

Factorization theorem and gluon poles

I.V. Anikin (JINR, Dubna)

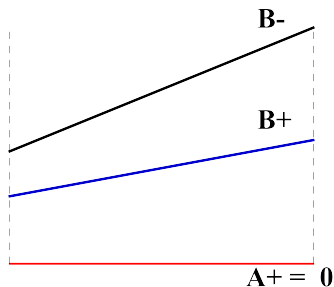
in collaboration with O.V. Teryaev

based on PLB690, 519 (2010); EPJC75, 184 (2015)

Brazil-JINR Forum, Dubna 2015

June 17, 2015

- Within the CF-procedure applied for DY- and DPP-process hadron tensor, new contributions to gluon poles have been found;
- The crucial step is to use the contour gauge conception, $[x_0, x] = 1$, which generates the usual axial-type gauges:



Factorization theorem, in a nutshell

Factorization theorem states that the **short (hard)** and **long (soft)** distance dynamics can be separated out provided large Q^2 , *i.e.*

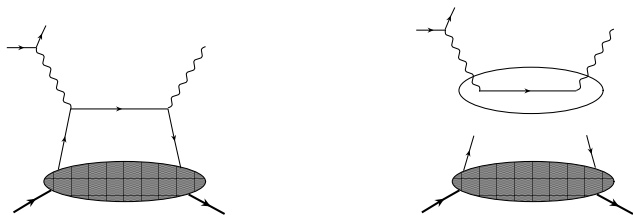
$$T_{\mu\nu} = \int d^4k \operatorname{tr}[E_{\mu\nu}(k)\Phi(k)] \xrightarrow{Q^2 \rightarrow \infty} \int dx \operatorname{tr}[E_{\mu\nu}(x)\Phi(x)] + \mathcal{O}(1/Q^2)$$

where $E_{\mu\nu}$ implies the product of propagators, while

$$\Phi(k) = - \int d^4z e^{i(k-\Delta/2)\cdot z} \langle p' | \psi(z) \bar{\psi}(0) | p \rangle ,$$

$$\Phi(x) = \int d^4k \delta(x - k \cdot n) \Phi(k) .$$

Schematically, this corresponds to



or, an alternative expression reads

$$\text{Amplitude} = \{\text{Hard part (pQCD)}\} \otimes \{\text{Soft part (npQCD)}\},$$

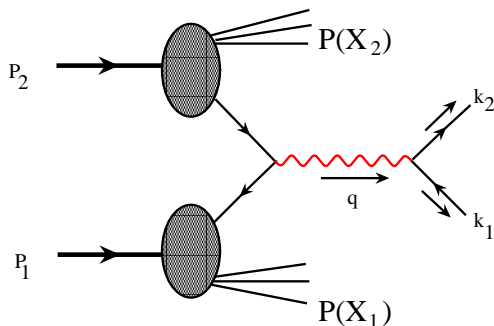
where both **hard** and **soft** parts are independent of each other, UV- and IR-renormalizable and, finally, parton distributions must possess the universality property.

Drell-Yan process

We study

$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \rightarrow \gamma^*(q) + X(P_X) \rightarrow \ell(l_1) + \bar{\ell}(l_2) + X(P_X),$$

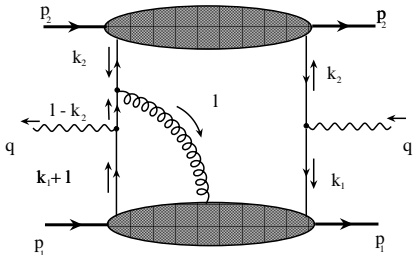
where $l_1 + l_2 = q$ has a large mass squared ($q^2 = Q^2$).



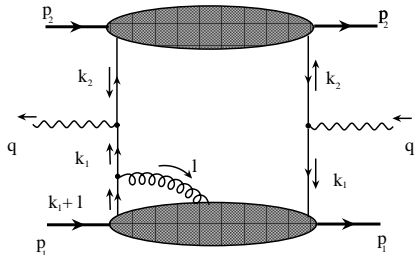
The cross-section reads (kinematics: $p_1 \sim n^{*+}$, $p_2 \sim n^-$)

$$d\sigma = (dP.S.)^2 \mathcal{L}_{\mu\nu} \mathcal{W}_{\mu\nu}^{GI},$$

where $\mathcal{L}_{\mu\nu}$ is a lepton tensor, and $\mathcal{W}_{\mu\nu}^{GI}$ – the QED gauge invariant **hadron** tensor.



