Lattice studies on proton spin structure

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Introduction

- The polarized DIS revealed that quarks (both valence and sea) carry only a small fraction, $\sim 20\%$. [EMC, J. Ashman *et al.*,1988].
- Subsequent experiments confirmed the EMC results, $20-30\%$.
- What are the other candidates for the missing proton spin? "Proton Spin Crisis"!!
- Spin sum rule: $$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma_q + L_q + J_g$$ [X. Ji, 1997]
- Controversy remains : $J_g \stackrel{?}{=} \Sigma_g + L_g$

  [Chen *et al.*, 2008; X. Ji, 2008; Wakamatsu, 2010; E. Leader, 2011; Y. Hatta, 2011; $\cdots$]

- Starts with QCD action in continuum

$$S = \int d^4x \overline{\psi}(\gamma^\mu D^\mu + m)\psi + \frac{1}{2}\mathrm{Tr}\int d^4x F^{\mu\nu}F_{\mu\nu}$$

- Euclidean rotation: $t \to -it$.
- $A_\mu \Rightarrow U_\mu(x) \equiv e^{igA_\mu(x+a\hat{\mu}/2)}$
- Fermionic fields: $\psi(x) \to \psi(na)$.
- Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ with the weight $[\mathrm{Det}\,M]\,e^{-s_g}$.
- Quenched Approximation: $\mathrm{Det}\,M = 1$.

$O(a)$-improved fermion actions [C. Alexandrou, 2014; K. Jansen, Lattice 2008]

Action	**Advantages**	**Disadvantages**
Clover improved Wilson	computationally fast	breaks chiral symmetry
Twisted Mass (TM)	computationally fast	breaks chiral symmetry
Staggered	computationally fast	four doublers (fourth root issue)
Domain wall (DW)	improved chiral symmetry	computationally expensive
Overlap	exact chiral symmetry	computationally expensive

Collaborations:

Clover	QCDSF, BMW, ALPHA, CLS, PACS-CS, NPQCD
Twisted Mass	ETMC
Staggered	MILC
Domain wall	RBC + UKQCD
Overlap	JLQCD

Primary Quantities:

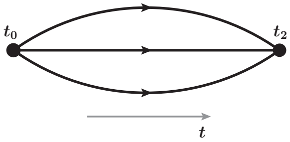
- Gauge configurations, U.
- Quark propagator, M^{-1}, on every configuration.

Construct:

- Two-point correlation functions.
- Three-point correlation functions.

Two-point Correlation Functions

$$G_{NN}(t_2,\vec{p}) = \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} \langle 0|T\left[\chi(x_2)\,\bar{\chi}(x_0)\right]|0\rangle$$



Inserting Energy eigenstates:

$$\mathrm{Tr}[\Gamma\, G_{NN}(t_2,\vec{p})] = A\, e^{-E_p(t_2-t_0)} + \text{Higher states}$$

Path Integral formalism and Wick Contraction:

$$\mathrm{Tr}[\Gamma\, G_{NN}(t_2,\vec{p})] = \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} \sum_{\{U\}} \mathrm{Tr}\{f\left[M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0);U\right]\}$$

Three-point Correlation Functions

The three point-function for an operator, $\mathcal{T}$,

$$G_{N\mathcal{T}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}',t_0) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}.\vec{x}_2} e^{i\vec{q}.\vec{x}_1} e^{i\vec{p}'.\vec{x}_0} \langle 0|\mathrm{T}(\chi(x_2)\mathcal{T}(x_1)\bar{\chi}(x_0))|0\rangle$$

$$= \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} e^{+i\vec{q}.(\vec{x}_1-\vec{x}_0)}$$

$$\sum_{\{U\}} \mathrm{Tr}\{f\ [M^{-1}\,M^{-1}\,M^{-1}\,M^{-1};U]\}$$

$$\mathrm{Tr}\left[\Gamma G_{N\mathcal{T}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}',t_0)\right] \xrightarrow{t_1 \gg t_0,\ t_2 \gg t_1} B\, e^{-E_p(t_2-t_1)} e^{-E_{p'}(t_1-t_0)} \mathcal{M}(\Delta^2)$$

$\mathcal{M}(\Delta^2) = \langle p,s|\mathcal{T}|p',s'\rangle$. Δ^2 is the momentum transfer squared.

