Glueballs based on the inner structure of gauge field

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Outline

- Structure of Non-Abelian gauge field
- Glueball
- Glueball-Quarkonium Mixing
- Conclusion

I. Decomposition of Gauge Potential

Inner Structure of Gauge Field

Abelian Decomposition - $U(1)$ subgroup

- Yi-shi Duan and Mo-lin Ge, Sci. Sinica 11 (**1979**) 1072

$$\vec{A}_\mu = A_\mu \vec{n} + \partial_\mu \vec{n} \times \vec{n} + \vec{b}_\mu, \qquad SU(2)$$

- Yongmin Cho, PRD 21 (**1980**) 1080

$$\vec{A}_\mu = A_\mu \vec{n} + \partial_\mu \vec{n} \times \vec{n} + \vec{X}_\mu, \qquad SU(2)$$

- L. Faddeev and A. Niemi, PRL 82 (**1999**) 1624

$$\vec{A}_\mu = C_\mu \vec{n} + \partial_\mu \vec{n} \times \vec{n} + \rho \partial_\mu \vec{n} + \sigma \partial_\mu \vec{n} \times \vec{n}$$

Vacuum Decomposition - pure gauge

- Xiangsong Chen et al., PRL 100 (**2008**) 232002

$$\vec{A}_\mu = \vec{A}_\mu^{pure} + \vec{A}_\mu^{phys}, \quad \vec{F}_{\mu\nu}^{pure} = 0$$

What is the Abelian Decompistion?

- Definition of covariant derivative

$$D_\mu \vec{n} := \partial_\mu \vec{n} + \vec{A}_\mu \times \vec{n}, \qquad \vec{n}^2 = 1$$

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- Abelian Decomposition:

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = A_\mu \vec{n} + \partial_\mu \vec{n} \times \vec{n} + \vec{X}_\mu$$

$$\hat{A}'_\mu = U \hat{A}_\mu U^{-1} + U \partial_\mu U^{-1}, \qquad \vec{X}'_\mu = U \vec{X}_\mu U^{-1}$$

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$\hat{A}_\mu$ is the solution of $D_\mu \vec{n} = 0$.

Properties of Abelian Decomposition

- Gauge field

$$\vec{G}_{\mu\nu} = (F_{\mu\nu} + H_{\mu\nu})\vec{n} + \vec{G}_{\mu\nu}(\vec{X})$$

where No Self-Interaction for $\hat{A}_\mu$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad H_{\mu\nu} = \vec{n} \cdot (\partial_\mu \vec{n} \times \partial_\nu \vec{n}) = \partial_\mu C_\nu - \partial_\nu C_\mu$$

For $\vec{X}_\mu$ (with $\hat{D}_\mu = \partial_\mu + \hat{A}_\mu \times$)

$$\vec{G}_{\mu\nu}(\vec{X}) = \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu, \quad \mathit{Self - Interaction}!$$

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- Two different type gluons

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu$$

$\hat{A}_\mu$ is the **binding gluon**, corresponding to neutral gluons (similar to photons)
$\vec{X}_\mu$ is the **valence gluon**, carrying color charges

