Hadronic light-by-light contribution to muon anomalous magnetic moment in nonlocal quark model

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9th APCTP-BLTP JINR Joint workshop
Almaty, Kazakhstan, 2 July 2015

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Plan

1. Motivation
2. Lagrangian of nonlocal quark model
3. External fields. Multi-photon vertices
4. Light-by-light hadronic contribution to the muon AMM in leading $1/N_c$ order
   4.1 Resonance contribution
   4.2 Contact contribution
5. Further extensions
6. Conclusions

Motivation

Cosmology tell us that 95% of matter is not described in text-books yet. Dark Matter surrounds us! Where it is ? *(A.Gladyshev talk)*

Two search strategies

1. High energy physics to excite heavy degrees of freedom. No any evidence till now. We live in LHC era!

2. Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.

Anomalous magnetic moment of the muon $(g-2)_\mu$ is most famous and stable example

Motivation

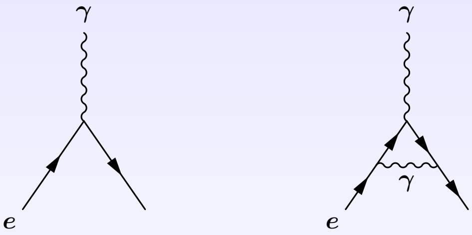
Dirac Equation Predicts for free point-like spin $\frac{1}{2}$ charged particle:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{p^2}{2m} - \frac{e}{2m}\left(\overrightarrow{L} + 2\,\overrightarrow{S}\right)\cdot\overrightarrow{B}\right]\psi$$

$g = 2$, $a = (g-2)/2 = 0$ (no anomaly at tree level)

a becomes nonzero due to interactions resulting in fermion substructure

Motivation. One loop QED radiative correction



$$\Gamma_\mu = e\gamma_\mu + a\frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

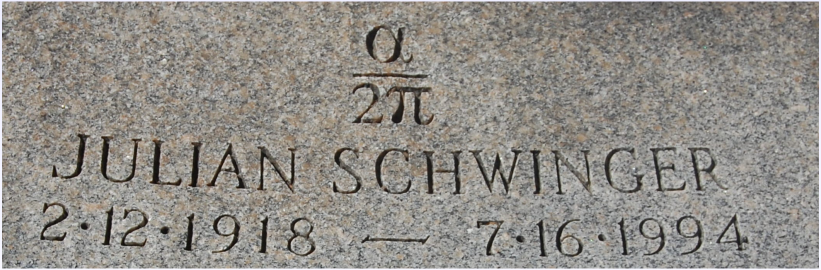
$$a_e = \frac{\alpha}{2\pi} = 0.001162,$$ [1]

$$a_e^{exp} = 0.001145 \pm 0.00004,$$ [2]

[1] J. S. Schwinger, Phys. Rev. **73** (1948) 416.

[2] H. M. Foley and P. Kusch, Phys. Rev. **72**, 1256 (1947).

Motivation. One loop QED radiative correction



Anomalous magnetic momentum of electron.

1. To measurable level a_e arises entirely from virtual electrons and photons

$$a_e^{\text{Harvard}} = 1\ 159\ 652\ 180.73\ (0.28) \times 10^{-12} \quad [0.24\ \text{ppb}].^3$$

2. In standard model

$$a_e = \left\{a_e^{\text{QED}} + a_e^{\text{weak}} + a_e^{\text{hadr}}\right\}^{\text{SM}}, \quad a_e^{\text{QED}} = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)},$$

3. This result leads to the determination of the fine structure constant α with the extraordinary precision [4]

$$\alpha^{-1} = 137.0359991570(29)(27)(18)(331)$$

where uncertainties are from the eighth-order, tenth-order, and hadronic and EW terms, and the measurement of a_e.

[3] D. Hanneke, S. Fogwell and G. Gabrielse, PRL **100** , 120801 (2008).

[4] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, PRD **91**, 033006 (2015).

Motivation. Anomalous magnetic momentum of muon.

1. Anomalous magnetic momentum of muon $a_\mu = (g-2)_\mu$ is measured in experiment E821 (BNL) with high precision[5]

$$a_\mu^{\rm exp} = 11\,659\,209.1(6.3)\cdot 10^{-10}$$

2. The nonzero lepton AMMs are induced by radiative corrections. In the SM are induced by QED, weak and strong (hadronic) interactions.

$$a_\mu = \left\{a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadr}\right\}^{\rm SM} + ???$$

3. Tenth-order QED contribution[6] to a_μ

$$a_\mu^{\rm QED} = 11\ 658\ 471.8951\ (0.0080)\times 10^{-10}$$

4. Weak contribution[7]

$$a_\mu^{\rm weak} = 15.36\ \ (0.1)\times 10^{-10}$$

[5] G.W.Bennett,et al.PRD73,072003(2006); P.J.Mohr,et al.RMP84,1527(2012).

[6] T.Aoyama, M.Hayakawa, T.Kinoshita, M.Nio, PRL 109, 111807 (2012).

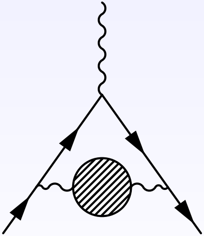
[7] A.Czarnecki, W.J.Marciano, A.Vainshtein, PRD67, 073006 (2003); C.Gnendiger, D.Stockinger and H.Stockinger-Kim, PRD88, 053005 (2013)

Motivation. Anomalous magnetic momentum. HVP

5. Strong contribution separated into three terms

$$a_\mu^{\rm hadr} = a_\mu^{\rm HVP,LO} + (a_\mu^{\rm HVP,NLO} + a_\mu^{\rm HVP,NNLO} + ..) + a_\mu^{\rm HLbL}$$

- Contribution of hadron vacuum polarization can be extracted from experimental data for process $e^+e^- \to$ in hadrons (or hadronic τ-lepton decays)



$$a_\mu^{\rm HVP,LO} = \begin{cases} 692.3\ \ (4.2) \times 10^{-10}, & ^{8} \\ 694.91\ \ (4.27) \times 10^{-10}. & ^{9} \end{cases}$$

