Massive higher spins in the frame-like multispinor formalism

Yurii Zinoviev

Institute for High Energy Physics, Protvino, Russia

with Maksim Khabarov

Outlook

1. Formalism
   - gauge invariant
   - frame-like
   - multispinor
2. Illustration: massless case
3. Massive case
   - Lagrangian
   - Gauge invariant objects
   - Unfolded equations
   - Skvortsov-Vasiliev formalism

Gauge invariance for massive fields

- Fields set: all the fields which are necessary for the description of the massless fields with spins $s, s-1, s-2, \dots (1/2)0$ (works both for the metric-like and frame-like formalism, including unfolded equations)
- We keep all the gauge symmetries of the initial massless fields $\Rightarrow$ correct number of physical degrees of freedom
- Works both in flat space as well as $(A)dS$ spaces, including all possible massless and partially massless limits
- It is possible to reproduce ordinary, non gauge invariant description by gauge fixing
- Admits a limit $s \to \infty$ to provide a description for infinite (continuous) spin fields

Frame-like formalism

- It is a generalization of the well-known frame formalism for gravity with its separation of world and local indices
- Allows to use a coordinate-free description for flat or $(A)dS$ space in terms of the background frame $e_\mu{}^a$ and Lorentz covariant derivative D_μ
- A whole set of the gauge invariant objects (curvatures) can be constructed
- The so-called extra fields effectively describe the higher derivatives of the physical fields, while their extra gauge parameters — higher derivatives of the main ones

Multispinor formalism

- In the frame-like formalism even in $d = 4$ one has to work with the mixed symmetry (spin-)tensors corresponding to Young tableau with two rows that makes calculations rather involved (especially for the fermions)
- In the multispinor formalism all such fields are described by the completely symmetric multispinors of the type

$$\Phi^{\alpha(k)\dot\alpha(l)} = \Phi^{(\alpha_1\alpha_2...\alpha_k)(\dot\alpha_1\dot\alpha_2...\dot\alpha_l)}$$

  where $\alpha = 1, 2$, $\dot\alpha = 1, 2$
- The only two operations we need are symmetrization and contraction.
- The description for the bosonic and fermionic cases become very similar and this make such formalism suitable for the supersymmetric theories

Massless case

- The frame-like description for the massless integer or half-integer spin-s case requires (world indices from now on omitted)

$$\Phi^{\alpha(2s-2-k)\dot{\alpha}(k)}, \quad 0 \le k \le 2s-2$$

- Each field has its own gauge transformation parameter

$$\delta\Phi \sim D\eta + e \wedge \eta$$

- For each field the corresponding gauge invariant object (curvature) can be constructed

$$\mathcal{R} \sim D\Phi + e \wedge \Phi$$

- The Lagrangian can be rewritten in the explicitly gauge invariant form

$$\mathcal{L} \sim \sum a_k \mathcal{R}_k \wedge \mathcal{R}_k$$

Unfolded equations

- To go on-shell we can consistently set to zero all the gauge invariant curvatures except the highest ones

$$\mathcal{R}^{\alpha(2s-2)} + h.c.$$

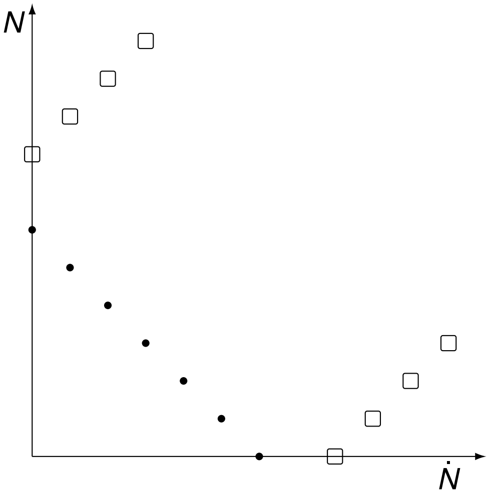
- The last two require introduction of the gauge invariant zero-forms (generalizing the Weyl tensor in gravity)