Ratios

$$\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{pol,unpol}} G_{N\mathcal{T}N}(\vec{p}, t_2; \vec{q}, t_1; \vec{p}\,', t_0)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}, t_2)\right]}$$

$$\times \sqrt{\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}\,', t_2 - t_1 + t_0)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}, t_2 - t_1 + t_0)\right]}}$$

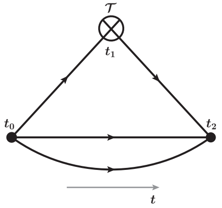
$$\times \sqrt{\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}, t_1)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}\,', t_1)\right]} \cdot \frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}, t_2)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}\,', t_2)\right]}}$$

$$\xrightarrow{t_1 \gg t_0,\; t_2 \gg t_1} \mathcal{M}(\Delta^2) f(E_p, E_{p'}, m)$$

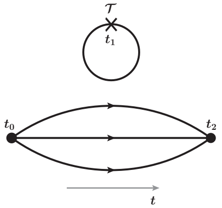
Connected and Disconnected Insertions

$$G_{N\mathcal{T}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}',t_0) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}.\vec{x}_2} e^{i\vec{q}.\vec{x}_1} e^{i\vec{p}'.\vec{x}_0} \langle 0|\mathrm{T}(\chi(x_2)\mathcal{T}(x_1)\bar{\chi}(x_0))|0\rangle$$

The three-point functions for quarks have two different contributions: connected (CI) and disconnected insertions (DI).



- CI three-point functions are relatively easier to compute and well studied.



- DI three-point functions are computationally challenging: Involves propagators from *all-to-all* points.
- *strange* quark contributions come only from DI. *up* and *down* quarks have contributions both from CI and DI.
- DI and disconnected diagrams are not the same.

$$G_{N\mathcal{T}N}|_{\mathrm{DI}} \equiv \langle \chi \mathcal{T} \bar{\chi} \rangle - \langle \mathcal{T} \rangle \langle \chi \bar{\chi} \rangle$$

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma_q + L_q + J_g$$

- Quark Spin: $\langle p, s|\overline{\psi}_q \gamma_\mu \gamma_5 \psi_q|p, s\rangle \sim \Sigma_q$.
- Angular momenta

$$J_q : \mathcal{T}^q_{\{4i\}} = \overline{\psi}_q \gamma_{\{4}(-i \overleftrightarrow{D})_{i\}} \psi_q \quad , \quad J_g : \mathcal{T}^g_{\{4i\}} = -i F_{4\alpha} F_{i\alpha}$$

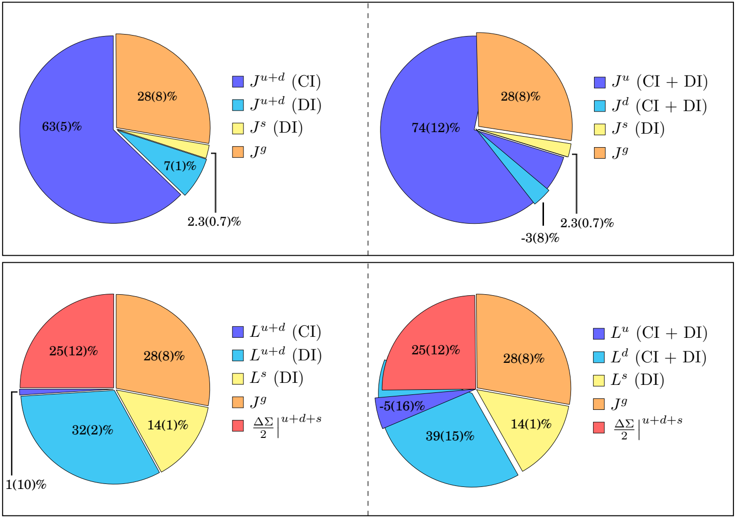
ME: $\langle p, s|\mathcal{T}^{q,g}_{\{4i\}}|p', s'\rangle \sim [a_1 T_1(\Delta^2) + a_2 T_2(\Delta^2) + a_3 T_3(\Delta^2)]$

