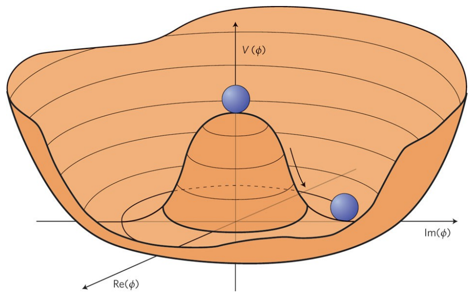
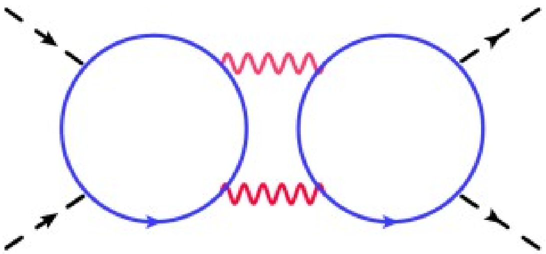
High-order corrections for the Vacuum Stability analysis of the SM



A.V. Bednyakov

BLTP JINR



(in collaboration with A.F. Pikelner, V.N. Velizhanin, B.A. Kniehl, and O.L. Veretin)

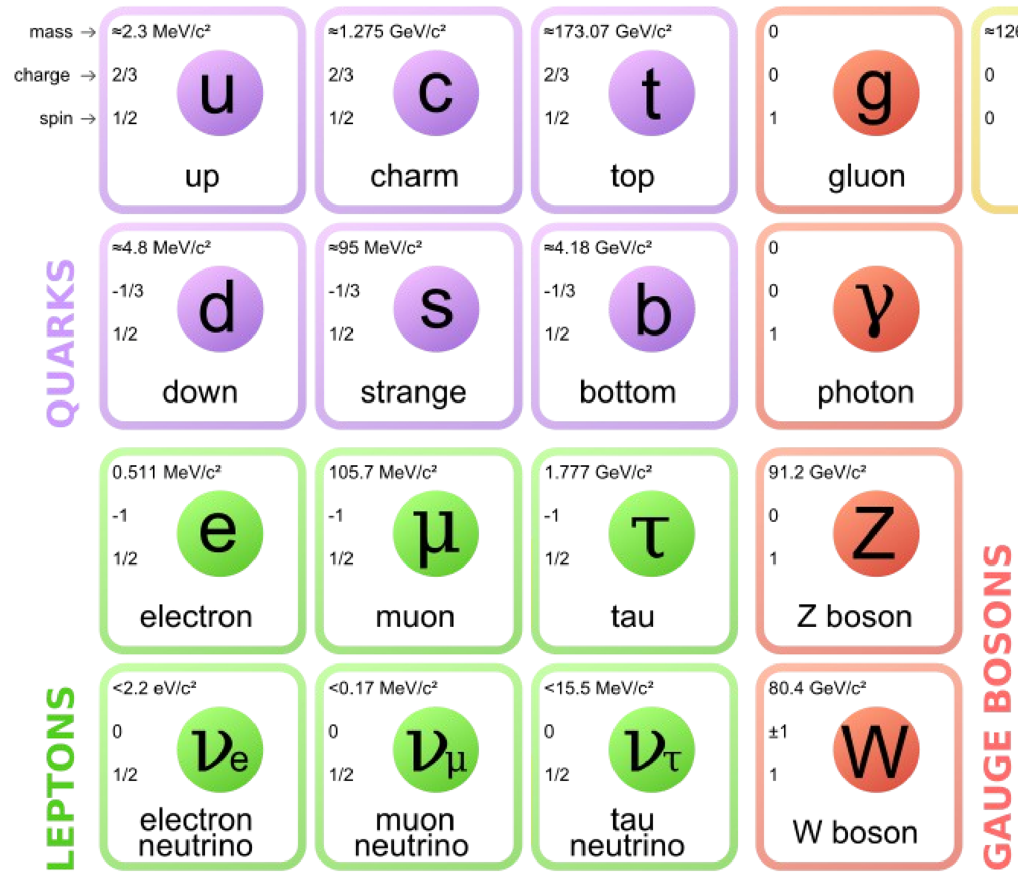
Brazil-JINR Forum, Dubna, 18 June 2015

Outline

- The Standard Model and the Higgs boson
- Effective potential and Vacuum Stability
- Renormalization Group Analysis of the SM
- Our contribution
- Conclusions

The Standard Model

$$\mathcal{L}_{\rm SM} =$$



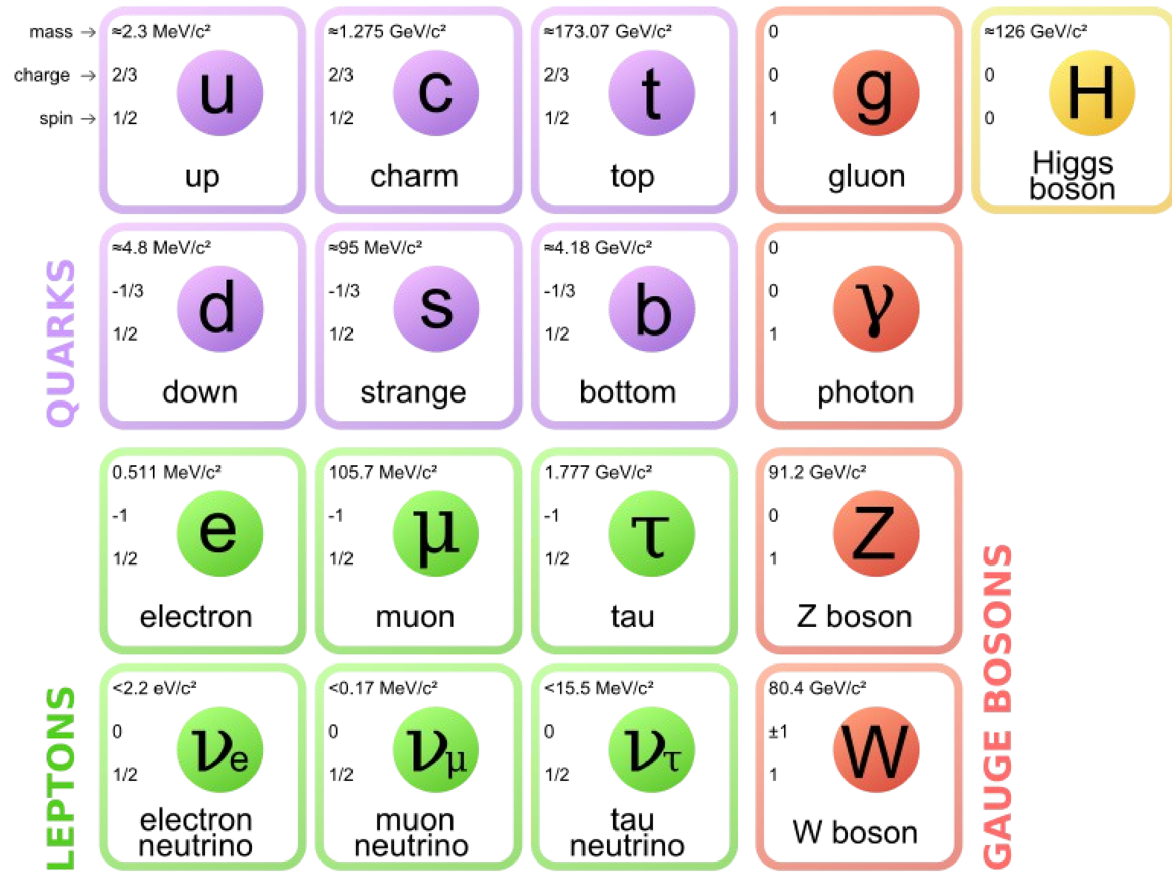
The SM was established as a QFT model in mid 70s of last century...

Turns out to be a perfect description of Physics at the electroweak scale...

The Standard Model

$$\mathcal{L}_{\rm SM} =$$

$$\mathcal{L}_{\rm Gauge}(g_1, g_2, g_S)$$



The SM was established as a QFT model in mid 70s of last century...

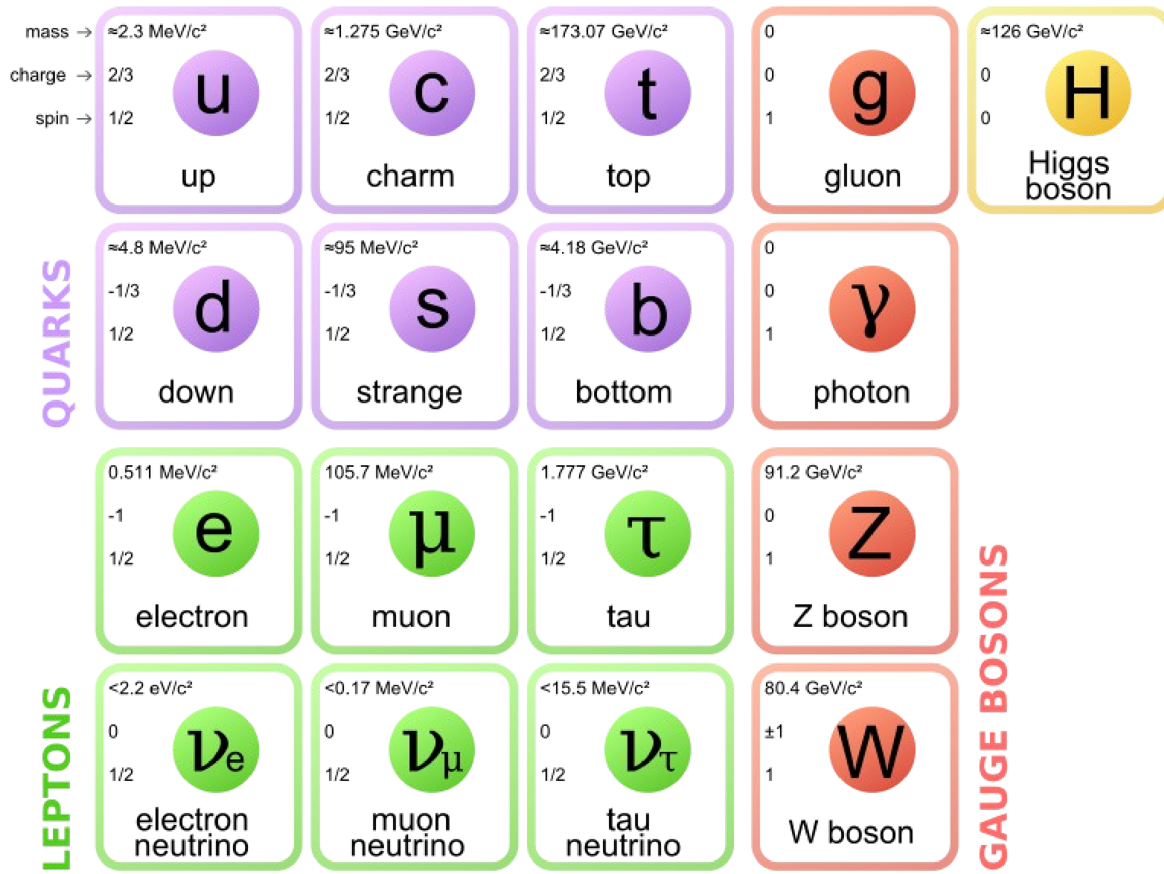
Turns out to be a perfect description of Physics at the electroweak scale...

The Standard Model

$$\mathcal{L}_{\rm SM} =$$

$$\mathcal{L}_{\rm Gauge}(g_1, g_2, g_S)$$

$$+\mathcal{L}_{\rm Yukawa}(Y_u, Y_d, Y_l)$$



The SM was established as a QFT model in mid 70s of last century...

Turns out to be a perfect description of Physics at the electroweak scale...