a)



b)

Single Spin Asymmetry

Any SSA are defined as

$$\text{SSA} \sim d\sigma^{(\uparrow)} - d\sigma^{(\downarrow)} \sim \mathcal{L}_{\mu\nu} H_{\mu\nu}.$$

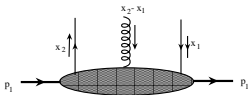
In our case, we deal with the unpolarized leptons, *i.e.*

$\mathcal{L}_{\mu\nu} \in \Re$. Therefore, the hadron tensor $H_{\mu\nu}$ should also be real one, *i.e.* $H_{\mu\nu} \in \Re$, provided, at the same time, one of hadrons is transversely polarized. Usually, it is possible if

$$H_{\mu\nu}^{(a)} \sim \Im[\text{Hard}] \otimes \left\{ \langle p_1, S_T | \mathcal{O}(\bar{\psi}, \psi, A) | S_T, p_1 \rangle \stackrel{\mathcal{F}}{\sim} i \varepsilon_{\alpha\beta S_T p_1} \Phi \right\},$$

$$H_{\mu\nu}^{(b)} \sim \text{Hard} \otimes \left\{ \langle p_1, S_T | \mathcal{O}(\bar{\psi}, \psi, A) | S_T, p_1 \rangle \stackrel{\mathcal{F}}{\sim} i \varepsilon_{\alpha\beta S_T p_1} \Im[\Phi] \right\}.$$

However, for the pure **real** B^V -function $\in \Re$, which parametrizes



$$\langle p_1, S^T | \bar{\psi}(\lambda_1 \tilde{n}) \gamma_\beta g A_\alpha^T(\lambda_2 \tilde{n}) \psi(0) | S^T, p_1 \rangle =$$

$$i \varepsilon_{\beta\alpha S^T p_1} \int dx_1 dx_2 e^{i x_1 \lambda_1 + i(x_2 - x_1) \lambda_2} B^V(x_1, x_2),$$

the diagram (b) does **NOT** contribute to the SSA.

As a result, we are faced to a problem with QED gauge invariance and, therefore, with the factorization breaking.

The inference on B^V -function $\in \mathfrak{Re}$ is based on the solution of the differential equation (within the gauge: $A^+ = 0$)

$$\partial^+ A_T^\alpha = G_T^{+\alpha},$$

one can get that

$$\begin{aligned} A^\mu(z) &= \int_{-\infty}^{\infty} d\omega^- \theta(z^- - \omega^-) G^{+\mu}(\omega^-) + A^\mu(-\infty) \\ &= - \int_{-\infty}^{\infty} d\omega^- \theta(\omega^- - z^-) G^{+\mu}(\omega^-) + A^\mu(\infty). \end{aligned}$$

Inserting the above-mentioned presentations into the corresponding m.e., we thus obtain that

$$\Phi_A^\alpha(x_1, x_2) = \delta(x_1 - x_2) \Phi_{A(-\infty)}^\alpha(x_1) + \frac{(-i) \Phi_G^\alpha(x_1, x_2)}{x_2 - x_1 - i\epsilon},$$

and

$$\Phi_A^\alpha(x_1, x_2) = \delta(x_1 - x_2) \Phi_{A(+\infty)}^\alpha(x_1) + \frac{(-i) \Phi_G^\alpha(x_1, x_2)}{x_2 - x_1 + i\epsilon}.$$

Calculation the plus and minus combinations leads to

$$\begin{aligned} \Phi_A^\alpha(x_1, x_2) &= \frac{1}{2} \Phi_A^\alpha(x_1, x_2) + \frac{1}{2} \Phi_A^\alpha(x_1, x_2) = \\ &= \frac{1}{2} \delta(x_1 - x_2) \left\{ \Phi_{A(-\infty)}^\alpha(x_1) + \Phi_{A(+\infty)}^\alpha(x_1) \right\} + \\ &= \frac{\mathcal{P}}{x_2 - x_1} (-i) \Phi_G^\alpha(x_1, x_2) \end{aligned}$$

and

$$\begin{aligned} 0 &= \Phi_A^\alpha(x_1, x_2) - \Phi_A^\alpha(x_1, x_2) = \\ &= \delta(x_1 - x_2) \left\{ \Phi_{A(+\infty)}^\alpha(x_1) - \Phi_{A(-\infty)}^\alpha(x_1) \right\} - \\ &= 2i \pi \delta(x_1 - x_2) (-i) \Phi_G^\alpha(x_1, x_2). \end{aligned}$$