different gluons

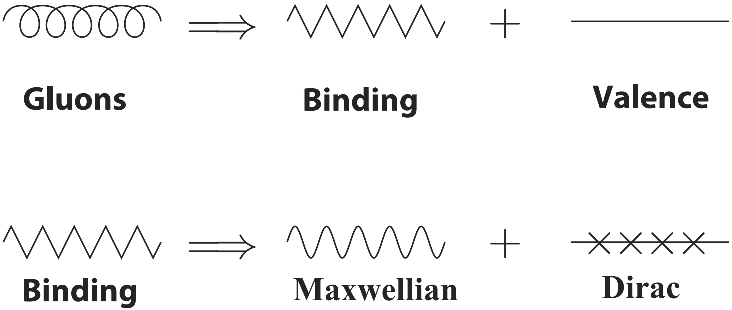

Figure: different gluons

Interaction based on Abelian Decomposition

Gauge field strength

$$\vec{G}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu$$

QCD Lagrangian can be reformulated by $\hat{A}_\mu$ and $\vec{X}_\mu$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 - \frac{1}{2}g\hat{F}_{\mu\nu}\cdot(\vec{X}_\mu \times \vec{X}_\nu) \\ & -\frac{1}{4}g^2(\vec{X}_\mu \times \vec{X}_\nu)^2 \\ & -\frac{1}{2}g(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)\cdot(\vec{X}_\mu \times \vec{X}_\nu).\end{aligned}$$

We can see 3,4 point vertex of QCD

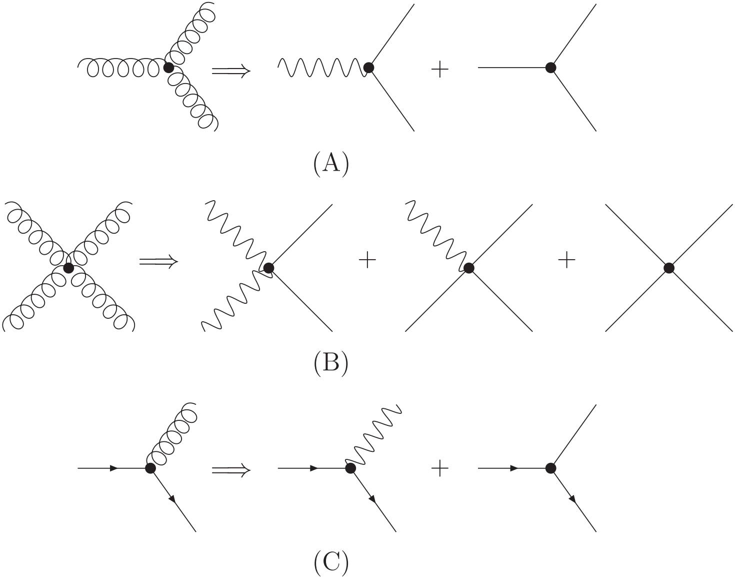

Figure: The possible vertices

Lattice results based on decomposition [1]

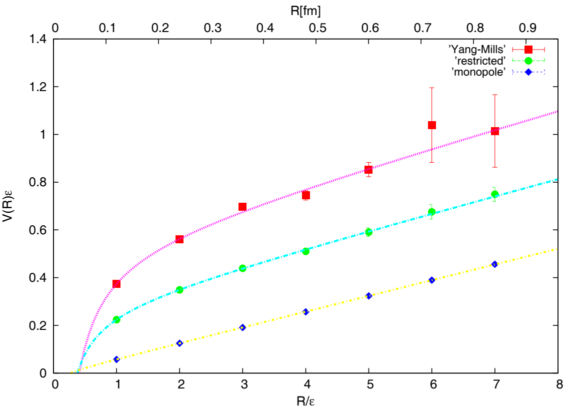

Figure: Potential $V(R)$ as function of R on 16^4 lattice at $\beta = 2.4$ where the Wilson loop with $T = 8$ was used for obtaining $V_{full}(R)$ and $T = 10$ for $V_{Ableian}(R)$ and $V_{mono}(R)$

[1] K.I. Kondo, et al., Phys. Rept. 579 (2015) 1

Related reference

1. Y. S. Duan and M. L. Ge, Sci. Sinica 11 (1979) 1072
2. Y. M. Cho, Phys. Rev. D 21 (1980) 1080; Phys. Rev. D 23 (1981) 2415
3. L. Faddeev and A. Niemi, Phys. Rev. Lett. 82 (1999) 1624
4. K.-I. Kondo, T. Murakami, and T. Shinohara, Prog. Theor. Phys. 115 (2006) 201
5. Y. M. Cho, F. H. Cho and J.H. Yoon, Phys. Rev. D 87, 085025 (2013).
6. K. -I. Kondo, Seikou Kato, Akihiro Shibata, Toru Shinohara, Phys. Rept. 579 (2015) 1.

