Ultracold Resonant Processes in Atomic and Molecular Traps

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Results were obtained in collaboration with

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Panagiotis Giannakeas	***ZOQ, Hamburg (now:Ch.Greene group)***
Shahpoor Saeidian	***IASBS, Zanjan, Iran***
Ji Il Kim	***Univ. São Paulo, Brazil***
Oksana and Eugene Koval	***BLTP JINR, Dubna***

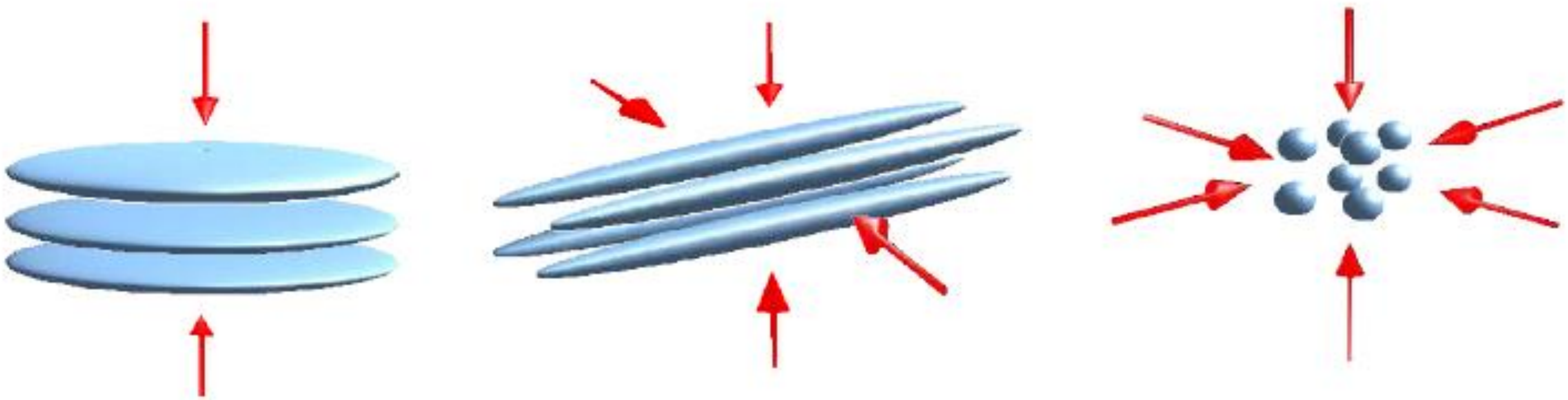
Innsbruck experiment:

Elmar Haller

Hans-Chrisroph Nägerl

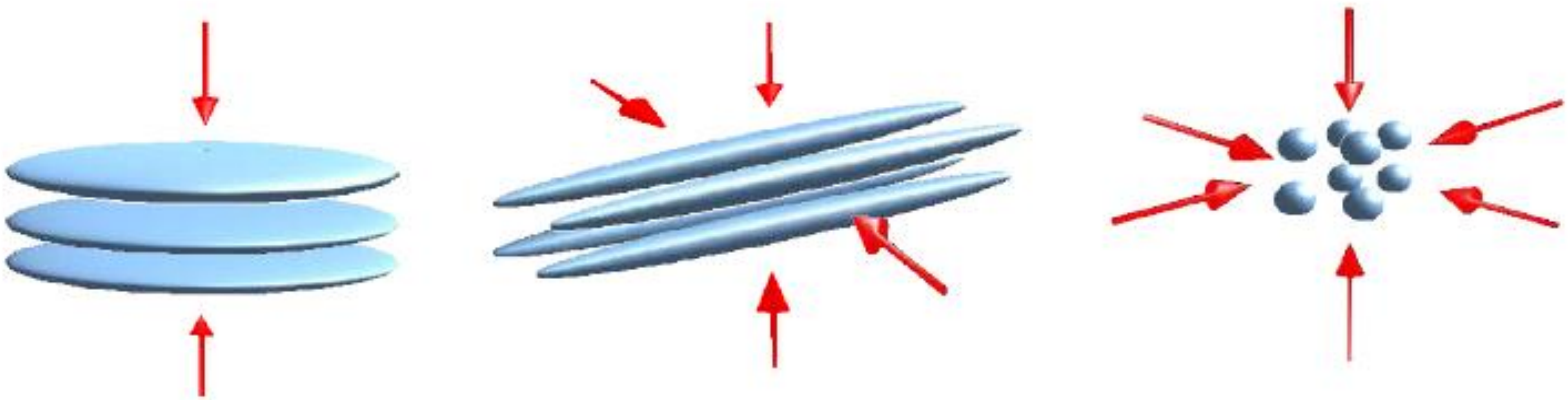
Outline

- Ultracold atoms in optical traps: why it is interesting?
- Confinement-induced resonances (CIRs)
- Feshbach resonances in quasi-1D atomic traps (bosons and fermions)
- Dipolar CIRs in quasi-1D traps
- Anisotropic quantum scattering in two dimensions
- Resonant molecule formation with energy transfer to CM excitation
- Conclusion and outlook



Lattices formed by applying orthogonal standing waves in one, two, and three directions.

2D **1D**



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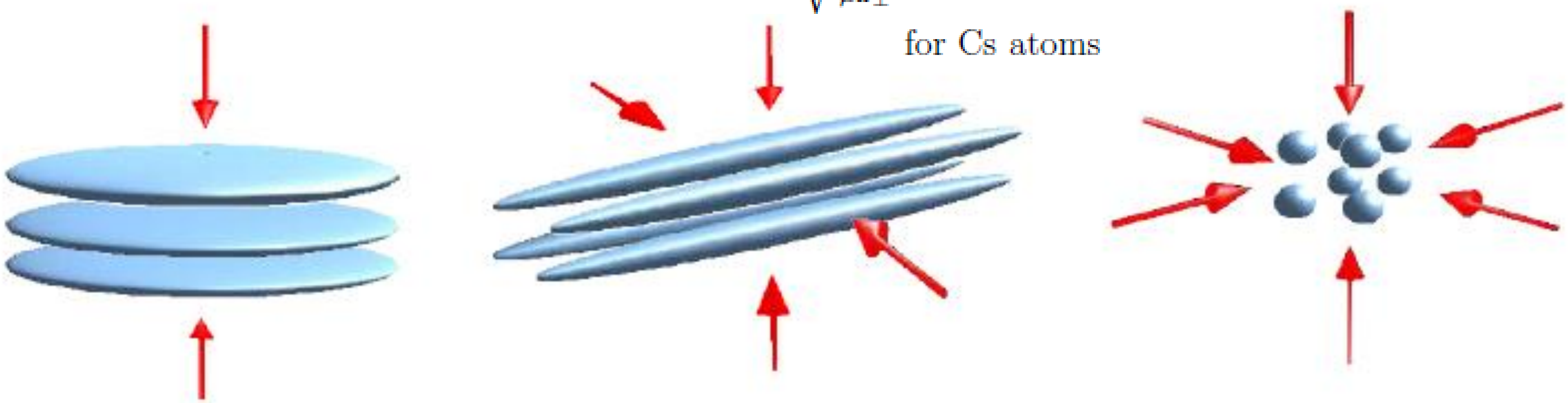
2D **1D**

$\omega_\perp \sim 14\text{kHz}$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}} \sim 70 - 100nm$$

for Cs atoms



Lattices formed by applying orthogonal standing waves in one, two, and three directions.

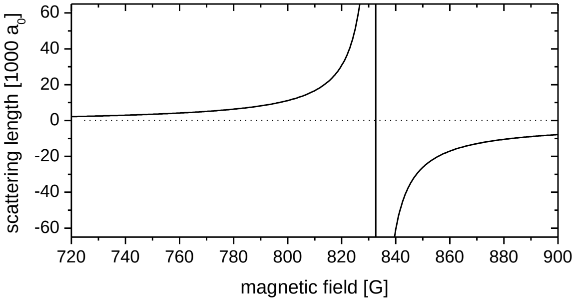
motivation in brief

experimental aspects

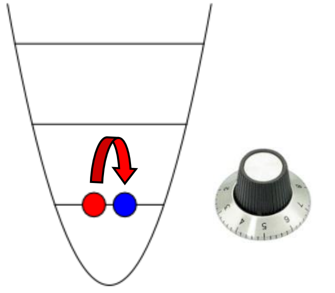
Experiments with deterministically prepared quantum systems

- **control interparticle interaction**

2 interacting particles in a 1D potential



magnetic Feshbach resonance



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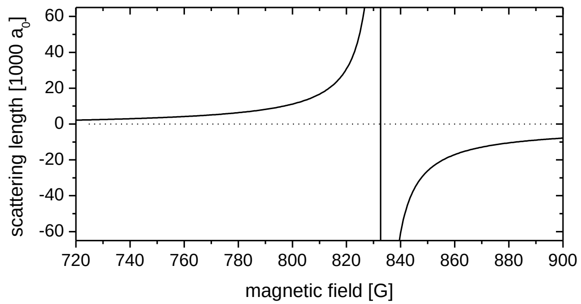

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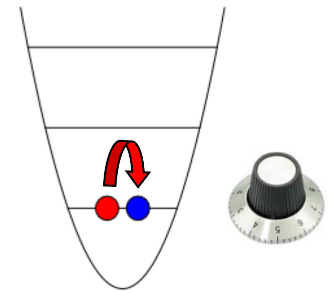
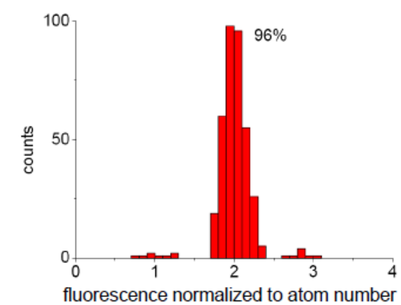


- **control over quantum states and particle number with long lifetime**



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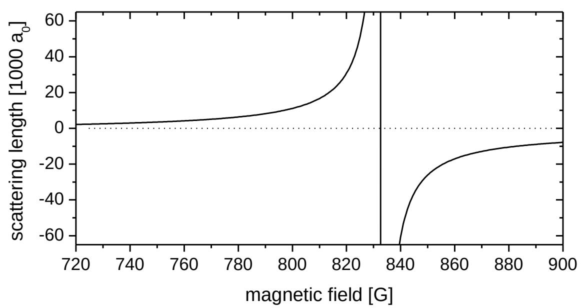

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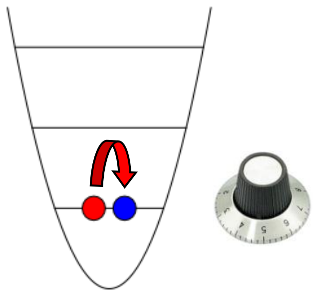

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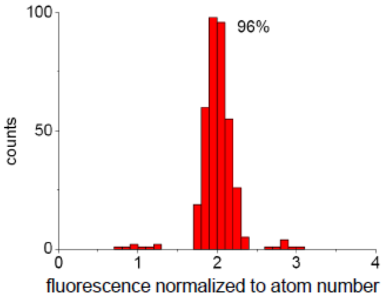

quantum simulation with fully controlled few-body systems

G.Zürn et. al. Phys. Rev. Lett. 108, 075303 (2012)

Quantum simulation with fully controlled few-body systems

control over: quantum states, particle number, interaction

- attractive interactions ⇒ BCS-like pairing in finite systems
- repulsive int.+splitting of trap → entangled pairs of atoms
  (quantum information processing)
- + periodic potential → quantum many-body physics
  (systems with low entropy to explore
  such as quantum magnetism)
- ...

Quantum simulation with fully controlled few-body systems

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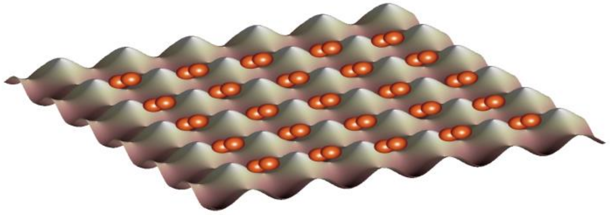
- ...

Bose-Hubbard Physics



R. P. Feynman's Vision

A Quantum Simulator to study the quantum dynamics of another system.