So, this ambiguity ultimately gives us the **standard-used** representation:

$$B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2),$$

$$T(x_1, x_2) \stackrel{\mathcal{F}}{\sim} \langle \bar{\psi} \gamma_\beta \tilde{n}_\nu G_{\nu\alpha} \psi \rangle \quad T(x, x) \neq 0.$$

provided the asymmetric boundary condition for gluons:

$$B_{A(\infty)}^V(x) = -B_{A(-\infty)}^V(x)$$

Thus, for the considered DY, a pure real $B^V(x_1, x_2)$ will lead to the problem with QED gauge invariance which means factorization breaking.

Actually, the B^V -function is **not** the real one. Indeed,

the sign of $i\epsilon$ in the propagator of the h. p. \implies
 θ -function in the gluon field repres. \implies
the **contour gauge** for gluons which demands

$$g(x) \equiv [x, x_0] = P \exp \left\{ ig \int_{\mathbb{P}(x_0, x)} d\omega \cdot A(\omega) \right\} = 1 \quad (\forall x \in \mathbb{R}^4),$$

where the final point at the minus infinity changes the fixed "starting" point x_0 , whereas the point z changes the point x .

For an arbitrary path connecting z and $-\infty$, one has

$$\begin{aligned} A_{\mu}^{\text{ax}}(z) &= [z, -\infty]^{-1} A_{\mu}(z) [z, -\infty] + \frac{i}{g} [z, -\infty]^{-1} \partial_{\mu} [z, -\infty] \\ &= \int_{-\infty}^z d\omega_{\alpha} \frac{\partial \omega_{\beta}}{\partial z_{\mu}} G_{\alpha\beta}(\omega) + A_{\mu}(-\infty). \end{aligned}$$

Choosing now the path in the form of the straight line:

$$\omega_{\alpha}(v) \Big|_z^{-\infty} = z_{\alpha} - \tilde{n}_{\alpha} \lim_{\epsilon \rightarrow 0} \frac{1 - e^{-\epsilon v}}{\epsilon} \Big|_0^{\infty},$$

we arrive at

$$A^{\mu}(z) = \int_{-\infty}^{\infty} d\omega^{-\theta}(z^{-} - \omega^{-}) G^{+\mu}(\omega^{-}) + A^{\mu}(-\infty).$$

Roughly speaking, the **hard** and **soft** parts are **NOT** fully independent:

Causal Prescrip. Hard Part $\overset{\text{C. g.}}{\longleftrightarrow}$ **Pole Prescrip. Soft Part**

Despite of this, factorization still works owing to the universal property of B -function.

All these fix (due to the T -reversal inv-ce, $B_{A(-\infty)}^V(x) = 0$)

$$B^V(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} + \delta(x_1 - x_2)B_{A(-\infty)}^V(x_1),$$

which leads to the **non-zero** contribution from the diagram (b).

Conclusions for DY:

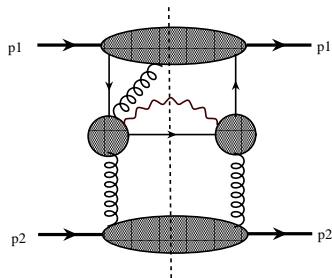
$$\mathbf{ISI} \Rightarrow \frac{1}{\ell^+ - i\epsilon} \Rightarrow [z^-, -\infty^-] \Rightarrow \text{b.c. } A_\mu(-\infty) \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Rightarrow \mathbf{GI}$$

Direct Photon Production in hadron collisions

We now dwell on the direct photon production in two hadron collisions:

$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \rightarrow \gamma(q) + X(P_X).$$

where $q^2 = Q^2$ is relatively **large**. The cross-section $d\sigma$ is defined by the hadron tensor as



