Vector meson dominance, axial anomaly and mixing

Yaroslav Klopot[1], Armen Oganesian[1,2] and Oleg Teryaev[1]

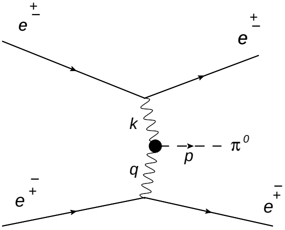
[1]Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, Dubna, Russia

[2]Institute of Theoretical and Experimental Physics,
Moscow, Russia

Brazil-JINR Forum
"Frontiers in Nuclear, Elementary Particle, and Condensed Matter Physics"
JINR, Dubna, June 15 - 19, 2015

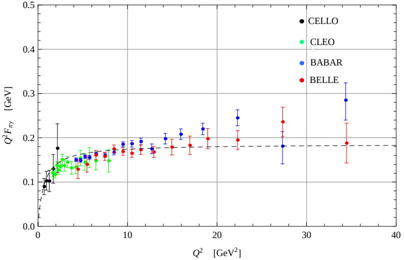
Outline

Transition form factors



π^0 TFF: theoretical and experimental status

Pion transition form factor: available data



- The current experimental status of the pion transition form factor (TFF) $F_{\pi\gamma}$ is rather controversial.
- The measurements of the BABAR collaboration [Aubert et al. '09] show a steady rise of $Q^2 F_{\pi\gamma}$, surpassing the pQCD predicted asymptote $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$, $f_\pi = 130.7$ MeV at $Q^2 \simeq 10$ GeV^2 and questioning the collinear factorization.

Axial anomaly: real and virtual photons

- Axial anomaly determines the $\pi^0 \to \gamma\gamma$ decay width: a unique example of a low-energy process, precisely predicted from QCD.
- The dispersive approach to axial anomaly leads to the *anomaly sum rule* (ASR) providing a handy tool to study the meson transition form factors – $M \to \gamma\gamma^*$ (even beyond the factorization hypothesis).

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)} \quad (1)$$

- Holds for any Q^2 and any m^2.
- It has neither α_s corrections (Adler-Bardeen theorem) nor non-perturbative corrections (t'Hooft's consistency principle).
- Exact nonperturbative relation – powerful tool.

Axial anomaly

In QCD, for a given flavor q, the divergence of the axial current $J_{\mu5}^{(q)} = \bar{q}\gamma_\mu\gamma_5 q$ acquires both electromagnetic and gluonic anomalous terms:

$$\partial_\mu J_{\mu5}^{(q)} = m_q\bar{q}\gamma_5 q + \frac{e^2}{8\pi^2}e_q^2 N_c F\tilde{F} + \frac{\alpha_s}{4\pi}N_c G\tilde{G}, \tag{2}$$

An octet of axial currents

$$J_{\mu5}^{(a)} = \sum_q \bar{q}\gamma_5\gamma_\mu\frac{\lambda^a}{\sqrt{2}}q$$

Singlet axial current $J_{\mu5}^{(0)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$:

$$\partial^\mu J_{\mu5}^{(0)} = \frac{1}{\sqrt{3}}(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d + m_s\overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi}C^{(0)}N_c F\tilde{F} + \frac{\sqrt{3}\alpha_s}{4\pi}N_c G\widetilde{G}, \tag{3}$$

The diagonal components of the octet of axial currents
$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$,
$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$
acquire an electromagnetic anomalous term only:

$$\partial^\mu J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(m_u\overline{u}\gamma_5 u - m_d\overline{d}\gamma_5 d) + \frac{\alpha_{em}}{2\pi}C^{(3)}N_c F\tilde{F}, \tag{4}$$

$$\partial^\mu J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d - 2m_s\overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi}C^{(8)}N_c F\tilde{F}. \tag{5}$$

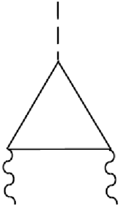
The electromagnetic charge factors $C^{(a)}$ are

$$C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}},$$

$$C^{(8)} = \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}},$$

$$C^{(0)} = \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \tag{6}$$

Anomaly sum rule



The matrix element for the transition of the axial current $J_{\alpha 5}$ with momentum $p = k + q$ into two real or virtual photons with momenta k and q is:

$$T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0| T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\}|0\rangle; \quad (7)$$

