# On massive higher spin N=1 supermultiplets in 4D AdS

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# Plan

1. Introduction and a setup
2. Free higher spin fields
3. Massless higher spin supermultiplets
4. Massive higher spin supermultiplets
5. Summary

The talk is based on arXiv:1901.09637
with I.L. Buchbinder, M.V. Khabarov and Yu.M. Zinoviev

In supersymmetric theory it is natural to start with description of supermultiplets.

- Massless higher spin $N = 1$ supermutiplets contain one bosonic $f$ and one fermionic $\Phi$ fields with spin $(k, k \pm 1/2)$. It is not hard to construct on-shell supertransformation leaving the free Lagrangin invariant $\delta(\mathcal{L}_f + \mathcal{L}_\Phi) = 0$

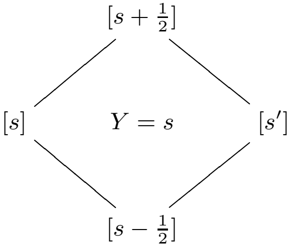
$$\delta f \sim \Phi\zeta \qquad \delta\Phi \sim \partial f \zeta$$

  Metric-like formulation T. Curtright 1979
  Frame-like formulation M.A. Vasiliev 1980

- Off-shell $N = 1$ superfield formulation of massless supermultiplets
  S.M. Kuzenko, A.G. Sibiryakov and V.V.Postnikov 1993

# Introduction

- Massive higher spin $N = 1$ supermultiplets contain pair bosonic and pair fermionic fields with spins




What the supertransformations leave free Lagrangian invariant?

$$\delta(\mathcal{L}_{[s]} + \mathcal{L}_{[s']} + \mathcal{L}_{[s+\frac{1}{2}]} + \mathcal{L}_{[s-\frac{1}{2}]}) = 0$$

The main problem is that massive higher spin fields are not gauge ones and Lagrangian description for them requires auxiliary fields L.P.S. Singh, C.R. Hagen 1974. This leads to introducing of non-trivial higher derivative correction in supertransformations.

Systematic solution was proposed by Yu.M. Zinoviev 2007 as generalization of gauge invariant formulation of massive fields to the case of massive supermultiplets in $4D$ Minkowski.

**Gauge invariant description of massive fields** Yu.M. Zinoviev 1983

Massive fields are described as deformation of system of massless ones coupled by Stuckelberg symmetries:

$$[s] \quad \xrightarrow{m=0} \quad \oplus \sum_{k=0}^{s} k \qquad \mathcal{L}_{[s]}(f_k)$$

$$[s+\frac{1}{2}] \quad \xrightarrow{m=0} \quad \oplus \sum_{k=0}^{s} (k+\frac{1}{2}) \qquad \mathcal{L}_{[s+\frac{1}{2}]}(\Phi_{k+\frac{1}{2}})$$

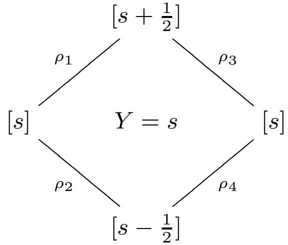
Massive supermultiplets as massive deformation of system of massless ones.

GOAL: extending supersymmetric gauge invariant formulation of massive models to construct $N=1$ higher spin supermultiplets in 4D AdS.

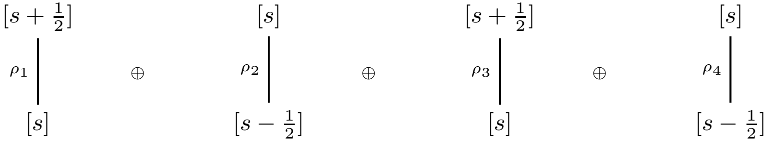
# Introduction

**General construction of massive supermultiplets**

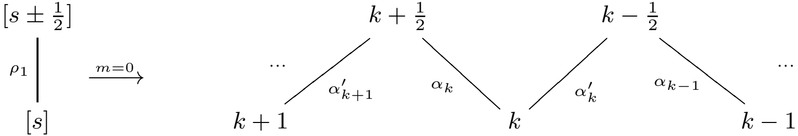
Massive supermultiplet




Massive superblocks



Massive suberblocks in gauge invariant form




Massless supermultiplets

**Frame-like multispinor formalism**

- 4D AdS background frame is described by 1-form $e^{\alpha\dot{\alpha}}$ $(\alpha, \dot{\alpha} = 1, 2)$. Double product is defined as

