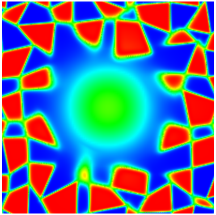
$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z},$$

Impact of electromagnetic fields on “QCD vacuum“.

- **Relativistic heavy ion collisions - strong electromagnetic fields**

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!
B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017.
M. D’Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)
G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

The domain model of QCD vacuum

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001)

Euclidean partition function is defined as

$$\mathcal{Z}(\theta) = \lim_{V,N\to\infty} \mathcal{N} \prod_{i=1}^{N} \int_{\mathcal{B}} dB_i \int_{\Omega_{\alpha,\vec{\beta}}} d\Omega_{\alpha,\vec{\beta}} \int_{\Psi^i} \mathcal{D}\psi^{(i)} \mathcal{D}\bar{\psi}^{(i)} \int_{\mathcal{Q}^i} \mathcal{D}\mu[Q^i]$$

$$\times e^{-S^{\rm QCD}_{V_i}\left[Q^{(i)}+B^{(i)},\psi^{(i)},\bar{\psi}^{(i)}\right]-i\theta Q_{V_i}[Q^{(i)}+B^{(i)}]}$$

$$\mathcal{D}\mu = \delta[D(B^{(i)})Q^{(i)}]\Delta_{\rm FP}[B^{(i)},Q^{(i)}]$$

The thermodynamic limit: $v^{-1} = N/V =$ const, as $V, N \to \infty$. Functional spaces $\mathcal{Q}^i$ and Ψ^i are specified by BCs at $(x - z_i)^2 = R^2$

$$\breve{n}_i Q^{(i)}(x) = 0,$$

$$i\,\not{\eta}_i(x) e^{i(\alpha+\beta^a\lambda^a/2)\gamma_5}\psi^{(i)}(x) = \psi^{(i)}(x),$$

$$\bar{\psi}^{(i)} e^{i(\alpha+\beta^a\lambda^a/2)\gamma_5} i\,\not{\eta}_i(x) = -\bar{\psi}^{(i)}(x),$$

$$\eta_i^\mu = \frac{(x-z_i)^\mu}{|x-z_i|}, \quad \breve{n}_i = n_i^a T^a, T^a\text{- adjoint representation}$$

Testing the domain model - characteristics of the domain network ensemble

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006)

Area law
Spontaneous chiral symmetry breaking
$U_A(1)$ is broken by anomaly
There is no strong CP violation

Hadronization

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006)

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \int_{\mathcal{Q}} \mathcal{D}Q \delta[D(B)Q] \Delta_{\mathrm{FP}}[B,Q] e^{-S^{\mathrm{QCD}}[Q+B,\psi,\bar{\psi}]} =$$

$$\int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left\{ \int dx \bar{\psi} \left(i \not\partial + g \not B - m \right) \psi \right\} W[j]$$

$$W[j] = \int_{\mathcal{Q}} \mathcal{D}Q \delta[D(B)Q] \Delta_{\mathrm{FP}}[B,Q] \exp\left\{ -\frac{1}{2} \int dx \ \mathrm{Tr} G^2 \left[B + Q \right] + g \int dx \ Q^a_\mu j^a_\mu \right\},$$

$$j^a_\mu = \bar{\psi} \gamma_\mu t^a \psi$$

Recalling the definition of Green's functions

$$G^{a_1 \dots a_n}_{\mu_1 \dots \mu_n}(x_1, \dots, x_n | B) = \frac{1}{g^n} \frac{\delta^n \ln W[j]}{\delta j^{a_1}_{\mu_1}(x_1) \dots \delta j^{a_n}_{\mu_n}(x_n)},$$

we obtain

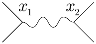
$$W[j] = \exp\left\{ \sum_n \frac{g^n}{n!} \int dx_1 \dots \int dx_n j^{a_1}_{\mu_1}(x_1) \dots j^{a_n}_{\mu_n}(x_n) G^{a_1 \dots a_n}_{\mu_1 \dots \mu_n}(x_1, \dots, x_n | B) \right\}$$