R.P. Feyman, Int. J. Theo. Phys. (1982)
R.P. Feynman, Found. Phys (1986)

motivation in brief

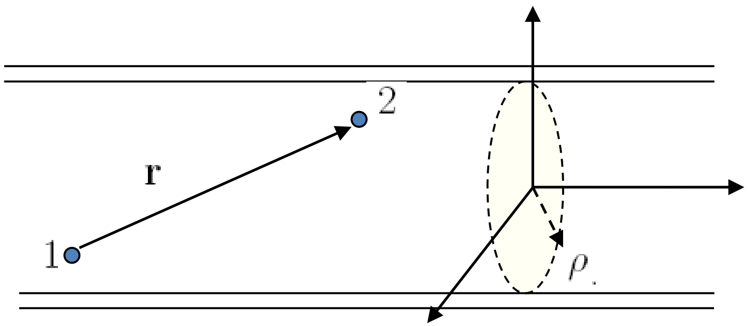
theoretical aspects

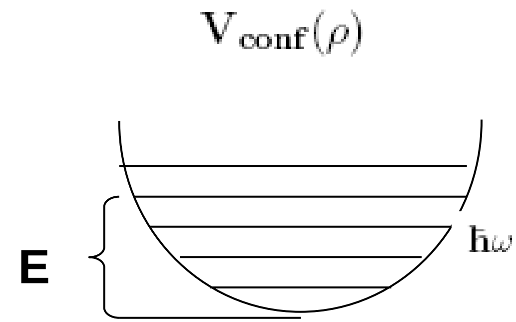
motivation in brief

theoretical aspects

3D free-space scattering theory is no longer valid and development of low-dimensional theory including influence of the trap is needed

- **What happens if atoms scatter in confined geometry (quasi-1D) ?**





- **What happens in collision of two distinguishable atoms in harmonic trap, or identical atoms in anharmonic trap ?**

$$H'(\boldsymbol{\rho_R}, \boldsymbol{r}) = -\frac{1}{2M}\left(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{4\rho_R^2}\right) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial\phi_R^2} + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + \underline{\mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos(\phi - \phi_R)} + V(r)$$

$$\mathbf{5D} \quad \rightarrow \quad \{r, \theta, \phi, \rho_R, \phi_R\}$$

non-separable two-body problem

Methods:

- **non-direct 2D discrete-variable representation (2D DVR)**

  1D DVR: J.C.Light et al J.Chem.Phys. 1985

  2D DVR: V.Melezhik Phys.Rev. A 1993
  Phys.Lett. 1997
  V.Melezhik & D.Baye Phys.Rev. C 1999
  V.Melezhik & P.Schmelcher Phys.Rev.Lett. 2000

- **multi-channel scattering problem as a boundary-value problem**

  V.Melezhik J.Comp.Phys. 1991
  V.Melezhik & C.-Y. Hu Phys.Rev.Lett. 2003
  S.Saeidian & V. Melezhik & P.Schmelcher Phys.Rev.A 2008

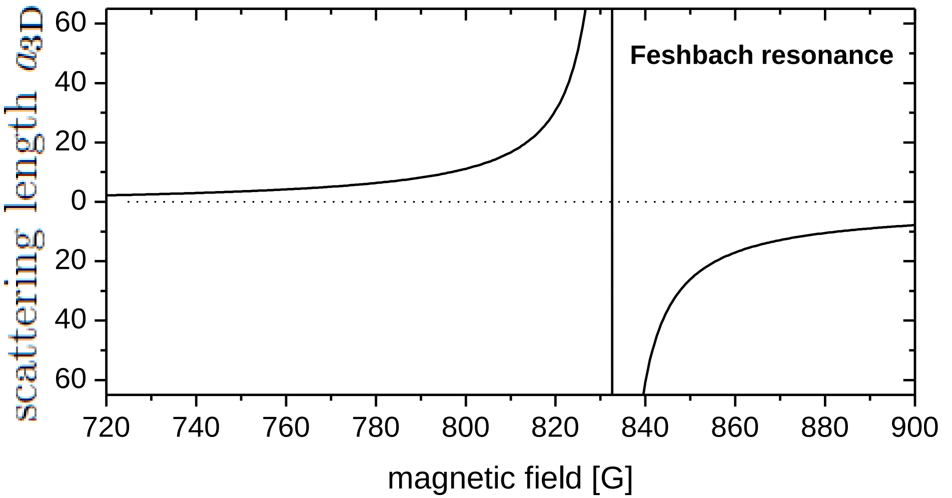
- **splitting-up method for time-dependent 3D and 4D Schrödinger eqs.**

  G.I.Marchuk 1971
  V.Melezhik Phys.Lett. 1997
  V.Melezhik & D.Baye Phys.Rev. C 1999
  J.I.Kim & V.Melezhik & P.Schmelcher Phys.Rev. A 2007
  V.Melezhik & P.Schmelcher New J. Phys 2009

confinement-induced resonances (CIRs)

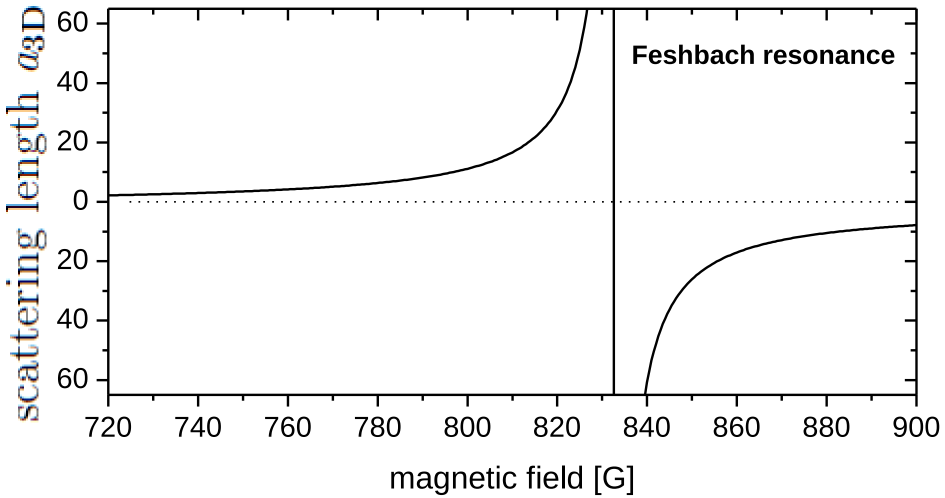
Tuning the interaction in 3D

3D



Tuning the interaction in 3D

3D

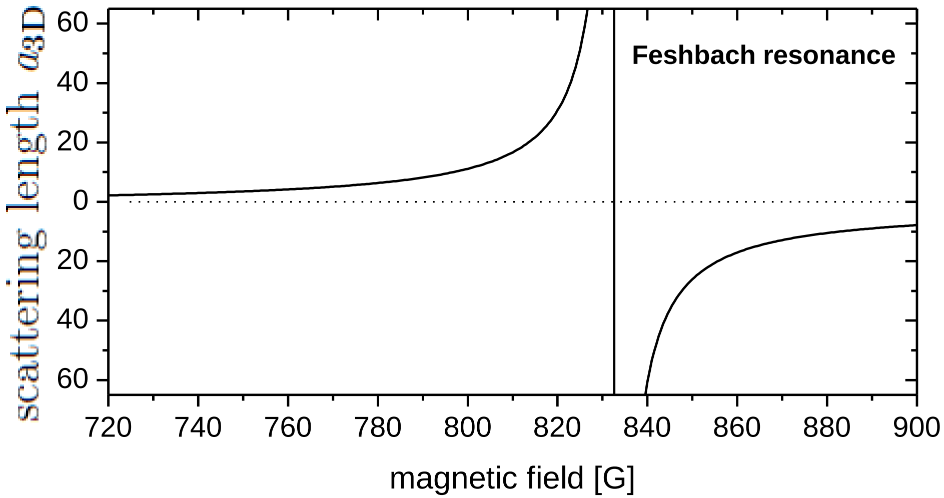


single-channel pseudopotential

$$\frac{2\pi\hbar^2 a_{3D}(B)}{\mu}\delta(\mathbf{r})$$

Tuning the interaction in 1D: B and ω

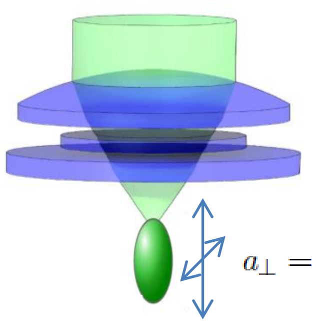
3D



single-channel pseudopotential

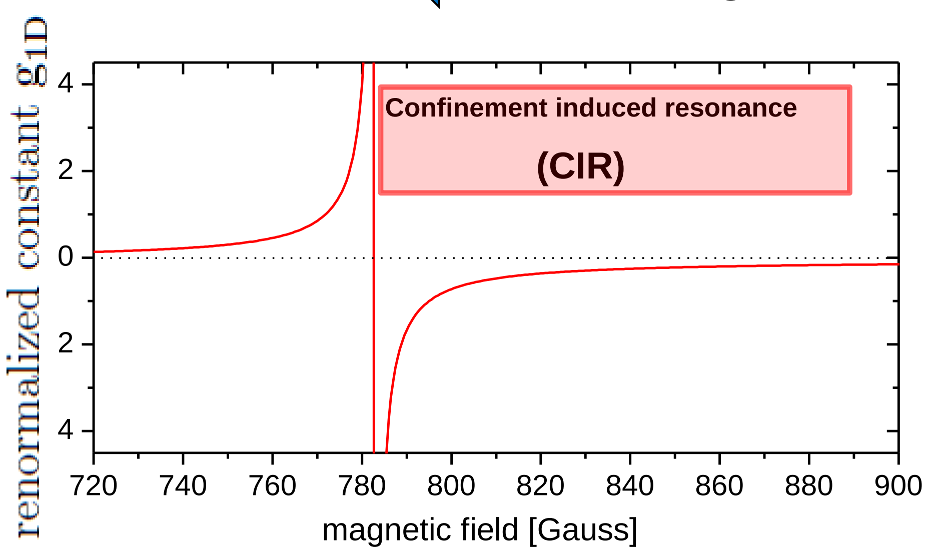
$$\frac{2\pi\hbar^2 a_{3D}(B)}{\mu}\delta(\mathbf{r})$$

strong confinement



$a_\perp = \sqrt{\hbar/(\mu\omega)}$

1D



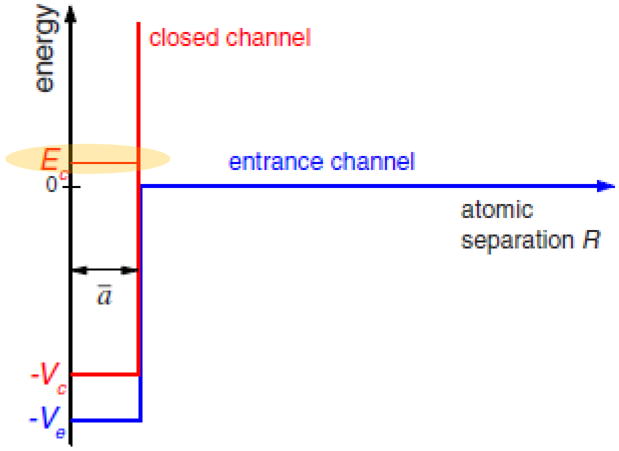
single-channel pseudopotential with renormalized interaction constant

$$g_{1D} = \frac{2\hbar^2 a_{3D}(B)}{\mu a_\perp^2}\frac{1}{1 - C a_{3D}/a_\perp}$$

M. Olshanii, PRL 81, 938 (1998).

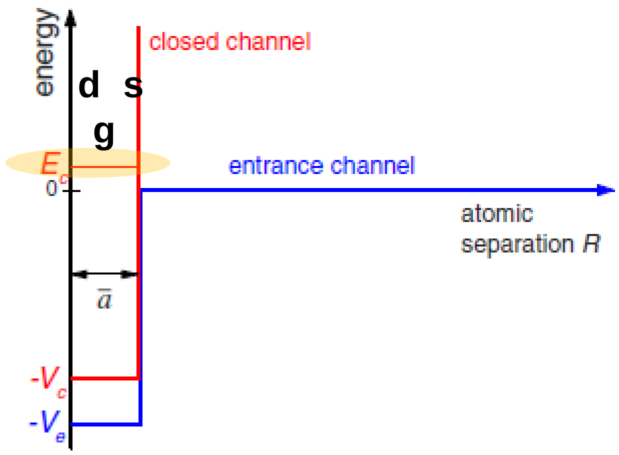
Feshbach resonances in quasi-1D atomic traps (bosons)

two-channel problem



two-channel problem

tensorial sructure of molecular state



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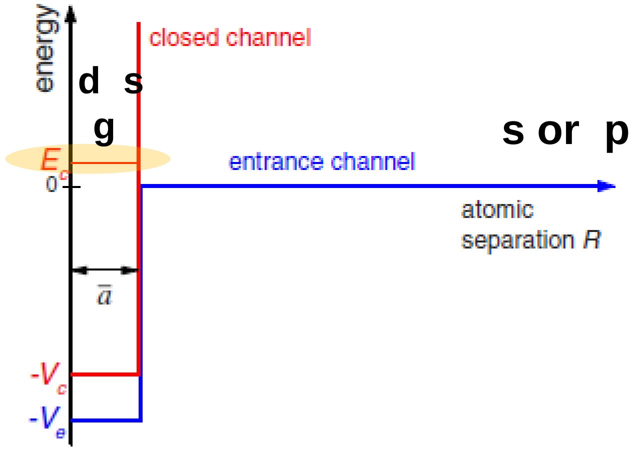