$$e^{\alpha\dot{\alpha}} \wedge e^{\beta\dot{\beta}} = \varepsilon^{\alpha\beta} E^{\dot{\alpha}\dot{\beta}} + \varepsilon^{\dot{\alpha}\dot{\beta}} E^{\alpha\beta}$$

- AdS covariant derivative D is normalized as

$$D^2 \xi^{\alpha\dot{\alpha}} = -2\lambda^2 (E^{\alpha}{}_{\beta} \xi^{\beta\dot{\alpha}} + E^{\dot{\alpha}}{}_{\dot{\beta}} \xi^{\alpha\dot{\beta}}), \qquad \lambda^2 > 0$$

- The parameter of global supertransformations $\zeta^{\alpha}, \zeta^{\dot{\alpha}}$ satisfies the relation

$$D\zeta^{\alpha} = -\lambda e^{\alpha}{}_{\dot{\beta}} \zeta^{\dot{\beta}}$$

- 4D Minkowski case is reproduced by limit $\lambda \to 0$ and $D \to d$.

**Massless (gauge) boson**
Frame-like formulation for spin $k \geq 2$: physical 1-forms $f^{\alpha(k-1)\dot{\alpha}(k-1)}$ and axillary 1-forms $\Omega^{\alpha(k)\dot{\alpha}(k-2)}$ and $h.c.$

$$\begin{aligned} \frac{(-1)^k}{i}\mathcal{L}_k &= k\Omega^{\alpha(k-1)\beta\dot{\alpha}(k-2)}E_\beta{}^\gamma\Omega_{\alpha(k-1)\gamma\dot{\alpha}(k-2)} \\ &\quad -(k-2)\Omega^{\alpha(k)\dot{\alpha}(k-3)\dot{\beta}}E_{\dot{\beta}}{}^{\dot{\gamma}}\Omega_{\alpha(k)\dot{\alpha}(k-3)\dot{\gamma}} \\ &\quad +2\Omega^{\alpha(k-1)\beta\dot{\alpha}(k-2)}e_\beta{}^{\dot{\beta}}df_{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta}} - h.c. \end{aligned}$$

$$\begin{aligned} \delta f^{\alpha(k-1)\dot{\alpha}(k-1)} &= d\xi^{\alpha(k-1)\dot{\alpha}(k-1)} \\ &\quad +e_\beta{}^{\dot{\alpha}}\eta^{\alpha(k-1)\beta\dot{\alpha}(k-2)} + e^\alpha{}_{\dot{\beta}}\eta^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\beta}} \\ \delta\Omega^{\alpha(k)\dot{\alpha}(k-2)} &= d\eta^{\alpha(k),\dot{\alpha}(k-2)} \end{aligned}$$

Schematically: $f_k$ is physical fields, $\Omega_k$ is axillary one

$$\mathcal{L}_k = \Omega_k\Omega_k + \Omega_k df_k, \qquad \delta f_k = d\xi_k + \eta_k, \quad \delta\Omega_k = d\eta_k$$

# Free higher spin boson

**Massive boson**

In gauge invariant form massive spin $s$ can be described as system of massless one $s \geq k \geq 0$ coupled by the Stuckelberg symmetries.

