Higher-Spin Theory

Yesterday, Today and Tomorrow

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MV arXiv:1804.06520 and I.Degtev, MV 1905.11267

Gelfond, MV arXiv:1805.11941 and 1908.?????

Didenko, Gelfond, Korybut, MV arXiv:1807.00001 and 1908.?????

SQS 19

Erevan, Aug 29, 2019

Quantum Gravity Challenge

QG effects should matter at ultra-high energies of Planck scale

$$M_P = 10^{19} GeV$$

A distinguished theoretic possibility is to conjecture that the regime of ultra high (transPlanckian) energies exhibits some high symmetries that are spontaneously broken at low energies

Idea: to understand what kind of higher symmetries can be introduced in relativistic theory and to see consequences

HS gauge theory:

theory of higher symmetries consistent with unitary QFT

Must be beautiful

and can affect fundamental concepts of gravity and quantum mechanics

Fronsdal Fields

Fronsdal fields 1978

All $m=0$ HS fields are gauge fields

$\varphi_{n_1\ldots n_s}$ is a rank s symmetric tensor obeying $\varphi^k{}_k{}^m{}_{mn_5\ldots n_s}=0$

Gauge transformation:

$$\delta\varphi_{n_1\ldots n_s}=\partial_{(n_1}\varepsilon_{n_2\ldots n_s)}\,,\qquad \varepsilon^m{}_{mn_3\ldots n_{s-1}}=0$$

In 60-70th it was argued (Weinberg, Coleman-Mandula) that

HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

Green light: AdS background with $\Lambda\neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda\to 0$ is singular

HS Theories and String Theory

HS theories: $\Lambda \neq 0$, $m = 0$, **symmetric fields** $s = 0, 1, 2, \ldots \infty$

First Regge trajectory

String Theory: $\Lambda = 0$, $m \neq 0$ **except for a few zero modes**

Infinite set of Regge trajectories

What is a HS symmetry of a string-like extension of HS theory?

MV 2012, 2018, Gaberdiel and Gopakumar 2014-2018

String Theory as spontaneously broken HS theory?! $(s > 2; m > 0)$.

Non-Locality of HS Gauge Theory

HS symmetries do not commute with space-time symmetries

$$[T^n, T^{HS}] = T^{HS}, \qquad [T^{nm}, T^{HS}] = T^{HS}$$

Riemann geometry is not appropriate for HS theory

HS interactions contain higher derivatives:

A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984)

Infinite towers of spins imply infinite towers of derivatives.

How (non)local is HS gauge theory?

The mildest possibility: each vertex with fields of definite spins is local.

All vertices we have found so far up to the quintic order are spin-local:

local in the spinor space.

Most Urgent Problems

Appropriate scheme leading to (spin-local) choice of variables.

Didenko, Gelfond, Korybut, MV 2016 - 2019

String-like and Tensor-like HS theory!?

MV 1804.06520; Degtev, MV 1905.11267

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to \mathrm{d}\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\Omega_{n_1 \ldots n_p}(x)$$

$$\mathrm{d}\mathbf{W}^\Omega(\mathbf{x}) = \mathbf{G}^\Omega(\mathbf{W}(\mathbf{x}))\,, \qquad \mathrm{d} = \mathbf{dx^n}\partial_\mathbf{n} \qquad \textbf{MV} \quad \textbf{1988}$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1 \ldots \Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Phi(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Phi} \equiv 0$$

L_∞, A_∞, Q-**manifolds, etc**

Central On-Shell Theorem

Infinite set of integer spins

$\omega(y,\bar{y}\mid x)\,,\quad C(y,\bar{y}\mid x)\quad f(y,\bar{y})=\sum_{n,m=0}^{\infty}\frac{1}{n!m!}f_{\alpha_1\ldots\alpha_n,\dot{\alpha}_1\ldots\dot{\alpha}_m}y^{\alpha_1}\ldots y^{\alpha_n}\bar{y}^{\dot{\alpha}_1}\ldots\bar{y}^{\dot{\alpha}_m}$