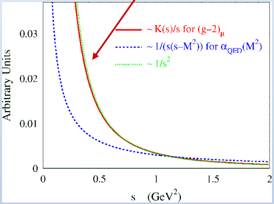
[8] M.Davier, A.Hoecker, B.Malaescu, Z.Zhang, EPJC 71, 1515 (2011)

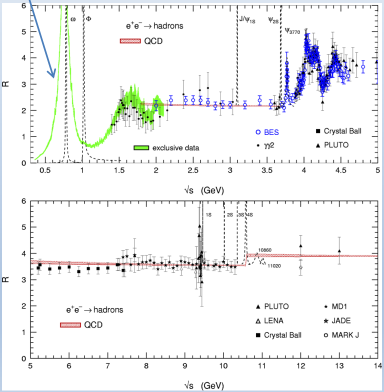
[9] K.Hagiwara,R.Liao,A.D.Martin,D.Nomura,T.Teubner, JPG 38, 085003 (2011)

II. Leading Order Hadronic contributions

$$a_\mu^{\mathrm{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} \, R^{(0)}(s)$$

Dispersion relation, uses unitarity (optical theorem) and analyticity (Bouchiat and Michel, 1961)





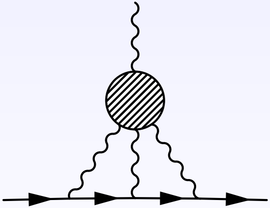
$$R(s) = \frac{\sigma\left[e^+e^- \to \mathrm{hadrons}\right]}{\sigma\left[e^+e^- \to \mu^+\mu^-\right]}$$

Motivation. Anomalous magnetic momentum. HLbL

6. Higher orders hadronic contribution to HVP [10, 11]

$$
\begin{aligned}
a_\mu^{\text{HVP,NLO}} &= -9.84(0.06)(0.04) \times 10^{-10}, \\
a_\mu^{\text{HVP,NNLO}} &= 1.24(0.01) \times 10^{-10}.
\end{aligned}
$$

7. The "guessed" value for the hadronic light-by-light contribution [12]



$$a_\mu^{\text{HLbL}}(\text{Guess}) = 10.5\ \ (2.6) \times 10^{-10}$$

[10] K.Hagiwara,R.Liao,A.D.Martin,D.Nomura,T.Teubner, JPG 38, 085003 (2011)

[11] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, PLB 734, 144 (2014).

[12] J.Prades, E.de Rafael, A.Vainshtein, in *Advanced series on directions in high energy physics, Vol. 20* [arXiv:0901.0306 [hep-ph]].

Motivation. Anomalous magnetic momentum.

8. Combining all SM contributions one obtains

$$a_\mu^{\rm SM} = 116\ 591\ 84.1\ (5.0) \times 10^{-10}$$

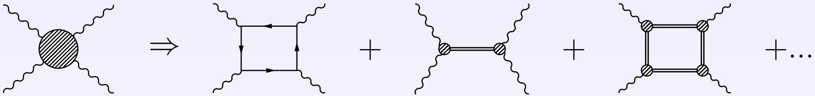
9. The resulting difference between the experimental result and the full SM prediction is

$$a_\mu^{\rm BNL} - a_\mu^{\rm SM} = 25.0\ \ (8.04) \times 10^{-10},$$

which signals an 3.11 σ discrepancy between theory and experiment.
10. The new $(g-2)$ experiments E989 (at Fermilab) and g-2/EDM (E34) (at J-PARC) are expected more precise than BNL.

Motivation. Anomalous magnetic momentum. HLbL

The SM theoretical error is dominated by the hadronic contributions. Theoretical predictions of HVP and HLbL contributions to a_μ should be of the same level or better than the precision of planed experiments.



LbL scattering amplitude is a complicated object. It is a sum of different diagrams, the quark loop, the meson exchanges, the meson loops and the iterations of these processes. However, there is hierarchy connected to existence of two small parameters: the inverse number of colors $1/N_c$ and the ratio of the characteristic internal momentum to the chiral symmetry parameter $m_\mu/(4\pi f_\pi) \sim 0.1$.

Lagrangian

The Lagrangian of the nonlocal model has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH}$$

$$\mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - m_c)q(x)$$

m_c – current quark mass matrix with diagonal elements $m_c^u = m_c^d$, m_c^s

$$\begin{aligned} \mathcal{L}_{4q} &= \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)] \\ \mathcal{L}_{tH} &= -\frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_P^a(x)J_P^b(x)J_P^c(x)] \end{aligned}$$

Nonlocal quark currents are

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x-x_1)\,\Gamma_M^a q(x+x_2),$$

where $M = S, P$ and $\Gamma_S^a = \lambda^a$, $\Gamma_P = i\gamma^5\lambda^a$, and $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum.

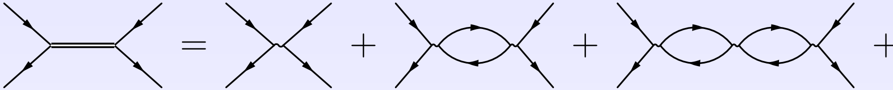
Lagrangian

The model can be bosonized using the stationary phase approximation which leads to the system of gap equations for the dynamical quark masses $m_{d,i} \quad (i = u, d, s)$

$$
\begin{aligned}
m_{d,u} + G S_u + \frac{H}{2} S_u S_s &= 0, \\
m_{d,s} + G S_s + \frac{H}{2} S_u^2 &= 0, \\
S_i = -8 N_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{f^2(k^2) m_i(k^2)}{D_i(k^2)} &,
\end{aligned}
$$

where $m_i(k^2) = m_{c,i} + m_{d,i} f^2(k^2)$ is the dynamical quark mass, $D_i(k^2) = k^2 + m_i^2(k^2)$, $f(k^2)$ is the nonlocal form factor in the momentum representation.