Kinematics:

$$k^2 = 0, Q^2 = -q^2$$

The VVA triangle graph amplitude can be presented as a tensor decomposition

$$T_{\alpha\mu\nu}(k,q) = F_1\,\varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\,\varepsilon_{\alpha\mu\nu\rho}q^\rho + F_3\,k_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_5\,k_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\,q_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma, \quad (8)$$

$F_j = F_j(p^2, k^2, q^2; m^2)$, $p = k + q$.
Dispersive approach to axial anomaly leads to [Hořejší, Teryaev'95]:

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2)ds = \frac{1}{2\pi}N_c C^{(a)}\ , \quad (9)$$

$$A_3 \equiv \frac{1}{2}Im(F_3 - F_6),\ N_c = 3;$$

$$C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}},$$
$$C^{(8)} = \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}},$$
$$C^{(0)} = \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \quad (10)$$

ASR and meson contributions

Saturating the l.h.s. of the 3-point correlation function (7) with the resonances and singling out their contributions to ASR (1) we get the (infinite) sum of resonances with appropriate quantum numbers:

$$\pi \sum f_M^a F_{M\gamma} = \int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)}, \tag{11}$$

where the coupling (decay) constants f_M^a:

$$\langle 0 | J_{\alpha 5}^{(a)}(0) | M(p) \rangle = i p_\alpha f_M^a \ , \tag{12}$$

and form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \to M$ are:

$$\int d^4 x e^{ikx} \langle M(p) | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \tag{13}$$

- Sum of finite number of resonances decreasing $F_{M\gamma}^{\mathrm{asymp}}(Q^2) \propto \frac{f_M}{Q^2}$- infinite number of states are needed to saturate ASR (collective effect). [Y.K.,A.Oganesian,O.Teryaev'10]

Isovector channel: π^0

$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$, $C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}$.

- π^0 + higher contributions ("continuum"):

$$\pi f_\pi F_{\pi\gamma}(Q^2) + \int_{s_0}^{\infty} A_3(s, Q^2) = \frac{1}{2\pi} N_c C^{(3)}. \quad (14)$$

The spectral density $A_3(s, Q^2)$ can be calculated from VVA triangle diagram:

$$A_3(s, Q^2) = \frac{1}{2\sqrt{2}\pi}\frac{Q^2}{(Q^2+s)^2}. \quad (15)$$

The pion TFF:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_0}{s_0+Q^2}. \quad (16)$$

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} \qquad (17)$$

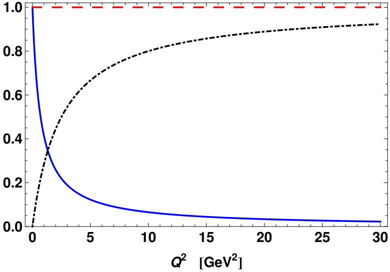
The limit $Q^2 \to \infty$ + pQCD prediction $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi$ gives

$$s_0 = 4\pi^2 f_\pi^2 = 0.67\,GeV^2$$

– fits perfectly the value extracted from SVZ (two-point) QCD sum rules $s_0 = 0.7\,GeV^2$ [Shifman,Vainshtein,Zakharov'79].
– reproduces BL interpolation formula[Brodsky,Lepage'81]:

$$F^{\mathrm{BL}}_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{1}{1 + Q^2/\left(4\pi^2 f_\pi^2\right)}. \qquad (18)$$

Corrections interplay



- The full integral is exact

$$\frac{1}{2\pi} = \int_0^\infty A_3(s, Q^2)ds = I_\pi + I_{cont}$$

- The continuum contribution $I_{cont} = \int_{s_0}^\infty A_3(s, Q^2)ds$ may have perturbative as well as power corrections.
- $\delta I_\pi = -\delta I_{cont}$: small relative correction to continuum – due to exactness of ASR – must be compensated by large relative correction to the pion contribution!