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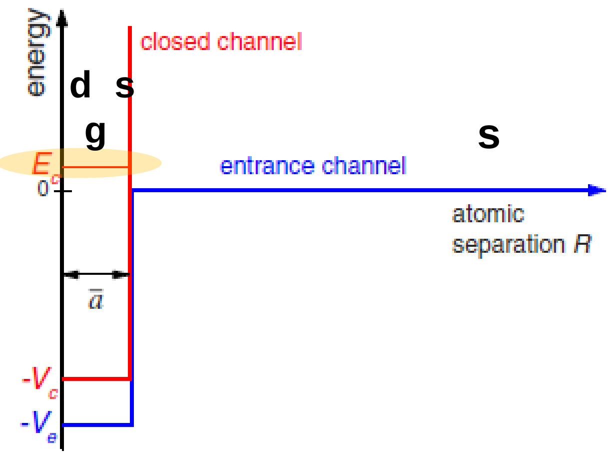
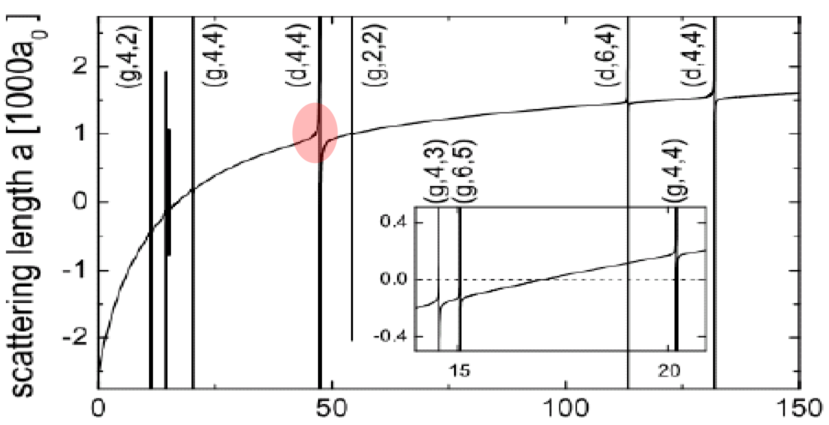


tensorial sructure of molecular state

Innsbruck experiment with Cs atoms:

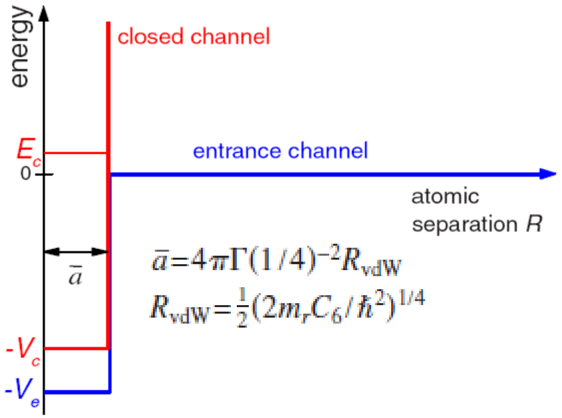
Feshbach Resonanzen



two-channel model of Lange et. al. Phys.Rev.79,013622(2009)

$$\hat{H}(r) = -\frac{\hbar^2}{2\mu}\hat{I}\frac{d^2}{d^2 r} + \hat{V}(r)$$

$$\hat{V} = \begin{bmatrix} -V_c & \hbar\Omega \\ \hbar\Omega & -V_e \end{bmatrix} \quad (\text{for } R < \bar{a}) = \begin{bmatrix} \infty & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{for } R > \bar{a})$$



$$\bar{a} = 4\pi\Gamma(1/4)^{-2} R_{vdW}$$

$$R_{vdW} = \frac{1}{2}(2m_r C_6/\hbar^2)^{1/4}$$

3 fitting parameters:

$$\frac{1}{a-\bar{a}} = \frac{1}{a_{bg}-\bar{a}} + \frac{\Gamma/2}{\bar{a}E_c}$$

$$E_c = \delta\mu(B - B_c)$$

$\longrightarrow$ $V_e \; V_c \; \Omega$

$$\Gamma = \delta\mu\Delta$$

TABLE I. Fitting parameters for the s-, d-, and g-wave Feshbach resonances, determining the scattering length in the magnetic-field range of interest; see Fig. 3. The background scattering length $a_{bg}=1875a_0$, the mean scattering length of cesium, $\bar{a}=95.7a_B$, and the bare s-wave state magnetic moment $\delta\mu_1=2.50\mu_B$ [28] are set constant. Poles $B_{0,i}$ and zeros B_i^* of the scattering length are derived; see text. Uncertainties in the parentheses are statistical. The systematic uncertainty of the magnetic field is 10 mG.

Res.	Γ_i/h (MHz)	$\delta\mu_i/\mu_B$	$B_{c,i}$ (G)	$B_{0,i}$ (G)	B_i^* (G)
s-wv.	11.6(3)	2.50	19.7(2)	−11.1(6)	18.1(6)
d-wv.	0.065(3)	1.15(2)	47.962(5)	47.78(1)	47.944(5)
g-wv.	0.0042(6)	1.5(1)	53.458(3)	53.449(3)	53.457(3)

extension of
two-channel model of Lange et. al. to 1D geometry

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)

A four-channel square-well potential

$$\hat{V} = \begin{pmatrix} -V_{c,3} & 0 & 0 & \hbar\Omega_3 \\ 0 & -V_{c,2} & 0 & \hbar\Omega_2 \\ 0 & 0 & -V_{c,1} & \hbar\Omega_1 \\ \hbar\Omega_3 & \hbar\Omega_2 & \hbar\Omega_1 & -V_e \end{pmatrix}$$

$$|\psi\rangle = \sum_\alpha \psi_\alpha(\mathbf{r})|\alpha\rangle = \sum_\alpha \phi_\alpha(r) Y_{l_\alpha 0}(\hat{r})|\alpha\rangle$$

$\omega_\perp = 0$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l_\alpha(l_\alpha+1)}{2\mu r^2} + B_{\alpha\alpha}\right]\phi_\alpha(r) + \sum_\beta V_{\alpha\beta}(r)\phi_\beta(r) = E\phi_\alpha(r)$$

4-coupled radial equations

$$\psi_e(\mathbf{r}) \to \exp\{ikz\} + f(k,\theta)/r \, \exp\{ikr\}, \qquad \psi_{c,i}(\mathbf{r}) \to 0$$

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$\omega_\perp \neq 0$

$$\left(\left[-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{1}{2}\mu\omega_\perp^2\rho^2\right]\hat{I} + \hat{B} + \hat{V}(r)\right)|\psi\rangle = E|\psi\rangle$$

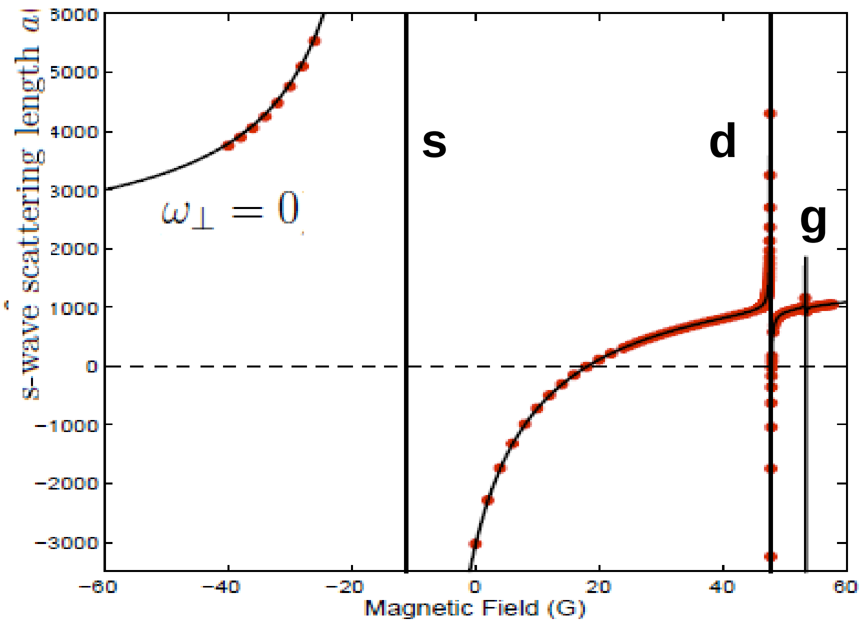
4-coupled 2D equations in the plane $\{r, \theta\}$

$$\psi_e(\mathbf{r}) = [\cos(k_0 z) + f_e \exp\{ik_0|z|\}]\Phi_0(\rho), \quad \psi_{c,i}(\mathbf{r}) \to 0$$

$$T(B) = |1 + f_e(B)|^2$$

Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)



Feshbach resonances in the Cs ultracold gas in the 3D free space

$$H(r,\theta) = [-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{1}{2}\mu\omega_\perp^2\rho^2]\hat{I} + \hat{V}(r)$$

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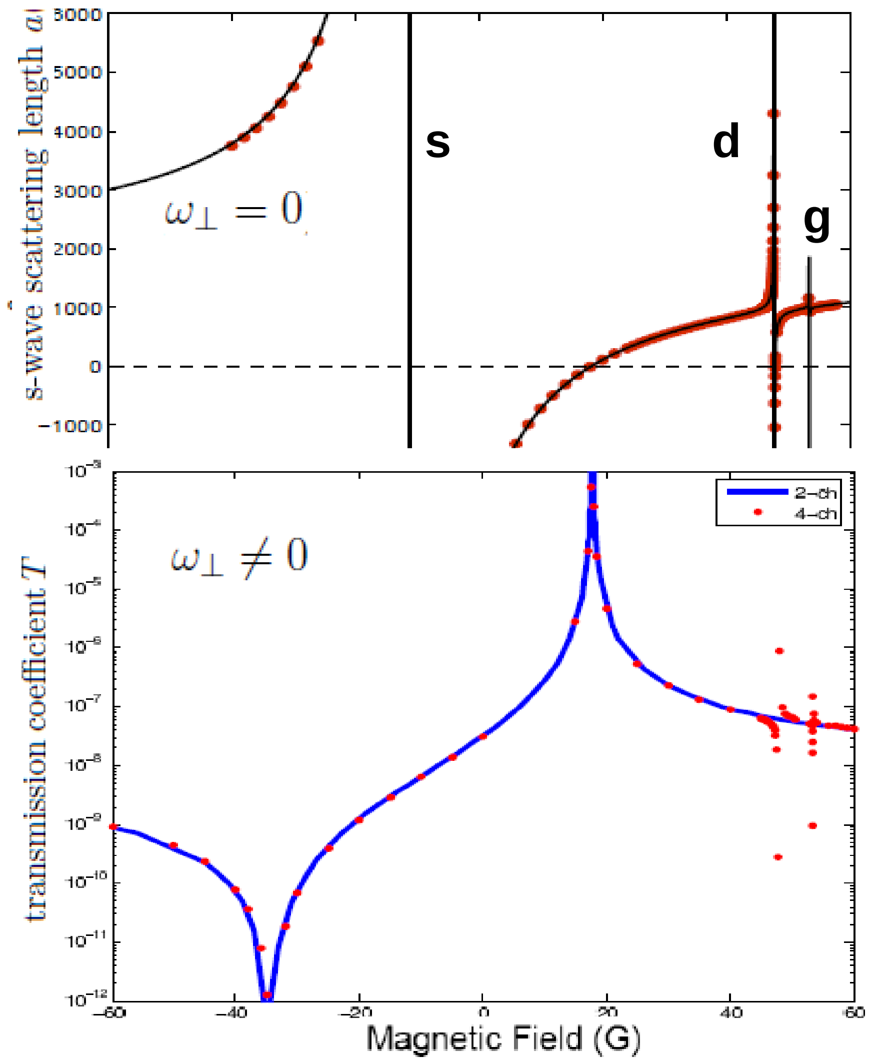