T matrix



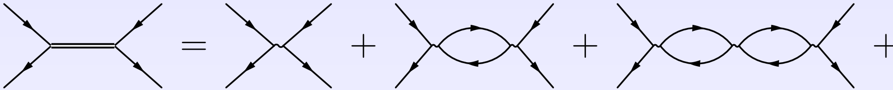
The vertex functions and the meson masses can be found from the Bethe-Salpeter equation. For the separable interaction the quark-antiquark scattering matrix in pseudoscalar channel becomes

$$\mathbf{T} = \hat{\mathbf{T}}(p^2)\delta^4\left(p_1 + p_2 - (p_3 + p_4)\right)\prod_{i=1}^{4} f(p_i^2),$$

$$\hat{\mathbf{T}}(p^2) = i\gamma_5\lambda_k\left(\frac{1}{-\mathbf{G}^{-1} + \mathbf{\Pi}(p^2)}\right)_{kl} i\gamma_5\lambda_l,$$

where p_i are the momenta of external quark lines, $\mathbf{G}$ and $\mathbf{\Pi}(p^2)$ are the corresponding matrices of the four-quark coupling constants and the polarization operators of pseudoscalar mesons $(p = p_1 + p_2 = p_3 + p_4)$.

T matrix



The meson masses can be found from the zeros of determinant $\det(\mathbf{G}^{-1} - \mathbf{\Pi}(-M^2)) = 0$. The $\hat{\mathbf{T}}$-matrix for the system of mesons in each neutral channel can be expressed as

$$\hat{\mathbf{T}}_{ch}(P^2) = \sum_{M_{ch}} \frac{\overline{V}_{M_{ch}}(P^2) \otimes V_{M_{ch}}(P^2)}{-(P^2 + \mathrm{M}^2_{M_{ch}})},$$

where M_M are the meson masses, $V_M(P^2)$ are the vertex functions $\left(\overline{V}_M(p^2) = \gamma^0 V_M^\dagger(P^2)\gamma^0\right)$. The sum is over full set of light mesons: $(M_{PS} = \pi^0, \eta, \eta')$ in the pseudoscalar channel and $(M_S = a_0(980), f_0(980), \sigma)$ in the scalar one.

External fields

The gauge-invariant interactions with external photon field can be introduced with Schwinger phase factor

$$q(y) \to Q(x,y) = \mathcal{P}\exp\left\{ i \int\limits_x^y dz^\mu V_\mu^a(z) T^a \right\} q(y),$$

apart from kinetic term the additional terms in nonlocal interations are generated

$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{Q}(x - x_1, x)\, \Gamma_I\, Q(x, x + x_2)$$

External fields

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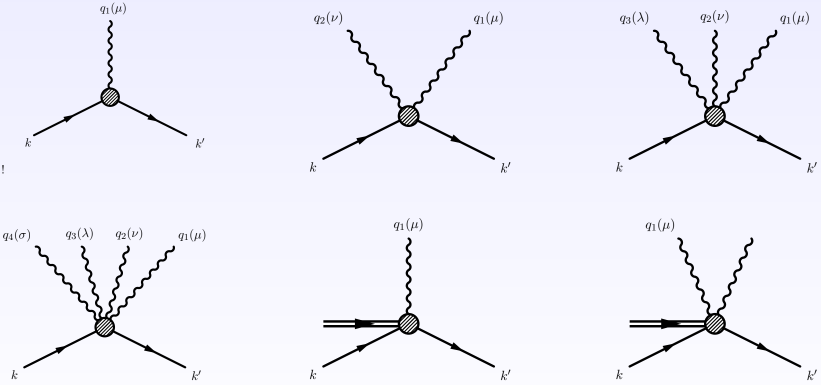
The following equations are used for obtaining of nonlocal vertices

$$\frac{\partial}{\partial y^\mu} \int\limits_x^y dz^\nu \; F_\nu(z) = F_\mu(y), \qquad \delta^{(4)}(x - y) \int\limits_x^y dz^\nu \; F_\nu(z) = 0.$$

Nonlocal vertices

As a result the nonlocal vertices with arbitrary number of photon fields are generated



One-photon–quark–antiquark vertex

Quark-antiquark nonlocal vertex with one photon line

$$\Gamma_\mu(q_1) = -(k+k_1)_\mu m^{(1)}(k,k_1)$$

Here, and below $k_1 = k + q_1$, $k_{ij..k} = k + q_i + q_j + .. + q_k$ and eQ omitted. k is a momentum of incoming quark, and q_i momenta of incoming photons. The first-order finite difference is introduced

$$f^{(1)}(a,b) = \frac{f(a+b) - f(a)}{(a+b)^2 - a^2}$$

Combination with local vertex γ_μ satisfies the Ward identity for dynamical quarks.

Two-photons–quark–antiquark vertex

With two lines

$$\begin{aligned}\Gamma_{\mu\nu}(q_1,q_2) &= 2g_{\mu\nu}m^{(1)}(k,k_{12}) \\ &\quad + (k+k_1)_\mu(k_1+k_{12})_\nu m^{(2)}(k,k_1,k_{12}) \\ &\quad + (k+k_2)_\nu(k_2+k_{12})_\mu m^{(2)}(k,k_2,k_{12})\end{aligned}$$

$$f^{(2)}\left(a,b_1,b_2\right) = \frac{f^{(1)}\left(a,b_1\right) - f^{(1)}\left(a,b_2\right)}{\left(a+b_1\right)^2 - \left(a+b_2\right)^2},$$

Three-photons–quark–antiquark vertex

with three lines

$$\Gamma_{\mu\nu\rho}(q_1, q_2, q_3) = .. + 2g_{..}(k + k_b)_{..} m^{(2)}(k, k_b, k_{123})..$$
$$.. + 2g_{..}(k_b + k_{123})_{..} m^{(2)}(k, k_b, k_{123})..$$
$$.. + (k + k_b)_{..}(k_b + k_c)_{..}(k_c + k_{123})_{..} m^{(3)}(k, k_b, k_c, k_{123})..$$

$$f^{(n)}\left(a, \{b_i\}, b_1, b_2\right) = \frac{f^{(n-1)}\left(a, \{b_i\}, b_1\right) - f^{(n-1)}\left(a, \{b_i\}, b_2\right)}{\left(a + b_1\right)^2 - \left(a + b_2\right)^2}.$$