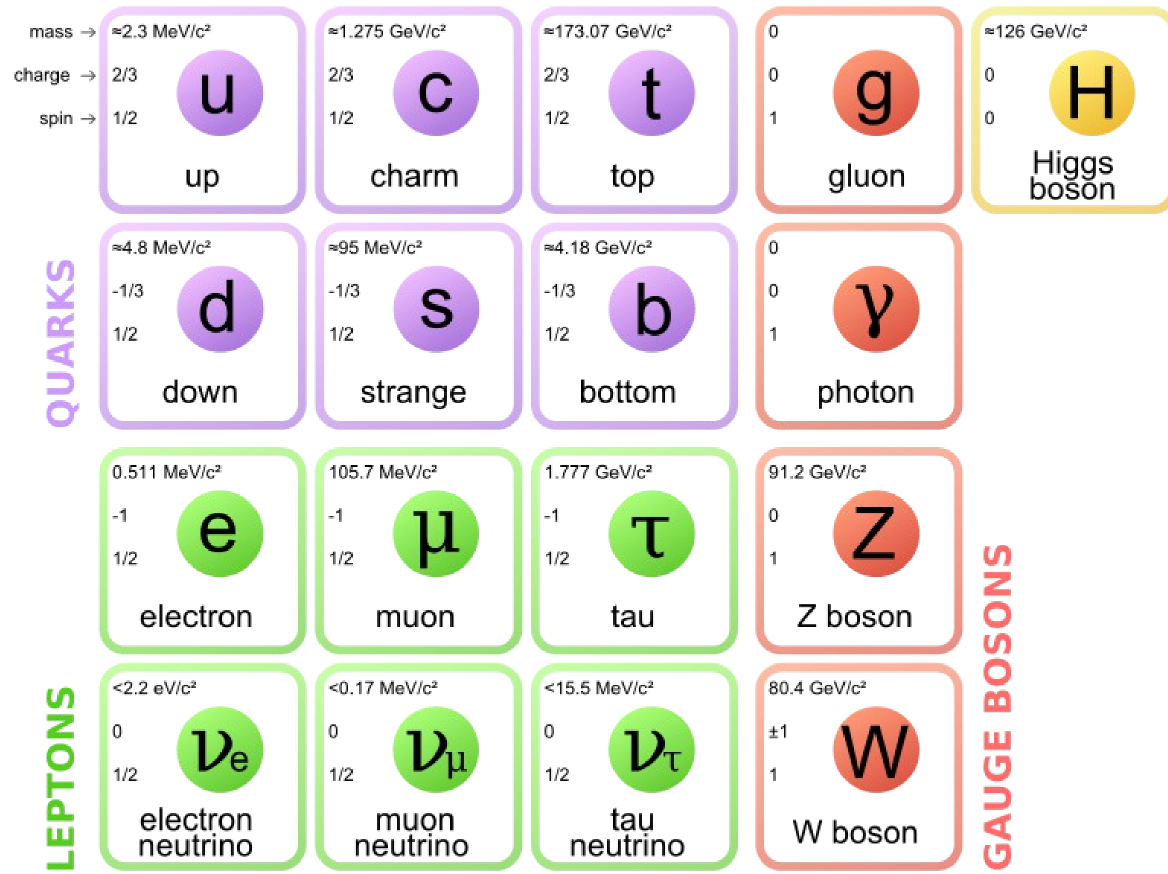
The Standard Model

$$\mathcal{L}_{\rm SM} =$$

$$\mathcal{L}_{\rm Gauge}(g_1, g_2, g_S)$$

$$+\mathcal{L}_{\rm Yukawa}(Y_u, Y_d, Y_l)$$

$$+\mathcal{L}_{\rm Higgs}(\lambda, m^2)$$

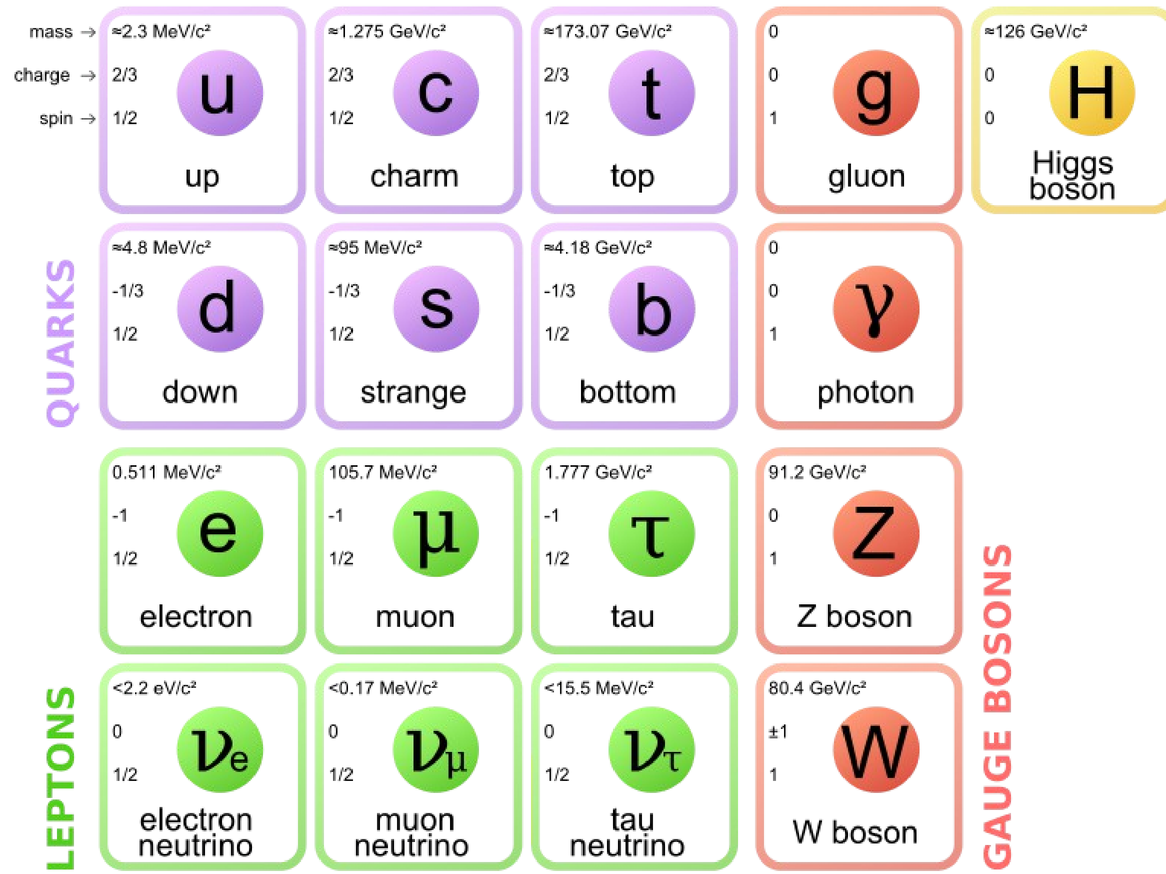


The SM was established as a QFT model in mid 70s of last century...

Turns out to be a perfect description of Physics at the electroweak scale...

The Standard Model

$$\mathcal{L}_{\rm SM} =$$
$$\mathcal{L}_{\rm Gauge}(g_1, g_2, g_S)$$
$$+\mathcal{L}_{\rm Yukawa}(Y_u, Y_d, Y_l)$$
$$+\mathcal{L}_{\rm Higgs}(\lambda, m^2)$$
$$+\mathcal{L}_{\rm Gauge-fixing}$$
$$+\mathcal{L}_{\rm Ghost}$$

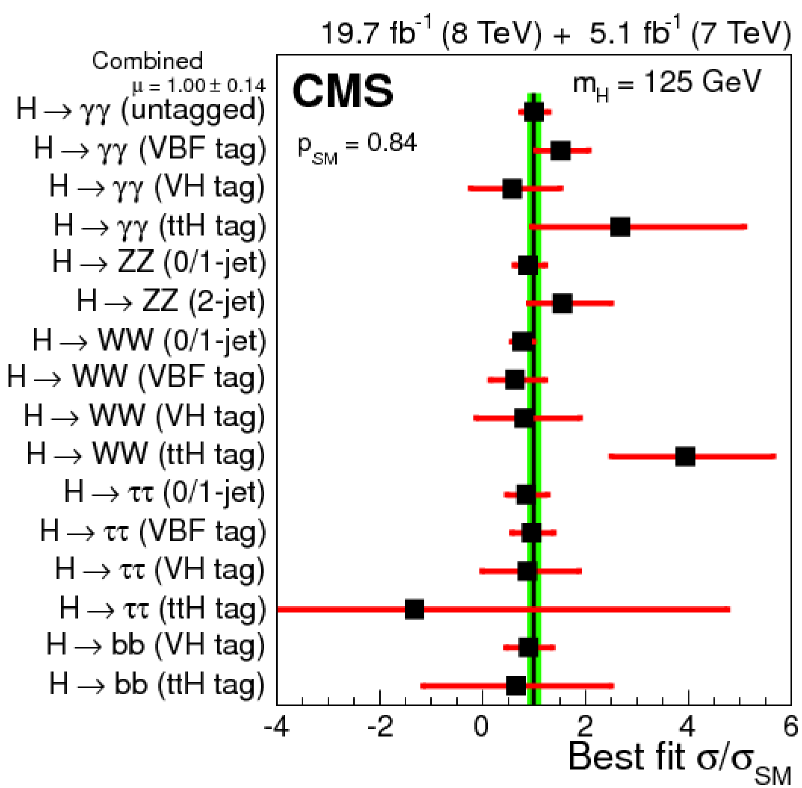


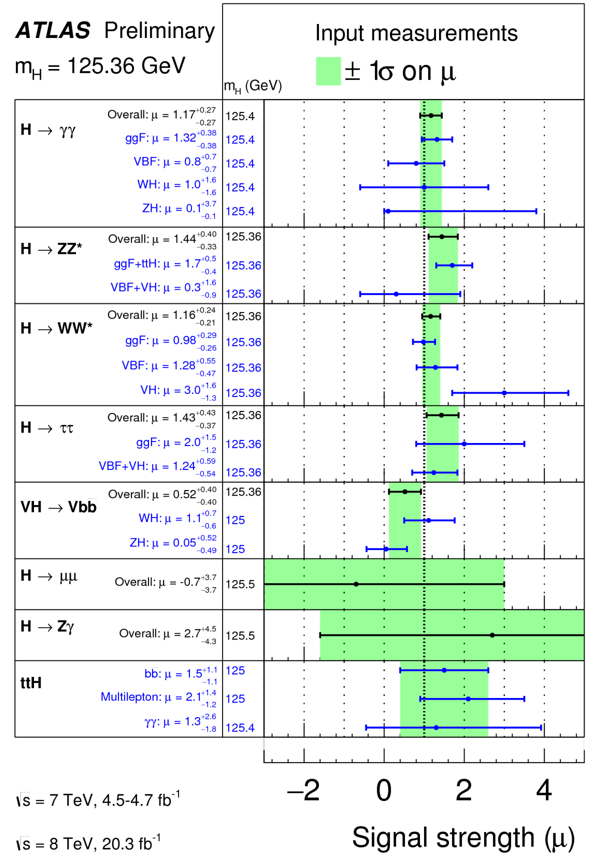
The SM was established as a QFT model in mid 70s of last century...

Turns out to be a perfect description of Physics at the electroweak scale...

...and the Higgs boson

On 4 July 2012 it was announced that a new Higgs-like particle was discovered experimentally at the LHC by ATLAS and CMS collaborations.





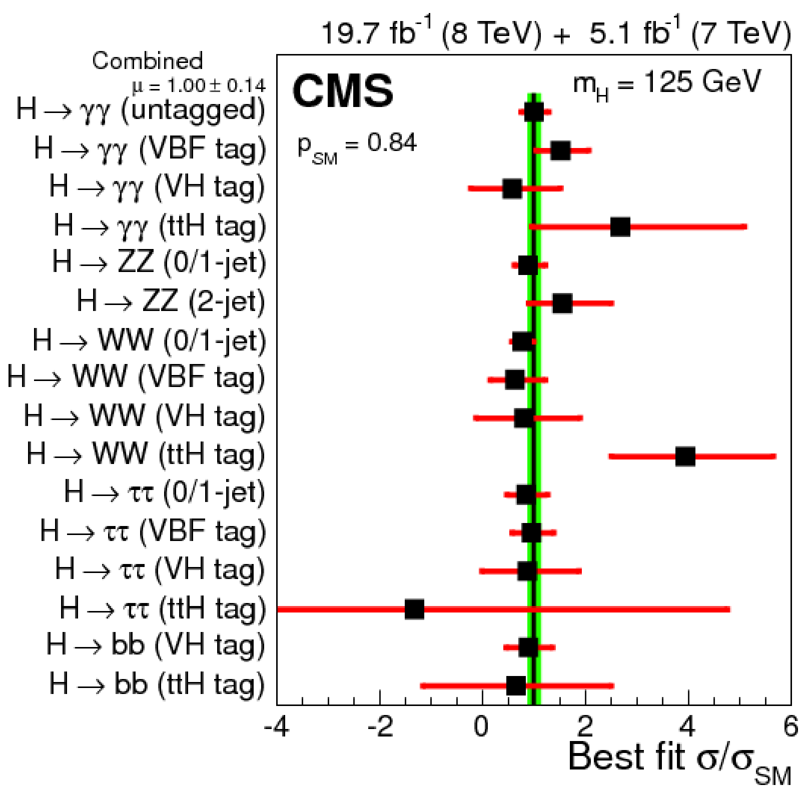
CMS-HIG-14-009

ATLAS-CONF-2015-007

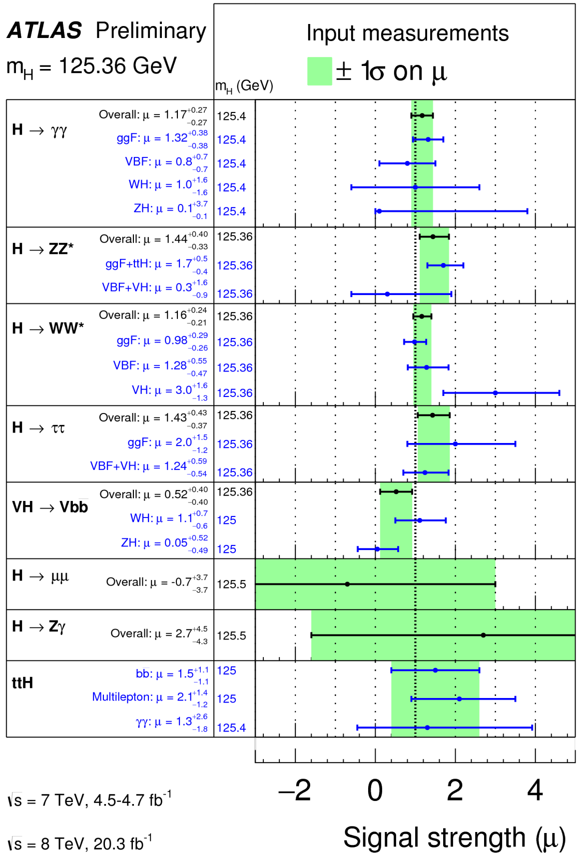
...and the Higgs boson

On 4 July 2012 it was announced that a new Higgs-like particle was discovered experimentally at the LHC by ATLAS and CMS collaborations.

During 2013-2014 it was confirmed that the particle looks very much like the Higgs boson, the last missing ingredient of the SM.