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in harmonic waveguides

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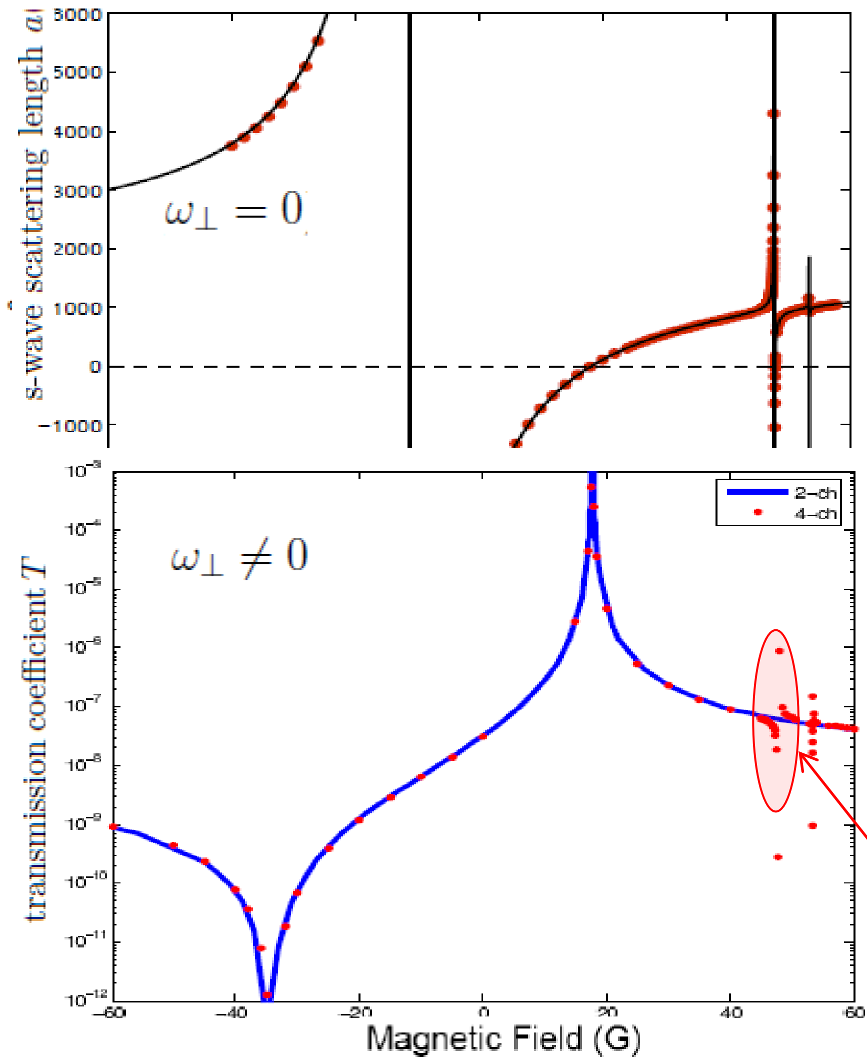

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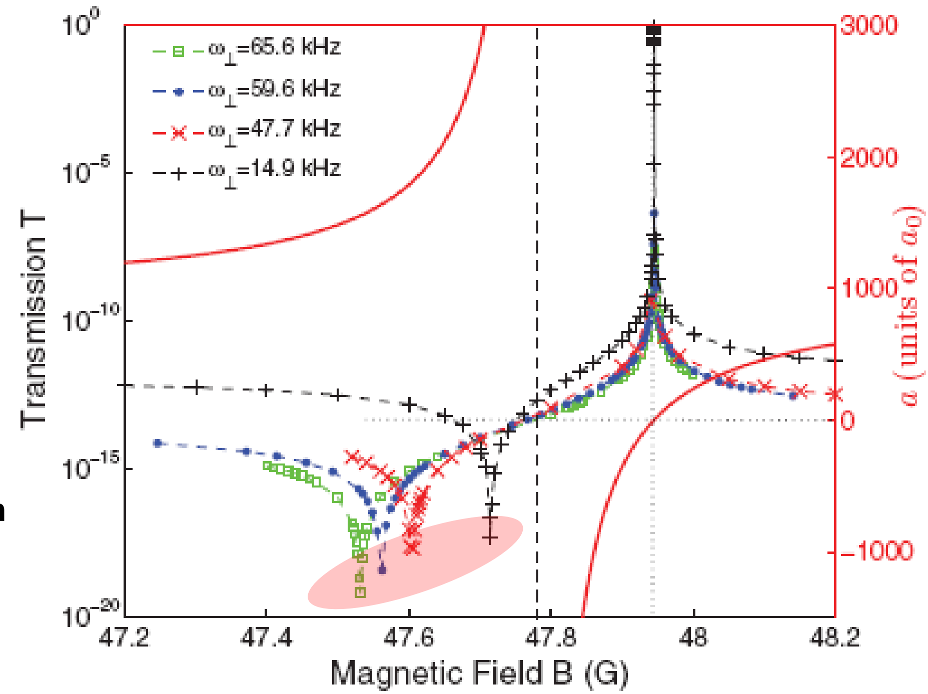
in harmonic waveguides

region of Innsbruck experiment
(d-wave Feshbach resonance)

Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)

$$a_\perp = \sqrt{\hbar/(m\omega_\perp)}$$



d-wave FR at 47.8G develops in waveguide as depending on $\omega_\perp$ minimums and stable maximum of transmission coefficient T

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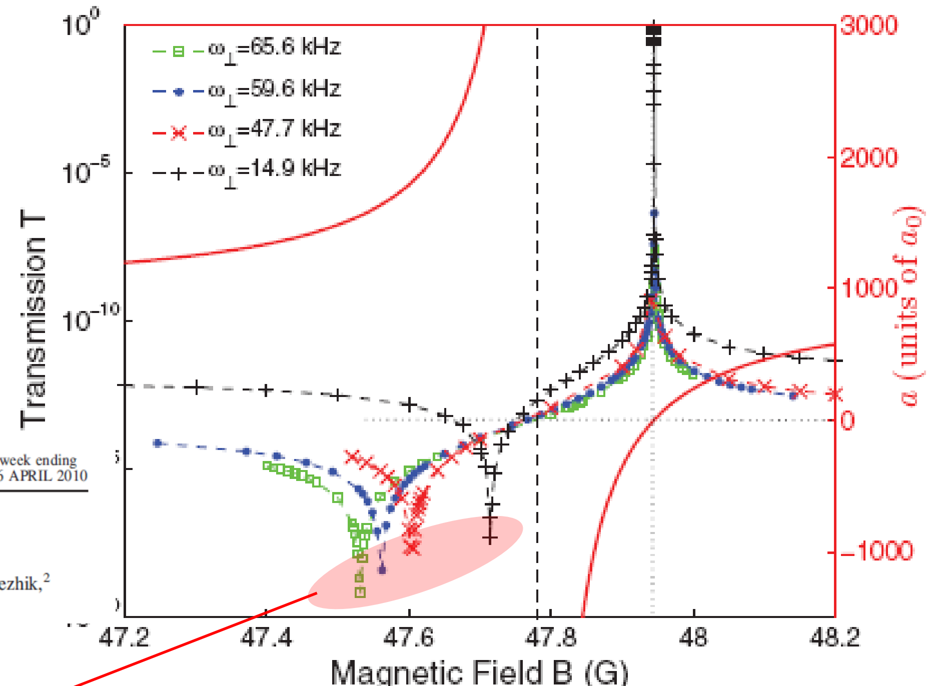
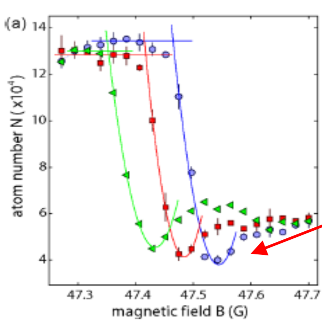


PRL 104, 153203 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

Confinement-Induced Resonances in Low-Dimensional Quantum Systems

Elmar Haller,[1] Manfred J. Mark,[1] Russell Hart,[1] Johann G. Danzl,[1] Lukas Reichsöllner,[1] Vladimir Melezhik,[2] Peter Schmelcher,[3] and Hanns-Christoph Nägerl[1]



experiment

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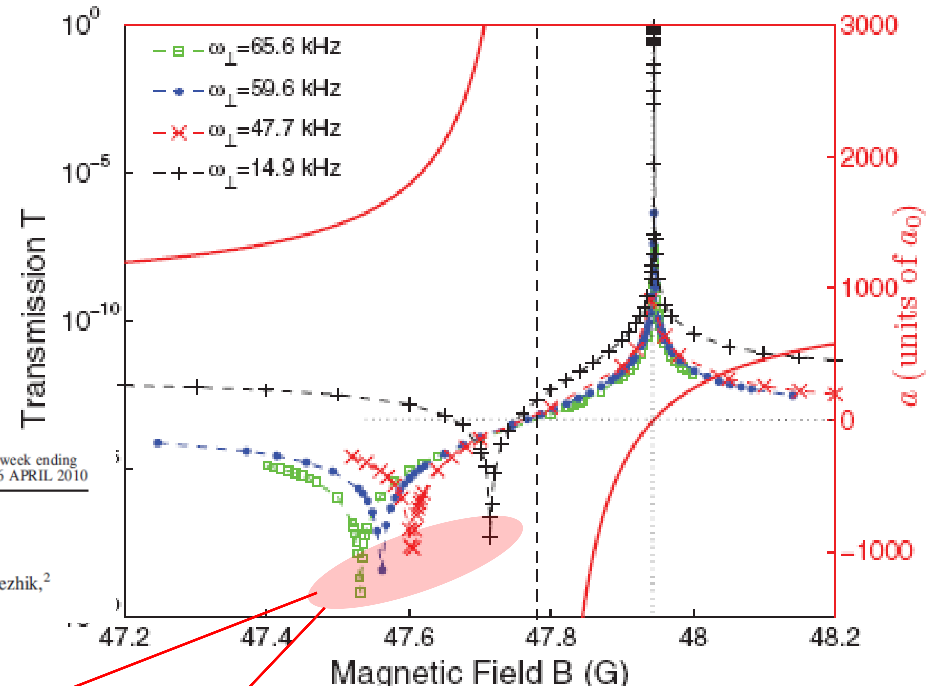

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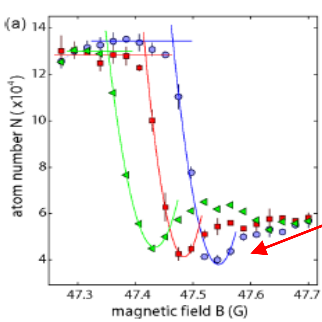
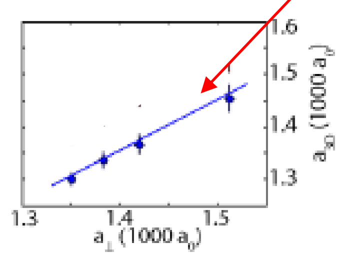


experiment

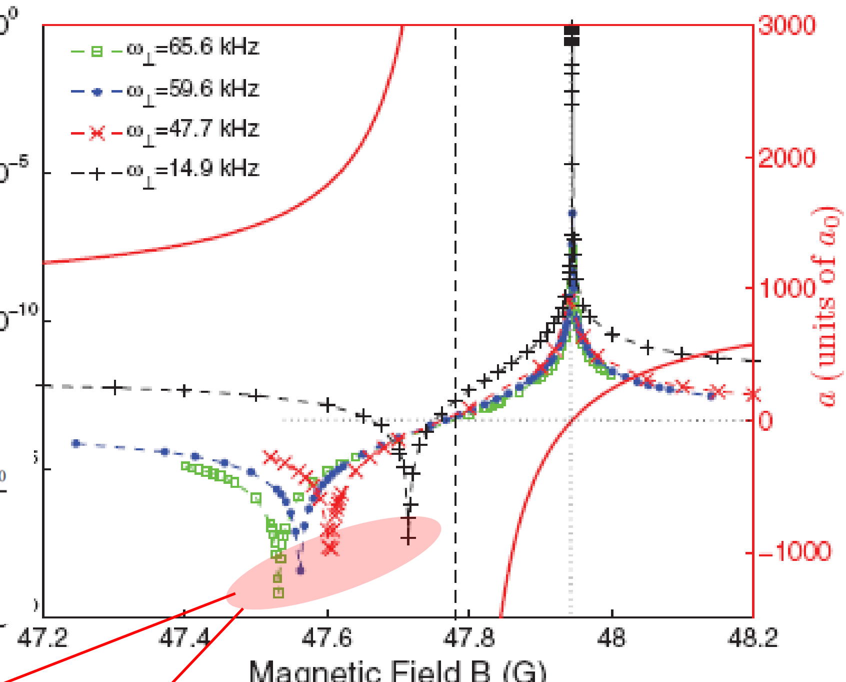
theory



Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)

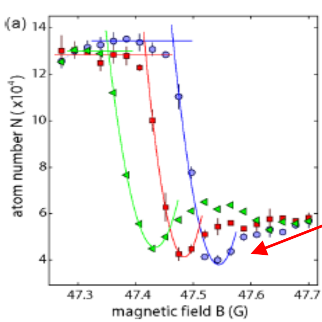
$$a_\perp = \sqrt{\hbar/(m\omega_\perp)}$$



PRL 104, 153203 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

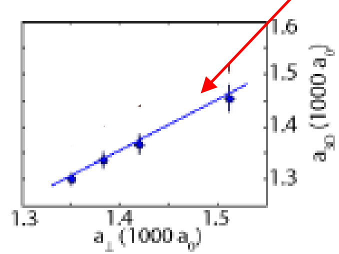
Confinement-Induced Resonances in Low-Dimensional Quantum Systems

Elmar Haller,[1] Manfred J. Mark,[1] Russell Hart,[1] Johann G. Danzl,[1] Lukas Reichsöllner,[1] Vladimir Melezhik,[2] Peter Schmelcher,[3] and Hanns-Christoph Nägerl[1]



experiment

theory



Olshanii formula works for s,d,and g FRs

$$a = 0.68 a_\perp$$

Feshbach resonances in quasi-1D atomic traps (fermions)