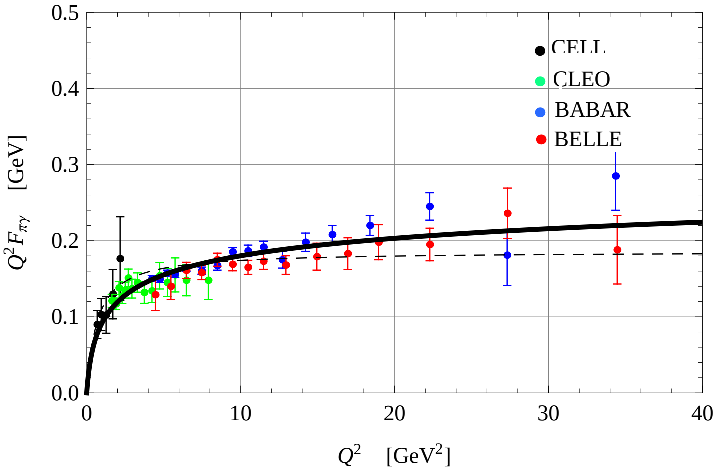
Possible corrections to A_3

- Perturbative two-loop corrections to spectral density A_3 are zero [Jegerlehner&Tarasov'06]
- Nonperturbative corrections to A_3 are possible: vacuum condensates, instantons, short strings.
- General requirements for the correction $\delta I = \int_{s_0}^{\infty} \delta A_3(s, Q^2) ds$: $\delta I = 0$
  - at $s_0 \to \infty$ (the continuum contribution vanishes),
  - at $s_0 \to 0$ (the full integral has no corrections),
  - at $Q^2 \to \infty$ (the perturbative theory works at large Q^2),
  - at $Q^2 \to 0$ (anomaly perfectly describes pion decay width).

$$\delta I = \frac{1}{2\sqrt{2}\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma), \quad (19)$$

$$\delta F_{\pi\gamma} = \frac{1}{\pi f_\pi} \delta I_\pi = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma). \quad (20)$$

Correction vs. experimental data



CELLO+CLEO+BABAR: $\lambda = 0.14$, $\sigma = -2.43$, $\chi^2/n.d.f. = 1.08$

Octet channel (η,η')

$$J^{(8)}_{\alpha 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s),$$

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2)ds = \frac{1}{2\pi} N_c C^{(8)}, \tag{21}$$

$$C^{(8)} \equiv \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}$$

ASR in the octet channel:

$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0 + Q^2}. \tag{22}$$

- Significant mixing.
- η' decays into two real photons, so it should be taken into account explicitly along with η meson.

Large Q^2

$$Q^2 F^{as}_{\eta\gamma} = 2(C^{(8)} f^8_\eta + C^{(0)} f^0_\eta) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \tag{23}$$

$$Q^2 F^{as}_{\eta'\gamma} = 2(C^{(8)} f^8_{\eta'} + C^{(0)} f^0_{\eta'}) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \tag{24}$$

$\phi^{as}(x) = 6x(1-x)$. Then the ASR at $Q^2 \to \infty$:

$$4\pi^2((f^8_\eta)^2 + (f^8_{\eta'})^2 + 2\sqrt{2}[f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'}]) = s_0 \tag{25}$$

$Q^2 = 0$

The ASR takes the form:

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}, \qquad (26)$$

where

$$F_{M\gamma}(0) = \sqrt{\frac{4\Gamma_{M\to\gamma\gamma}}{\pi\alpha^2 m_M^3}}.$$

Additional constraint - $R_{J/\Psi}$.
The radiative decays $J/\Psi \to \eta(\eta')\gamma$ are dominated by non-perturbative gluonic matrix elements, and the ratio of the decay rates $R_{J/\Psi} = (\Gamma(J/\Psi) \to \eta'\gamma)/(\Gamma(J/\Psi) \to \eta\gamma)$ can be expressed as follows [Novikov'79]:

$$R_{J/\Psi} = \left| \frac{\langle 0 \mid G\widetilde{G} \mid \eta' \rangle}{\langle 0 \mid G\widetilde{G} \mid \eta \rangle} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3, \quad (27)$$

where $p_{\eta(\eta')} = M_{J/\Psi}(1 - m^2_{\eta(\eta')}/M^2_{J/\Psi})/2$.