CMS-HIG-14-009

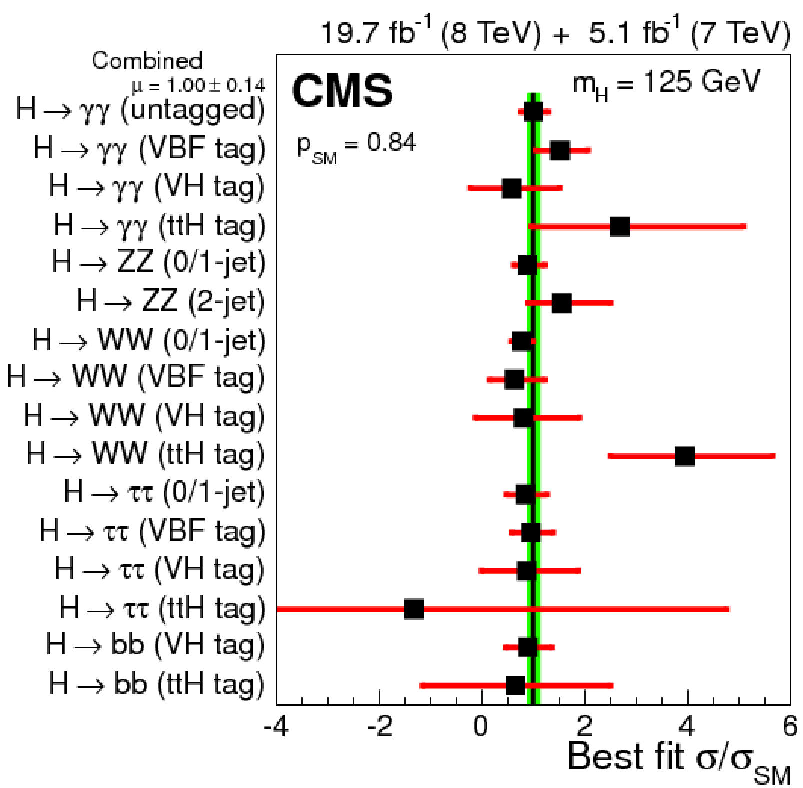


ATLAS-CONF-2015-007

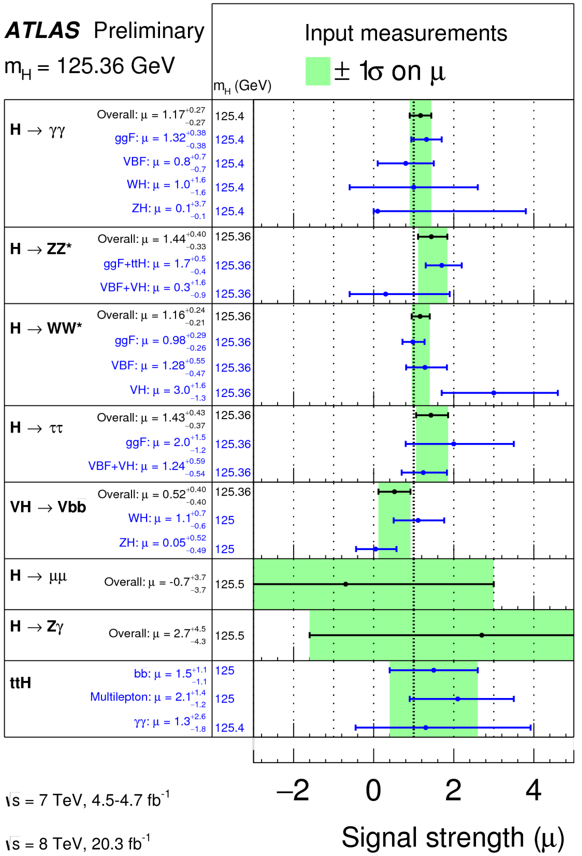
...and the Higgs boson

On 4 July 2012 it was announced that a new Higgs-like particle was discovered experimentally at the LHC by ATLAS and CMS collaborations.

During 2013-2014 it was confirmed that the particle looks very much like the Higgs boson, the last missing ingredient of the SM.



CMS-HIG-14-009



ATLAS-CONF-2015-007

...and the Higgs boson

P. Higgs

F. Englert and R. Brout

Broken Symmetry and the Mass of Gauge Vector Mesons
F. Englert and R. Brout
Phys. Rev. Lett. 13, 321 (1964)

Broken Symmetries and the Masses of Gauge Bosons
Peter W. Higgs
Phys. Rev. Lett. 13, 508 (1964)

NB: G. S. Guralnik, C. R. Hagen, and T. W. Kibble, Phys. Rev. Lett. 13, 585 (1964)

...and the Higgs boson

P. Higgs

F. Englert and R. Brout

2013

Broken Symmetry and the Mass of Gauge Vector Mesons
F. Englert and R. Brout
Phys. Rev. Lett. 13, 321 (1964)

Broken Symmetries and the Masses of Gauge Bosons
Peter W. Higgs
Phys. Rev. Lett. 13, 508 (1964)

NB: G. S. Guralnik, C. R. Hagen, and T. W. Kibble, Phys. Rev. Lett. 13, 585 (1964)

...and the Higgs boson

P. Higgs

F. Englert and R. Brout

Broken Symmetry and the Mass of Gauge Vector Mesons
F. Englert and R. Brout
Phys. Rev. Lett. 13, 321 (1964)

2013

Broken Symmetries and the Masses of Gauge Bosons
Peter W. Higgs
Phys. Rev. Lett. 13, 508 (1964)

50th anniversary
In 2014

NB: G. S. Guralnik, C. R. Hagen, and T. W. Kibble, Phys. Rev. Lett. 13, 585 (1964)

...and the Higgs boson

P. Higgs

F. Englert and R. Brout

Broken Symmetry and the Mass of Gauge Vector Mesons
F. Englert and R. Brout
Phys. Rev. Lett. 13, 321 (1964)

2013

Broken Symmetries and the Masses of Gauge Bosons
Peter W. Higgs
Phys. Rev. Lett. 13, 508 (1964)

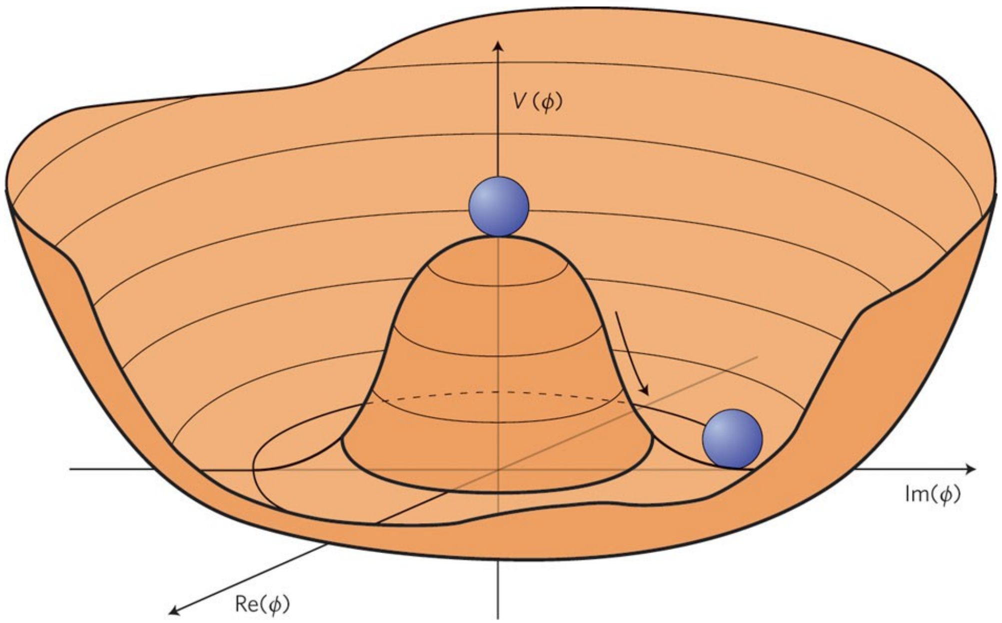
50th anniversary
In 2014

NB: G. S. Guralnik, C. R. Hagen, and T. W. Kibble, Phys. Rev. Lett. 13, 585 (1964)

Spontaneous symmetry breaking in the SM

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

$$V(\Phi) = m^2|\Phi|^2 + \lambda|\Phi|^4$$



Would-be goldstone

"eaten" by W-bosons

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(\phi + i\chi) \end{pmatrix}$$

Neutral higgs field

"eaten" by Z-boson

Would-be goldstone

Spontaneous symmetry breaking in the SM

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

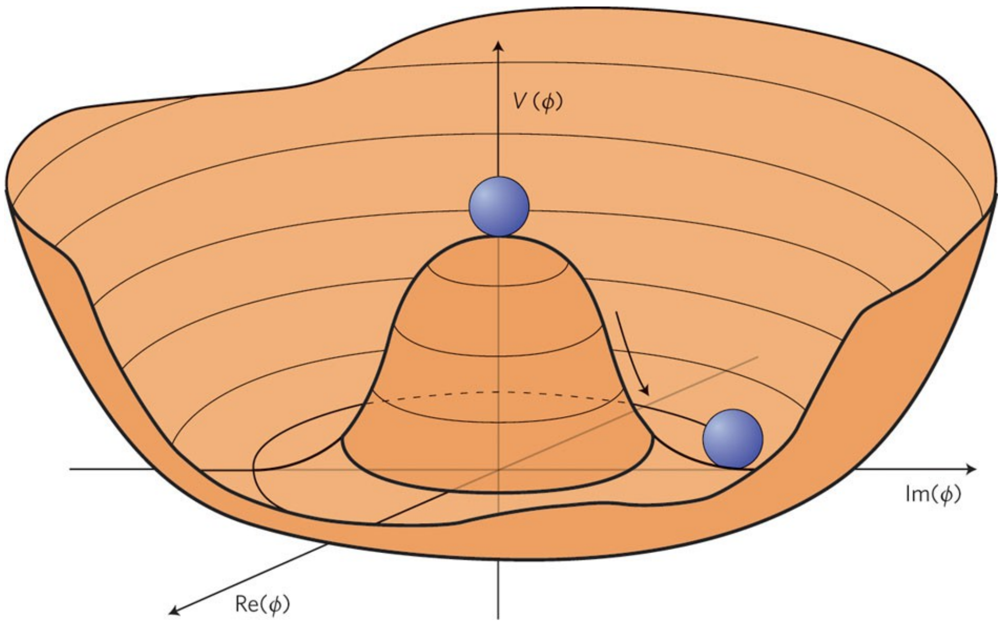
The electroweak vacuum state is characterized by vacuum expectation value

$$\langle\phi\rangle = v \neq 0$$

with $v \simeq 246\ \mathrm{GeV}$

at tree level $$v = \sqrt{\frac{-m^2}{\lambda}}$$

$$M_h^2 = 2\lambda v^2$$



$$\left.\frac{\partial V(\phi)}{\partial \phi}\right|_{\phi=v} = 0$$

The SM parameters in the broken phase

Gauge and Yukawa couplings are connected to (observed) particle masses:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

$$M_f = \frac{Y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, coupling can be extracted from this relations

"Muon-Electron-Decay". Licensed under CC BY-SA 3.0 via Wikipedia

The SM parameters in the broken phase

Gauge and Yukawa couplings are connected to (observed) particle masses:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

$$M_f = \frac{Y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, coupling can be extracted from this relations

And v.e.v. can be related to the Fermi constant by considering Fermi Theory as a low-energy effective approximation of the SM valid for energies $E \ll M_W$

"Muon-Electron-Decay". Licensed under CC BY-SA 3.0 via Wikipedia

The SM parameters in the broken phase

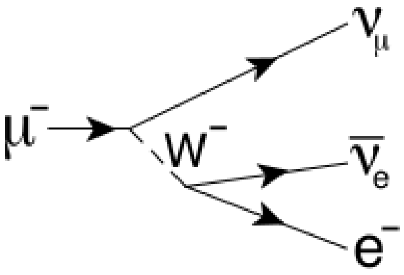
Gauge and Yukawa couplings are connected to (observed) particle masses:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

$$M_f = \frac{Y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, coupling can be extracted from this relations

And v.e.v. can be related to the Fermi constant by considering Fermi Theory as a low-energy effective approximation of the SM valid for energies $E \ll M_W$



"Muon-Electron-Decay". Licensed under CC BY-SA 3.0 via Wikipedia

The SM parameters in the broken phase

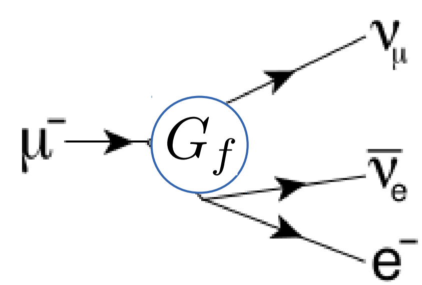
Gauge and Yukawa couplings are connected to (observed) particle masses:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

$$M_f = \frac{Y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, coupling can be extracted from this relations

And v.e.v. can be related to the Fermi constant by considering Fermi Theory as a low-energy effective approximation of the SM valid for energies $E \ll M_W$



$$\frac{-g_2^2}{q^2 - M_W^2} \xrightarrow{q \to 0} \frac{g_2^2}{M_W^2} \propto \frac{1}{v^2}$$

"Muon-Electron-Decay". Licensed under CC BY-SA 3.0 via Wikipedia

The SM parameters in the broken phase

Gauge and Yukawa couplings are connected to (observed) particle masses:

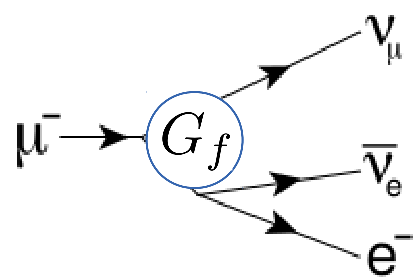
$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

$$M_f = \frac{Y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, coupling can be extracted from this relations