The full unfolded system for free bosonic fields is 1989

$$\star\qquad R_1(y,\bar{y}\mid x)=\frac{i}{4}\left(\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\beta}}}\,C(0,\bar{y}\mid x)+\bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,C(y,0\mid x)\right)$$

$$\star\star\qquad \tilde{D}_0C(y,\bar{y}\mid x)=0$$

Vacuum: $sp(4)\sim o(3,2)$

$$\mathbf{R}_{\alpha\beta}:=\mathbf{d}\omega_{\alpha\beta}+\omega_{\alpha\gamma}\omega_{\beta}{}^{\gamma}-\mathbf{H}_{\alpha\beta}=\mathbf{0}\,,\qquad \mathbf{R}_{\alpha\dot{\beta}}:=\mathbf{d}+\omega_{\alpha\gamma}\mathbf{h}^{\gamma}{}_{\dot{\beta}}+\overline{\omega}_{\dot{\beta}\dot{\delta}}\mathbf{h}_{\alpha}{}^{\dot{\delta}}=\mathbf{0}$$

$$\mathbf{H}^{\alpha\beta}:=\mathbf{h}^{\alpha\dot{\alpha}}\wedge\mathbf{h}^{\beta}{}_{\dot{\alpha}}\,,\qquad \overline{\mathbf{H}}^{\dot{\alpha}\dot{\beta}}:=\mathbf{h}^{\alpha\dot{\alpha}}\wedge\mathbf{h}_{\alpha}{}^{\dot{\beta}}$$

$$R_1(y,\bar{y}\mid x)=D_0^{ad}\omega(y,\bar{y}\mid x)\qquad D_0^{ad}=D^L-h^{\alpha\dot{\beta}}\left(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}+\frac{\partial}{\partial y^{\alpha}}\bar{y}_{\dot{\beta}}\right)$$

$$\tilde{D}_0=D^L+h^{\alpha\dot{\beta}}\left(y_\alpha\bar{y}_{\dot{\beta}}+\frac{\partial^2}{\partial\mathbf{y}^{\alpha}\partial\bar{\mathbf{y}}^{\dot{\beta}}}\right)\qquad D^L=\mathsf{d}_x-\left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^{\beta}}+\bar{\omega}^{\dot{\alpha}\dot{\beta}}\bar{y}_{\dot{\alpha}}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}\right)$$

$\star\star$ **implies that higher-order terms in y and $\bar{y}$ describe higher-derivative descendants of the primary HS fields**

Spin-Locality

Free fields of different spins

$$\omega(\mu y, \mu \bar{y}) = \mu^{2(s-1)} \omega(y, \bar{y})\,, \qquad C(\mu y, \mu^{-1} \bar{y}) = \mu^{\pm 2s} C(y, \bar{y})$$

ω: **finite number of components (derivatives) for definite spin**

C: **infinite number of components (derivatives) for definite spin**

Nonlinear corrections have the form

$$F(P^{ij}, \bar{P}^{kl}) C(Y_1) \dots C(Y_n)\,, \qquad P^{ij} := \frac{\partial}{\partial y_i^\alpha} \frac{\partial}{\partial y_{j\alpha}}\,, \qquad \bar{P}^{ij} := \frac{\partial}{\partial \bar{y}_i{}^{\dot{\alpha}}} \frac{\partial}{\partial \bar{y}_{j\dot{\alpha}}},$$

with some nonpolynomial functions $F(P^{ij}, \bar{P}^{kl})$

Spin-locality: Polynomiality of $F(P^{ij}, \bar{P}^{kl})$ **in either** P or $\bar{P}$

Projector on fixed spins relates degree in P^{ij} **and** $\bar{P}^{kl}$ **to each other!**

Relation with space-time locality

In the lowest orders relation is direct by virtue of the unfolded equations

In the higher orders it gets more involved due to nonlinear corrections

Conceptual problem with the space-time definition of locality in AdS.