Four-photons–quark–antiquark vertex

with four lines

$$\Gamma_{\mu\nu\rho\tau}(q_1, q_2, q_3, q_4) = .. + 4g_{..}g_{..}m^{(2)}(k, .., k_{1234})$$
$$+ 2g_{..}(k + k_b)_{..}(k_b + k_c)_{..}m^{(3)}(k, k_b, k_c, k_{1234})$$
$$+ 2g_{..}(k + k_b)_{..}(k_c + k_{1234})_{..}m^{(3)}(k, k_b, k_c, k_{1234})$$
$$+ 2g_{..}(k_b + k_c)_{..}(k_c + k_{1234})_{..}m^{(3)}(k, k_b, k_c, k_{1234})$$
$$+ (k + k_b)_{..}(k_b + k_c)_{..}(k_c + k_d)_{..}(k_d + k_{1234})_{..}m^{(4)}(k, k_b, k_c, k_d, k_{1234})$$

Multi-photon vertices. Gauge simplification.

$$
\begin{aligned}
\Gamma_{\mu} q_1^{\mu} =& m_k - m_{k_1} \\
\Gamma^{\mu\nu} q_1^{\mu} q_2^{\nu} =& m_k + m_{k_{12}} - m_{k_1} - m_{k_2} \\
\Gamma^{\mu\nu\rho} q_1^{\mu} q_2^{\nu} q_3^{\rho} =& m_k - m_{k_{123}} - m_{k_1} - m_{k_2} - m_{k_3} \\
& + m_{k_{12}} + m_{k_{13}} + m_{k_{23}} \\
\Gamma^{\mu\nu\rho\tau} q_1^{\mu} q_2^{\nu} q_3^{\rho} q_4^{\tau} =& m_k + m_{k_{1234}} + m_{k_{12}} + m_{k_{13}} \\
& + m_{k_{14}} + m_{k_{34}} + m_{k_{23}} + m_{k_{24}} \\
& - m_{k_1} - m_{k_2} - m_{k_3} - m_{k_4} \\
& - m_{k_{123}} - m_{k_{124}} - m_{k_{134}} - m_{k_{234}}
\end{aligned}
$$

N_c counting rules.

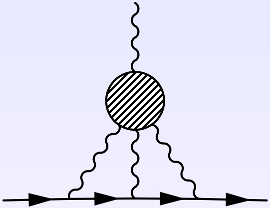
In order to have correspondence with QCD the quark mass should scale as N_c^0 for large number of colors

$$m_d = GN_c \cdot 8 \int \frac{d_E^4 k}{(2\pi)^4} f^2(k) \frac{m(k)}{k^2 + m^2(k)}$$

This means that four-quark coupling constant should scales as $G \sim 1/N_c$. As a result meson propagator leads to $1/N_c$ suppression of diagrams

$$\mathrm{D}_p^{\mathrm{M}} = \frac{1}{-G^{-1} + \Pi_p^{\mathrm{M}}} \rightarrow \frac{1}{N_c},$$

Light-by-light hadronic contribution to the muon AMM



Muon AMM can be extracted by using the projection

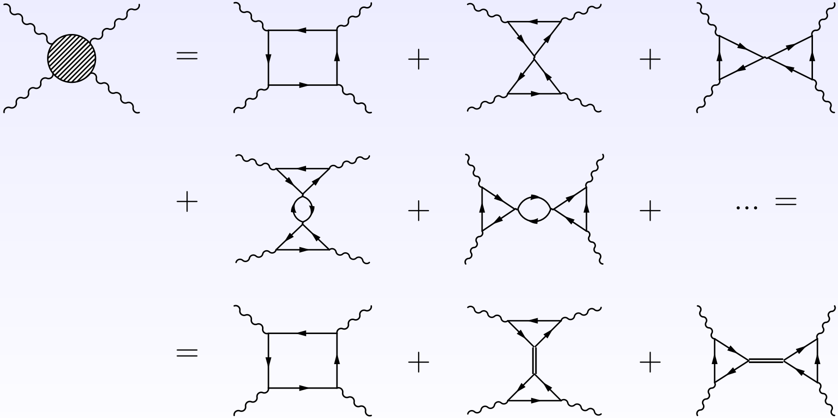
$$a_\mu^{\mathrm{HLbL}} = \frac{1}{48 m_\mu} \mathrm{Tr}\left((\hat{p} + m_\mu)[\gamma^\rho, \gamma^\sigma](\hat{p} + m_\mu)\Pi_{\rho\sigma}(p,p) \right),$$

$$\Pi_{\rho\sigma}(p', p) = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_2^2 (q_1 + q_2)^2 (q_1 + k)^2} \times$$
$$\times \gamma^\mu \frac{\hat{p}' - \hat{q}_2 + m_\mu}{(p' - q_2)^2 - m_\mu^2} \gamma^\nu \frac{\hat{p} + \hat{q}_1 + m_\mu}{(p + q_1)^2 - m_\mu^2} \gamma^\lambda \times$$
$$\times \frac{\partial}{\partial k_\rho} \Pi_{\mu\nu\lambda\sigma}(q_2, -(q_1 + q_2), k + q_1, -k),$$

m_μ is the muon mass, $k_\mu = (p' - p)_\mu$, static limit $k_\mu \to 0$.

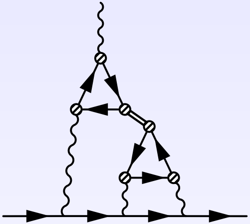
Four-rank polarization tensor

To the leading $1/N_c$ order four-rank polarization tensor $\Pi_{\mu\nu\lambda\sigma}$ can be represented in the form [13]



[13]The nonlocal multi-photon vertices are not shown for simplicity.

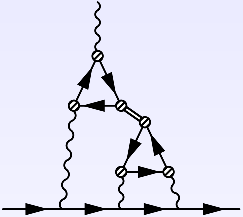
HLbL resonance contribution to the muon AMM



HLbL resonance contribution to the muon AMM

$$\Pi^{\mu\nu\lambda\rho}(q_1,q_2,q_3) =$$
$$i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)\Delta^{\lambda\rho}(q_1+q_2,q_3,q_1+q_2+q_3)}{(q_1+q_2)^2-M^2}+$$
$$+\,i\frac{\Delta^{\mu\rho}(q_2+q_3,q_1,q_1+q_2+q_3)\Delta^{\nu\lambda}(q_2+q_3,q_2,q_3)}{(q_2+q_3)^2-M^2}+$$
$$+\,i\frac{\Delta^{\mu\lambda}(q_1+q_3,q_1,q_3)\Delta^{\nu\rho}(q_1+q_3,q_2,q_1+q_2+q_3)}{(q_1+q_3)^2-M^2},$$