$$\mathcal{R}^{\alpha(2s-2)} = E_{\beta(2)} W^{\alpha(2s-2)\beta(2)}$$

- This in turn leads to the infinite chain of the gauge invariant zero-forms $W^{\alpha(2s+k)\dot{\alpha}(k)}$, $0 \le k < \infty$ + $h.c.$ with the equations

$$0 = DW^{\alpha(2s+k)\dot{\alpha}(k)} + e_{\beta\dot{\beta}} W^{\alpha(2s+k)\beta\dot{\alpha}(k)\dot{\beta}} + \Lambda e^{\alpha\dot{\alpha}} W^{\alpha(2s+k-1)\dot{\alpha}(k-1)}$$

Complete spectrum



Lagrangian formulation

- Set of fields in the bosonic case
  - physical fields: $\Phi_\mu^{\alpha(k)\dot{\alpha}(k)}$, $1 \leq k \leq s-1$, W_μ, W
  - auxiliary fields: $\Omega_\mu{}^{\alpha(k+1)\dot{\alpha}(k-1)} + h.c.$, $1 \leq k \leq s-1$, $W^{\alpha(2)} + h.c.$, $W^{\alpha\dot{\alpha}}$
- Fermionic case
  - physical fields: $\Psi_\mu^{\alpha(k+1)\dot{\alpha}(k)} + h.c.$, $\psi^\alpha + h.c.$
- Gauge invariant Lagrangians:

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass} + \mathcal{L}_{cross}$$

$$\mathcal{L}_{cross} \sim \sum_k \Phi_k \Phi_{k-1}$$

- All coefficients are determined by the (appropriately modified) gauge transformations and ”boundary conditions”

Gauge invariant curvatures

- To construct a complete set of gauge invariant objects we have to introduce:
  - all extra one-forms needed for all massless $s \geq 2(3/2)$ components
  - a similar set of extra zero-forms

  so that there is a one to one correspondence between one-forms and zero-forms
- Each one forms is a gauge field, while each zero-form is a Stueckelberg field

$$\delta\Phi = D\eta + \dots, \qquad \delta W = \eta$$

- For each field a corresponding gauge invariant object (two-form or one form) can be constructed

$$\mathcal{R} = D\Phi + \dots, \qquad \mathcal{C} = DW - \Phi + \dots$$

Lagrangian in terms of curvatures

- We can rewrite the Lagrangian using curvatures

$$\mathcal{L} \sim \sum [\mathcal{R}\mathcal{R} + \mathcal{R}e\mathcal{C} + \mathcal{C}E\mathcal{C}]$$

- Extra field decoupling conditions do not fix all the coefficients
- We have differential identities:

$$\begin{aligned} D\mathcal{R} &\sim e\mathcal{R} + E\mathcal{C} \\ D\mathcal{C} &\sim \mathcal{R} + e\mathcal{C} \end{aligned}$$

$$0 = D[\mathcal{R}\mathcal{C}] \sim [\mathcal{R}\mathcal{R} + \mathcal{R}e\mathcal{C} + \mathcal{C}E\mathcal{C}]$$

- Using this freedom the Lagrangian can be reduced to the form used by Ponomarev-Vasiliev (both for bosons and fermions)

Unfolded equations

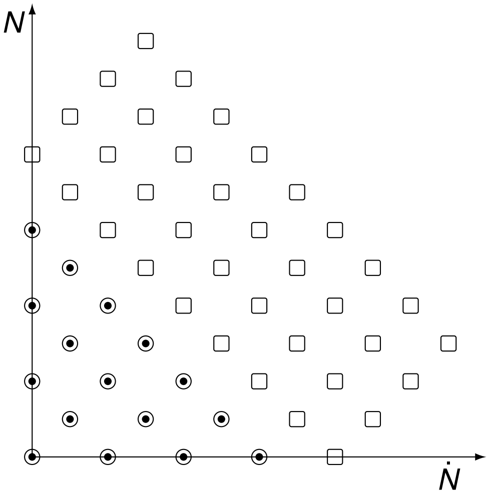
- On-shell we can consistently set to zero most of the gauge invariant curvatures except some highest ones:

$$\mathcal{R}^{\alpha(2s-2)} + h.c., \qquad \mathcal{C}^{\alpha(2s-k)\dot\alpha(k)}, \quad 0 \le k \le 2s-2$$