Fields variables

$$\sum_{k=0}^{s}(f_k, \Omega_k) = \sum_{k=2}^{s}(f^{\alpha(k-1)\dot{\alpha}(k-1)}, \Omega^{\alpha(k)\dot{\alpha}(k-2)}) + \text{spin } 1 + \text{spin } 0$$

Lagrangian

$$\mathcal{L}_{[s]} = \sum_k [\mathcal{L}_k + a_k(\Omega_k f_{k-1} + f_k \Omega_{k-1}) + b_k(f_k f_k)]$$

Gauge transformations

$$\begin{aligned} \delta f_k &= D\xi_k + \eta_k + (a_k \xi_{k-1} + a_{k+1}\xi_{k+1}) \\ \delta \Omega_k &= D\eta_k + (a_k \eta_{k-1} + a_{k+1}\eta_{k+1}) + b_k \xi_k \end{aligned}$$

$$\begin{aligned} b_k &= \frac{2s(s+1)}{k(k-1)(k+1)} M^2, \qquad M^2 = m^2 + s(s-1)\lambda^2 \\ a_k &= \sqrt{\frac{4(s-k+1)(s+k)}{(k-1)(k-2)}[M^2 - k(k-1)\lambda^2]} \end{aligned}$$

# Free higher spin fermion

**Massless (gauge) fermion**

Frame-like formulation for spin $k+1/2,\ (k \geq 1)$: physical 1-forms $\Phi^{\alpha(k)\dot\alpha(k-1)}$ and $h.c.$

$$(-1)^k \mathcal{L}_{k+\frac{1}{2}} = \Psi_{\alpha(k-1)\beta\dot\alpha(k-1)} e^{\beta}{}_{\dot\beta} d\Psi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta}$$

$$\delta\Psi^{\alpha(k)\dot\alpha(k-1)} = d\xi^{\alpha(k)\dot\alpha(k-1)}$$

Schematically: $\Phi_{k+\frac{1}{2}}$ is physical fields

$$\mathcal{L}_{k+\frac{1}{2}} = \Phi_{k+\frac{1}{2}} d\Phi_{k+\frac{1}{2}}, \qquad \delta\Phi_{k+\frac{1}{2}} = d\xi_{k+\frac{1}{2}}$$

**Massive fermion**

In gauge invariant form massive spin $s+\frac{1}{2}$ can be described as system of massless one $s+\frac{1}{2} \geq k+\frac{1}{2} \geq 1/2$ coupled by the Stueckelberg symmetries.

Fields variables

$$\sum_{k=0}^{s} \Phi_{k+\frac{1}{2}} = \sum_{k=1}^{s} \Phi^{\alpha(k)\dot{\alpha}(k-1)} + \text{spin } \frac{1}{2}$$

Lagrangian

$$\mathcal{L}_{[s+\frac{1}{2}]} = \sum_{k} [\mathcal{L}_{k+\frac{1}{2}} + c_k(\Phi_{k+\frac{1}{2}}\Phi_{k-\frac{1}{2}}) + d_k(\Phi_{k+\frac{1}{2}}\Phi_{k+\frac{1}{2}})]$$

Gauge transformations

$$\delta\Phi_{k+\frac{1}{2}} = D\xi_{k+\frac{1}{2}} + (c_k\xi_{k-\frac{1}{2}} + c_{k+1}\xi_{k+\frac{3}{2}}) + d_k\xi_{k+\frac{1}{2}}$$

$$d_k = \pm\frac{(s+1)}{2k(k+1)}M_1, \qquad {M_1}^2 = {m_1}^2 + s^2\lambda^2$$

$$c_k = \sqrt{\frac{(s-k+1)(s+k+1)}{k^2}[{M_1}^2 - k^2\lambda^2]}$$

**Massless supermultiplet** $(k, k+\frac{1}{2})$

$$\delta(\mathcal{L}_k + \mathcal{L}_{k+\frac{1}{2}}) = 0$$

Supertransformations

$$\begin{aligned} \delta f^{\alpha(k-1)\dot\alpha(k-1)} &= \alpha_{k-1}\Phi^{\alpha(k-1)\beta\dot\alpha(k-1)}\zeta_\beta - \bar\alpha_{k-1}\Phi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta}\zeta_{\dot\beta} \\ \delta\Phi^{\alpha(k)\dot\alpha(k-1)} &= \beta_{k-1}\Omega^{\alpha(k)\dot\alpha(k-2)}\zeta^{\dot\alpha} \end{aligned}$$