II. Glueball

QCD: $SU(3)$

Decomposition of $SU(3)$ gauge potential

$$\vec{A}_\mu = \sum_{a=3,8} \underbrace{A_{\mu a}\vec{m}_a + \partial_\mu \vec{m}_a \times \vec{m}_a}_{\hat{A}_\mu} + \vec{X}_\mu,$$

For $SU(3)$, there are two Killing vectors: m_3 and m_8 satisfied

$$\hat{D}_\mu \vec{m}_a = (\partial_\mu + \hat{A}_\mu)\vec{m}_a = 0$$

$\hat{A}_\mu$ restricted gauge potential (Binding Gluons): 2 types
$\vec{X}_\mu$ Gauge-covariant part of gauge potential (Valence Gluons): 6 types

Chormons (Valence gluons)

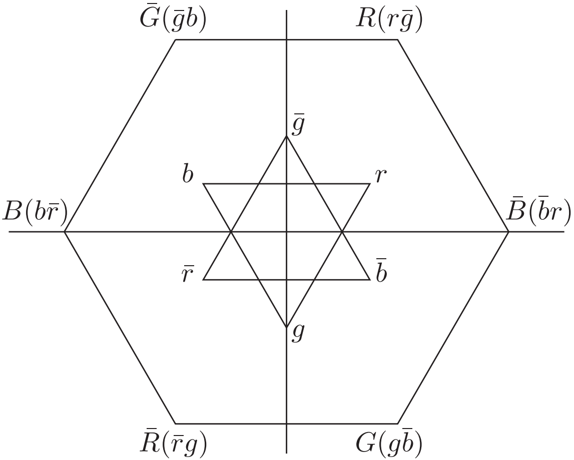

Figure: (r, g, b) are the colors of quarks, and $(R, B, G, \bar{R}, \bar{B}, \bar{G})$ are the colors of valence gluons

Usual Definition of Glueball

- NonAbelian gauge field theory suggests glueballs

$$G^a_{\mu\nu} G^{a\mu\nu}, \quad G^a_{\mu\nu} \tilde{G}^{a\mu\nu}, \quad f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}, \quad G^a_{\mu\nu} D_\delta G^a_{\alpha\beta}, \ldots$$

- States as $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ are forbidden in quark model, but theoretical study shows $g\bar{g}$ or ggg glueballs could have these quantum states. Lowlying $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ meson states could be pure glueballs.

$$\boxed{g \to G^a_{\mu\nu}}$$

Alternative Definition of Glueball

- From $\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu$, valence part $\vec{X}_\mu$ is covariant
- we have new gauge invariant construction:

$$X^a_\mu X^a_\mu, \quad \hat{D}_\mu X^a_\nu \hat{D}_\mu X^a_\nu, \quad f^{abc} X^a_\mu X^b_\nu \hat{D}_\mu X^c_\nu, \; \ldots$$

for $g\bar{g}$, ggg states.

$$\boxed{g \rightarrow X^a_\mu}$$

- Binding gluons $\hat{A}_\mu$ are responsible for the QCD confinement by monopole condensation.

New Glueball State: $g\bar{g}$, ggg

In the adjoint representation of SU(3) group, we can have six valence gluons $r\bar{b}(R)$, $b\bar{g}(B)$, $g\bar{r}(G)$, $\bar{r}b(\bar{R})$, $\bar{b}g(\bar{B})$, $\bar{g}r(\bar{G})$, two binding gluons $(r\bar{r} - b\bar{b})/\sqrt{2}$, $(r\bar{r} - b\bar{b} - 2g\bar{g})/\sqrt{6}$.
In detail, we have a clear picture to construct color singlet glueball

$$|g\bar{g}>= \frac{|R_\mu \bar{R}_\nu > + |B_\mu \bar{B}_\nu > + |G_\mu \bar{G}_\nu >}{\sqrt{3}}$$

$$|ggg>= \frac{\sum_{(RGB)} |R_\mu G_\nu B_\rho >}{\sqrt{6}}$$

Glueballs from the structure of gauge field

There may be three types of glueballs in QCD.