HLbL resonance contribution to the muon AMM



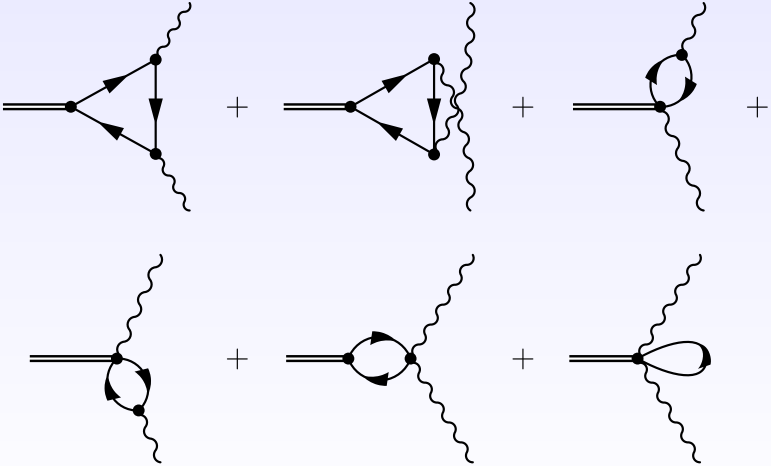
$$\frac{\partial}{\partial k^\rho}\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2) =$$

$$i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)}{(q_1+q_2)^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\lambda\sigma}(q_1+q_2,-q_1-q_2,k)$$

$$+\, i\frac{\Delta^{\nu\lambda}(-q_1,q_2,-q_1-q_2)}{q_1^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\mu\sigma}(-q_1,q_1,k)$$

$$+\, i\frac{\Delta^{\mu\lambda}(-q_2,q_1,-q_1-q_2)}{q_2^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\nu\sigma}(-q_2,q_2,k)+O(k)$$

Meson–photon–photon transition amplitude



Two-photon–pseudoscalar meson. I

Triangular diagram with external pseudoscalar meson and two photon legs with arbitrary virtualities can be written as

$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)} \to P^*_{(p)}\right) = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma \mathrm{F}_{P^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right),$$

$$\mathrm{F}_{\pi_0^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = g_\pi(p^2)F_u\left(p^2;q_1^2,q_2^2\right),$$

$$\mathrm{F}_{\eta^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_\eta(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right) - 2F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2) -$$

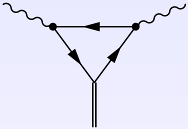
$$-\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right) + F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)\Big],$$

$$\mathrm{F}_{\eta'^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_{\eta'}(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right) - 2F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2) +$$

$$+\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right) + F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)\Big].$$

Two-photon–pseudoscalar meson. II



$$F_i\left(p^2;q_1^2,q_2^2\right) = 8\int\frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k_2^2)}{D_i(k_1^2)D_i(k_2^2)D_i(k^2)}\times$$

$$\times\left[m_i(k^2) - \mathrm{m}_i^{(1)}(k_1,k)J_1 - \mathrm{m}_i^{(1)}(k_2,k)J_2\right],$$

$$J_1 = k^2 + \frac{q_2^2(kq_1)(k_1q_1) - q_1^2(kq_2)(k_1q_2)}{q_1^2q_2^2-(q_1q_2)^2},$$

$$J_2 = k^2 + \frac{q_1^2(kq_2)(k_2q_2) - q_2^2(kq_1)(k_2q_1)}{q_1^2q_2^2-(q_1q_2)^2},$$

where $k_1 = k + q_1,\ k_2 = k - q_2$.

Two-photon–scalar meson. I

Triangular diagram with external scalar meson and two photon legs with arbitrary virtualities can be written as

$$A\left(\gamma^*_{(q_1,\mu)}\gamma^*_{(q_2,\nu)} \to S^*_{(p)}\right) = e^2 \Delta^{\mu\nu}_{S^*\gamma^*\gamma^*}(q_3,q_1,q_2) =$$

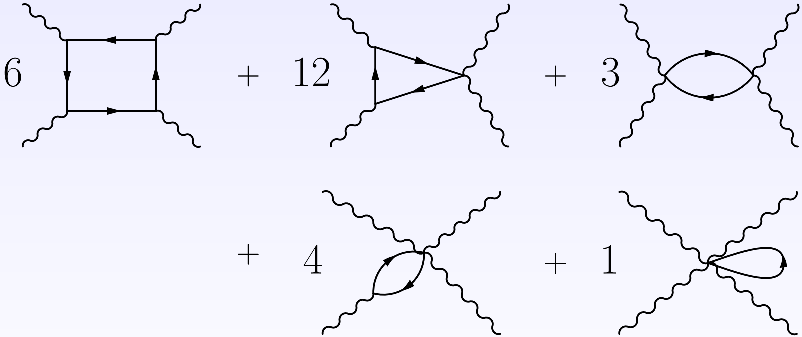
$$= e^2 \Big[A_{S^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) T_A^{\mu\nu}(q_1,q_2)$$

$$+ B_{S^*\gamma^*\gamma^*}(p^2;q_1^2,q_2^2) T_B^{\mu\nu}(q_1,q_2) \Big],$$

$$T_A^{\mu\nu}(q_1,q_2) = \left(g^{\mu\nu}(q_1\cdot q_2) - q_1^\nu q_2^\mu\right)$$

$$T_B^{\mu\nu}(q_1,q_2) = \left(q_1^2 q_2^\mu - (q_1\cdot q_2) q_1^\mu\right)\left(q_2^2 q_1^\nu - (q_1\cdot q_2) q_2^\nu\right),$$

HLbL contact contribution to the muon AMM



HLbL contact contribution to muon AMM. Crosscheck

Analytical check of gauge invariance

$$\Pi^{\mu\nu\lambda\rho}(q_1, q_2, q_3) q_1^\mu q_2^\nu q_3^\lambda (q_1 + q_2 + q_3)^\rho = \int d^4 l \sum_i F_i(l, q_1, q_2, q_3)$$

By shifting l one can show that $\sum_i F_i(l, q_1, q_2, q_3) = 0$ for any quark mass function.