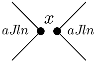
$W[j]$ is truncated up to the term including two-point gluon correlation function

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left\{ \int dx \bar{\psi}\left(i\not{\partial} + g\not{B} - m\right)\psi \right.$$
$$\left. + \frac{g^2}{2} \int dx_1 dx_2 G^{a_1 a_2}_{\mu_1 \mu_2}(x_1, x_2|B) j^{a_1}_{\mu_1}(x_1) j^{a_2}_{\mu_2}(x_2) \right\}$$
$$\int dz dx G(z|B) J^{aJ}(x,z) J^{aJ}(x,z)$$

α_s  $= \alpha_s(0)$  $\left[1 + \Pi^{\mathrm{R}}(p^2)\right]; \ \Pi^{\mathrm{R}}(0) = 0$

0  $z \ \to \ \dfrac{e^{-\frac{1}{4}Bz^2}}{4\pi^2 z^2}$

$\int dx_1\, dx_2$ 

$= \int dx \sum_{aJln}$ 

$$J^{aJ}(x,z) = \sum_{nl} \left(z^2\right)^{l/2} f^{nl}_{\mu_1 \dots \mu_l}(z) J^{aJln}_{\mu_1 \dots \mu_l}(x), \quad J^{aJln}_{\mu_1 \dots \mu_l}(x) = \bar{q}(x) V^{aJln}_{\mu_1 \dots \mu_l}\left(\frac{\overleftrightarrow{D}(x)}{B}\right) q(x),$$

$$f^{nl}_{\mu_1 \dots \mu_l} = L_{nl}\left(z^2\right) T^{(l)}_{\mu_1 \dots \mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}},$$

$$\int_{\Omega} \frac{d\omega}{2\pi^2} T^{(l)}_{\mu_1 \dots \mu_l}(n_z) T^{(k)}_{\nu_1 \dots \nu_k}(n_{z'}) = \frac{1}{2^l(l+1)} \delta^{lk} \delta_{\mu_1 \nu_1} \dots \delta_{\mu_l \nu_l}.$$

$T^{(l)}_{\mu_1 \dots \mu_l}$ are irreducible tensors of four-dimensional rotational group

$$T^{(l)}_{\mu_1 \dots \mu \dots \nu \dots \mu_l}(n_z) = T^{(l)}_{\mu_1 \dots \nu \dots \mu \dots \mu_l}(n_z), \quad T^{(l)}_{\mu \dots \mu \dots \mu_l}(n_z) = 0,$$

$\int^{\infty}$ ... e^{-Bz^2}

Effective meson action for composite colorless fields:

$$Z = \mathcal{N} \lim_{V\to\infty} \int D\Phi_{\mathcal{Q}} \exp\left\{ -\frac{B}{2}\frac{h^2_{\mathcal{Q}}}{g^2 C_{\mathcal{Q}}} \int dx \Phi^2_{\mathcal{Q}}(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad \mathcal{Q} = (aJln)$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(-M^2_{\mathcal{Q}}|B), \quad h^{-2}_{\mathcal{Q}} = \frac{d}{dp^2} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(p^2)|_{p^2=-M^2_{\mathcal{Q}}}.$$

$$W_k[\Phi] = \sum_{\mathcal{Q}_1\dots\mathcal{Q}_k} h_{\mathcal{Q}_1}\dots h_{\mathcal{Q}_k} \int dx_1 \dots \int dx_k \Phi_{\mathcal{Q}_1}(x_1)\dots\Phi_{\mathcal{Q}_k}(x_k) \Gamma^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k|B)$$

$$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} = \overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)} - \Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1} G^{(1)}_{\mathcal{Q}_2}},$$

$$\Gamma^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3} = \overline{G^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3}(x_1,x_2,x_3)} - \frac{3}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2) G^{(1)}_{\mathcal{Q}_3}(x_3)}$$

$$+\frac{1}{2}\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(1)}_{\mathcal{Q}_2}(x_2) G^{(1)}_{\mathcal{Q}_3}(x_3)},$$

$$\Gamma^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4} = \overline{G^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_1,x_2,x_3,x_4)} - \frac{4}{3}\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(3)}_{\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_2,x_3,x_4)}$$

$$-\frac{1}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2) G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$