1. Electric glueball, the color singlet bound state of valence gluons. $(g\bar{g}, \quad ggg)$

2. Neutral glueball, made of binding gluons. since they do not carry color charge (like photon in QED), the interaction between them should be weak, so that they are not likely to form bound states.

3. Magnetic glueball, quasi-particles in monopole condensation.

III. Glueball-Quarkonium mixing

Mixture

In general, Glueball can be mixed with quarkoniums

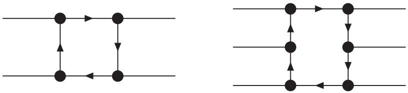

Figure: The possible glueball-quarkonuim mixing

it may not exist as mass eigenstates, which make it difficult to be identified in experiments.

However, there are states cannot easily be identified as $q\bar{q}$ states, e.g. $f_0(1500), f_0(1710), \eta(1405), \ \eta(1760)$...

Mixing matrix ($\bar{q}q$ mesons)

The standard mixing matrix for octet-singlet $\bar{q}q$ mesons is

$$M^2 = \begin{pmatrix} \langle 8|H|8\rangle & \langle 8|H|1\rangle \\ \langle 1|H|8\rangle & \langle 1|H|1\rangle \end{pmatrix} = \begin{pmatrix} E + \dfrac{2}{3}\Delta & -\dfrac{\sqrt{2}}{3}\Delta \\ -\dfrac{\sqrt{2}}{3}\Delta & E + \dfrac{1}{3}\Delta + 3A \end{pmatrix}.$$

By introducing the mixing between glueball $|G>$ and quarkonium $q\bar{q}$, the mass matrix is

$$M^2 = \begin{pmatrix} E + \frac{2}{3}\Delta & -\frac{\sqrt{2}}{3}\Delta & 0 \\ -\frac{\sqrt{2}}{3}\Delta & E + \frac{1}{3}\Delta + 3A & \nu \\ 0 & \nu & G \end{pmatrix},$$

$G = (2\mu)^2$, μ is the **constituent gluon mass**.

0^{++}

We choose $f_0(1500)$ and $f_0(1710)$ as the input states.

μ	A	ν	m_3	$m_1 = f_0(1500)$			$m_2 = f_0(1710)$			$m_3 = f_0(1370)$		
				$u+d$	s	g	$u+d$	s	g	$u+d$	s	g
0.76	0.27	0.18	1.40	0.07	0.00	**0.93**	0.73	0.20	0.07	0.19	0.80	0.00
0.78	0.23	0.31	1.40	0.26	0.01	0.73	0.59	0.16	0.25	0.15	0.83	0.02
0.80	0.18	0.36	1.39	0.44	0.01	0.54	0.45	0.12	0.43	0.11	0.87	0.02
0.82	0.14	0.35	1.39	0.62	0.02	0.36	0.30	0.08	0.62	0.09	0.90	0.01
0.84	0.09	0.29	1.39	0.79	0.02	0.18	0.15	0.04	0.80	0.05	0.93	0.01
0.86	0.04	0.07	1.39	0.96	0.03	0.01	0.01	0.00	**0.99**	0.03	0.97	0.00

$\mu = 0.76$, $f_0(1500)$ becomes mainly the **glueball state**

$\mu = 0.86$, $f_0(1710)$ becomes mainly the **glueball state**

2^{++}

We choose $f_2(1270)$ and $f_2(1950)$ as the input states.