S.Saeidian, V.S.Melezhik and P.Schmelcher, J. Phys B (2015) (in press)

p-wave magnetic Feshbach resonances of ^{40}K atoms emerging in harmonic waveguides as p-wave CIRs

bosons: only s-wave in entrance channel $\frac{a}{a_\perp} = 0.68$

fermions: only p-wave in entrance channel $\frac{V_p}{V_\perp} = ?$

$$V_\perp = a_\perp^3$$

$$k^3 cot\delta_1(k,B) = -\frac{1}{V_p(B)}$$

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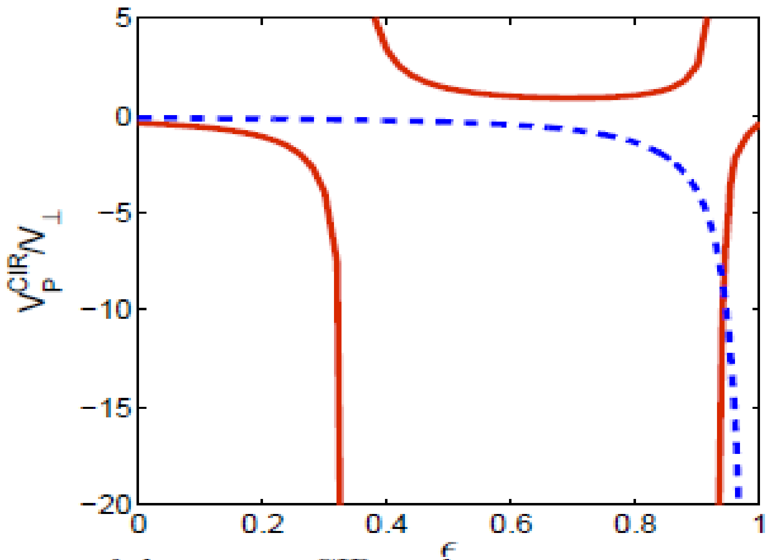

position of the p-wave CIR in the state $|F=9/2, m_F=-7/2\rangle$ of Potassium atoms as a function of the the rescaled energy ϵ

~~$\frac{a}{a_\perp} = 0.68$~~

$$\frac{V_p}{V_\perp} \neq const$$

$$k^3 cot\delta_1(k,B) = -\frac{1}{V_p(B)}$$

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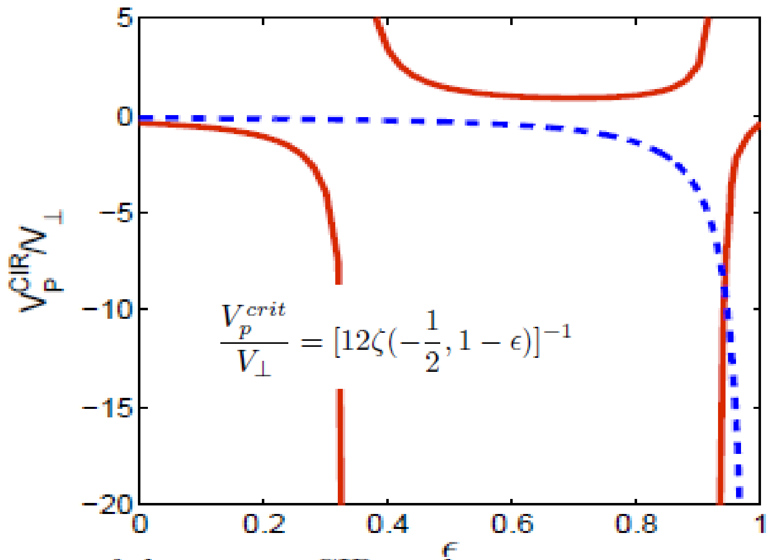

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B.E. Granger and D. Blume, Phys. Rev. Lett. **92**, 133202 (2004).

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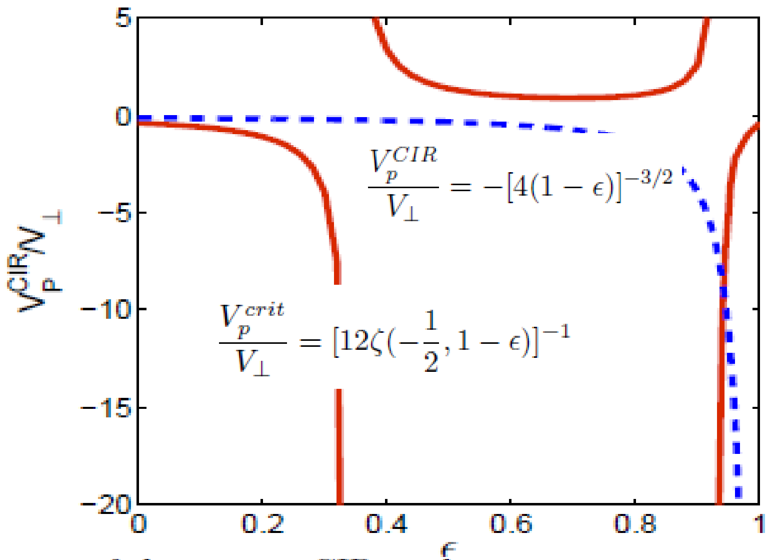

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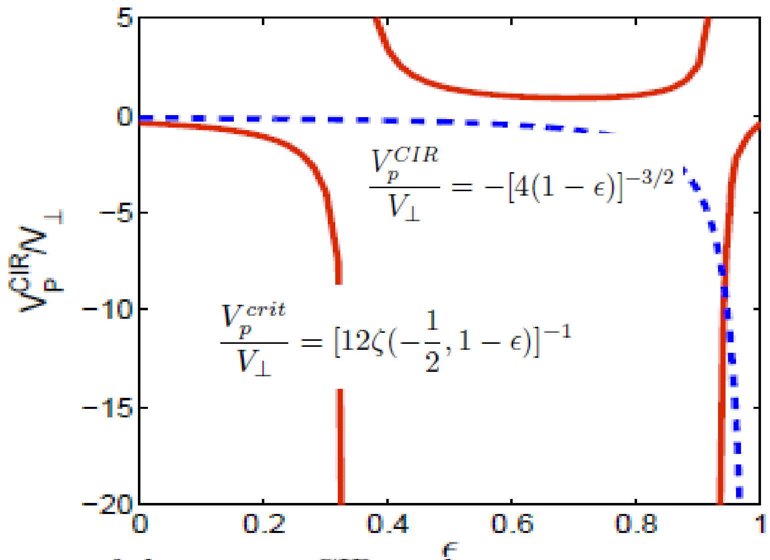

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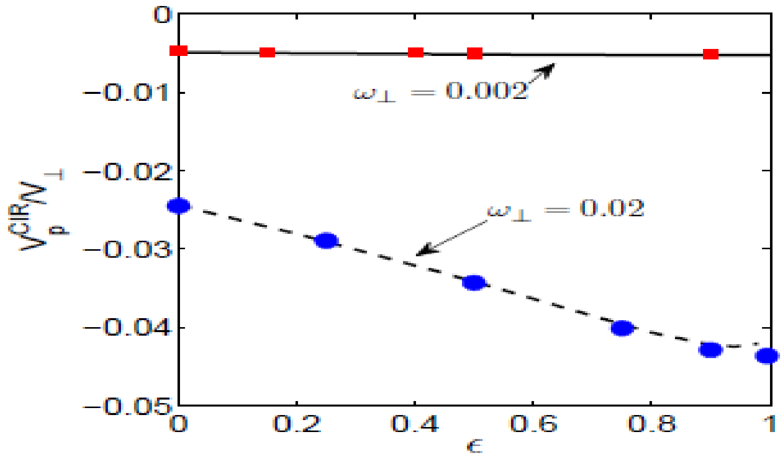

$$\frac{V_p^{crit}(k)}{V_\perp} = [\frac{2a_\perp}{R} + 12\zeta(-\frac{1}{2}, 1-\epsilon)]^{-1}$$

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Shi-Guo Peng, S. Tan, and K. Jiang, arXiv:1312.3392v2

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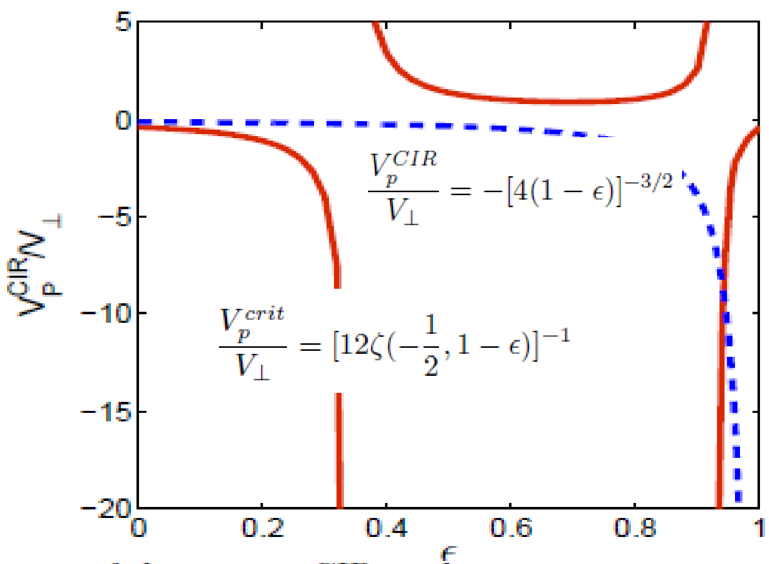

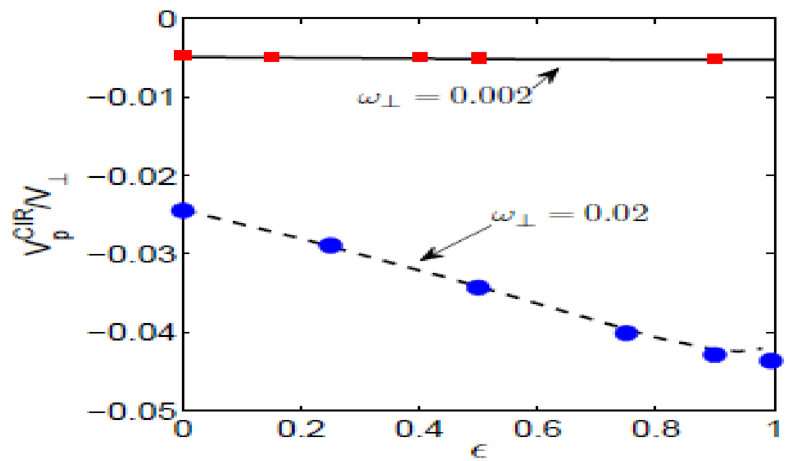

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Dipolar confinement-induced resonances in waveguides

P.Giannakeas, V. Melezhik & P.Schmelcher, PRL,111(2013)

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \underbrace{\frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}}_{V_{sr}} + \underbrace{\frac{d^2}{r^3}[1-3(\hat{z}\cdot\hat{r})]}_{V_{dd}}$$

$$\underline{K}^{3\mathrm{D}} = \begin{pmatrix} K_{ss} & K_{sd} & 0 \\ K_{ds} & K_{dd} & K_{dg} \\ 0 & K_{gd} & K_{gg} \end{pmatrix}$$

$$a_{ll'} = -\frac{K_{ll'}}{k}$$

$$l_d = \frac{\mu d^2}{\hbar^2}$$

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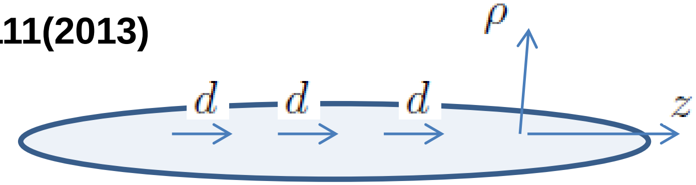

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{\mu}{2}\omega_\perp^2\rho^2 + \underbrace{\frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}}_{V_{sr}} + \underbrace{\frac{d^2}{r^3}[1 - 3(\hat{z}\cdot\hat{r})]}_{V_{dd}}$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

$$\underline{\tilde{K}}_{oo}^{1D} = \underline{K}_{oo}^{1D} + i\underline{K}_{oc}^{1D}(\mathcal{I} - i\underline{K}_{cc}^{1D})^{-1}\underline{K}_{co}^{1D}$$

$$\underline{K}^{3D} = \begin{pmatrix} K_{ss} & K_{sd} & 0 \\ K_{ds} & K_{dd} & K_{dg} \\ 0 & K_{gd} & K_{gg} \end{pmatrix}$$

$$\det(\mathcal{I} - i\underline{K}_{cc}^{1D}) = 0$$

$$a_{ll'} = -\frac{K_{ll'}}{k} \qquad \bar{a}_{ll'} = \frac{a_{ll'}}{a_\perp}$$

$$l_d = \frac{\mu d^2}{\hbar^2} \qquad \bar{l}_d = \frac{l_d}{a_\perp}$$

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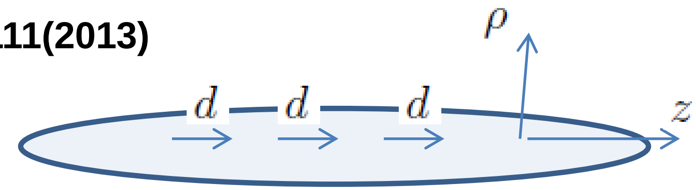