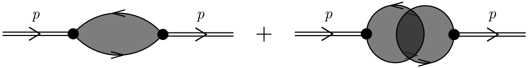
$$+\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(1)}_{\mathcal{Q}_2}(x_2) G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$

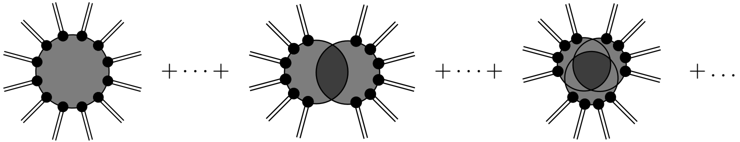
$$-\frac{1}{6}\Xi_4(x_1,x_2,x_3,x_4)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(1)}_{\mathcal{Q}_2}(x_2) G^{(1)}_{\mathcal{Q}_3}(x_3) G^{(1)}_{\mathcal{Q}_4}(x_4)}.$$

$$\overline{G^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k)} = \int dB_j \mathrm{Tr} V_{\mathcal{Q}_1}\left(x_1|B^{(j)}\right) S\left(x_1,x_2|B^{(j)}\right)\dots$$
$$\dots V_{\mathcal{Q}_k}\left(x_k|B^{(j)}\right) S\left(x_k,x_1|B^{(j)}\right)$$

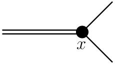
$$\overline{G^{(l)}_{\mathcal{Q}_1\dots\mathcal{Q}_l}(x_1,\dots,x_l) G^{(k)}_{\mathcal{Q}_{l+1}\dots\mathcal{Q}_k}(x_{l+1},\dots,x_k)} =$$
$$\int dB_j \mathrm{Tr}\left\{V_{\mathcal{Q}_1}\left(x_1|B^{(j)}\right) S\left(x_1,x_2|B^{(j)}\right)\dots V_{\mathcal{Q}_k}\left(x_l|B^{(j)}\right) S\left(x_l,x_1|B^{(j)}\right)\right\}$$
$$\times \mathrm{Tr}\left\{V_{\mathcal{Q}_{l+1}}\left(x_{l+1}|B^{(j)}\right) S\left(x_{l+1},x_{l+2}|B^{(j)}\right)\dots V_{\mathcal{Q}_k}\left(x_k|B^{(j)}\right) S\left(x_k,x_{l+1}|B^{(j)}\right)\right\},$$

Bar denotes integration over all configurations of the background field with measure dB_j.

$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} =$ 

$\Gamma^{(n)}_{\mathcal{Q}_1\mathcal{Q}_2\dots\mathcal{Q}_n} =$ 

Meson-quark vertex operators $\Leftarrow J^{aJln}_{\mu_1\dots\mu_l} = \bar{q}(x) V^{aJln}_{\mu_1\dots\mu_l} q(x)$



$$V^{aJln}_{\mu_1\dots\mu_l}(x) = M^a \Gamma^J \left\{ \left\{ F_{nl}\left(\frac{\overleftrightarrow{D}^2(x)}{B^2} \right) T^{(l)}_{\mu_1\dots\mu_l}\left(\frac{1}{i}\frac{\overleftrightarrow{D}(x)}{B} \right) \right\} \right\},$$

$$F_{nl}(s) = s^n \int_0^1 dt t^{n+l} \exp(st) = \int_0^1 dt t^{n+l} \frac{\partial^n}{\partial t^n} \exp(st),$$

$$\overleftrightarrow{D} = \overleftarrow{D} \xi_{f'} - \overrightarrow{D} \xi_f, \xi_f = \frac{m_f}{m_f + m_{f'}}$$