$J_{q,g} = \frac{1}{2}[T_1(0) + T_2(0)]$

- Quark orbital angular momenta: $L_q = J_q - \frac{1}{2}\Sigma_q$

Decomposition of Total and Orbital Angular Momenta

χQCD Collaboration [M. Deka, *et al.*, 2015]

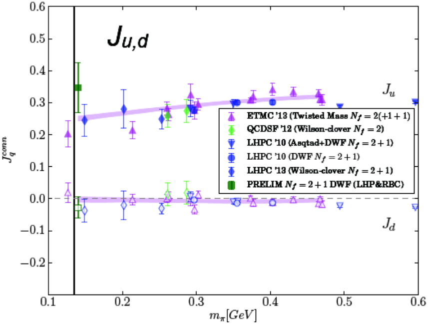


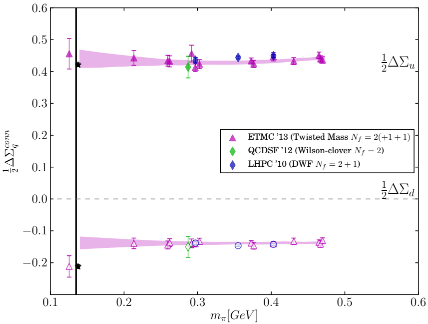
	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
$T_1(0)$	0.413(38)	0.150(19)	0.565(43)	0.038(7)	0.024(6)	0.334(55)
$T_2(0)$	0.286(108)	-0.220(77)	0.062(21)	-0.002(2)	-0.001(3)	-0.056(51)
$2J$	0.700(123)	-0.069(79)	0.628(49)	0.036(7)	0.023(7)	0.278(75)
$\Delta\Sigma$ (Dong et al., 1995)	0.91(11)	-0.30(12)	0.62(9)	-0.12(1)	-0.12(1)	–
$2L$	-0.21(16)	0.23(15)	0.01(10)	0.16(1)	0.14(1)	–

Few drawbacks:

- Quenched approximation ⇒ Gluon contributions likely being supressed
- Higher pion masses, lowest being 478 (4) MeV
- Smaller lattice, $16^3 \times 24$ ⇒ Finite volume effect

Quark Angular Momenta and Spin: Connected Insertions

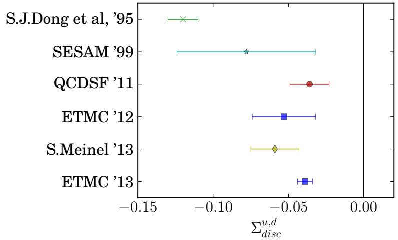


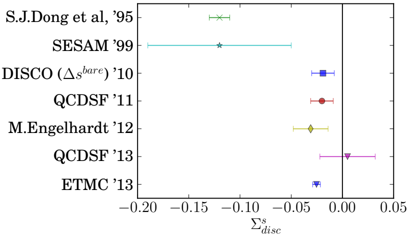


[Syritsyn, USQCD All-Hands Meeting, 2015]

Quark Spin: Disconnected Insertions

[Syritsyn, Lattice 2013]





[D. de Florian, *et al.*, 2009]

Spin	Global Analysis
Δu	0.814(28)(34)
Δd	−0.456(35)(25)
Δs	−0.112(28)(31)
$\Delta\Sigma$	0.245(42)(62)

$$g_A^{(3)} = \Delta u - \Delta d = 1.270(3)$$

[Beringer et al., 2012]

$$g_A^{(8)} = \Delta u + \Delta d - 2\Delta s = 0.58(3)$$

[Close and Roberts, 1993]

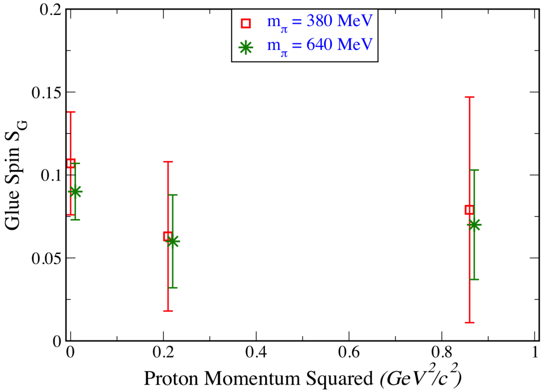
Quark Spin: Preliminary

χQCD Collaboration [Y. B. Yang, *et al.*, Lattice 2014]