$$\partial_\mu J^8_{\mu 5} = \frac{1}{\sqrt{6}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d - 2m_s \overline{s}\gamma_5 s), \quad (28)$$

$$\partial_\mu J^0_{\mu 5} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{1}{2\sqrt{3}} \frac{3\alpha_s}{4\pi} G\widetilde{G}. \quad (29)$$

$$R_{J/\Psi} = \left(\frac{f^8_{\eta'} + \sqrt{2} f^0_{\eta'}}{f^8_\eta + \sqrt{2} f^0_\eta} \right)^2 \left(\frac{m_{\eta'}}{m_{\eta'}} \right)^4 \left(\frac{p_{\eta'}}{p_\eta} \right)^3. \quad (30)$$

From experiment this ratio is: $R_{J/\Psi} = 4.67 \pm 0.15$ [PDG 2012].

Mixing

Octet-singlet basis (of currents):

$$J^{(8)}_{\mu 5}=\frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s),\ J^{(0)}_{\mu 5}=\frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s). \quad (31)$$

$$\langle 0|J^{(a)}_{\alpha 5}(0)|M(p)\rangle = ip_\alpha f^a_M\ , \quad (32)$$

Matrix of decay constants (32)

$$\mathbf{F}=\begin{pmatrix} f^8_\eta & f^8_{\eta'} \\ f^0_\eta & f^0_{\eta'} \end{pmatrix} \quad (33)$$

- *Octet-singlet (SU(3)) mixing scheme*: $f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'} = 0.$

$$\mathbf{F}=\begin{pmatrix} f_8\cos\theta & f_8\sin\theta \\ -f_0\sin\theta & f_0\cos\theta \end{pmatrix}. \quad (34)$$

For quark-flavour basis one explores the definitions of axial currents with decoupled light and strange quark composition:

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d),\ J^s_{\mu 5} = \bar{s}\gamma_\alpha\gamma_5 s, \tag{35}$$

$$\begin{pmatrix} J^8_{\mu 5} \\ J^0_{\mu 5} \end{pmatrix} = \mathbf{V}(\alpha)\begin{pmatrix} J^q_{\mu 5} \\ J^s_{\mu 5} \end{pmatrix},\ \mathbf{V}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{36}$$

where $\tan\alpha = \sqrt{2}$.

- *Quark-flavour mixing scheme*: [Feldmann,Kroll,Stech'97]

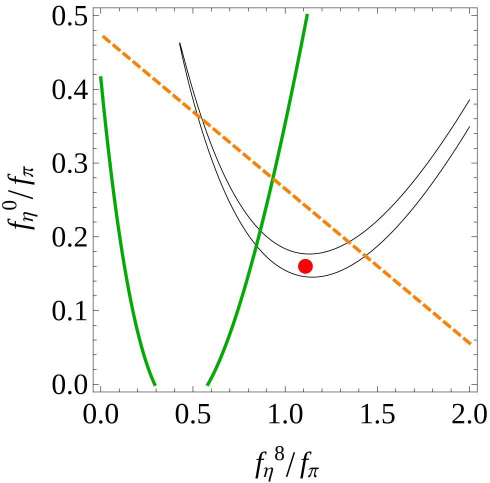
$$f^q_\eta f^s_\eta + f^q_{\eta'} f^s_{\eta'} = 0.$$

$$\mathbf{F_{qs}} = \begin{pmatrix} f_q\cos\phi & f_q\sin\phi \\ -f_s\sin\phi & f_s\cos\phi \end{pmatrix}. \tag{37}$$

$$4\pi^2((f_\eta^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0]) = s_0 \tag{38}$$

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0 + Q^2} \tag{39}$$

$$R_{J/\Psi} = \left(\frac{f_{\eta'}^8 + \sqrt{2} f_{\eta'}^0}{f_\eta^8 + \sqrt{2} f_\eta^0}\right)^2 \left(\frac{m_{\eta'}}{m_{\eta'}}\right)^4 \left(\frac{p_{\eta'}}{p_\eta}\right)^3. \tag{40}$$