Quark propagator in homogeneous Abelian (anti-)self-dual field

$$\longrightarrow = \frac{\rightarrow}{m(0)} \left[1 + \Sigma^{\mathrm{R}}(p^2) \right]; \ \Sigma^{\mathrm{R}}(0) = 0 \qquad S(x,y) = \exp\left(-\frac{i}{2} x_\mu B_{\mu\nu} y_\nu \right) H(x-y),$$

$$\widetilde{H}_f(p|B) = \frac{1}{\upsilon B^2} \int_0^1 ds e^{(-p^2/\upsilon B^2)s} \left(\frac{1-s}{1+s} \right)^{m_f^2/2\upsilon B^2} \left[p_\alpha \gamma_\alpha \pm is\gamma_5\gamma_\alpha \frac{B_{\alpha\beta}}{\upsilon B^2} p_\beta + \right.$$

$$\left. + m_f \left(P_\pm + P_\mp \frac{1+s^2}{1-s^2} - \frac{i}{2}\gamma_\alpha \frac{B_{\alpha\beta}}{\upsilon B^2}\gamma_\beta \frac{s}{1-s^2} \right) \right]$$

The parameters of the model are

$$\alpha_s(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R$$

$$\langle \alpha_s F^2 \rangle = \frac{B^2}{\pi} \qquad \chi_{\mathrm{YM}} = \frac{B^4 R^4}{128\pi^2}$$

Quadratic part of the effective action for colorless composite fields is

$$I_2 = -\frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\varphi_\mu^k(p)\left[B\delta^{aa'}\delta_{\mu\mu'} - G_{Jln}G_{J'l'n'}\Pi^{kk'}_{\mu\mu'}(p^2)\right]\varphi^{k'}_{\mu'}(p).$$

$$k = (aJln), \quad \mu = (\mu_1\dots\mu_l), \quad G_{Jln} = g\sqrt{C_J\frac{l+1}{2^l n!(n+l)!}}, \quad C_{V/A} = \frac{1}{18}, \quad C_{S/P} = \frac{1}{9}.$$

$$\widetilde{\varphi}^n(p) = O^{nn'}(p)\varphi^{n'}(p), \quad \widetilde{\Pi}^{nn'}(p^2) = \delta^{nn'}\widetilde{\Pi}^{n'}(p^2).$$

Mass and coupling constant of a meson with radial number n are found from

$$1 = \frac{g^2\widetilde{\Pi}^n(-M^2)}{B}, \quad h^{-2}_{aJln} = \frac{d}{dp^2}\widetilde{\Pi}^n(p^2)|_{p^2=-M^2}.$$

Polarization operator

Polarization operation for $l=0$:

$$\Pi_J^{nn'}\left(-M^2;m_f,m_{f'};B\right)=$$

$$\frac{B}{4\pi^2}\mathrm{Tr}_\upsilon\int\limits_0^1 dt_1\int\limits_0^1 dt_2\int\limits_0^1 ds_1\int\limits_0^1 ds_2\left(\frac{1-s_1}{1+s_1}\right)^{m_f^2/4\upsilon B}\left(\frac{1-s_2}{1+s_2}\right)^{m_{f'}^2/4\upsilon B}\times$$

$$\times\, t_1^n t_2^{n'}\frac{\partial^n}{\partial t_1^n}\frac{\partial^{n'}}{\partial t_2^{n'}}\frac{1}{\Phi_2^2}\left[\frac{M^2}{B}\frac{F_1^{(J)}}{\Phi_2^2}+\frac{m_f m_{f'}}{B}\frac{F_2^{(J)}}{(1-s_1^2)(1-s_2^2)}+\frac{F_3^{(J)}}{\Phi_2}\right]\exp\left\{\frac{M^2}{2\upsilon B}\frac{\Phi_1}{\Phi_2}\right\}.$$

$$\Phi_1=s_1s_2+2\left(\xi_1^2s_1+\xi_2^2s_2\right)(t_1+t_2)\upsilon,$$

$$\Phi_2=s_1+s_2+2(1+s_1s_2)(t_1+t_2)\upsilon+16(\xi_1^2s_1+\xi_2^2s_2)t_1t_2\upsilon^2,$$

$$F_1^{(P)}=(1+s_1s_2)\left[2(\xi_1s_1+\xi_2s_2)(t_1+t_2)\upsilon+\right.$$
$$\left.4\xi_1\xi_2(1+s_1s_2)(t_1+t_2)^2\upsilon^2+s_1s_2(1-16\xi_1\xi_2t_1t_2\upsilon^2)\right],$$