And v.e.v. can be related to the Fermi constant by considering Fermi Theory as a low-energy effective approximation of the SM valid for energies $E \ll M_W$

$$G_f = \frac{1}{\sqrt{2} v^2}$$



$$\frac{-g_2^2}{q^2 - M_W^2} \xrightarrow{q \to 0} \frac{g_2^2}{M_W^2} \propto \frac{1}{v^2}$$

"Muon-Electron-Decay". Licensed under CC BY-SA 3.0 via Wikipedia

The SM parameters in the broken phase

Gauge and Yukawa couplings are connected to **(observed)** particle masses:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

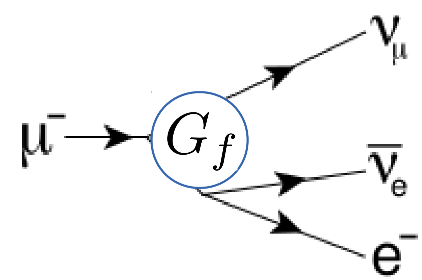
$$M_f = \frac{Y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, coupling can be extracted from this relations

And v.e.v. can be related to the Fermi constant by considering Fermi Theory as a low-energy effective approximation of the SM valid for energies $E \ll M_W$

$$G_f = \frac{1}{\sqrt{2} v^2}$$

NB: VALID AT THE LEADING ORDER!



$$\frac{-g_2^2}{q^2 - M_W^2} \xrightarrow{q \to 0} \frac{g_2^2}{M_W^2} \propto \frac{1}{v^2}$$

"Muon-Electron-Decay". Licensed under CC BY-SA 3.0 via Wikipedia

Spontaneous symmetry breaking and the Higgs effective potential

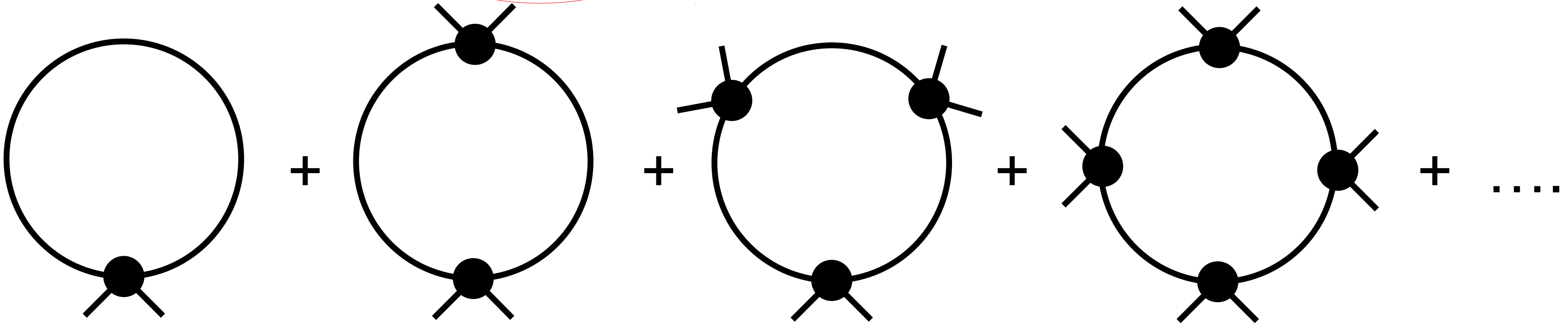
A proper way to study the symmetry breaking in the SM is to consider the effective potential for the background Higgs field which takes into account vacuum fluctuations

$$V(\phi) \to V_{\rm eff}(\phi) = V(\phi) + \Delta V(\phi)$$

Coleman, E.Weinberg, '73
Jackiw, '74
See also, M.Sher' 89

Loop expansion:

$$\Delta V(\phi) = \Delta^{(1)} V(\phi) + \Delta^{(2)} V(\phi) + \Delta^{(3)} V(\phi) + \ldots$$



Spontaneous symmetry breaking and the Higgs effective potential

A proper way to study the symmetry breaking in the SM is to consider the effective potential for the background Higgs field which takes into account vacuum fluctuations

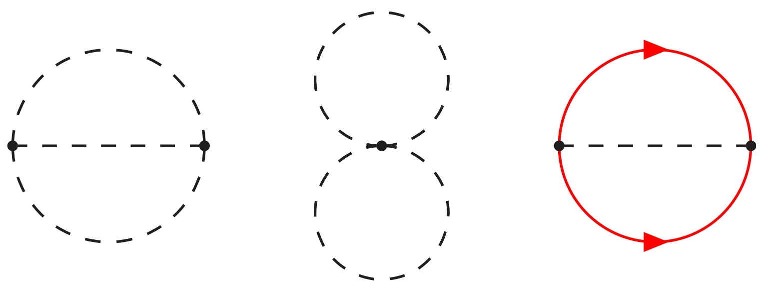
$$V(\phi) \to V_{\text{eff}}(\phi) = V(\phi) + \Delta V(\phi)$$

Coleman, E.Weinberg, '73
Jackiw, '74
See also, M.Sher' 89

Loop expansion:

$$\Delta V(\phi) = \Delta^{(1)} V(\phi) + \Delta^{(2)} V(\phi) + \Delta^{(3)} V(\phi) + \ldots$$

Ford, Jack, Jones, '92,'97
S. Martin, 2002



Example two-loop diagrams

Spontaneous symmetry breaking and the Higgs effective potential

A proper way to study the symmetry breaking in the SM is to consider the effective potential for the background Higgs field which takes into account vacuum fluctuations

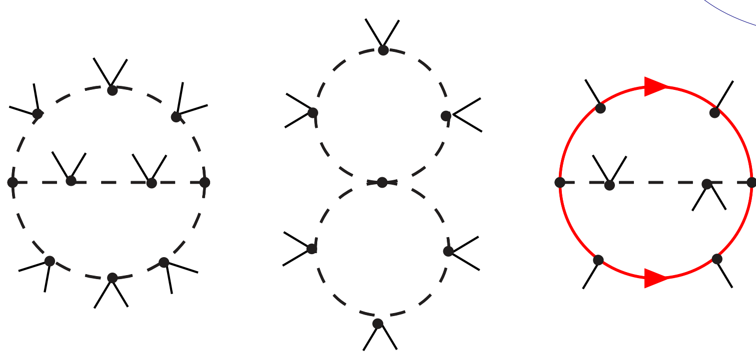
$$V(\phi) \to V_{\rm eff}(\phi) = V(\phi) + \Delta V(\phi)$$

Coleman, E.Weinberg, '73
Jackiw, '74
See also, M.Sher' 89

Loop expansion:

$$\Delta V(\phi) = \Delta^{(1)} V(\phi) + \Delta^{(2)} V(\phi) + \Delta^{(3)} V(\phi) + \dots$$

Ford, Jack, Jones, '92,'97
S. Martin, 2002



$$M_t(\phi) = y_t \phi / \sqrt{2}$$

Example two-loop diagrams with field-dependent masses, e.g.

Spontaneous symmetry breaking and the Higgs effective potential

A proper way to study the symmetry breaking in the SM is to consider the effective potential for the background Higgs field which takes into account vacuum fluctuations

$$V(\phi) \to V_{\text{eff}}(\phi) = V(\phi) + \Delta V(\phi)$$

Coleman, E.Weinberg, '73
Jackiw, '74
See also, M.Sher' 89

Loop expansion:

$$\Delta V(\phi) = \Delta^{(1)} V(\phi) + \Delta^{(2)} V(\phi) + \Delta^{(3)} V(\phi) + \ldots$$

Ford, Jack, Jones, '92,'97
S. Martin, 2002

S. Martin, only g_s and y_t 3-loop contributions, October, 2013

Spontaneous symmetry breaking and the Higgs effective potential

A proper way to study the symmetry breaking in the SM is to consider the effective potential for the background Higgs field which takes into account vacuum fluctuations

$$V(\phi) \to V_{\text{eff}}(\phi) = V(\phi) + \Delta V(\phi)$$

Loop expansion:

$$\Delta V(\phi) = \Delta^{(1)} V(\phi) + \Delta^{(2)} V(\phi) + \Delta^{(3)} V(\phi) + \ldots$$

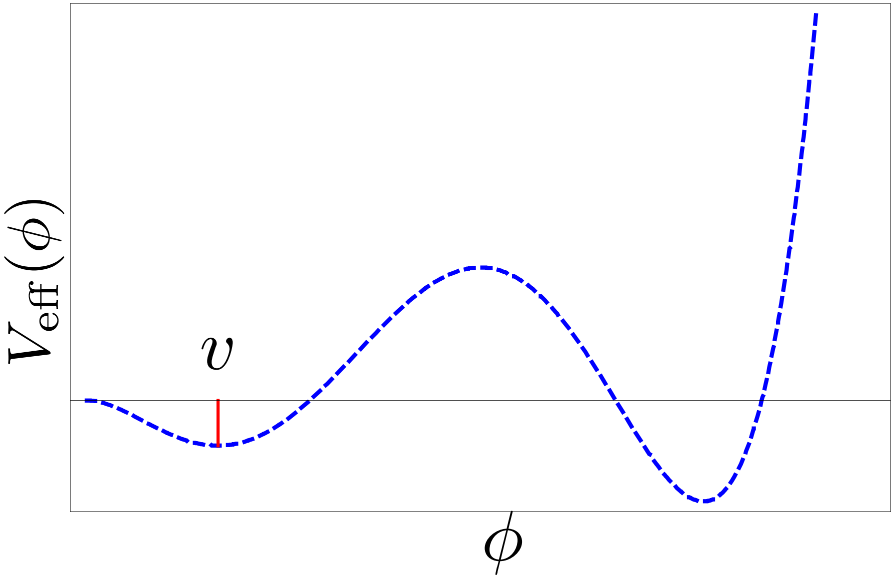
We should study the solutions of

$$\frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} = 0$$

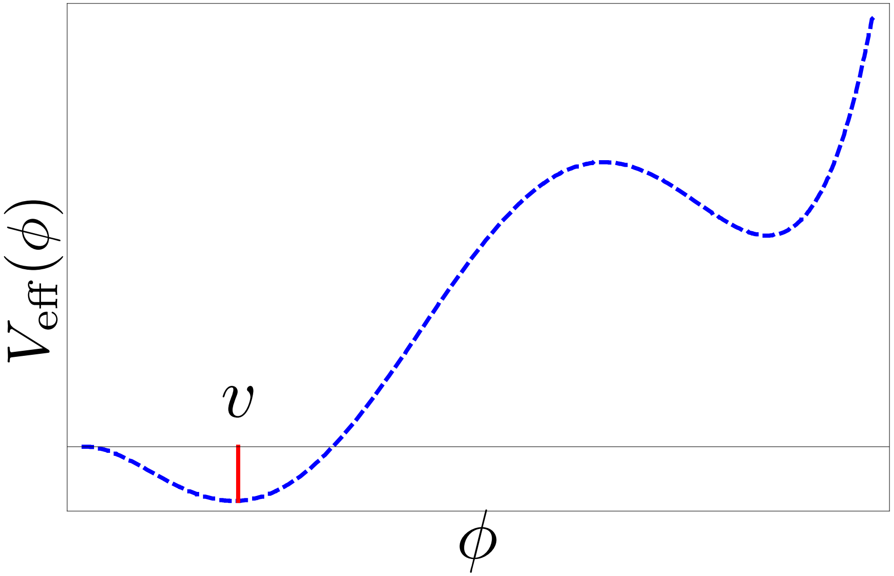
Questions:

1. Is the SM effective potential **bounded from below**?
2. Does the electroweak vacuum correspond to **the global minimum** of the effective potential or we are living in a false vacuum?

The Higgs field effective potential (schematic view)



Meta-stability?



Stability?

from Zoller,'13

The Higgs field effective potential

But … n-loop corrections to the tree-level potential involve logarithms of the form

$$\alpha^{n+1}(\mu)\left[\ln\frac{\phi^2}{\mu^2}\right]^n$$

with $\alpha(\mu)$ being some SM coupling constant defined at the normalization scale μ. The latter inevitably appears in perturbative calculations beyond the leading order.

The Higgs field effective potential

But … n-loop corrections to the tree-level potential involve logarithms of the form

$$\alpha^{n+1}(\mu)\left[\ln\frac{\phi^2}{\mu^2}\right]^n$$

with $\alpha(\mu)$ being some SM coupling constant defined at the normalization scale μ. The latter inevitably appears in perturbative calculations beyond the leading order.

This kind of terms can spoil the analysis of the potential since for fixed μ one can always find such large values of ϕ that render loop expansion invalid.

The Higgs field effective potential

But … n-loop corrections to the tree-level potential involve logarithms of the form

$$\alpha^{n+1}(\mu)\left[\ln\frac{\phi^2}{\mu^2}\right]^n$$

with $\alpha(\mu)$ being some SM coupling constant defined at the normalization scale μ. The latter inevitably appears in perturbative calculations beyond the leading order.

This kind of terms can spoil the analysis of the potential since for fixed μ one can always find such large values of ϕ that render loop expansion invalid.

This issue can be addressed by means of renormalization group improvement which basically corresponds to the choice $\mu^2 \sim \phi^2$

The Higgs field effective potential

At large values of the Higgs field the full effective potential can be approximated by the following expression:

$$V_{\rm eff}(\phi \gg v) \simeq \frac{\lambda(\mu=\phi)}{4}\phi^4$$

See Ford,Jack,Jones'92,'97

with "running" self-coupling $\lambda(\mu)$ evaluated at the scale $\mu = \phi$ .This effectively resumms dangerous contributions.