**Massless supermultiplet** $(k, k-\frac{1}{2})$

$$\delta(\mathcal{L}_k + \mathcal{L}_{k-\frac{1}{2}}) = 0$$

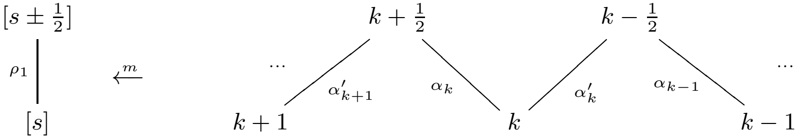
Supertransformations

$$\begin{aligned} \delta f^{\alpha(k-1)\dot\alpha(k-1)} &= \alpha'_{k-1}\Phi^{\alpha(k-1)\dot\alpha(k-2)}\zeta^{\dot\alpha} - \bar\alpha'_{k-1}\Phi^{\alpha(k-2)\dot\alpha(k-1)}\zeta^{\alpha} \\ \delta\Phi^{\alpha(k-1)\dot\alpha(k-2)} &= \beta'_{k-1}\Omega^{\alpha(k-1)\beta\dot\alpha(k-2)}\zeta_\beta \end{aligned}$$

where

$$\alpha_{k-1} = i\frac{(k-1)}{4}\bar\beta_{k-1}, \quad \alpha'_{k-1} = \frac{i}{4(k-1)}\bar\beta'_{k-1}$$

**Massive superblocks**




$$\begin{aligned}
\delta f^{\alpha(k-1)\dot\alpha(k-1)} &= \alpha_{k-1}\Phi^{\alpha(k-1)\beta\dot\alpha(k-1)}\zeta_\beta - \bar\alpha_{k-1}\Phi^{\alpha(k-1)\dot\alpha(k-1)\dot\beta}\zeta_{\dot\beta} \\
&\quad + \alpha'_{k-1}\Phi^{\alpha(k-1)\dot\alpha(k-2)}\zeta^{\dot\alpha} - \bar\alpha'_{k-1}\Phi^{\alpha(k-2)\dot\alpha(k-1)}\zeta^{\alpha} \\
\delta\Phi^{\alpha(k)\dot\alpha(k-1)} &= \beta_{k-1}\Omega^{\alpha(k)\dot\alpha(k-2)}\zeta^{\dot\alpha} + \beta'_k\Omega^{\alpha(k)\beta\dot\alpha(k-1)}\zeta_\beta
\end{aligned}$$

where $k \leq s$. Boundary conditions:

$$\alpha_s = \beta_s = 0, \qquad \alpha'_s = \beta'_s = 0 \tag{1}$$

in the case $[s+\frac{1}{2}]$ and

$$\alpha_{s-1} = \beta_{s-1} = 0, \qquad \alpha'_s = \beta'_s = 0 \tag{2}$$

in the case $[s-\frac{1}{2}]$

**Massive superblocks**

Massive deformation

$$\sum_k (\mathcal{L}_k + \mathcal{L}_{k+\frac{1}{2}}) \quad \xrightarrow{m} \quad \mathcal{L}_{[s]} + \mathcal{L}_{[s\pm\frac{1}{2}]}$$

$$\delta_m \Phi^{\alpha(k)\dot{\alpha}(k-1)} = \gamma_{k-1} f^{\alpha(k-1)\dot{\alpha}(k-1)} \zeta^{\alpha} + \gamma'_k f^{\alpha(k)\dot{\alpha}(k-1)\dot{\beta}} \zeta_{\dot{\beta}}$$

The requirement

$$\delta(\mathcal{L}_{[s]} + \mathcal{L}_{[s\pm\frac{1}{2}]}) = 0$$

gives

- The expressions $\gamma_k \sim \beta_k, \qquad \gamma'_k \sim \beta'_k$
- The recurrent equations on the parameters $\beta_k, \beta'_k$. There are four solution both for $(s, s+\frac{1}{2})$ and $(s, s-\frac{1}{2})$ superblocks.