$$F_1^{(V)}=\left(1-\frac{1}{3}s_1s_2\right)\left[s_1s_2+16\xi_1\xi_2t_1t_2\upsilon^2+2(\xi_1s_1+\xi_2s_2)(t_1+t_2)\upsilon\right]+$$
$$4\xi_1\xi_2(1-s_1^2s_2^2)(t_1-t_2)^2\upsilon^2,$$

$$F_2^{(P)}=(1+s_1s_2)^2,\quad F_2^{(V)}=(1-s_1^2s_2^2),$$

$$F_3^{(P)}=4\upsilon(1+s_1s_2)(1-16\xi_1\xi_2t_1t_2\upsilon^2),\ F_3^{(V)}=2\upsilon(1-s_1s_2)(1-16\xi_1\xi_2t_1t_2\upsilon^2).$$

Masses of radially excited mesons: light mesons

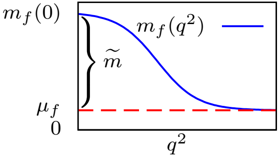
$m_{u/d}$, MeV	m_s, MeV	m_c, MeV	m_b, MeV	Λ, MeV	α_s	R, fm
145	376	1566	4879	416	3.45	1.12

$$\chi_{\rm YM} = \frac{B^4 R^4}{128\pi^2} = (604\ {\rm MeV})^4 \qquad \frac{\alpha_s}{\pi}\langle F^2\rangle = \frac{B^2}{\pi^2} = 0.04\ {\rm GeV}^4 \qquad q = \frac{B^2 R^4}{16} = 0.2$$

Asymptotic relation for spectrum (Regge trajectories):

$$M_n^2 \sim Bn,\ n \gg 1$$
$$M_l^2 \sim Bl,\ l \gg 1$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)



$$\widetilde{m} = 136\ \text{MeV}$$
$$\mu_{u/d} = m_{u/d} - \widetilde{m}$$
$$\mu_s = m_s - \widetilde{m}$$
$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$

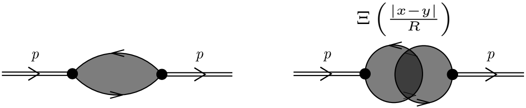
Masses of radially excited mesons: light mesons

meson	n	M, MeV [*]	M, MeV	$\widetilde{M}$, MeV
π	0	140	140	0
$\pi(1300)$	1	1300	1310	1301
$\pi(1800)$	1	1812	1503	1466
K	0	494	494	0
$K(1460)$	1	1460	1302	1301
K	2		1655	1466
ρ	0	775	775	769
$\rho(1450)$	1	1450	1571	1576
ρ	2	1720	1946	2098
K^*	0	892	892	769
$K^*(1410)$	1	1410	1443	1576
$K^*(1717)$	1	1717	1781	2098
ϕ	0	1019	1039	769
$\phi(1680)$	1	1680	1686	1576
ϕ	2	2175	1897	2098

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Masses of radially excited mesons: η and η'

Resolution of $U_A(1)$ problem.



meson	n	M, MeV [*]	M, MeV	$\widetilde{M}$, MeV
η	0	548	621	0
η'	0	958	958	872
$\eta(1295)$	1	1294	1138	1361
$\eta(1475)$	1	1476	1297	1516

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Masses of radially excited mesons: heavy-light mesons

Asymptotic formula:

$$M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q),\ m_Q \gg \sqrt{B},\ m_Q \gg m_q$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)

meson	n	M, MeV	M, MeV
D	0	1864 [*]	1715
D	1	2579 [†]	2274
D	2		2508
D_s	0	1968 [*]	1827
D_s	1	2670 [†]	2521
D_s	2	2670 [†]	2808
B	0	5279 [*]	5041
B	1	5883 [†]	5535
B	2		5746
B_s	0	5366 [*]	5135
B_s	1	5971 [†]	5746
B_s	2		5988
B_c	0	6277 [*]	5952
B_c	1	6842 [‡]	6904
B_c	2		7233

meson	n	M, MeV	M, MeV
D^*	0	2010 [*]	1944
D^*	1	2629 [†]	2341
D^*	2		2564
D_s^*	0	2112 [*]	2092
D_s^*	1	2716 [†]	2578
D_s^*	2		2859
B^*	0	5325 [*]	5215
B^*	1	5898 [†]	5578
B^*	2		5781
B_s^*	0	5415 [*]	5355
B_s^*	1	5984 [†]	5783
B_s^*	2		6021