Numerical check of gauge invariance

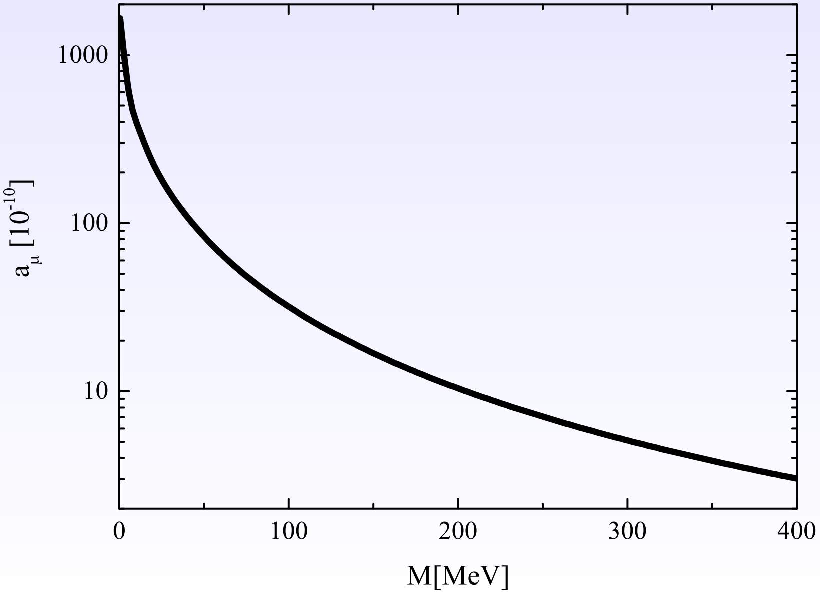
Longitudinal projection of derivative of polarization tensor should be machine zero.

$$\frac{\partial}{\partial k^\rho} \Pi^{\mu\nu\lambda\sigma}(q_1, q_2, k - q_1 - q_2) q_1^\mu q_2^\nu (q_1 + q_2)^\lambda \Big|_{k \to 0} = 0$$

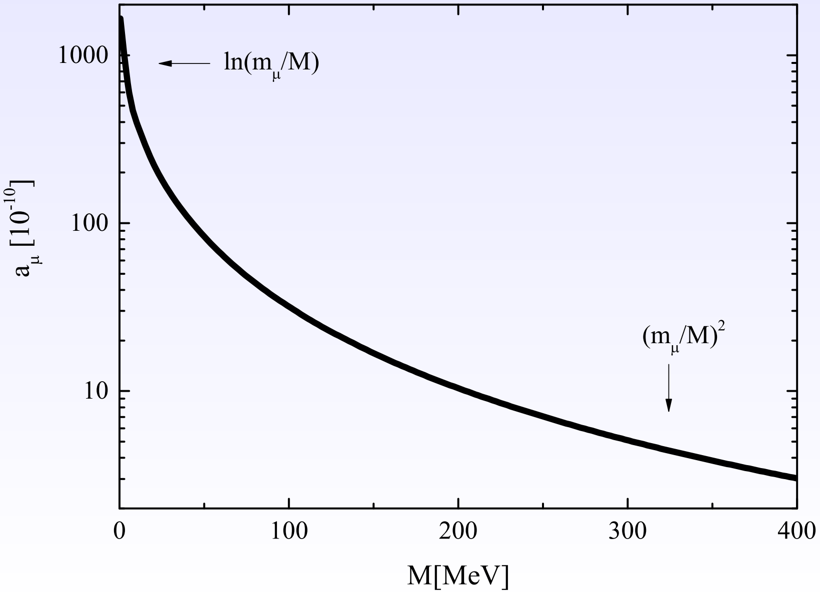
Local limit crosscheck

In the local limit $\Lambda \to \infty$ (or $m_d \to 0$) one should reproduce the result of the usual lepton loop LbL contribution, e.g. e, μ, τ contribution.

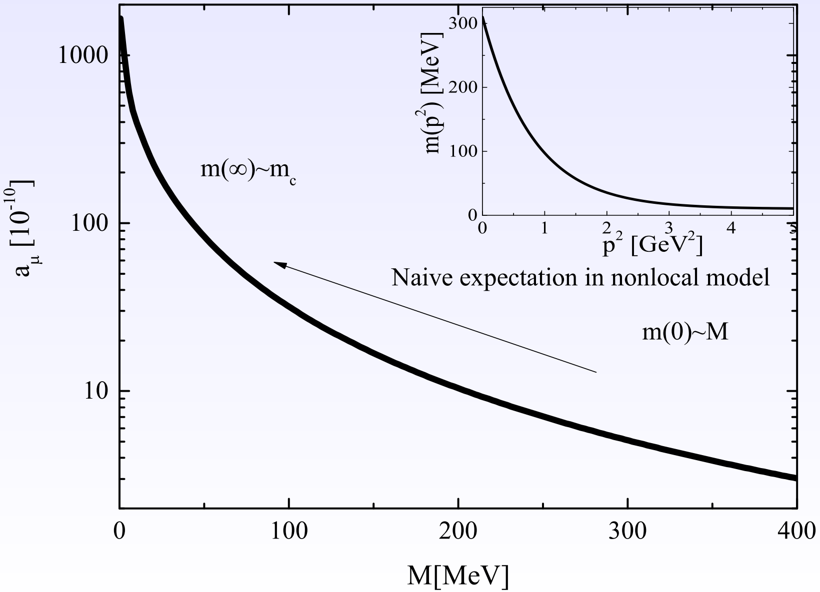
HLbL local contact contribution to muon AMM