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$$a_{ll'} = -\frac{K_{ll'}}{k} \qquad \bar{a}_{ll'} = \frac{a_{ll'}}{a_\perp}$$

we obtained resonance condition:

$$\bar{a}_{ss}(ka_\perp, d) = \mathcal{F}(\{\bar{a}_{\ell\ell'}(ka_\perp, d)\})$$

$$l_d = \frac{\mu d^2}{\hbar^2} \qquad \bar{l}_d = \frac{l_d}{a_\perp}$$

$$\mathcal{F}_{\mathrm{BA}} = -\frac{1 + \eta_1\bar{l}_d + \eta_2\bar{l}_d^2 + \eta_3\bar{l}_d^3}{\sigma_0 + \sigma_1\bar{l}_d + \sigma_2\bar{l}_d^2}$$

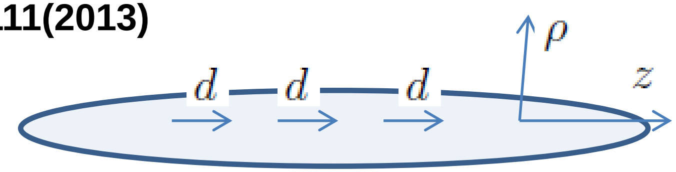
For $l_d = 0$, the resonance condition $\bar{a}_{ss} = \mathcal{F}_{\mathrm{BA}}$

reduces to $\bar{a}_s = -1/\sigma_0 = 0.68$

$$a_s = 0.68a_\perp$$

Dipolar confinement-induced resonances in waveguides

P.Giannakeas, V. Melezhik & P.Schmelcher, PRL,111(2013)



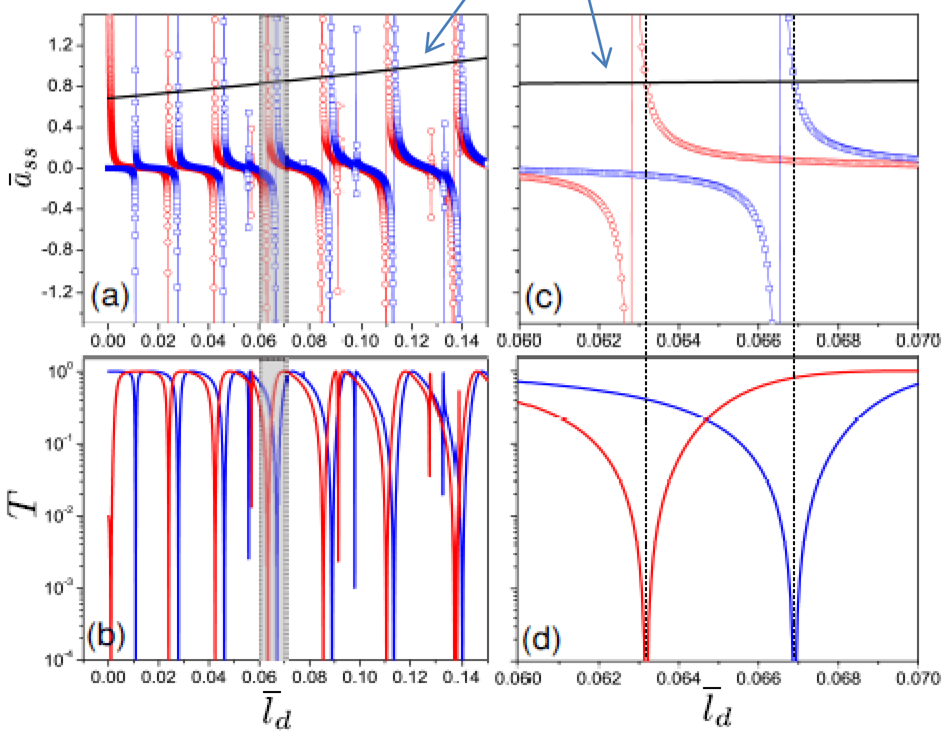
$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{\mu}{2}\omega_\perp^2\rho^2 + \underbrace{\frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}}_{V_{sr}} + \underbrace{\frac{d^2}{r^3}[1-3(\hat{z}\cdot\hat{r})]}_{V_{dd}}$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

$a_s \gg l_{\mathrm{vdW}}$ (−○−)

$a_s \ll l_{\mathrm{vdW}}$ (−□−)

$$\mathcal{F}_{\mathrm{BA}} = -\frac{1+\eta_1\bar{l}_d+\eta_2\bar{l}_d^2+\eta_3\bar{l}_d^3}{\sigma_0+\sigma_1\bar{l}_d+\sigma_2\bar{l}_d^2}$$



$$\bar{a}_{ll'} = \frac{a_{ll'}}{a_\perp} \qquad a_{ll'} = -\frac{K_{ll'}}{k}$$

$$\bar{l}_d = \frac{l_d}{a_\perp} \qquad l_d = \frac{\mu d^2}{\hbar^2}$$

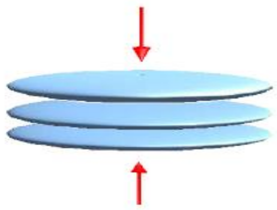
For $l_d = 0$, the resonance condition $\bar{a}_{ss} = \mathcal{F}_{\mathrm{BA}}$ reduces to $\bar{a}_s = -1/\sigma_0 = 0.68$

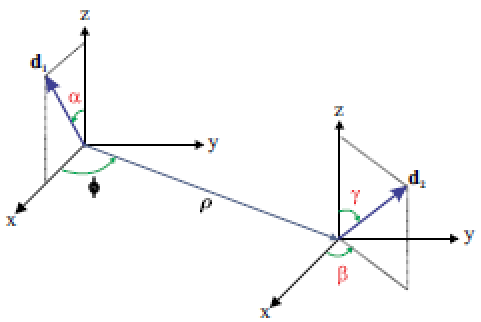
$$a_s = 0.68 a_\perp$$

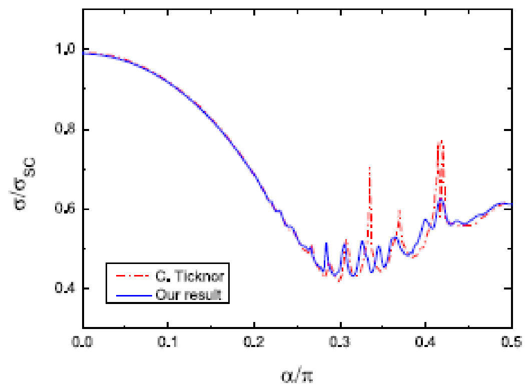
analytically derived resonant condition $\bar{a}_{ss} = \mathcal{F}_{\mathrm{BA}}$ **predict position of** **dipolar confinement-induced resonances**

Anisotropic quantum scattering in two dimensions

E.Koval, O.Koval, V.Melezhik, Phys.Rev.A89,052710 (2014)







$$(\alpha = \gamma;\ \beta = 0)$$

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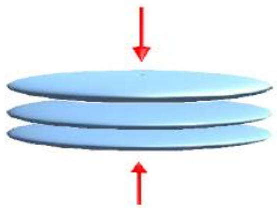

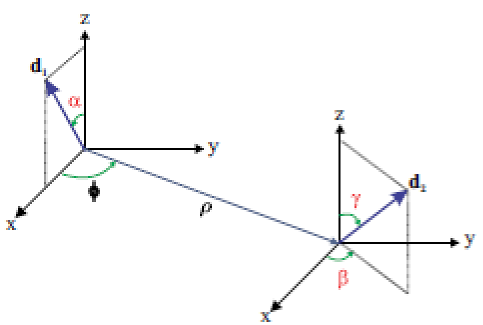

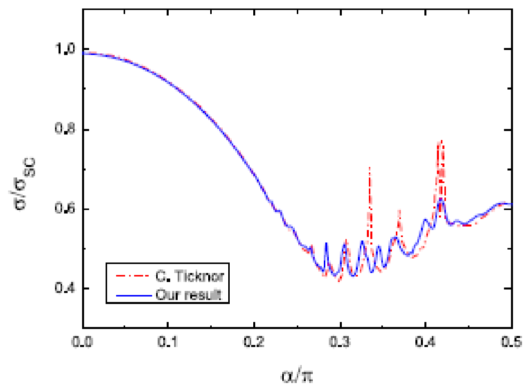

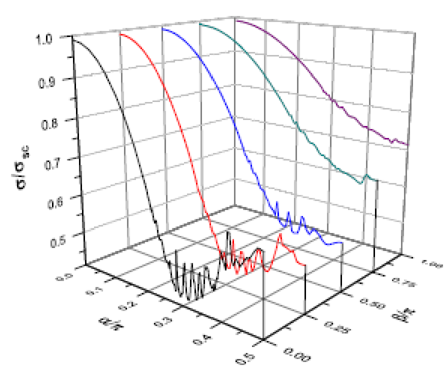

FIG. 8. (Color online) The total cross sections σ in the units of σ_{SC} as a function of the dipole tilt angle $\alpha = \gamma$ and the rotational angle β calculated for potential (23) at $D = 1$, $Dq = 10$.

$\pi/2$.

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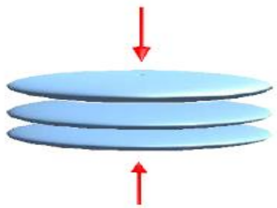

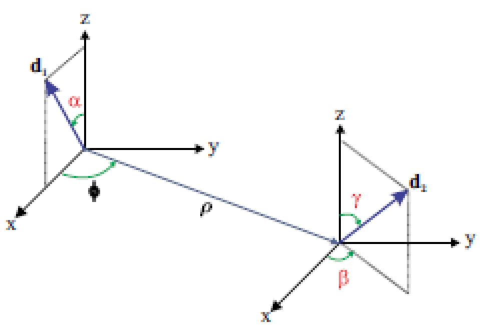

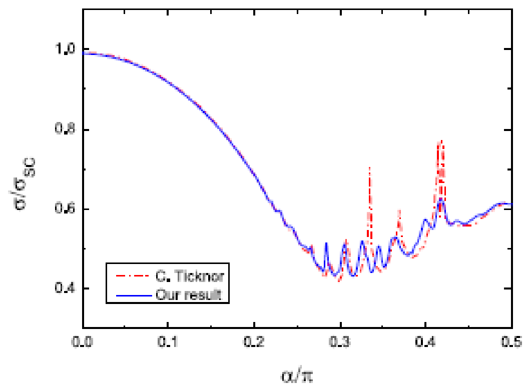

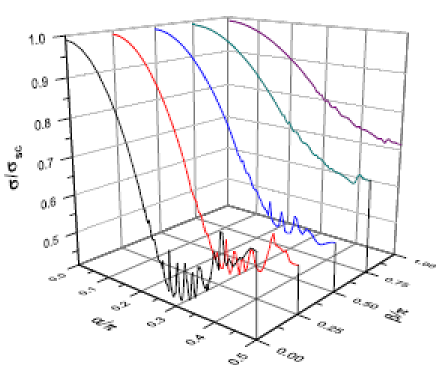

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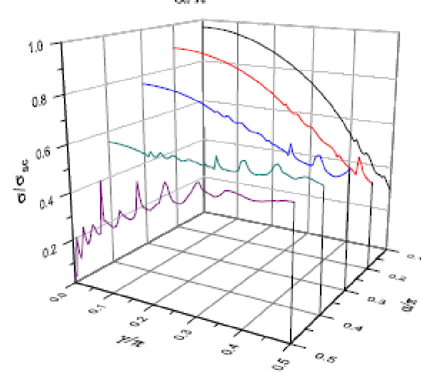

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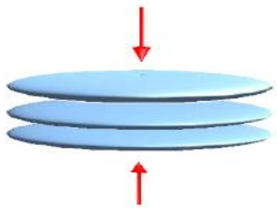

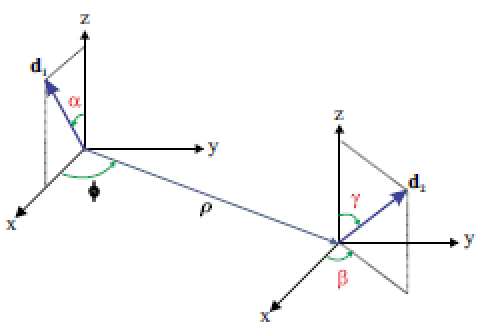

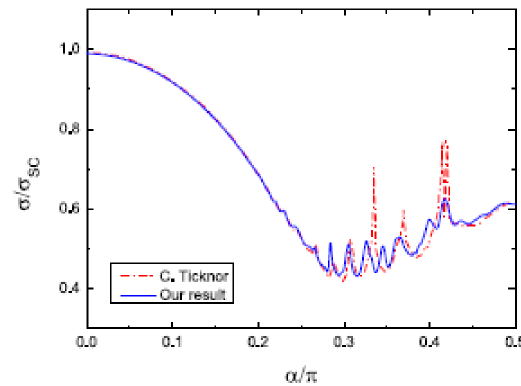

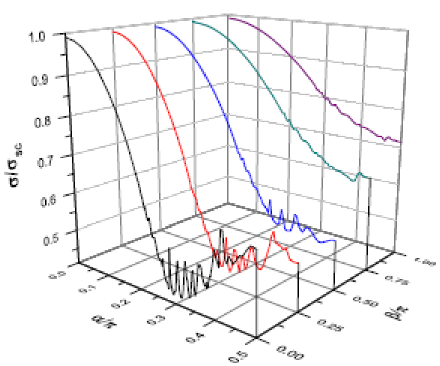

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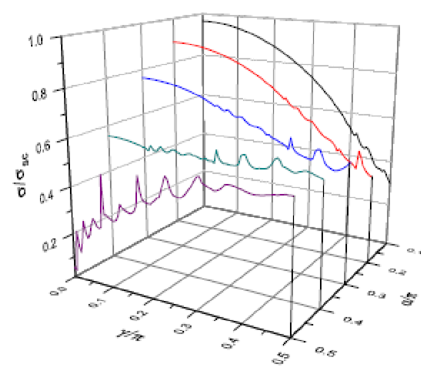