- Similarly to the massless case this leads to the introduction of $2s+1$ infinite chains of the gauge invariant zero-forms
- The general unfolded equation for them looks like:

$$\begin{aligned} 0 \;=\; & DW^{\alpha(k)\dot\alpha(l)} + e_{\beta\dot\beta} W^{\alpha(k)\beta\dot\alpha(l)\dot\beta} + \alpha_{k,l} e^{\alpha}{}_{\dot\beta} W^{\alpha(k-1)\dot\alpha(l)\dot\beta} \\ & + \beta_{k,l} e_{\beta}{}^{\dot\alpha} W^{\alpha(k)\beta\dot\alpha(l-1)} + \gamma_{k,l} e^{\alpha\dot\alpha} W^{\alpha(k-1)\dot\alpha(l-1)} \end{aligned}$$

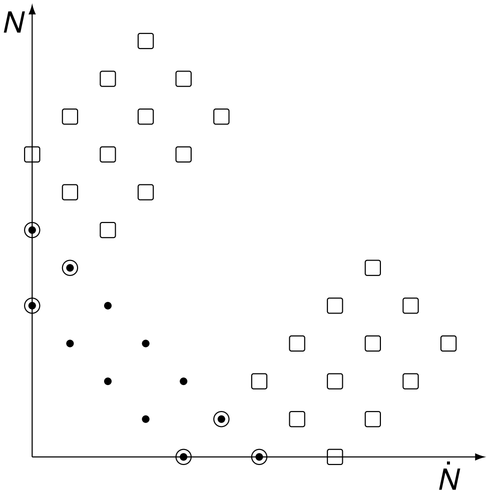
Massive case



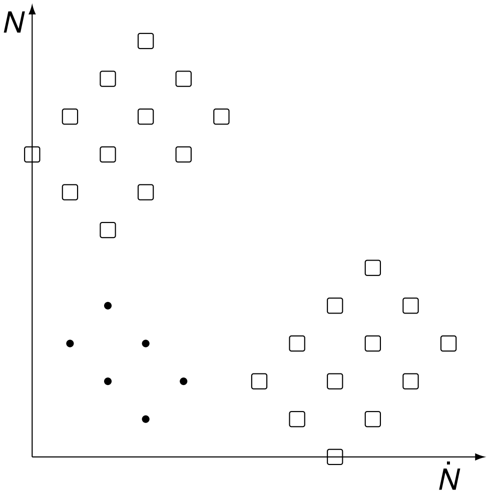
Partially massless limits

- Partially massless cases in de Sitter space $\Lambda > 0$ correspond to the specific values of the mass parameter:
  - bosons
  $$m_k{}^2 = (s+k-1)(s-k)\Lambda$$
  - fermions
  $$m_k{}^2 = (s+k-1)(s-k+1)\Lambda$$
- The Lagrangian decomposes into two disconnected parts
  $$\mathcal{L} = \mathcal{L}_{high} + \mathcal{L}_{low}$$
  with the helicities $k \leq h \leq s$ and $0 \leq h < k$
- Decoupling takes place also for the extra fields as well as for the gauge invariant zero-forms in the unfolded equations

Partially massless case



Skvortsov-Vasiliev case



Practical application

Construction of the higher spin $N = 1$ supermultiplets
(joint work with I.L. Buchbinder and T.V. Snegirev)

- massive finite spin ones
  Nucl. Phys. B942 (2019) 1, arXiv:1901.09637
- partially massless ones
  JHEP 08 (2019) 116, arXiv:1904.01959
- massless infinite spin ones
  Nucl. Phys. B946 (2019) 114717, arXiv:1904.05580