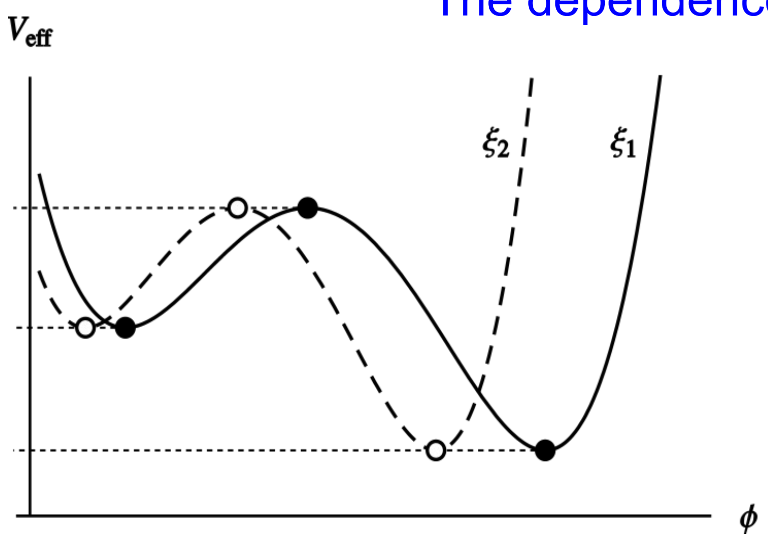
As a consequence, the stability of the electroweak vacuum is related to the behavior of the running Higgs self-coupling constant at large values of the renormalization scale.

If at some point $\lambda(\phi) < 0$, the potential there is much deeper than our vacuum and the stability of the latter should be questioned..

The Higgs field effective potential. Gauge-dependence issue

In order to quantize the SM we introduce gauge-fixing terms in the SM Lagrangian parametrized by auxiliary ξ_i for each gauge field of the model.

At general field values the effective potential is gauge-dependent.
The dependence is governed by Nielsen Identities:



$$\frac{\partial V_{eff}}{\partial \xi} = -C(\phi, \xi) \frac{\partial V_{eff}}{\partial \phi}$$

Which tell us that only at extrema
the effective potential is gauge-independent

from Patel, Ramsey-Musolf,'11

Renormalization group equations (RGE) in the SM

The running of the SM coupling constants is given by the system of coupled Renormalization Group Equations, which basically describe how different SM charges are screened (or anti-screened) with scale variation.

The (anti)screening is due to emission and absorption of virtual particles

$$\mu^2 \frac{da_i}{d\mu^2} = \beta_{a_i}(a_j)$$

$$(4\pi)^2 a_i = \{g_1^2, g_2^2, g_s^2, y_b^2, y_t^2, y_\tau^2, \lambda\}$$

The beta-functions are calculated in perturbation theory

$$\beta_{a_i} = \beta_{a_i}^{(1)} + \beta_{a_i}^{(2)} + \beta_{a_i}^{(3)} + \dots$$

Renormalization group equations (RGE) in the SM

The running of the SM coupling constants is given by the system of coupled Renormalization Group Equations, which basically describe how different SM charges are screened (or anti-screened) with scale variation.

The (anti)screening is due to emission and absorption of virtual particles

$$\mu^2 \frac{da_i}{d\mu^2} = \beta_{a_i}(a_j)$$

$$(4\pi)^2 a_i = \{g_1^2, g_2^2, g_s^2, y_b^2, y_t^2, y_\tau^2, \lambda\}$$

The beta-functions are calculated in perturbation theory

$$\beta_{a_i} = \beta_{a_i}^{(1)} + \beta_{a_i}^{(2)} + \beta_{a_i}^{(3)} + \dots$$

$\overline{MS}$ renormalization scheme

Our group contributed to the calculation of three-loop RGEs for all SM couplings

RGE in the SM: initial conditions

In order to solve RGE one needs to provide initial conditions at some scale.

One needs to perform matching – find relations between Lagrangian parameters and observables and solve them for the former.

$$2^{1/2} M_f = Y_f v(1+\bar{\delta}_f), \quad 4M_W^2 = g_2^2 v^2 (1+\bar{\delta}_W), \quad 4M_Z^2 = \left(g_1^2 + g_2^2\right) v(1+\bar{\delta}_Z),$$
$$M_h^2 = 2\lambda v^2 (1+\bar{\delta}_h), \quad 2^{1/2} G_f = v^{-2}(1+\bar{\delta}r), \quad (4\pi)^2 \alpha_s^{(5)}(\mu) = g_s^2 (1+\bar{\delta}\alpha_s)$$

RHS depends on "running" parameters (which enter RGE) and the scale μ
Various deltas correspond to quantum corrections and for consistent L-loop RGE analysis one needs to know $\bar{\delta}(\mu)$'s at (L-1)-loop level.

Effective theories allow one to separate physics at different scales...

G_f describes muon decay in Fermi theory and absorbs "hard" fluctuations due to heavy SM particles (W,Z,t)

$\alpha_s^{(5)}(\mu)$ parametrize QCD without top-quark and also absorbs "hard" fluctuations due to the latter.

The SM at *very* high energies

The boundary conditions for RGE can be found from the given "measured" quantities

$$\alpha^{-1} = 137.035999074(44)$$
$$\alpha_s(M_Z) = 0.1184(7)$$
$$G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$$
$$\mathrm{M}_t = 173.5(0.9)\ \mathrm{GeV}$$
$$\mathrm{M}_Z = 91.1876(21)\ \mathrm{GeV}$$
$$\mathrm{M}_W = 80.385(15)\ \mathrm{GeV}$$
$$\mathrm{m}_b(m_b) = 4.18(3)\ \mathrm{GeV}$$
$$\mathrm{M}_\tau = 1.77682(16)\ \mathrm{GeV}$$
$$\mathrm{M}_h = 125.5(0.4)\ \mathrm{GeV}$$

With the help of two-loop **matching** one obtains the boundary values

$$a_i(\mu \simeq v)$$

from PDG

The SM at *very* high energies

The boundary conditions for RGE can be found from the given "measured" quantities

$$\alpha^{-1} = 137.035999074(44)$$
$$\alpha_s(M_Z) = 0.1184(7)$$
$$G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$$
$$y_t \longleftarrow \mathrm{M}_t = 173.5(0.9)\ \mathrm{GeV}$$
$$\mathrm{M}_Z = 91.1876(21)\ \mathrm{GeV}$$
$$\mathrm{M}_W = 80.385(15)\ \mathrm{GeV}$$
$$\mathrm{m}_b(m_b) = 4.18(3)\ \mathrm{GeV}$$
$$\mathrm{M}_\tau = 1.77682(16)\ \mathrm{GeV}$$
$$\lambda \longleftarrow \mathrm{M}_h = 125.5(0.4)\ \mathrm{GeV}$$

With the help of two-loop **matching** one obtains the boundary values

$$a_i(\mu \simeq v)$$

from PDG

SM RGE: state of the art

In a series of papers our group calculated beta-functions for all the SM couplings at the three-loop level

Gauge couplings in ArXiv: 1210.6873 (JHEP1301)

Yukawa couplings in ArXiv: 1212.6829 (Phys.Lett.B722) (only for 3rd generation)

Higgs self-coupling in ArXiv: 1303.4364 (Nucl.Phys.B875)

AVB, A.F. Pikelner (BLTP JINR), V.N. Velizhanin (PNPI)

NB: All expressions can be found online as ancillary files of the arXiv preprints.

SM RGE: state of the art – boundary values for parameters

The running parameters at the electroweak scale can be extracted from observables by means of explicit gauge-independent matching procedure, which fully account for two-loop electroweak (EW) corrections

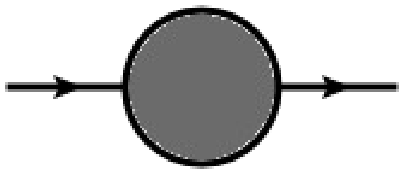
To the strong coupling in ArXiv: 1410.7603 (Phys.Lett.B741) by AVB

To all other couplings in ArXiv: 1503.02138 (Nucl.Phys.B896) by B.A. Kniehl, A.F. Pikelner, O.L. Veretin

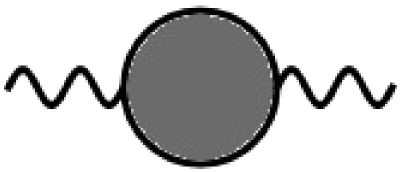
(see also references therein)

NB: All the relevant expressions can be found online either as ancillary files of the arXiv preprints or numerical routines ready to be used in RGE analysis.

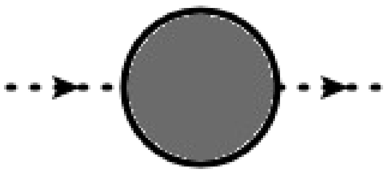
Some details of the calculation



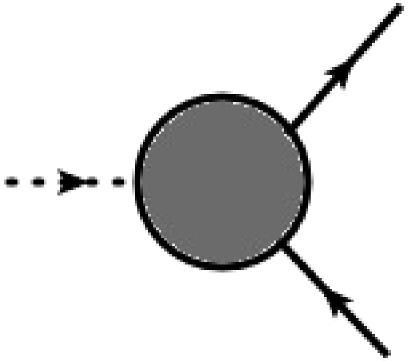
Fermions



Gauge bosons

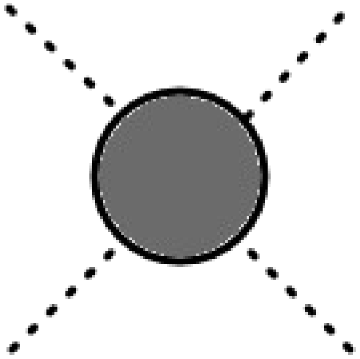


Higgs boson



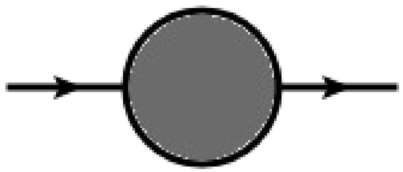
Yukawa vertices

Three-loop Feynman
diagrams contributing
to these Green functions
were evaluated...

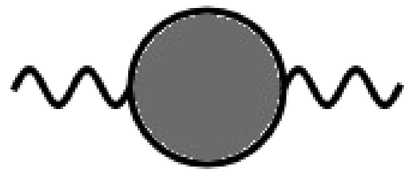


Higgs vertices

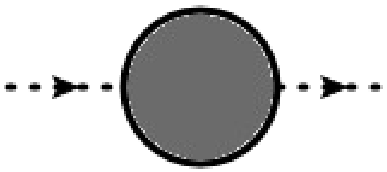
Some details of the calculation



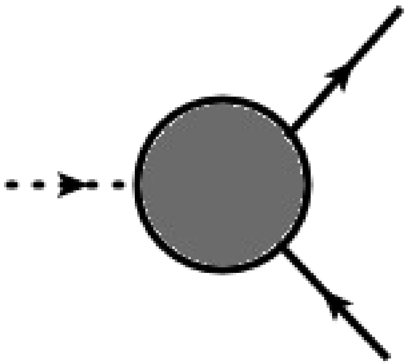
Fermions



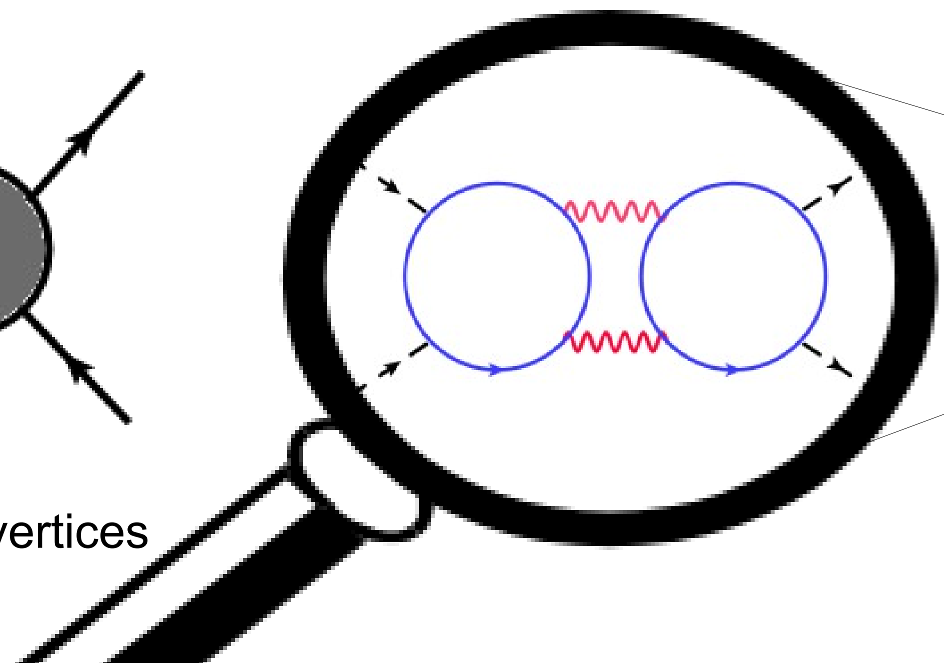
Gauge bosons

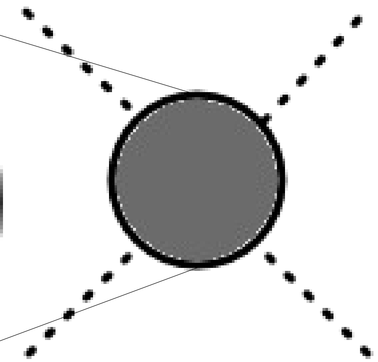


Higgs boson



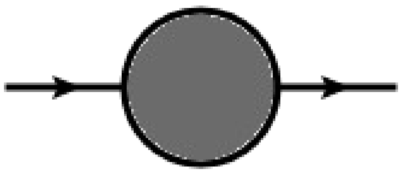
Yukawa vertices





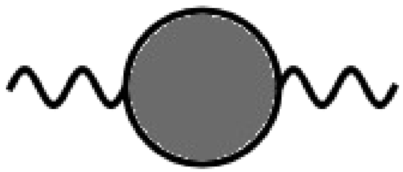
Higgs vertices

Some details of the calculation



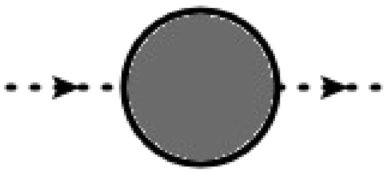
Fermions

$\mathcal{O}\left(10^4\right)$



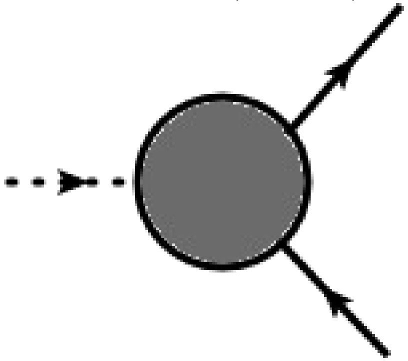
Gauge bosons

$\mathcal{O}\left(10^4\right)$



Higgs boson

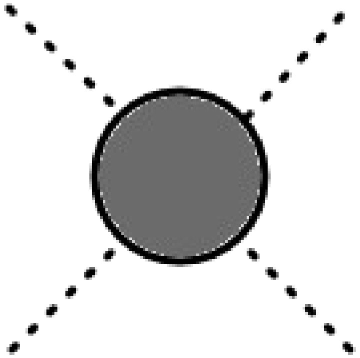
$\mathcal{O}\left(10^4\right)$



Yukawa vertices

$\mathcal{O}\left(10^5\right)$

Approximate number of three-loop diagrams



Higgs vertices

$\mathcal{O}\left(10^6\right)$

Impossible to evaluate by hand

Three-loop SM RGE: Fair play

The same results from two Karlsruhe groups:

L.Mihaila, J.Salomon, M.Steinhauser, PRL108 (2012)
– full three-loop gauge beta-functions *for the first time*

K. Chetyrkin, M.Zoller, JHEP1206 (2012)
– three-loop beta-functions for λ and y_t *for the first time*

(all couplings but g_s, λ, *and* y_t are neglected)

K. Chetyrkin, M.Zoller, JHEP1304 (2013)
– full three-loop self-coupling beta-function *for the first time*

(we made the results public a week later)

Perfect agreement was obtained. Slightly different setup was used ...

Two-loop matching: Fair play

Similar results were obtained in:

G. Degrassi et al, JHEP1208 (2012) 091,
D. Buttazzo et al, JHEP1312 (2013) 098

- lack of explicit control of gauge-parameter dependence
  - Landau gauge is employed everywhere
- No public code available to cross-check the results.

Numerical agreement was found at fixed scale.

Still some discrepancy in uncertainties....

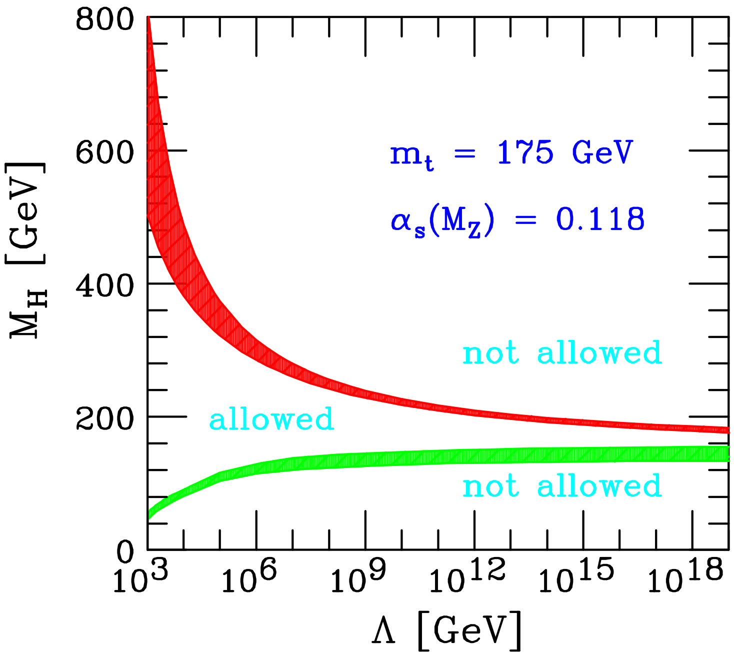
Slightly different setup was used ...

Back to physics:
The evolution of self-coupling

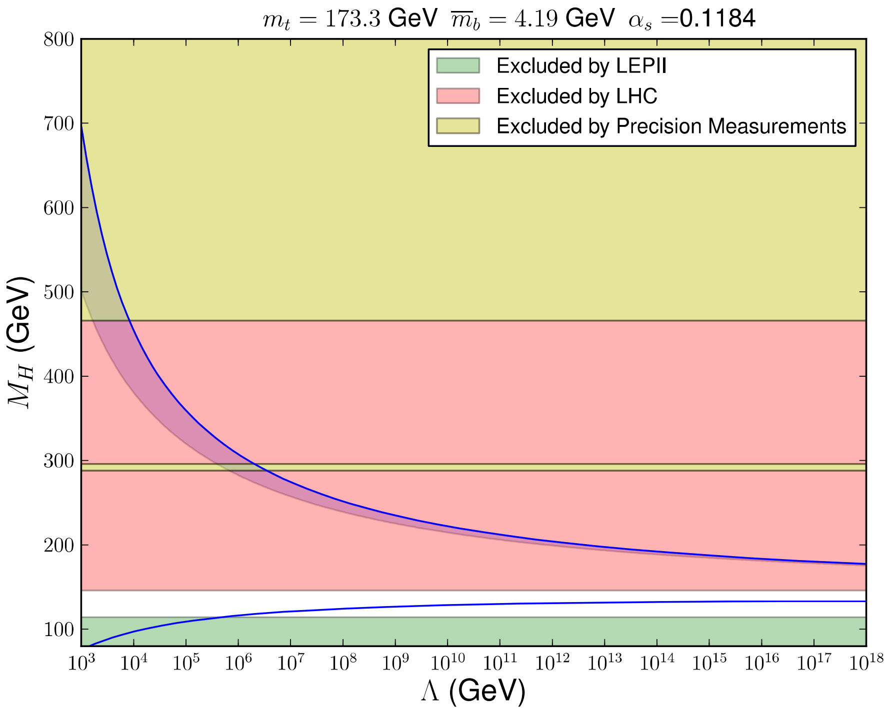
$$(4\pi)^2\mu^2\frac{d\lambda(\mu)}{d\mu^2} \simeq \beta_\lambda^{(1)} = 12\lambda^2 - 3y_t^4 + 6\lambda y_t^2 + \dots$$

screening → $12\lambda^2$

antiscreening → $-3y_t^4$



Hambye, Riesselmann, '97

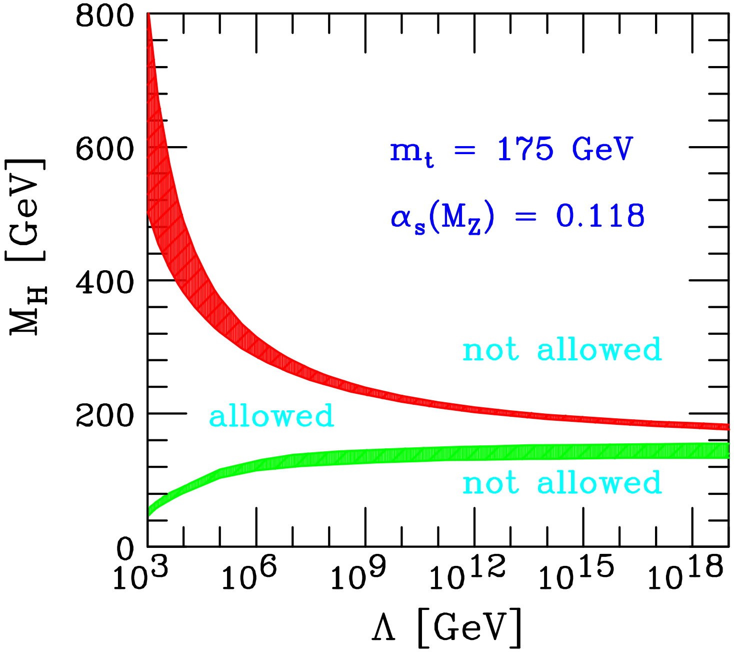


Wingerter,2011

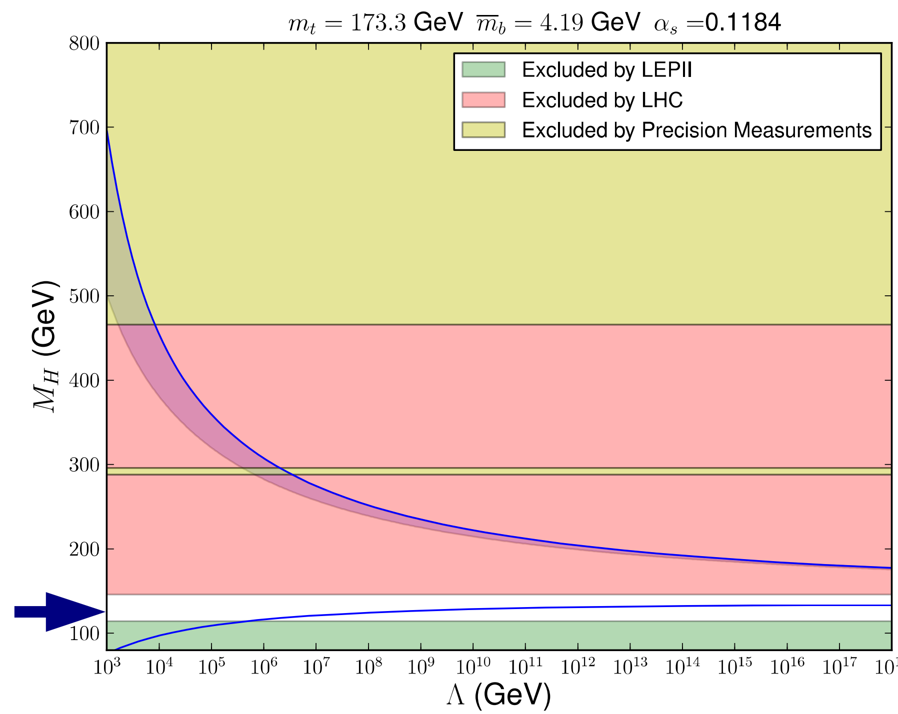
Back to physics:
The evolution of self-coupling

$$(4\pi)^2 \mu^2 \frac{d\lambda(\mu)}{d\mu^2} \simeq \beta_\lambda^{(1)} = 12\lambda^2 - 3y_t^4 + 6\lambda y_t^2 + \ldots$$

screening → $12\lambda^2$; antiscreening → $3y_t^4$



Hambye, Riesselmann, '97



Wingerter,2011

What if?

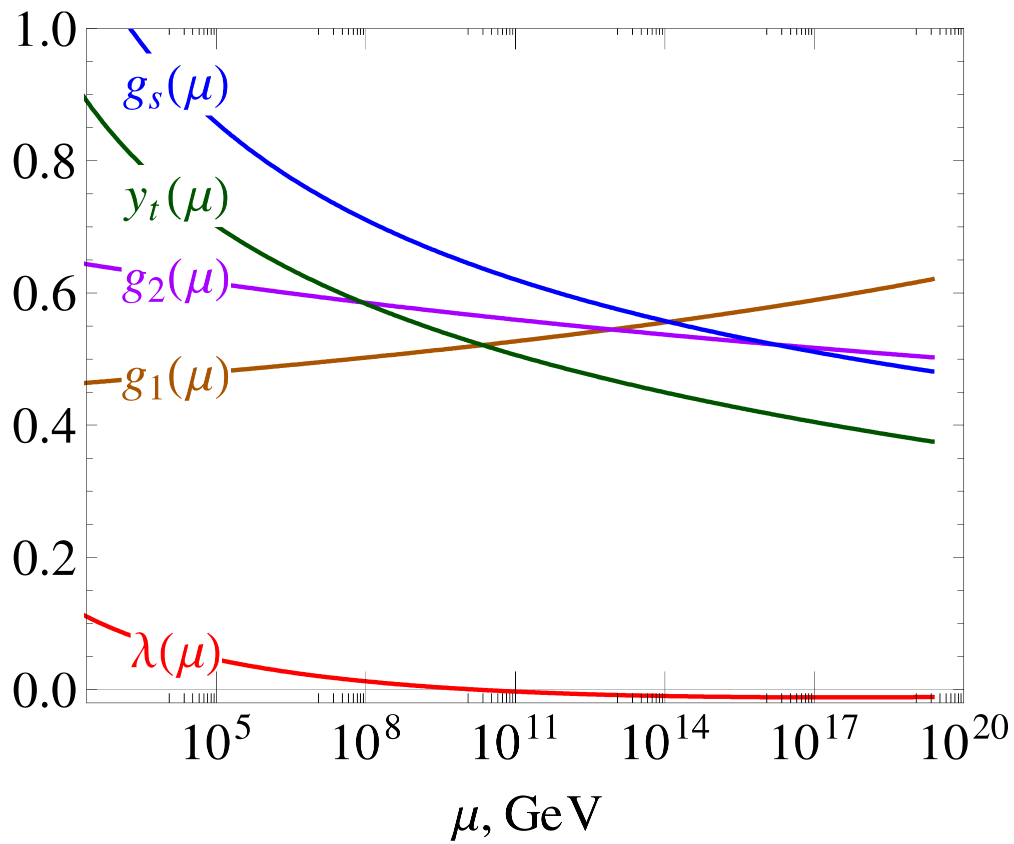
There is no New Physics up to *very* high energies, e.g. up to the Planck scale...

$$M_P = \sqrt{\frac{\hbar c}{G}} \simeq 10^{19}\ \mathrm{GeV}$$

This possibility can be explored in a precise analysis based on the obtained three-loop RGE and recent experimental results.