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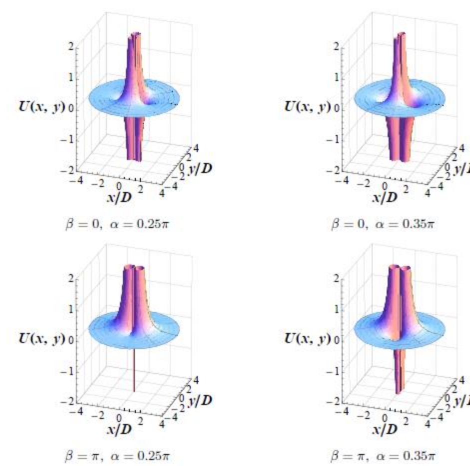


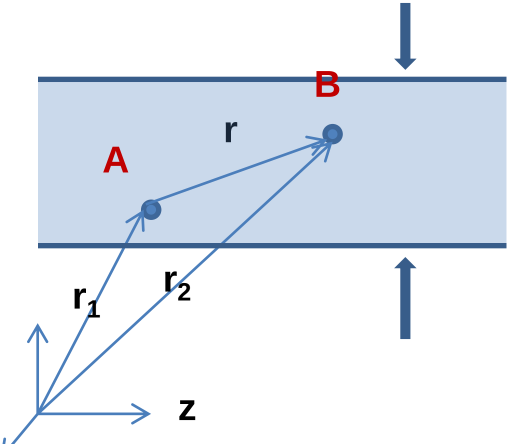
Mechanism of molecule formation with transferring the energy release to CM excitation of forming molecule was considered in:

E.Bolda et.al. Phys.Rev. A71,033404 (2004) (in anharmonic lattices)

V.Melezhik &P.Schmelcher, New J.Phys.11,073031 (2009) (distinguishable atoms in harmonic waveguides)

non-separability of two-body problem in trap
(distinguishable atoms in harmonic trap or identical atoms in anharmonic trap)

V. Melezhik & P. Schmelcher, New J. of Phys. 11, 073031 (2009)



$$V(r) + \frac{1}{2}m_1\omega_1^2\rho_1 + \frac{1}{2}m_2\omega_2^2\rho_2$$

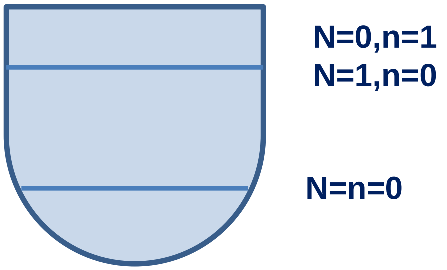
$$i\frac{\partial}{\partial t}\psi(\rho_R,\mathbf{r},t) = H(\rho_R,\mathbf{r})\psi(\rho_R,\mathbf{r},t)$$

$$H(\rho_R,\mathbf{r}) = H_{\mathrm{CM}}(\rho_R) + H_{\mathrm{rel}}(\mathbf{r}) + W(\rho_R,\mathbf{r})$$

$$H_{\mathrm{CM}} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{\rho_R^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{4\rho_R^2}\right) + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$H_{\mathrm{rel}} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + V(r)$$

$$\frac{L^2(\theta,\phi)}{2\mu r^2} = -\frac{1}{2\mu r^2\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right)$$



$$W(\rho_R,\mathbf{r}) = \mu(\omega_1^2 - \omega_2^2)r\rho_R\sin\theta\cos\phi$$ → **4D TDSE:** ρ_R, r, θ, ϕ

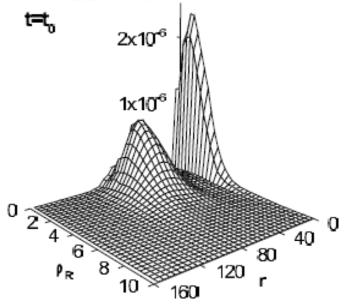
$A_{n1=0}$ + $B_{n2=0}$ → $(AB)_{n=0,N=1}$

$A_{n1=0}$ + $B_{n2=0}$ → $(AB)_{n=0,N=1}$

Time evolution of the probability density distribution during collision

$$W(\rho_R, r, t) = \int |\psi(\rho_R, r, \theta, \phi, t)|^2 (r^2 \rho_R)^{-1} \sin\theta \, d\theta \, d\phi$$

CM coupling with interatomic motion:



$$\omega_1 \neq \omega_2$$

$A_{n1=0} + B_{n2=0} \rightarrow (AB)_{n=0,N=1}$

Time evolution of the probability density distribution during collision

$$W(\rho_R, r, t) = \int |\psi(\rho_R, r, \theta, \phi, t)|^2 (r^2 \rho_R)^{-1} \sin\theta \, d\theta \, d\phi$$

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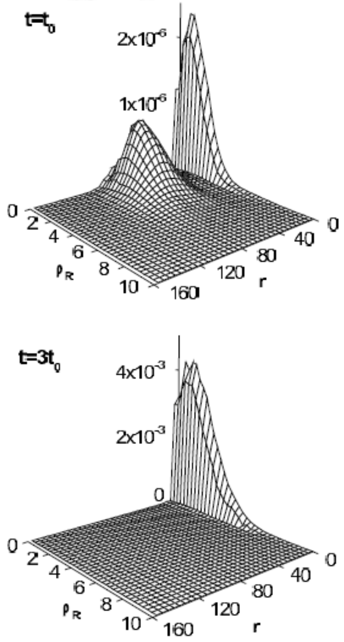

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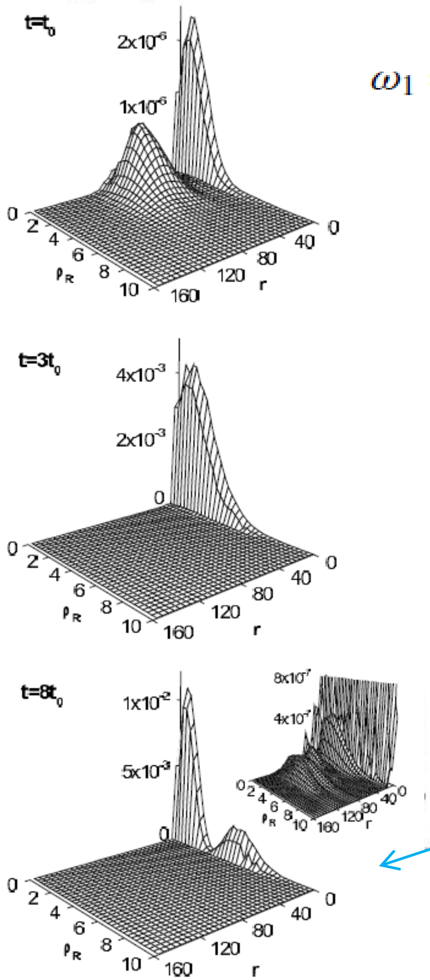

molecule formation

$A_{n1=0} + B_{n2=0} \rightarrow (AB)_{n=0,N=1}$

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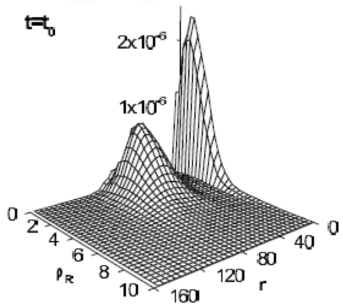
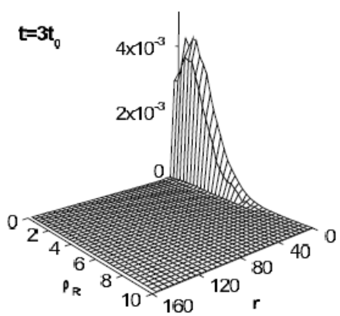
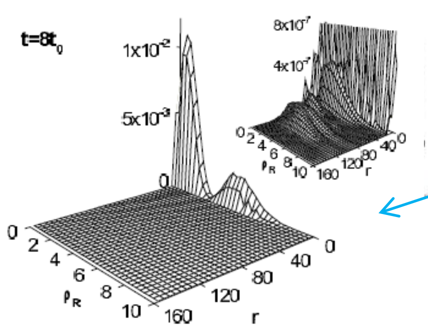


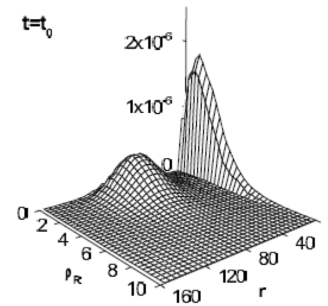


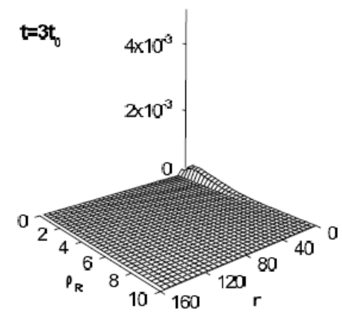


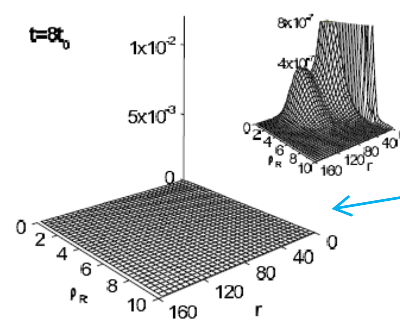
molecule formation

CM decouples from interatomic motion:

$\omega_1 = \omega_2$







no molecule

Resonant Formation of Ultracold Molecules in Waveguides

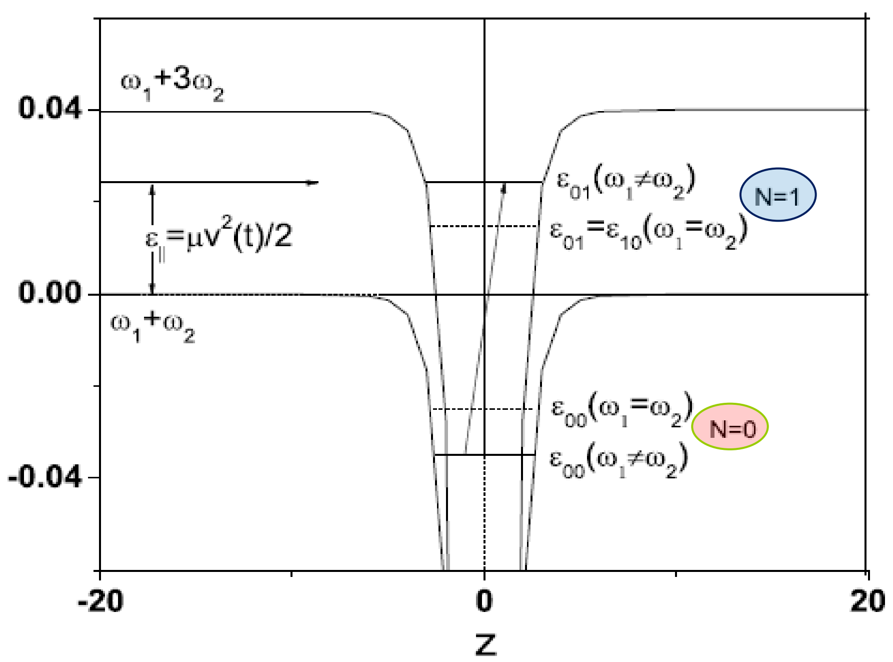
V. Melezhik & P. Schmelcher, New J. of Phys. 11, 073031 (2009)

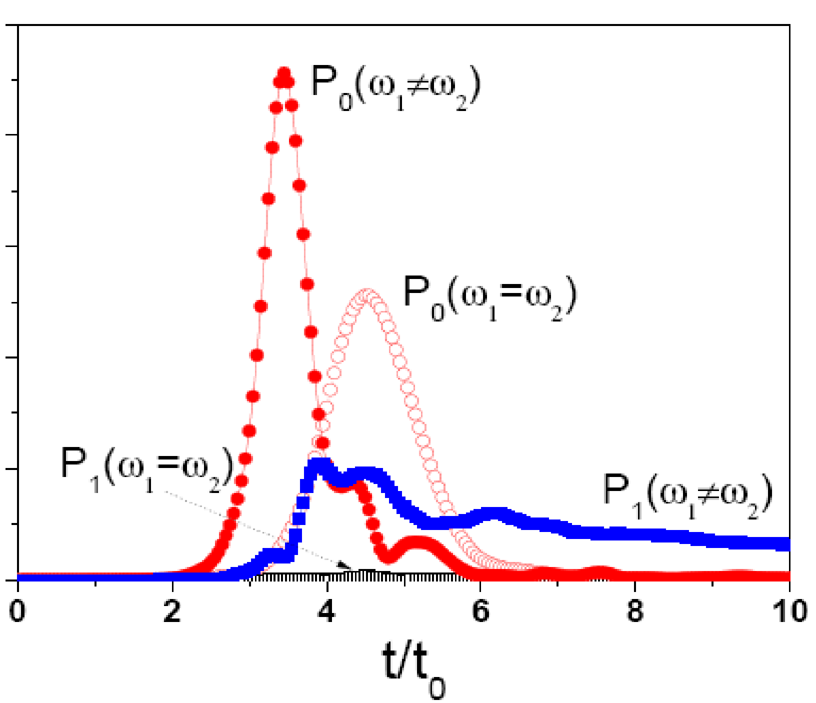
coupling of the deatomic continuum with the CM of excited molecule at (N=1) in closed transverse channels:

if the atoms in the colliding pair are identical, then coupling term goes to zero and the effect disappears.