HLbL local contact contribution to muon AMM



HLbL local contact contribution to muon AMM



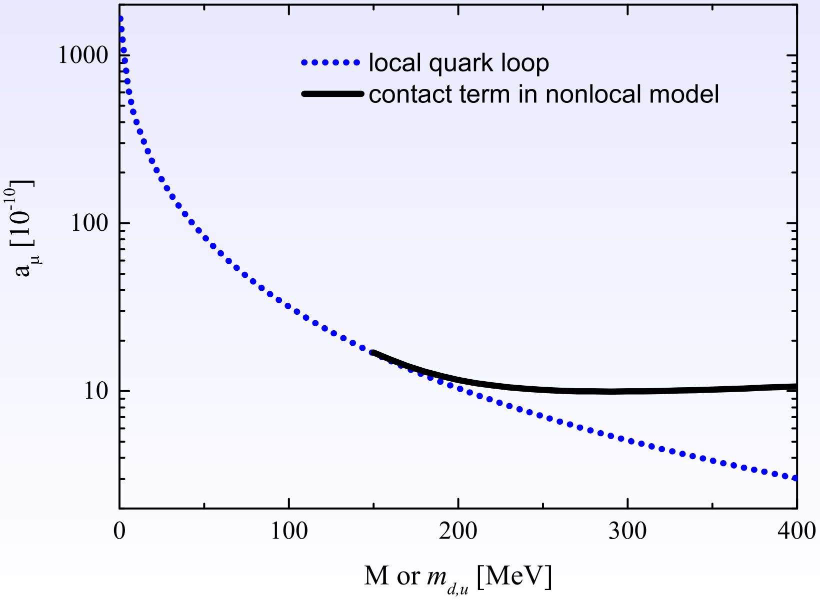
Light-by-light hadronic contribution to the muon AMM. Estimation of model dependence.

- Nonstrange contributions (u, d-quark loop, π and σ mesons) can be estimated in the framework of $SU(2)$ model[14]. Model has only three parameters: current quark mass $m_{c,u}$, dynamical quark mass $m_{d,u}$ and nonlocality parameter Λ. We fix model parameters to M_{π^0}, two-photon pion width $\Gamma_{\pi^0\gamma\gamma}$ and vary dynamical mass in region 200–350 MeV.
- For strange sector (s-quark loop, η,η', $a_0(980)$,$f_0(980)$) in $SU(3)$ model we fit two additional parameters ($m_{c,s}$ and $m_{d,s}$) to kaon mass M_{K^0} and $\Gamma_{\eta\gamma\gamma}$ (with reasonable M_η).
- Check results for model parameters fitted in other works [15].

[14] We take Gaussian form-factor $f(k^2) = \exp(-k^2/\Lambda^2)$.

[15] A.Scarpettini,D.Gomez Dumm,N.N.Scoccola, PRD 69, 114018 (2004).

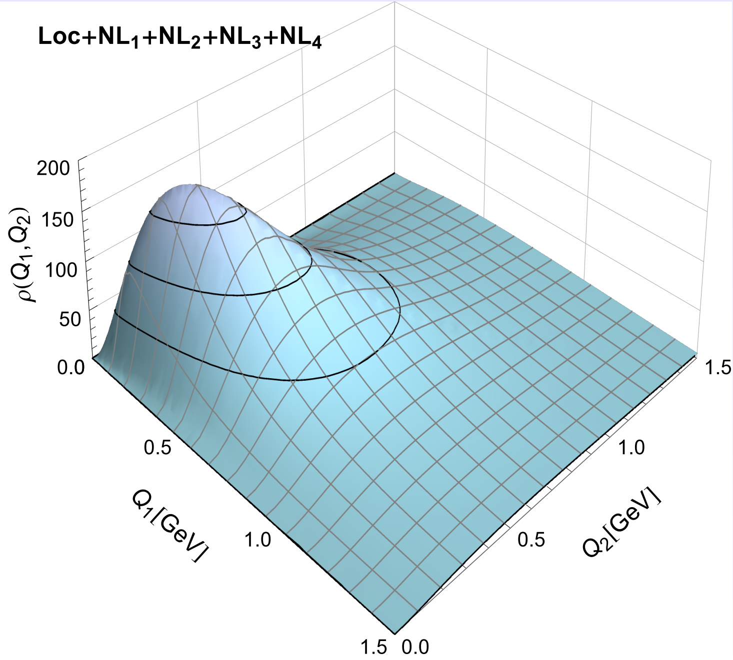
HLbL contact contribution to muon AMM



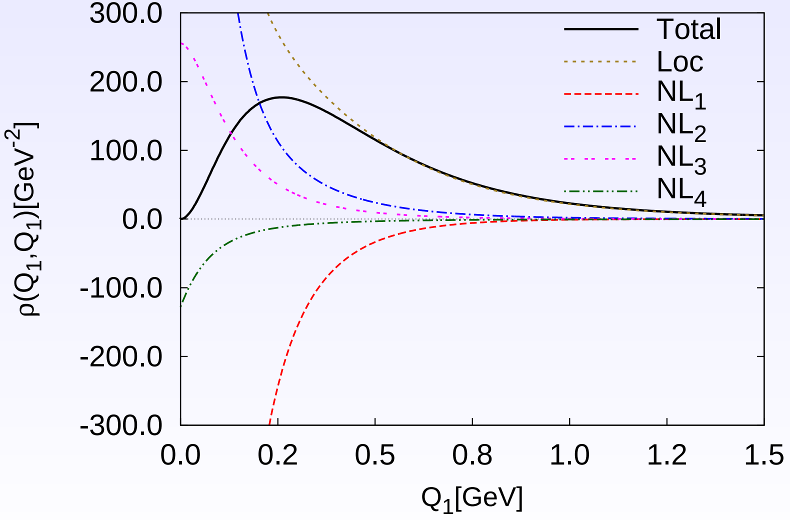
HLbL contact contribution to muon AMM. Density

It is instructive to investigate "density" which is defined by

$$a_\mu^{\mathrm{LbL}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \quad \rho^{\mathrm{LbL}}(Q_1, Q_2)$$


Loc+NL1+NL2+NL3+NL4
200
150
100
50
0.0
ρ(Q1,Q2)
0.5
1.0
1.5
Q1[GeV]
0.0
0.5
1.0
1.5
Q2[GeV]