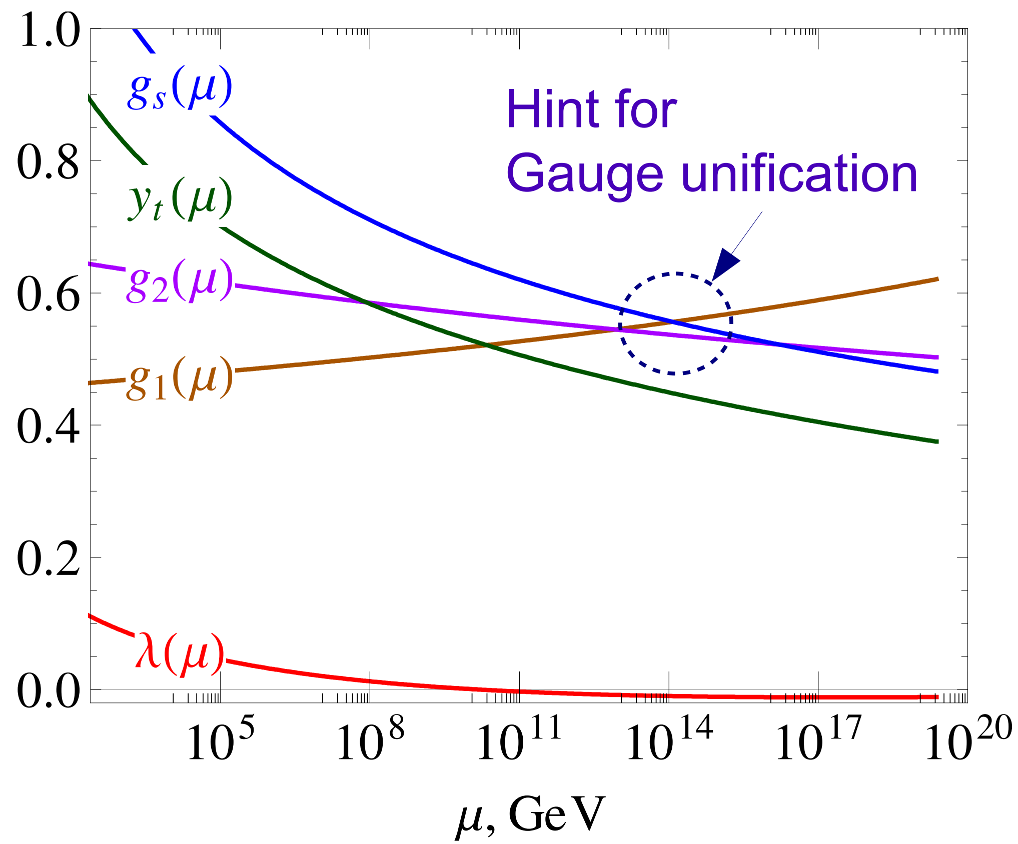
Evolution of the SM couplings

The initial conditions at the electroweak scale
are by means of
relations presented in
Kniehl, Pikelner, Veretin,2015



Evolution of the SM couplings

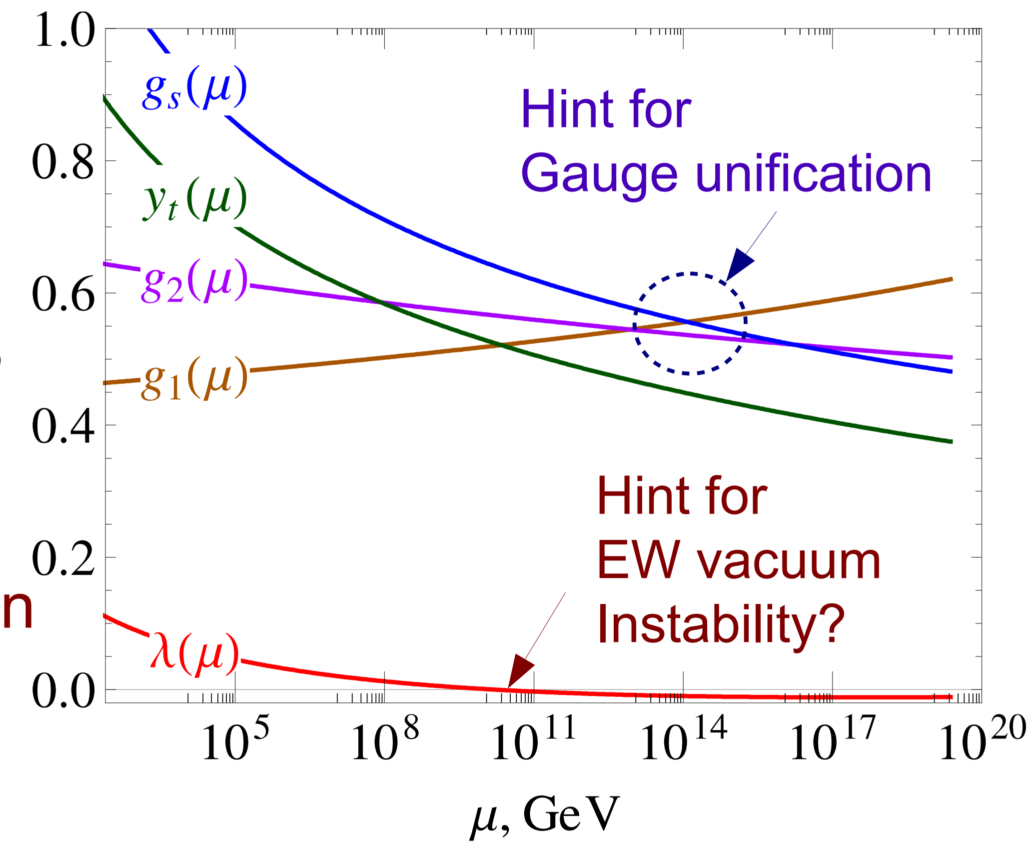
The initial conditions at the electroweak scale are by means of relations presented in Kniehl, Pikelner, Veretin,2015



Evolution of the SM couplings

The initial conditions at the electroweak scale are by means of relations presented in Kniehl, Pikelner, Veretin,2015

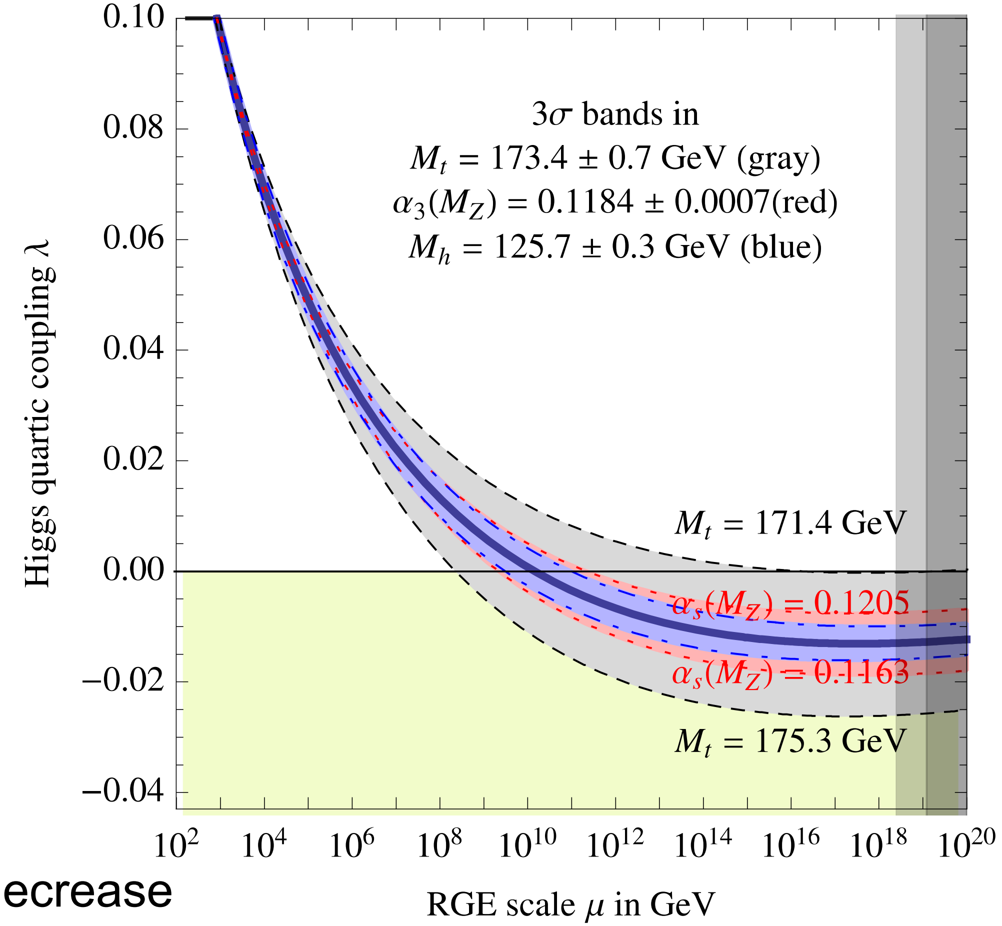
We are interested in the self-coupling evolution



Evolution of the Higgs self-coupling

And the influence of experimental uncertainties in input parameters:

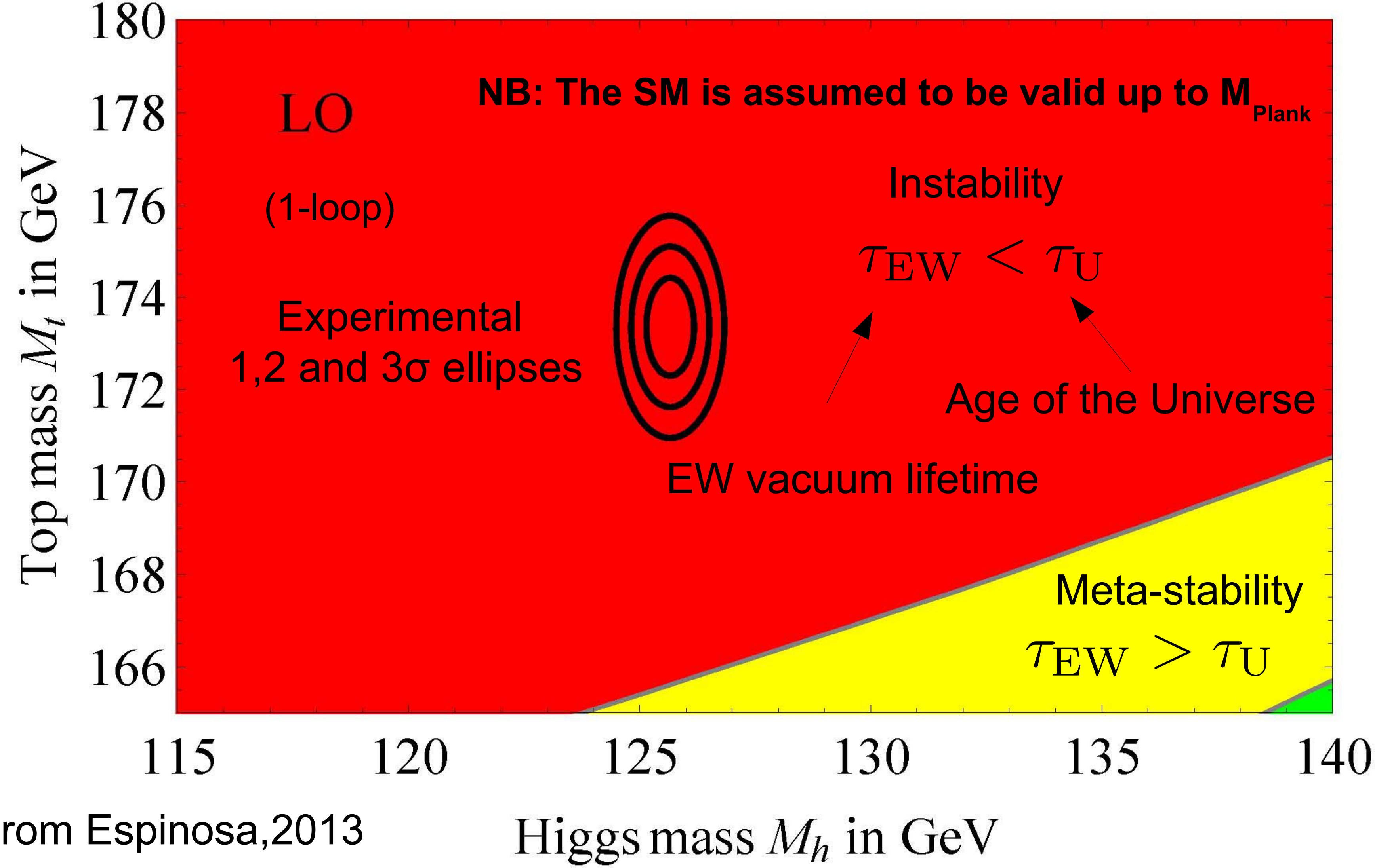
- The largest one is due to the top mass M_t (gray)
- The next one is due to the strong coupling α_s (pink)
- Uncertainty from M_h is given in light blue



NB: Strong interactions tend to decrease the top Yukawa coupling ...