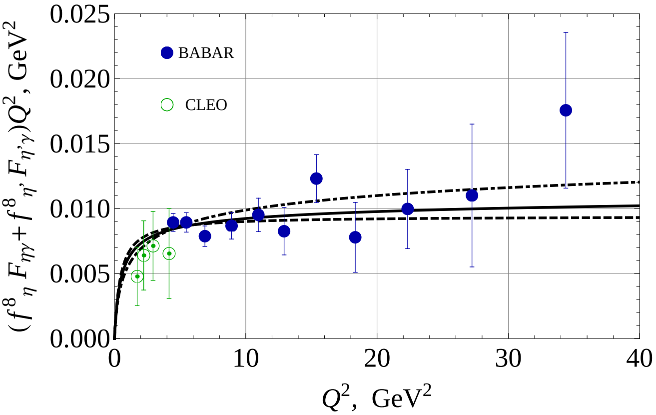


Black curve: $\chi^2/n.d.f. = 1$,
green curve: $f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0 = 0$,
orange curve: $f_\eta^q f_\eta^s + f_{\eta'}^q f_{\eta'}^s = 0$.

Octet-siglet scheme: mixing parameters

$$\begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} 1.11 & -0.42 \\ 0.16 & 1.04 \end{pmatrix} f_\pi. \tag{41}$$

ASR in octet channel (space-like region, $Q^2 > 0$ ($q^2 < 0$))



η, η' TFFs

Add the hypothesis: the TFFs of the state $|q\rangle \equiv \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle)$ is related to the pion form factor as $F_{q\gamma}(Q^2) = (5/3)F_{\pi\gamma}(Q^2)$ (numerical factor comes from the quark charges $(e_u^2 + e_d^2)/(e_u^2 - e_d^2) = 5/3$). QF mixing scheme:

$$|q\rangle = \cos\phi|\eta\rangle + \sin\phi|\eta'\rangle, |s\rangle = -\sin\phi|\eta\rangle + \sin\phi|\eta'\rangle. \tag{42}$$

Then one can relate the form factors:

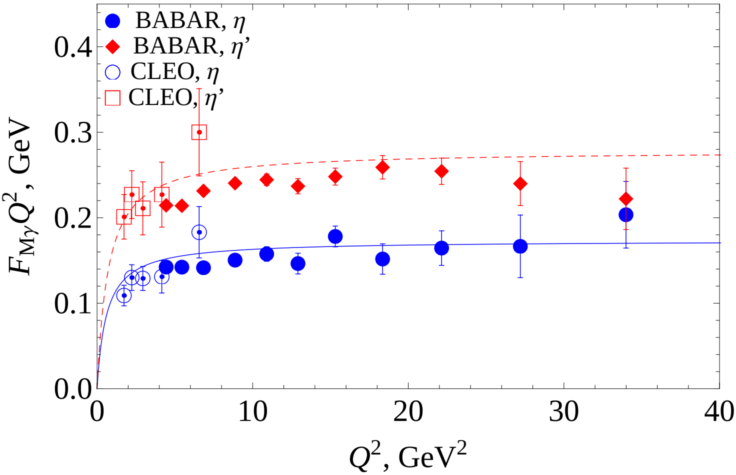
$$\frac{5}{3}F_{\pi\gamma} = F_{\eta\gamma}\cos\phi + F_{\eta'\gamma}\sin\phi. \tag{43}$$

$$F_{\eta\gamma}(Q^2) = \frac{5}{12\pi^2 f_s f_\pi}\frac{s_0^{(3)}(\sqrt{2}f_s\cos\phi - f_q\sin\phi)}{s_0^{(3)} + Q^2} + \frac{1}{4\pi^2 f_s}\frac{s_0^{(8)}\sin\phi}{s_0^{(8)} + Q^2}, \tag{44}$$

$$F_{\eta'\gamma}(Q^2) = \frac{5}{12\pi^2 f_s f_\pi}\frac{s_0^{(3)}(\sqrt{2}f_s\sin\phi + f_q\cos\phi)}{s_0^{(3)} + Q^2} - \frac{1}{4\pi^2 f_s}\frac{s_0^{(8)}\cos\phi}{s_0^{(8)} + Q^2}, \tag{45}$$

where $s_0^{(3)} = 4\pi^2 f_\pi^2$, $s_0^{(8)} = (4/3)\pi^2(5f_q^2 - 2f_s^2)$.