μ	A	ν	m_3	$m_1 = f_2(1270)$			$m_2 = f_2(1950)$			$m_3 = f_2'(1525)$			R_{21}	R_{31}
				$u+d$	s	g	$u+d$	s	g	$u+d$	s	g		
0.76	0.39	0.95	1.47	0.40	0.00	0.60	0.35	0.36	0.29	0.25	0.64	0.11	0.19	0.15
0.78	0.35	0.99	1.47	0.46	0.01	0.53	0.33	0.33	0.34	0.22	0.66	0.12	0.25	0.18
0.80	0.31	1.01	1.48	0.52	0.01	0.47	0.30	0.30	0.40	0.18	0.69	0.12	0.33	0.21
0.82	0.28	1.02	1.48	0.58	0.01	0.41	0.27	0.27	0.46	0.15	0.72	0.13	0.43	0.24
0.84	0.24	1.02	1.49	0.64	0.01	0.36	0.24	0.24	0.52	0.13	0.75	0.12	0.57	0.27
0.86	0.20	0.99	1.49	0.69	0.01	0.30	0.20	0.21	0.59	0.10	0.78	0.11	0.76	**0.30**

Experimentally,

$$R_{31} = R(f_2'(1525)/f_2(1270)) = 0.31 \pm 0.05$$

With above $q\bar{q}$ - gg mixing picture, glue mass $= 0.86$ GeV, $f_0(1710)$ is glueball, which agree with result of Vento (arXiv:1505.05355)

0^{-+}

Since $|ggg>$ glueballs can also have 0^{-+} states, so that we may need to generalize the mixing matrix to 4×4 matix to include the $|ggg>$ glueball.

$$M^2 = \begin{pmatrix} E + \frac{2}{3}\Delta & -\frac{\sqrt{2}}{3}\Delta & 0 & 0 \\ -\frac{\sqrt{2}}{3}\Delta & E + \frac{1}{3}\Delta + 3A & \nu & \nu' \\ 0 & \nu & G & \epsilon \\ 0 & \nu' & \epsilon & G' \end{pmatrix} \quad (1)$$

gluon mass μ, $G = 4\mu^2$, $G' = 9\mu^2$, $\nu' = 3/2\nu$.

Choosing $\eta'(958)$, $\eta(1405)$, $\eta(1760)$ as the input,

μ	m_4	$m_1 = \eta'(958)$				$m_2 = \eta(1405)$				$m_3 = \eta(1760)$				m_4			
		$u+d$	s	$2g$	$3g$	$u+d$	s	$2g$	$3g$	$u+d$	s	$2g$	$3g$	$u+d$	s	$2g$	$3g$
0.50	0.55	0.02	0.03	0.93	0.02	0.13	0.11	0.05	0.72	0.43	0.30	0.01	0.26	0.43	0.57	0.00	0.00
0.50	0.55	0.01	0.01	0.96	0.03	0.16	0.13	0.01	0.70	0.41	0.28	0.04	0.27	0.43	0.57	0.00	0.00
0.52	0.54	0.04	0.07	0.85	0.04	0.20	0.17	0.13	0.50	0.31	0.22	0.01	0.46	0.45	0.54	0.00	0.00
0.52	0.55	0.00	0.01	0.92	0.07	0.29	0.25	0.01	0.45	0.26	0.18	0.07	0.48	0.44	0.56	0.00	0.00
0.54	0.54	0.06	0.12	0.76	0.06	0.26	0.22	0.23	0.28	0.20	0.14	0.00	0.66	0.47	0.52	0.01	0.00
0.54	0.54	0.00	0.00	0.88	0.11	0.44	0.37	0.01	0.19	0.11	0.08	0.11	0.70	0.45	0.55	0.00	0.00

We may identify $m_4 = 0.55$ GeV as $\eta(548)$ [2].

[2] See detail, PRD 91, 114020 (2015) (arXiv:1503.08890)

Conclusion

- Gluons = 2 binging gluons + 6 valence gluons

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu$$

- 2 binding gluons are responsible for the confinement potential; 6 valence gluons play the same role as valence quarks

- Glueball can mix with quarkonium, with the mixing matrix we obtained **constituent gluon mass** is $0.86 GeV$ and suggested that $f_0(1710)$ can be the candidates of glueball.

THANKS!