Lorentz-covariant derivatives D_n commute to a constant: background AdS curvature

$$[D_n\,, D_m] = R_{nm}$$

giving a meaning to non-polynomial functions of D_n demands a particular ordering prescription

Example: $\delta(a_-)\delta(a_+)$ in the Weyl ordering becomes $\exp(a_-a_+)$ in the normal ordering

Y variables provide an appropriate ordering for D_n derivatives

Fields of the Nonlinear System

Nonlinear HS equations demand doubling of spinors and Klein operator

$$\omega(Y;K|x) \longrightarrow W(Z;Y;K|x)\,, \qquad C(Y;K|x) \longrightarrow B(Z;Y;K|x)$$

$Y^A = (y^\alpha, \bar{y}^{\dot\alpha}),\ Z^A = (z^\alpha, \bar{z}^{\dot\alpha})$

Some of the nonlinear HS equations determine the dependence on Z_A

in terms of "initial data" $\omega(Y;K|x)$ and $C(Y;K|x)$

$S(Z;Y;K|x) = dZ^A S_A(Z;Y;K|x)$ is a connection along Z^A

Klein operators $K = (k, \bar{k})$ generate chirality automorphisms

$$kf(A) = f(\tilde{A})k\,, \qquad A = (a_\alpha\,, \bar{a}_{\dot\alpha}): \qquad \tilde{A} = (-a_\alpha\,, \bar{a}_{\dot\alpha})$$

Nonlinear HS Equations

HS star product

$$(f \star g)(Z,Y) = \int dSdT \exp iS_A T^A f(Z+S, Y+S) g(Z-T, Y+T)$$

$$[Y_A, Y_B]_\star = -[Z_A, Z_B]_\star = 2iC_{AB}\,, \qquad Z-Y\,:\,Z+Y \text{ normal ordering}$$

Via relation between space-time derivatives and $Y; Z$-derivatives non-locality of the star product induces space-time non-locality

Inner Klein operators:

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \bar\kappa = \exp i\bar z_{\dot\alpha}\bar y^{\dot\alpha}\,, \qquad \kappa \star f = \tilde f \star \kappa\,, \qquad \kappa \star \kappa = 1$$

$$\begin{cases} \mathrm{d}W + W \star W = 0 \\ \mathrm{d}B + W \star B - B \star W = 0 \\ \mathrm{d}S + W \star S + S \star W = 0 \\ \mathbf{S} \star \mathbf{B} - \mathbf{B} \star \mathbf{S} = 0 \\ \mathbf{S} \star \mathbf{S} = \mathrm{i}(\mathrm{dZ^A dZ_A} + \eta \mathrm{dz}^\alpha \mathrm{dz}_\alpha \mathrm{B} \star \mathrm{k} \star \kappa + \bar\eta \mathrm{d\bar z}^{\dot\alpha} \mathrm{d\bar z}_{\dot\alpha} \mathrm{B} \star \mathrm{k} \star \bar\kappa) \end{cases} \qquad 1992$$

Dynamical content is located in the x-independent twistor sector

$\eta = \exp i\varphi$ is a free phase parameter suggesting 3d bosonization.

The non-zero curvature has the form of Z_2-Cherednik algebra

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = dZ^A Z_A\,, \qquad W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B$$

$$\mathrm{d}W_0 + W_0 \star W_0 = 0\,, \qquad w^{AB} : AdS_4$$

$$[\mathbf{S_0}\,, \mathbf{f}]_\star = -\mathbf{2i}\mathrm{d}_\mathbf{Z}\mathbf{f}\,, \qquad \mathrm{d}_\mathbf{Z} = \mathbf{dZ^A}\frac{\partial}{\partial \mathbf{Z^A}}$$