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014.
[†] D. Ebert, V. O. Galkin and R. N. Faustov, Phys. Rev. D 57, 5663 (1998) [Erratum-ibid. D 59, 019902 (1999)] [hep-ph/9712318]
[‡] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 71, 1825 (2011) [arXiv:1111.0454 [hep-ph]]

Masses of radially excited mesons: heavy quarkonia

Asymptotic spectrum:

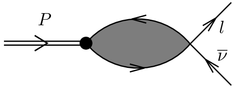
$$M_{Q\bar{Q}} = 2m_Q - \Delta^{(J)}_{Q\bar{Q}} + O(1/m_Q), \ m_Q \gg \sqrt{B}$$

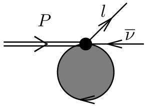
G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)

meson	n	M, MeV [*]	M, MeV
$\eta_c(1S)$	0	2981	2751
$\eta_c(2S)$	1	3639	3620
η_c	2		3882
$J/\psi(1S)$	0	3097	3097
$\psi(2S)$	1	3686	3665
$\psi(3770)$	2	3773	3810
$\Upsilon(1S)$	0	9460	9460
$\Upsilon(2S)$	1	10023	10102
$\Upsilon(3S)$	2	10355	10249

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Leptonic decay constants of pseudoscalar mesons



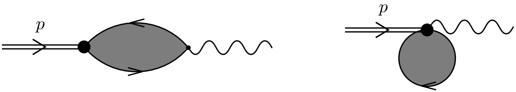


meson	n	f_P, MeV	f_P, MeV	$\widetilde{f}_P$, MeV
π	0	130 [*]	140	128
$\pi(1300)$	1	—	29	29
K	0	156 [*]	175	128
$K(1460)$	1	—	27	29
D	0	205 [*]	212	
D	1	—	51	
D_s	0	258 [*]	274	
D_s	1	—	57	
B	0	191 [*]	187	
B	1	—	55	
B_s	0	253 [†]	248	
B_s	1	—	68	
B_c	0	489 [†]	434	
B_c	1	—	135	

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

[†] T. W. Chiu *et al.* [TWQCD Collaboration], PoS LAT 2006, 180 (2007) [arXiv:0704.3495 [hep-lat]]

$g_{V\gamma}$



meson	n	$g_{V\gamma}$ [*]	$g_{V\gamma}$
ρ	0	0.2	0.2
ρ	1	—	0.034
ω	0	0.059	0.067
ω	1	—	0.011
ϕ	0	0.074	0.069
ϕ	1	—	0.025
J/ψ	0	0.09	0.057
J/ψ	1	—	0.024
Υ	0	0.025	0.011
Υ	1	—	0.0039

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

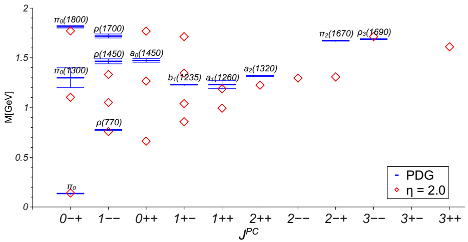
Comparison with Bethe-Salpeter approach

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **51**, no. 1, 10 (2015) [arXiv:1409.5076 [hep-ph]]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **50**, 126 (2014) [arXiv:1406.4370 [hep-ph]].