- $\Delta u(\mathrm{CI}) = 0.86(3), \Delta d(\mathrm{CI}) = -0.278(5)$.
- $\Delta u, \Delta d(\mathrm{DI}) = -0.12(3), \Delta s = -0.05(1)$.
- $\Delta u + \Delta d = 0.35(6)$ (CI + DI).
- $\Delta\Sigma = 0.30(6)$.
- $24^3 \times 64$ lattice. $m_\pi = 330$ MeV.
- $m_\pi \longrightarrow 140$ MeV.

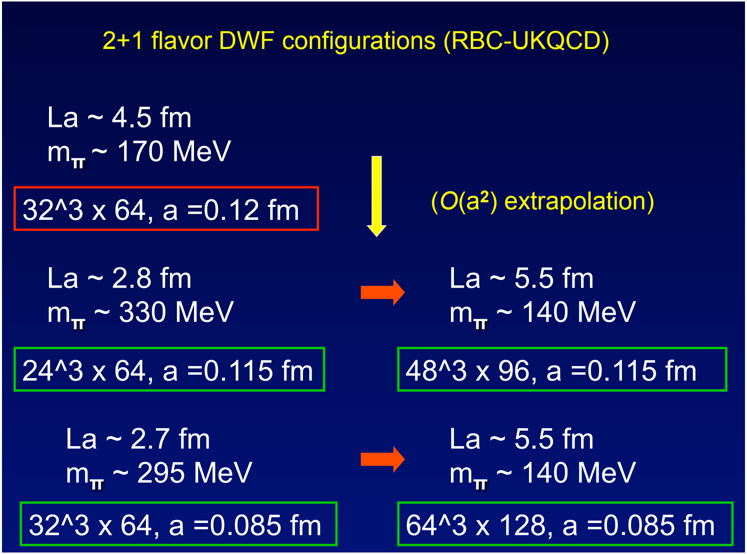
Gluon Spin: Preliminary

χQCD Collaboration [R. S. Sufian, *et al.*, Lattice 2014]



$S_G = 18(3)\%$ at $p = 0, m_\pi = 640$ MeV.

χQCD


2+1 flavor DWF configurations (RBC-UKQCD)
La ~ 4.5 fm
m_π ~ 170 MeV
32^3 x 64, a =0.12 fm
(O(a²) extrapolation)
La ~ 2.8 fm
m_π ~ 330 MeV
24^3 x 64, a =0.115 fm
La ~ 5.5 fm
m_π ~ 140 MeV
48^3 x 96, a =0.115 fm
La ~ 2.7 fm
m_π ~ 295 MeV
32^3 x 64, a =0.085 fm
La ~ 5.5 fm
m_π ~ 140 MeV
64^3 x 128, a =0.085 fm

LHP & NME

Scientific Objectives

Nucleon Structure with Isotropic Wilson Lattices

Goal : Compute Nucleon Structure and Quark Matrix Elements with high statistical precision and robust control of systematic errors

Wilson fermions are economical and permit

- higher statistics for better precision and noisy observables (TMDs, GPDs)
- experiments with newer techniques
  - controlling excited states
  - computing disconnected diagrams
  - exploring hadron states with high momentum

JLab Isotropic clover-improved Wilson lattices:

ID	a[fm]	Volume	m_π	$m_\pi L$	Traj. available	Conn.cost per conf.[NMEp]	%%	
D4	0.085	32^3x64	400	5.5	5100	500	~20%	Systematics study [NMEp]
D5	0.081	32^3x64	300	4.0	2600	825		
D6	0.080	48^3x96	190	3.7	700	7,125		
D7	0.080	64^3x128	190	4.9	900 (++ by 07/01)	32,055	~80%	proposed in [LHPp]
D8	0.080	64^3x128	140	4.1	Started	Next Year (hopefully)		

Thank you !!