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

Order-n equations containing S have the form

$$\mathrm{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|dZ) \qquad \mathrm{d}_Z V[U_{<n}](Z;Y|dZ) = 0$$

can be solved as

$$U_n(Z;Y|dZ) = \mathrm{d}^*_Z V[U_{<n}](Z;Y|dZ) + \mathbf{h(Y)} + \mathrm{d}_Z \epsilon(Z;Y|dZ)$$

$$\mathrm{d}^*_Z V(Z;Y|dZ) = (Z^A - \mathcal{C}^A)\frac{\partial}{dZ^A}\int_0^1 \frac{dt}{t} V(tZ + (1-t)\mathcal{C};Y|tdZ)$$

Interpretation

The resolution freedom encodes:

All possible gauge choices in dz-exact forms $\mathsf{d}_Z\epsilon(Z;Y|dZ)$

All possible choices of field variables in dz cohomology $\mathbf{h(Y)}$

How to single out the proper (e.g., minimally nonlocal) frames?

Spin-local limit: $\beta \to -\infty$ with $\mathcal{C}_A = \frac{\partial}{\partial Y^A}$

Didenko, Gelfond, Korybut, MV 1909.????

Local vertices up to the quintic order!

Talks by Gelfond and Didenko Today!

Coxeter HS Equations (Tomorrow)

Unfolded equations for 1804.06520 $\mathcal{C}$-HS theories remain the same except

$$iS \star S = dZ^{An} dZ_{An} + \sum_i \sum_{v \in \mathcal{R}_i} \eta_i B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v$$

κ_v are generators of $\mathcal{C}$ acting trivially on all elements except for dZ_n^A

$$\kappa_v \star dZ_A^n = R_v{}^n{}_m dZ_A^m \star \kappa_v \, .$$

η_i is a coupling constant on the conjugacy class $\mathcal{R}_i$ of $\mathcal{C}$.

In the important case of the Coxeter group B_p

$$iS \star S = dZ^{An} dZ_{An} + \sum_{v \in \mathcal{R}_1} \eta_1 B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v + \sum_{v \in \mathcal{R}_2} \eta_2 B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v$$

with arbitrary η_1 and η_2 responsible for the

HS and stringy/tensorial features, respectively

$\eta_2 \neq 0$ for $p \geq 2$

The framed construction leads to a proper massless spectrum.

Jacobi for Cherednik imply consistency of field equations.

Klein Operators and Single-Trace Operators

Enlargement of the field spectra of the rank- $p>1$ Coxeter HS models:
$C(Y^n_A;k_v)$ **depend on p copies of oscillators Y^n_A and Klein operators k_v.**

Qualitative agreement with enlargement of the boundary operators in tensorial boundary models

Klein operators of Coxeter reflections permute master field arguments

At $p=2$ the star product of two master fields $(C(Y_1;Y_2|x)k_{12})$ gives

$$(C(Y_1;Y_2|x)k_{12})\star(C(Y_1;Y_2|x)k_{12})=C(Y_1;Y_2|x)C(Y_2;Y_1|x)$$

$p=2$ **system: strings of fields with repeatedly permuted arguments**

$$C_{n\,string}:=\underbrace{C(Y_1;Y_2|x)\star C(Y_2|;Y_1|x)\star C(Y_1;Y_2|x)\ldots}_{n}$$

are analogous of the single-trace operators in AdS/CFT.

$C(Y_1;Y_2|x)$ and $C(Y_1;Y_2|x)C(Y_2;Y_1|x)$: **single-trace-like**

$C(Y_1;Y_2|x)C(Y_1;Y_2|x)$: **double-trace-like.**

For $p>2$ fields carry p arguments permuted by S_p generated by k_{ij}

Conclusion

Today

The shifted homotopy scheme is proposed leading to spin-local HS vertices derived from the nonlinear equations.

A class of new local vertices is found up to quintic order.

Didenko, Gelfond, Korybut, MV to appear

Tomorrow

Coxeter HS theories extend minimal HS theories to

String-like B_2 models and tensor-like B_p models of any rank p

The spectrum of the B_2 HS model is analogous to that of

String Theory with the infinite set of Regge trajectories.

$N = 4$ SYM is argued to be a natural dual of the B_2 HS model

Main problem on the agenda:

spontaneous breaking of HS symmetries in the Coxeter HS models