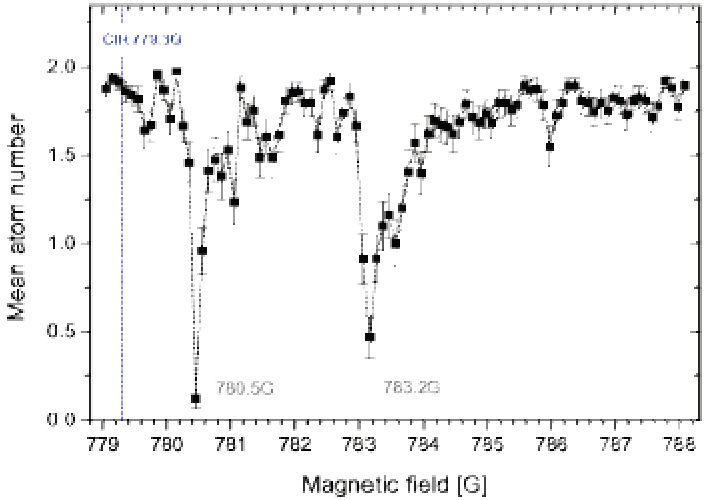
$$A_{n1=0} + B_{n2=0} \rightarrow (AB)_{n=0,N=1}$$

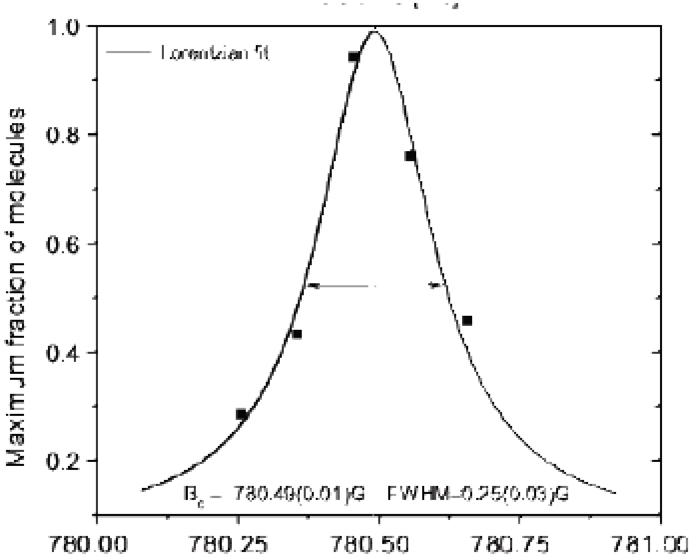
Time evolution of the molecular states (N=0 and 1) population $P_N(t)$ during a pair collision:





in Heidelberg experiment , S.Sala et. al. Phys.Rev.Lett.110,203202 (2013),
the mechanism of molecule formation with transferring
energy release to CM molecule excitation was observed
in anharmonic waveguide





signature of resonant molecule formation

Conclusion and outlook

- confinement-induced resonances in low-dimensional quantum systems

- s- and p-wave CIRs in quasi-1D traps
- resonant positions for dipolar CIRs in quasi-1D traps
- resonant mechanism for molecule formation in traps with energy transfer to CM excitation

Conclusion and outlook

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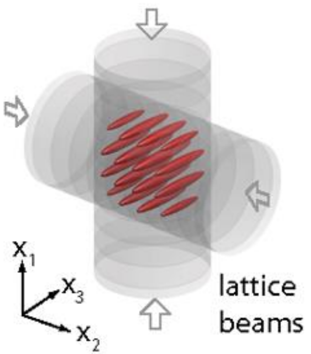
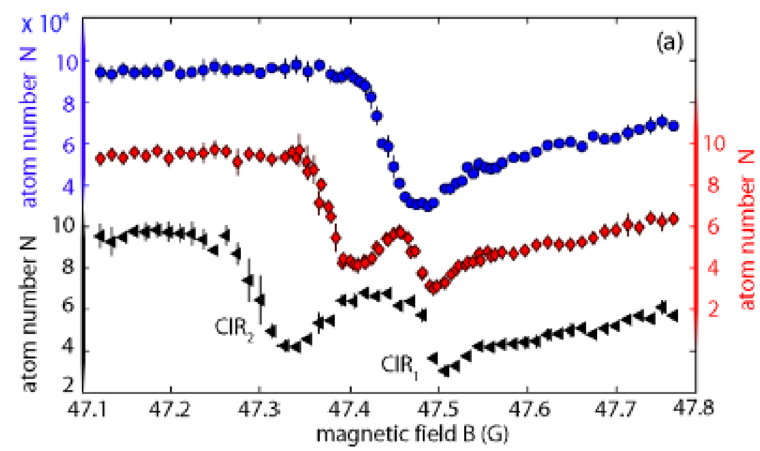


$$\omega_1 \neq \omega_2$$



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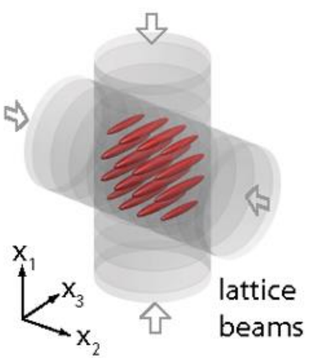
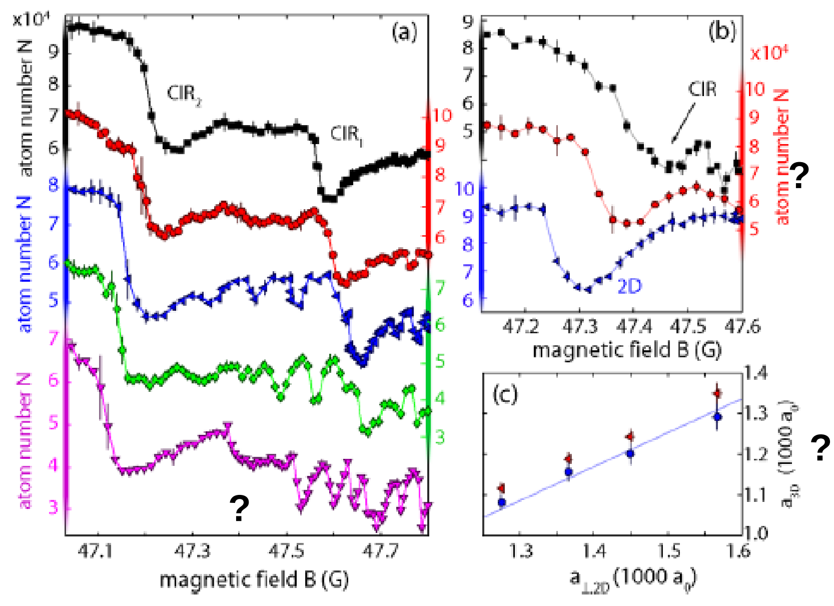


$$\omega_1 \gg \omega_2$$



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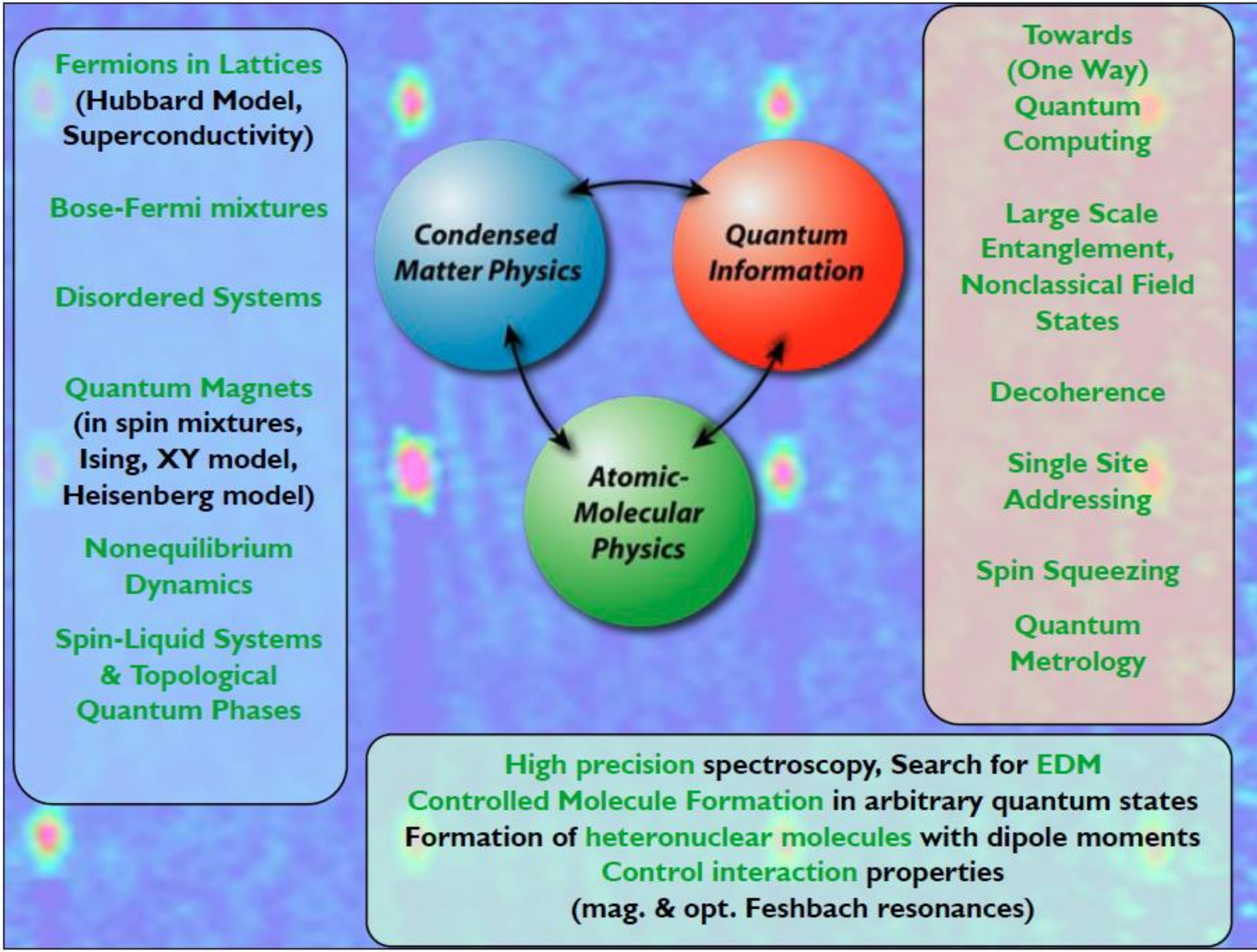
three-body collisional problem (Efimov resonances) in tight traps

non-linear time-dependent Schrödinger equation with CM coupling

Quantum simulation with fully controlled few-body systems

control over: quantum states, particle number, interaction


Fermions in Lattices
(Hubbard Model,
Superconductivity)
Bose-Fermi mixtures
Disordered Systems
Quantum Magnets
(in spin mixtures,
Ising, XY model,
Heisenberg model)
Nonequilibrium
Dynamics
Spin-Liquid Systems
& Topological
Quantum Phases
Condensed
Matter Physics
Quantum
Information
Atomic-
Molecular
Physics
Towards
(One Way)
Quantum
Computing
Large Scale
Entanglement,
Nonclassical Field
States
Decoherence
Single Site
Addressing
Spin Squeezing
Quantum
Metrology
High precision spectroscopy, Search for EDM
Controlled Molecule Formation in arbitrary quantum states
Formation of heteronuclear molecules with dipole moments
Control interaction properties
(mag. & opt. Feshbach resonances)