HLbL contact contribution to muon AMM. Density

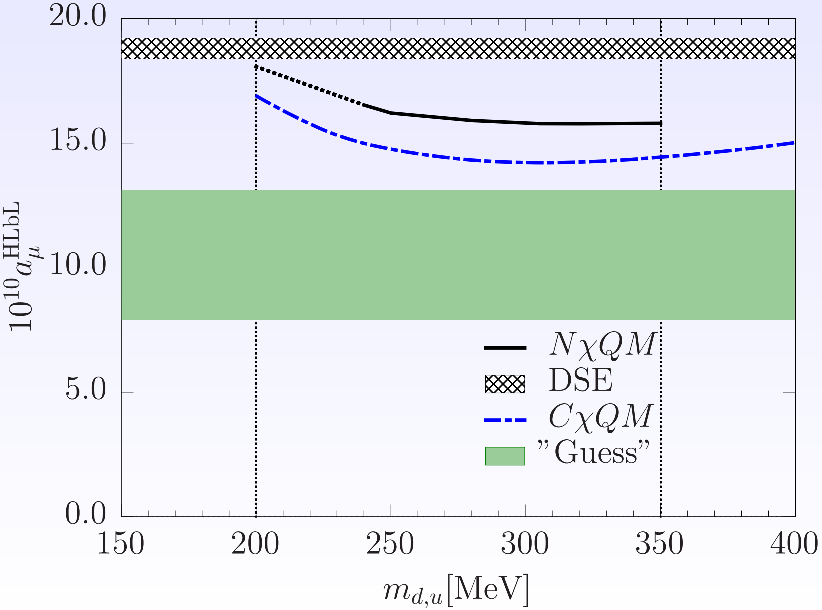


HLbL contribution to the muon AMM

Model	$a_\mu^{\rm HLbL}$	Reference
LMD+V	8.0(1.2)	(Knecht [1])
ENJL	8.3(3.2)	(Bijnens [2])
VMD	8.96(1.54)	(Hayakawa [3])
Guessed	10.5(2.6)	(Prades [4])
LENJL	10.77(1.68)	(Bartos [5])
oLMDV	11.6(4.0)	(Nyffeler [6])
(LMD+V)′	13.6(2.5)	(Melnikov [7])
Q-box	14.05	(Pivovarov [8])
CχQM	15.0(0.3)	(Greynat [9])
This work	16.8(1.25)	
DS	18.8(0.4)	(Goecke [10])

Table: Model estimates of the HLbL contribution to a_μ obtained in different works. All numbers are given in 10^{-10}. The errors do not include the systematic error of the models.

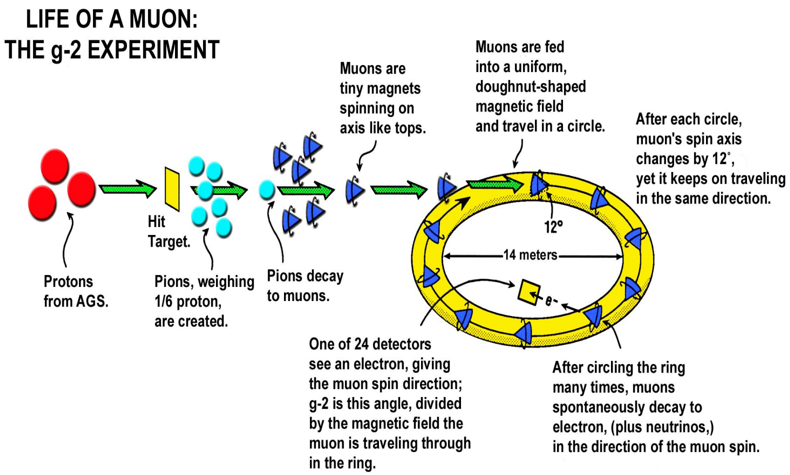
HLbL contribution to the muon AMM



HLbL contribution to $(g-2)_\mu$. Futher extensions.

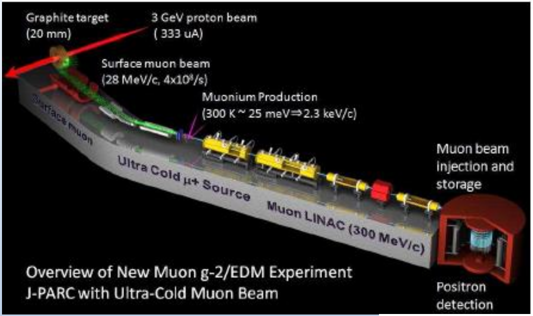
- Extension of model to vector – axial-vector sector:
  - Contribution of axial–vector “goat” exchange (suppressed due to large M_A)
  - Dressing of photon-quark vertex by vector mesons (possibly dangerous for contact term due to negative sign but small in nonlocal model)
- Extension of model to the next $1/N_c$ order (naively 30 % correction! but in practice expected to be smaller[16])
  - Pion loop contribution (possibly dangerous due to negative sign)
  - Dressing of “goat” exchange mesons by two-mesons intermediate state
  - Check stability of the model for next $1/N_c$ corrections

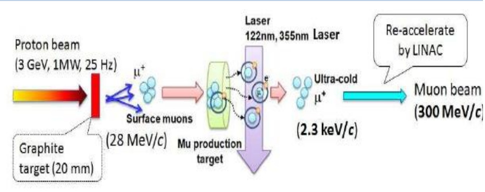
[16] AER, D. Blaschke, M. Buballa and M. K. Volkov, PRD 83, 116004 (2011)

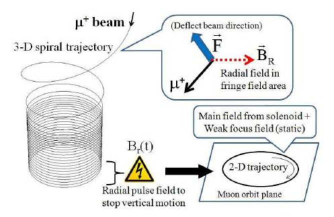

LIFE OF A MUON:
THE g-2 EXPERIMENT
Muons are tiny magnets spinning on axis like tops.
Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.
After each circle, muon's spin axis changes by 12˚, yet it keeps on traveling in the same direction.
Hit Target.
12°
14 meters
Protons from AGS.
Pions, weighing 1/6 proton, are created.
Pions decay to muons.
e-
One of 24 detectors see an electron, giving the muon spin direction; g-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.
After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.

www.facebook.com/pages/The-new-g-2-experiment-at-Fermilab/76812692423

Precise measurment of muon g-2/EDM at JPARC







Conclusions

1. Study of Electron AMM provides very precise value for the QED coupling α
2. Study of Muon AMM is sensitive to effects of Standard Model and New Physics
3. At present there is disagreement at the level 3σ between SM and BNL experiment. New experiments at FNAL and JPARC are promising
4. New experiments at VEPP2000, KLOE2, BESS III on cross section will further diminish the error for HVP contribution.
5. Our result decrease difference between experimental result and theoretical estimations. Further investigations are highly desirable.

THANKS !

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- [9] D. Greynat and E. de Rafael, JHEP **1207**, 020 (2012).
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