η, η' TFF in the space-like region ($Q^2 > 0$ ($q^2 < 0$))



Time-like region: $q^2>0(Q^2<0)$ and VMD

The ASR for time-like q^2 is given by the double dispersive integral:

$$\int_0^\infty ds \int_0^\infty dy \frac{\rho^{(a)}(s,y)}{y-q^2+i\epsilon} = N_c C^{(a)},\ a=3,8. \qquad (46)$$

The real and imaginary parts of the ASR read:

$$p.v. \int_0^\infty ds \int_0^\infty dy \frac{\rho^{(a)}(s,y)}{y-q^2} = N_c C^{(a)}, \qquad (47)$$

$$\int_0^\infty ds \rho^{(a)}(s,q^2) = 0,\ a=3,8. \qquad (48)$$

$$ReF_{\pi\gamma}(q^2) = \frac{N_c C^{(3)}}{2\pi^2 f_\pi}\left[p.v. \int_0^{s_3} ds \int_0^\infty dy \frac{\rho^{(a)}(s,y)}{y-q^2}\right] = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_0}{s_0-q^2}.$$

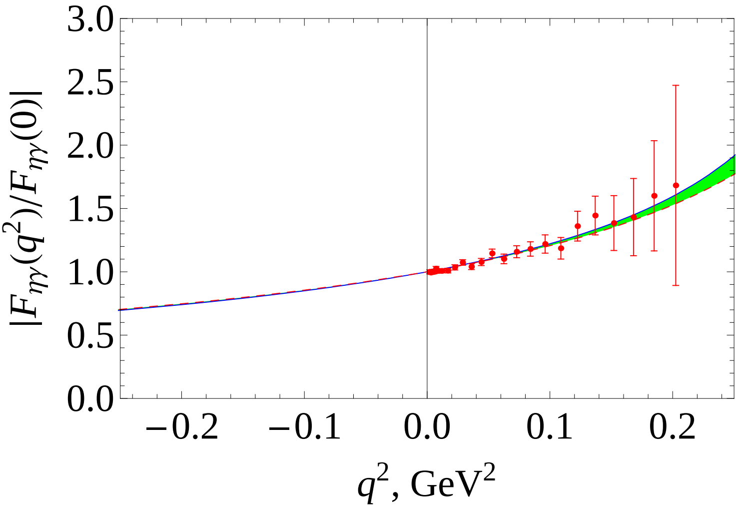
The TFF in the time-like region at $q^2=s_3$ has a pole, which is numerically close to the ρ meson mass squared, $m_\rho^2 \simeq 0.59$ GeV^2 — **VMD model**.

$$F_{\eta\gamma}(q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_3(\sqrt{2} f_s \cos\phi - f_q \sin\phi)}{s_3 - q^2} + \frac{1}{4\pi^2 f_s} \frac{s_8 \sin\phi}{s_8 - q^2}, \quad (49)$$

$$F_{\eta'\gamma}(q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_3(\sqrt{2} f_s \sin\phi + f_q \cos\phi)}{s_3 - q^2} - \frac{1}{4\pi^2 f_s} \frac{s_8 \cos\phi}{s_8 - q^2}, \quad (50)$$

$s_3 = 4\pi^2 f_\pi^2$, $s_8 = (4/3)\pi^2(5f_q^2 - 2f_s^2)$.

η TFF in the time-like region vs. data



Summary

- Meson TFFs are unique quantities which link (seemingly different) important ideas: axial anomaly, mixing and VMD model.
- The ASR in the isovector channel gives the anomaly-based ground for the BL interpolation formula for the pion TFF in the space-like region, and for the VMD model in the time-like region.
- In order to describe the BABAR data on pion TFF, ASR requires a new nonperturbative correction to the spectral density. At the same time, the BELLE data can be described well without such a correction. This correction is absent in the local OPE and possibly originates from instantons or short strings.
- Mixing parameters of η and η' meson can be extracted from TFFs using the ASR.
- ASR in the time-like region of TFFs substantiate the VMD model.

Thank you for your attention!