It is convenient to fix the dominant light-cone directions as

$$p_1 = \sqrt{\frac{S}{2}} n^*, \quad p_2 = \sqrt{\frac{S}{2}} n, \quad \text{with}$$
$$n_\mu^* = (1/\sqrt{2}, \mathbf{0}_T, 1/\sqrt{2}), \quad n_\mu = (1/\sqrt{2}, \mathbf{0}_T, -1/\sqrt{2}).$$

The final on-shell photon and quark(anti-quark) momenta can be presented as

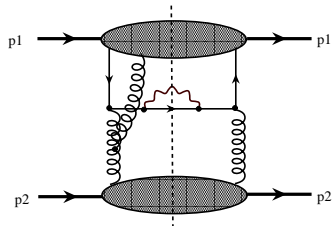
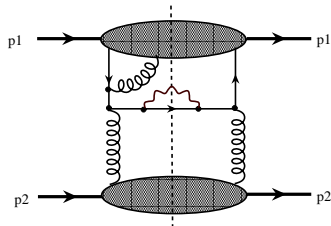
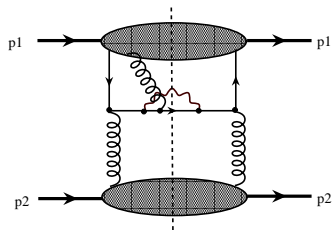
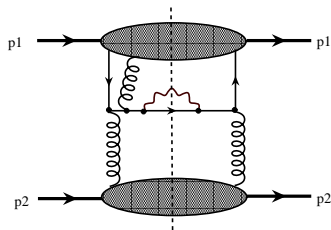
$$q = y_B \sqrt{\frac{S}{2}} n - \frac{q_\perp^2}{y_B \sqrt{2S}} n^* + q_\perp,$$
$$k = x_B \sqrt{\frac{S}{2}} n^* - \frac{k_\perp^2}{x_B \sqrt{2S}} n + k_\perp.$$

The Mandelstam variables for the process and subprocess are defined as

$$S = (p_1 + p_2)^2, \quad T = (p_1 - q)^2, \quad U = (q - p_2)^2,$$
$$\hat{s} = (x_1 p_1 + y p_2)^2 = x_1 y S,$$
$$\hat{t} = (x_1 p_1 - q)^2 = x_1 T, \quad \hat{u} = (q - y p_2)^2 = y U.$$

QCD gauge invariance

To study the QCD gauge invariance, we consider the following diagrams:



The quark-gluon correlator reads

$$\begin{aligned}\Phi_{\rho}^{\perp}(k_1, \ell) &= - \int (d^4 \eta_1 d^4 z) e^{-ik_1 \eta_1 - i\ell z} \langle p_1 | \bar{\psi}(0) \gamma^+ \psi(\eta_1) A_{\rho}^{\perp}(z) | p_1 \rangle \\ &= -\varepsilon_{\rho}^{\perp} \int (d^4 \eta_1) e^{-ik_1 \eta_1} \langle p_1 | \bar{\psi}(0) \gamma^+ \psi(\eta_1) a^+(\ell) | p_1 \rangle.\end{aligned}$$

Factorization procedure gives us

$$\begin{aligned}\Phi_{\rho}^{\perp}(x_1, x_2) &= \int (d^4 k_1 d^4 \ell) \delta(x_1 - k_1 n) \delta(x_2 - \ell n) \Phi_{\rho}^{\perp}(k_1, \ell) = \\ &= -\varepsilon_{\rho}^{\perp} \int (d\lambda_1) e^{-ix_1 \lambda_1} \langle p_1 | \bar{\psi}(0) \gamma^+ \psi(\lambda_1 n) \int (d^4 \ell) \delta(x_2 - \ell n) a^+(\ell) | p_1 \rangle.\end{aligned}$$

- For checking of the QCD gauge invariance, we make a replacement: $\hat{\varepsilon}^{\perp} \Rightarrow \hat{\ell}_L$ in the diagrams.