S. M. Dorkin, L. P. Kaptari and B. Kampfer, arXiv:1412.3345 [hep-ph]
S. M. Dorkin, L. P. Kaptari, T. Hilger and B. Kampfer, Phys. Rev. C **89**, no. 3, 034005 (2014) [arXiv:1312.2721 [hep-ph]]

$$S^{-1}(p) = Z_2 S_0^{-1}(p) + 4\pi Z_2^2 C_F \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k+p)\gamma^\nu (\delta_{\mu\nu} - k_\mu k_\nu / k^2)\ \frac{\alpha_{\rm eff}(k^2)}{k^2},$$

$$\alpha_{\rm eff}(q^2) = \pi\eta^7 x^2 e^{-\eta^2 x} + \frac{2\pi\gamma_m\left(1-e^{-y}\right)}{\ln\left[e^2 - 1 + (1+z)^2\right]},\ x = q^2/\Lambda^2,\ y = q^2/\Lambda_t^2,\ z = q^2/\Lambda_{\rm QCD}^2$$



S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]

Comparison with soft-wall AdS/QCD

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [hep-ph/0602229]

T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **82**, 074022 (2010) [arXiv:1008.0268 [hep-ph]].

$$\mathrm{S}_\Phi = \frac{(-1)^J}{2}\int d^dx\, dz \left(\frac{R}{z}\right)^{d+1} e^{-\kappa^2 z^2}\left(\partial_N \Phi_J \partial^N \Phi_J - \mu_J^2(z)\Phi_J\Phi^J\right)$$

$$M_{nJ} = 4\kappa^2\left(n + \tfrac{l+J}{2}\right)$$

$$\Phi_J(x,z) = \sum_n \phi_{nJ}(z)\Phi_{nJ}(x),$$

$$\phi_{nJ} = R^{J-(d-1)/2}\kappa^{1+l} z^{l-J+2} L_n^l(\kappa^2 z^2)$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51, 176 (1995)

$$J^{aJ}(x,z) = \sum_{nl} \left(z^2\right)^{l/2} f^{nl}(z) J^{aJln}(x), \quad J^{aJln}(x) = \bar{q}(x) V^{aJln}(x)\, q(x)$$

$$n \gg 1:\ M_n^2 \propto Bn$$
$$l \gg 1:\ M_l^2 \propto Bl$$

$$f^{nl}_{\mu_1\dots\mu_l} = L_n^l\left(z^2\right) T^{(l)}_{\mu_1\dots\mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}},$$

$$\int_0^\infty du \rho_l(u) L_n^l(u) L_{n'}^l(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u}.$$

Summary

Starting with

$$\lim_{V\to\infty}\frac{1}{V}\int_V d^4x g^2 F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)\neq 0.$$

one arrives at the importance of the lumpy structured gluon configurations (almost everywhere homogeneous abelian (anti-)self-dual field) and correctly implemented:

- **Domain wall network as QCD vacuum:** almost everywhere homogeneous Abelian (anti-)selfdual gluon fields.
- **Domain wall network as QCD vacuum:** deconfinement occurs in two stages: $\langle|\tilde{F}F|\rangle{=}\langle FF\rangle \to \langle|\tilde{F}F|\rangle \ll \langle FF\rangle \to \langle|\tilde{F}F|\rangle = \langle FF\rangle = 0.$
- **Confinement of both static and dynamical quarks** $\longrightarrow W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu}\rangle,$

  $S(x,y) = \langle\psi(y)\mathrm{P}e^{i\int_y^x dz_\mu \hat{A}_\mu}\bar{\psi}(x)\rangle$
- **Dynamical Breaking of** $SU_L(N_f)\times SU_R(N_f)$ $\longrightarrow \langle\bar{\psi}(x)\psi(x)\rangle$
- $U_A(1)$ **Problem** $\longrightarrow \eta'$, χ, Axial Anomaly
- **Strong CP Problem** $\longrightarrow \lim_{V\to\infty}\partial^n_\theta Z_V(\theta)\neq\partial^n_\theta\lim_{V\to\infty}Z_V(\theta)$
- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes: spectrum, formfactors (**Light** mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**)
- **QCD vacuum is characterized as heterophase mixed state with corresponding phase transition mechanism.**
- **Impact of a strong electromagnetic field as a trigger of deconfinement is indicated.**
- Basic meson wavefunctions in the domain model practically coincide with meson wavefunctions in soft-wall AdS/QCD with dilaton field $\phi=\kappa^2z^2$.