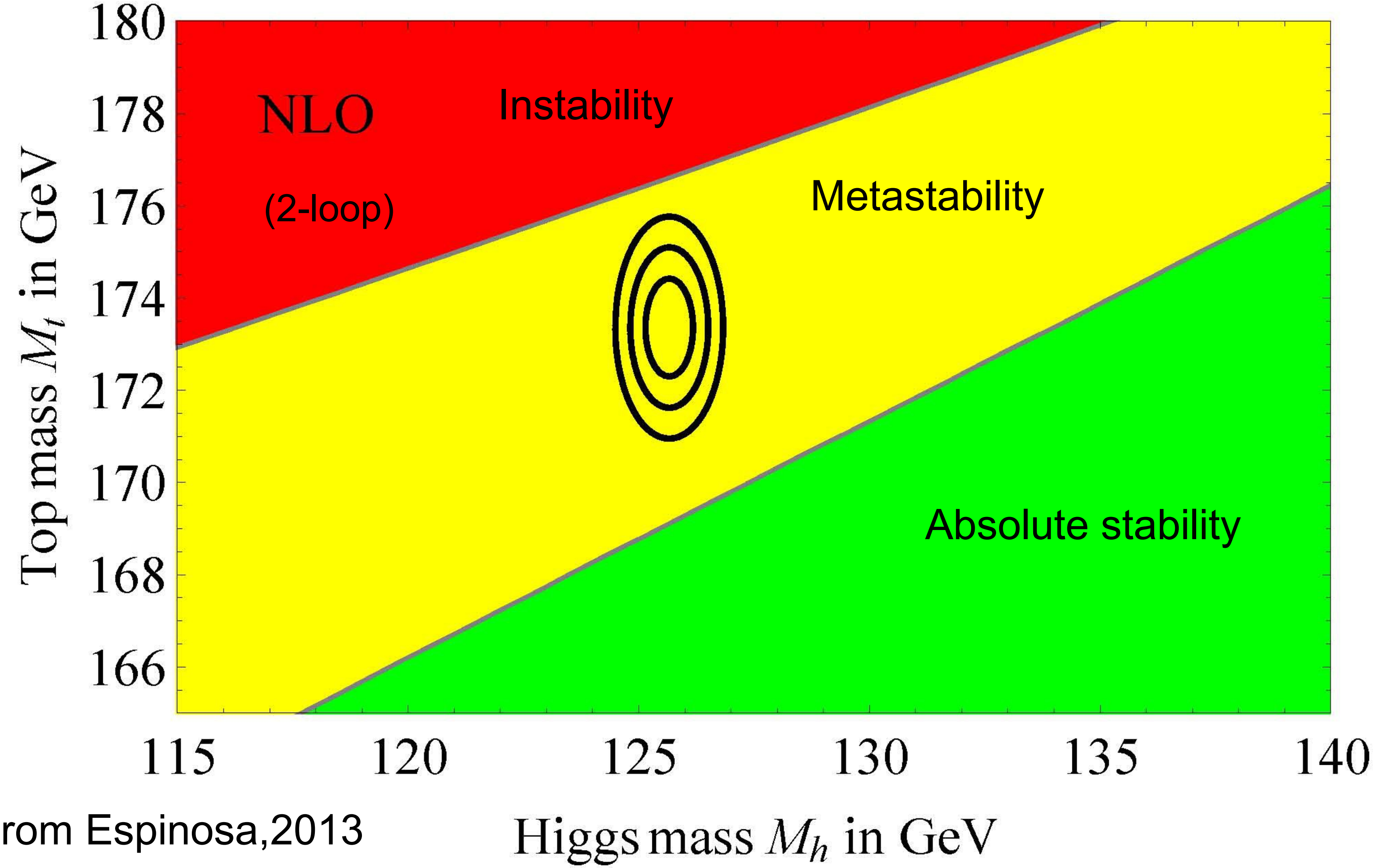
From Butazzo et al, 2013

Stable, unstable or metastable?



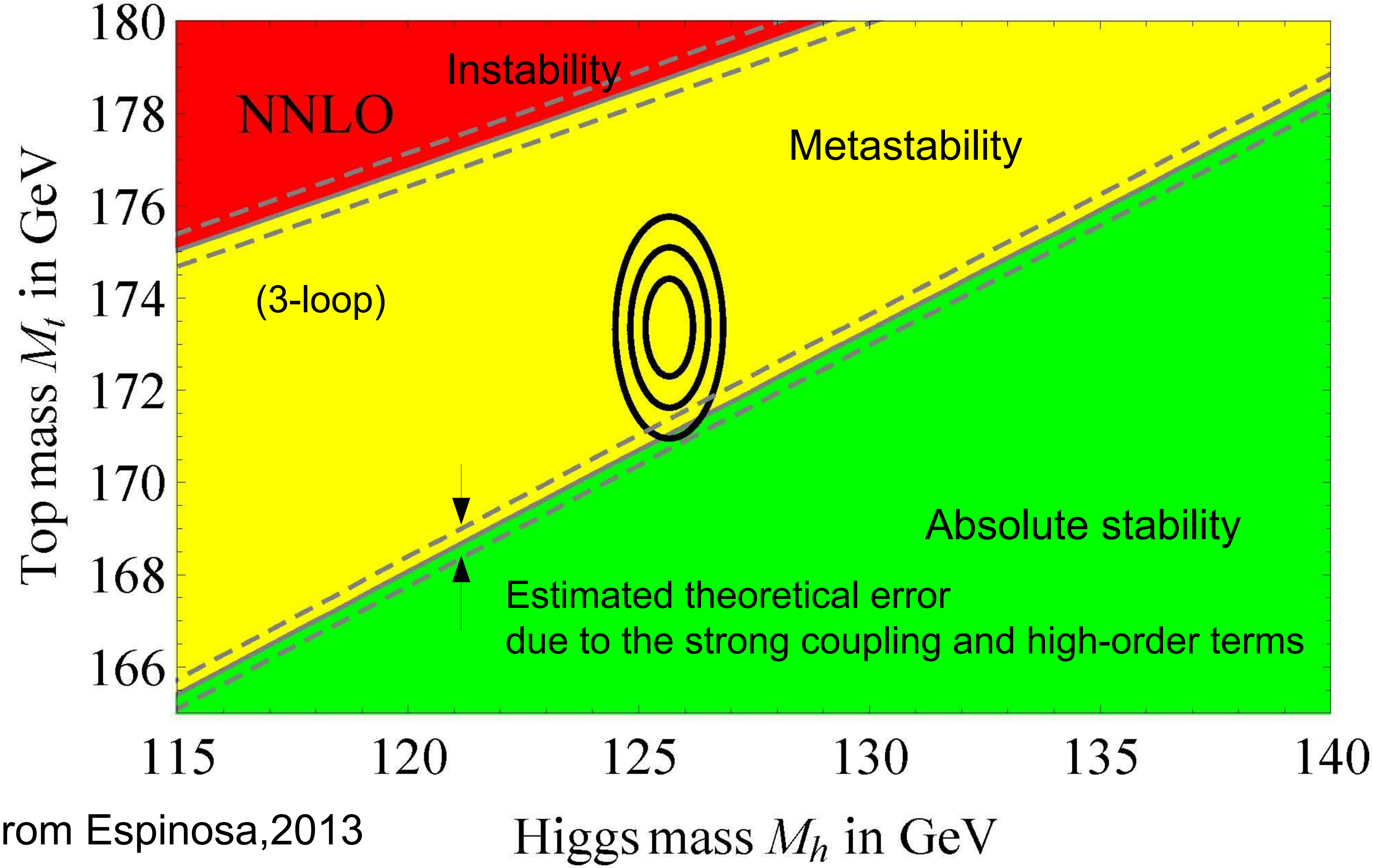
From Espinosa,2013

Stable, unstable or metastable?



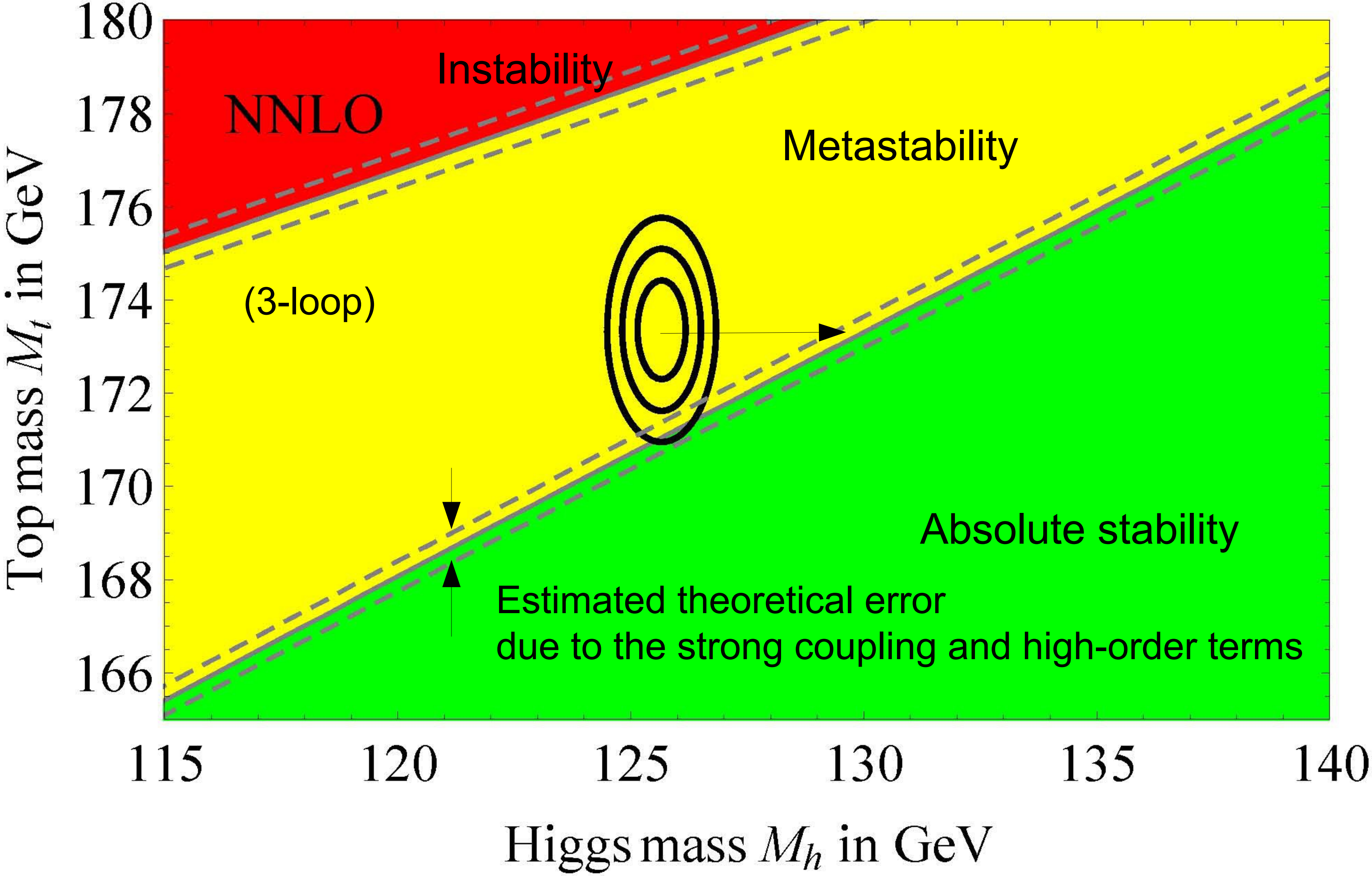
From Espinosa,2013

Stable, unstable or metastable?



From Espinosa,2013

$$M_h^{crit} = 129.30^{+0.72}_{-0.34} + 1.79\left(\frac{M_t - 173.21}{0.87}\right) - 0.48\left(\frac{\alpha_s^{(5)} - 0.1185}{0.0006}\right)$$



Conclusions ...

- Under assumption that there is no New Physics up to the Plank scale the stability of the EW vacuum is studied with the help of the "state of the art" 3-loop RGE.
- Our vacuum is most likely to be metastable with $\tau_{\rm EW} \gg \tau_{\rm U}$ but with current uncertainty in the top quark mass, it is still possible to have absolute stability within the SM

… Issues …

- The dominant uncertainty is due to the top-quark mass. Strictly speaking, this quantity is not a well-defined one (no free quarks), so better understanding of theoretical error in the top mass determination would be desirable in addition to a more precise experimental measurement.

See Alekhin et al, 2013 and Juste et al, 2013

- Our analysis showed that high-order terms reduce theoretical uncertainties due to missing terms, but still they can be non-negligible.
- Contributions from Planck-suppressed non-renormlizable operators...

See V. Branchina and E. Messina, 2013

In view of this, the theoretical uncertainty
can be underestimated in literature

Thank you for your attention!

Further references can be found in the next slide

Some references

On matching procedures and vacuum stability analysis:

1) F.Bezrukov, M.Y. Kalmykov, B.A. Kniehl, and M.Shaposhnikov, JHEP1210 (2012) 140;
2) G. Degrassi, S. Di Vita, J. Elias-Miro, .R Espinosa, G.F. Guidice, G. Isidori and A. Strumia, JHEP1208 (2012) 098;
3) D. Buttazzo, G. Degrassi, P.P Giardino, G.F. Guidice, F. Sala, A. Salvio and A. Strumia, JHEP12 (2013) 089;
4) V. Branchina and E. Messina, Phys. Rev. Lett 111 (2013) 241801;
5) A. Andreassen, W. Frost and M.D. Schwartz, Phys. Rev. Lett 113 (2014) 241801;

On the top-quark mass:

6) S. Alekhin, A. Djouadi and S. Moch, Phys. Lett, B716 (2012) 214;
7) I. Masina, Phys Rev. D87 (2013) 5, 053001;
8) A. Juste, S. Mantry, A. Mitov, A. Penin, P. Skands, E. Varnes, M. Vos and S. Wimpenny [arXiv:1310.0799];
9) S. Moch et al, arXiv:1405.4781