Initial and Final states interactions

In the process we consider, we have both ISI and FSI:

$$\begin{aligned} \text{ISI} &\Rightarrow \frac{1}{\ell^+ - i\epsilon} \Rightarrow [z^-, -\infty^-] \Rightarrow \text{b.c. } A_\mu(-\infty) \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \\ \text{FSI} &\Rightarrow \frac{1}{\ell^+ + i\epsilon} \Rightarrow [+ \infty^-, z^-] \Rightarrow \text{b.c. } A_\mu(+\infty) \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon} \end{aligned}$$

QCD gauge invariance: final stage

$$\overline{W^{(1)}} \sim \mathbf{C}_2 \frac{1}{x_1} \int dx_2 \frac{x_2 - x_1}{x_2} \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon},$$

$$\overline{W^{(2)}} \sim \mathbf{C}_2 \frac{1}{x_1} \int dx_2 \frac{1}{x_2} \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon},$$

$$\overline{W^{(3)}} \sim \mathbf{C}_1 \frac{1}{x_1^2} \int dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon},$$

$$\overline{W^{(4)}} \sim \mathbf{C}_3 \frac{1}{x_1^2} \int dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon},$$

where \mathbf{C}_i are corresponding colour factors. After calculation of imaginary parts, we get

$$+\mathbf{C}_2 - \mathbf{C}_1 - \mathbf{C}_3 = -[t^a, t^b] t^b t^a - if^{abc} t^c t^a t^b = 0$$

Hadron tensor of the DPP: new contributions

The full expression for the hadron tensor can be split into two groups:

(i) the first type, before factorization, takes the following form

$$\mathcal{W}(\text{diag.H}) = \int \frac{d^3 \vec{q}}{(2\pi)^3 2E} \frac{d^3 \vec{k}}{(2\pi)^3 2\varepsilon} C_H \int (d^4 k_1)(d^4 k_2) \times \\ \delta^{(4)}(k_1 + k_2 - q - k) \Phi_g^{\alpha\beta}(k_2) \int (d^4 \ell) \Phi_{\perp}^{[\gamma^+], \rho}(k_1, \ell) H^{\alpha\beta, \rho}(k_1, k_2, \ell),$$

(ii) the second type can be presented as

$$\mathcal{W}(\text{diag.D}) = \int \frac{d^3 \vec{q}}{(2\pi)^3 2E} \frac{d^3 \vec{k}}{(2\pi)^3 2\varepsilon} C_D \int (d^4 k_1)(d^4 k_2) \times \\ \delta^{(4)}(k_1 + k_2 - q - k) \Phi_g^{\alpha\beta}(k_2) \text{tr}_D [\Phi^{(1)}(k_1) D^{\alpha\beta}(k_1, k_2)].$$

where the twist-3 quark distribution which is given by

$$\Phi^{(1)}(k_1) = \frac{\gamma^+ \gamma_\perp^\rho \gamma^-}{2k_1^+ + i\epsilon} \int (d^4 \eta_1) e^{ik_1 \eta_1} \times \\ \langle p_1, S^T | \bar{\psi}(0) \gamma^+ A_\perp^\rho(0) \psi(\eta_1) | S^T, p_1 \rangle,$$

We now perform the factorization procedure, we obtain

$$\begin{aligned}
 d\mathcal{W}(\text{diag.H}) &= \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp) C_H \times \\
 &\int dx_1 dy \delta(x_1 - x_B) \delta(y - y_B) \frac{2}{S} \mathcal{F}^g(y) g_\perp^{\alpha\beta} \times \\
 &\int dx_2 \Phi_\perp^{[\gamma^+],\rho}(x_1, x_2) H^{\alpha\beta,\rho}(x_1, x_2),
 \end{aligned}$$

for the first type of contributions;

and

$$d\mathcal{W}(\text{diag.}D) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp) \mathbf{C}_D \times \\ \int dx_1 dy \delta(x_1 - x_B) \delta(y - y_B) \frac{2}{S} \mathcal{F}^g(y) g_\perp^{\alpha\beta} \text{tr}_D[\Phi^{(1)}(x_1) D^{\alpha\beta}(x_1)],$$

for the second type of contributions.

Kinematical constraints

To simplify our calculations without losing generality, we may impose the frame where $q_{\perp}^2 \ll S$. The Mandelstam variable defined for the subprocess, \hat{u} , is a small variable and can be neglected. It means that the Bjorken fraction y_B becomes independent of x_B , and one can write $y_B = -T/S$ (due to $\hat{s} + \hat{t} + \hat{u} = 0$).

DPP Hadron Tensor: final stage

After computing the corresponding traces and performing simple algebra within the frame we are choosing, it turns out that the only nonzero contributions to the hadron tensor come from the diagrams H1, H7, D4 and H10:

$$d\mathcal{W}(\text{diag.H1}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp) C_2 \times$$
$$\int dx_1 dy \delta(x_1 - x_B) \delta(y - y_B) \mathcal{F}^g(y) \times$$
$$\int dx_2 \frac{2S^2 x_1 y^2}{[x_2 y S + i\epsilon][x_1 y S + i\epsilon]^2} \frac{\varepsilon^{q_\perp + S_\perp -}}{p_1^+} B_-^V(x_1, x_2),$$

$$\begin{aligned}
d\mathcal{W}(\text{diag.H7}) &= \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\epsilon} \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp) \mathbf{C}_1 \times \\
&\int dx_1 dy \delta(x_1 - x_B) \delta(y - y_B) \mathcal{F}^g(y) \times \\
&\int dx_2 \frac{(-2) S T x_1 (y - 3y_B)}{[x_2 T + i\epsilon][x_1 T + i\epsilon]^2} \frac{\epsilon^{q_\perp + s_\perp -}}{p_1^+} B_+^V(x_1, x_2),
\end{aligned}$$

$$\begin{aligned}
d\mathcal{W}(\text{diag.D4}) &= \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\epsilon} \delta^{(1)}(\vec{k}_\perp + \vec{q}_\perp) \mathbf{C}_1 \times \\
&\int dx_1 dy \delta(x_1 - x_B) \delta(y - y_B) \frac{2}{S} \mathcal{F}^g(y) \times \\
&\frac{2S^2 x_1 (y - 2y_B)}{[x_1 T + i\epsilon]^2} \frac{\epsilon^{q_\perp + S_\perp -}}{2x_1 p_1^+ + i\epsilon} \int dx_2 B_+^V(x_1, x_2),
\end{aligned}$$

$$d\mathcal{W}(\text{diag.H10}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp) C_3 \times$$

$$\int dx_1 dy \delta(x_1 - x_B) \delta(y - y_B) \mathcal{F}^g(y) \times$$

$$\int dx_2 \frac{2T(x_1 - x_2)(2T + Sy)}{[x_1 T + i\epsilon][x_2 T + i\epsilon][(x_1 - x_2)yS + i\epsilon]} \frac{\varepsilon^{q_\perp + S_\perp -}}{p_1^+} B_+^V(x_1, x_2).$$

Here, $C_1 = C_F^2 N_c$, $C_2 = -C_F/2$, $C_3 = C_F N_c C_A/2$. The other diagram contributions disappear owing to the following reasons: (i) the γ -algebra gives $(\gamma^-)^2 = 0$; (ii) the common pre-factor $T + yS$ goes to zero, (iii) the diagrams H2 and H5 cancel each other.

Analysing the results for the diagrams H1, H7, D4 and H10, we can see that

$$d\mathcal{W}(\text{dia.H1}) + d\mathcal{W}(\text{dia.H7}) + d\mathcal{W}(\text{dia.D4}) = d\mathcal{W}(\text{dia.H10}).$$

In other words, as similar to the Drell-Yan process, the new ("non-standard") contributions generated by the diagrams H1, H7 and D4 result again in the factor of 2 compared to the "standard" diagram H10 contribution to the corresponding hadron tensor. **This is our principle result.**

Conclusions and Discussions

- ▶ **Drell-Yan process:** (i) It is mandatory to include a contribution of the extra diagram which naively does not have an imaginary part; (ii) This additional contribution emanates from the complex gluonic pole prescription in the representation of the twist 3 correlator $B^V(x_1, x_2)$ which, in its turn, is directly related to the complex pole prescription in the quark propagator forming the hard part of the corresponding hadron tensor; (iii) The causal prescription in the quark propagator, involved in the hard part of the diagram on Fig.(a), selects from the physical axial gauges the contour gauge.

- ▶ **Direct Photon Production:** In contrast to DY, this process includes both **ISI** and **FSI** that leads to the different gluonic pole prescriptions in the diagrams under our consideration; In turn, the different gluonic pole prescriptions ensure the QCD gauge invariance.
- ▶ We observed the universality breaking, which spoils the standard factorization. However, the factorization procedure we proposed can still be applied for calculations.
- ▶ We find that the “non-standard” new terms, which exist in the case of the complex twist-3 B^V -function with the corresponding prescriptions, do contribute to the hadron tensor exactly as the “standard” term known previously. This is another important result of our work. We also observe that this is exactly similar to the case of Drell-Yan process.