Let us denote boson with mass $M$ and fermion with mass $M_1$ as

$$[s]^{\pm}_{M}, \qquad [s+\frac{1}{2}]^{\pm}_{M_1}$$

here $\pm$ corresponds to parity-even/parity-odd for boson and sign of $d_k$ for fermion.

# Massive higher spin supermultiplets

**Massive superblocks** $(s, s+\frac{1}{2})$

$$\begin{pmatrix} [s]^+_M \\ [s+\frac{1}{2}]^+_{M_1} \end{pmatrix} \qquad \begin{pmatrix} [s]^-_M \\ [s+\frac{1}{2}]^-_{M_1} \end{pmatrix}$$
$$\begin{pmatrix} [s]^+_{M'} \\ [s+\frac{1}{2}]^-_{M_1} \end{pmatrix} \qquad \begin{pmatrix} [s]^-_{M'} \\ [s+\frac{1}{2}]^+_{M_1} \end{pmatrix}$$

$$\beta_{k-1} = \sqrt{\frac{(s+k+1)(M_1 \pm k\lambda)}{(k-1)}}\rho, \quad \beta'_{k-1} = \pm\sqrt{(k-1)(s-k+1)(M_1 \mp k\lambda)}\rho$$

where

$$M^2 = M_1(M_1 - \lambda), \quad M'^2 = M_1(M_1 + \lambda), \quad \rho = \pm\bar{\rho}$$

**Massive superblocks** $(s, s-\frac{1}{2})$

$$\begin{pmatrix} [s]^+_M \\ [s-\frac{1}{2}]^+_{M_2} \end{pmatrix} \qquad \begin{pmatrix} [s]^-_M \\ [s-\frac{1}{2}]^-_{M_2} \end{pmatrix}$$

$$\begin{pmatrix} [s]^+_{M'} \\ [s-\frac{1}{2}]^-_{M_2} \end{pmatrix} \qquad \begin{pmatrix} [s]^-_{M'} \\ [s-\frac{1}{2}]^+_{M_2} \end{pmatrix}$$

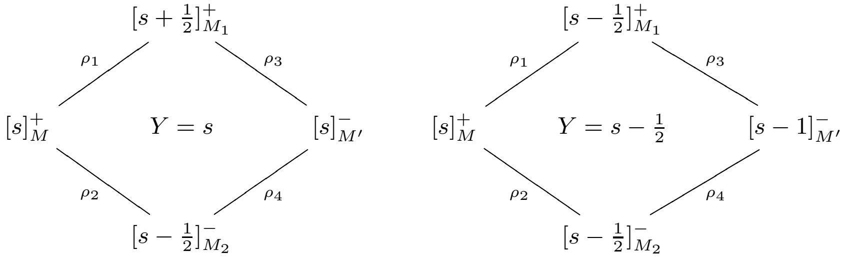
$$\beta_{k-1} = \sqrt{\frac{(s-k)(M_2 \mp k\lambda)}{(k-1)}}\rho, \quad \beta'_{k-1} = \pm\sqrt{(k-1)(s+k)(M_2 \pm k\lambda)}\rho$$

where

$$M^2 = M_2(M_2+\lambda), \quad M'^2 = M_2(M_2-\lambda), \quad \rho = \pm\bar{\rho}$$

**Massive supermultiplets**

To construct complete massive supermultiplets we choose appropriate solutions for each superblock and adjust their parameters so that the algebra of these supertransformations is closed.




$$M^2 = M_1(M_1-\lambda)$$
$$M'^2 = M_1(M_1+\lambda)$$
$$M_2 = M_1$$

$$M^2 = M'^2 = M_1(M_1+\lambda)$$
$$M_2 = M_1+\lambda$$

$${\rho_1}^2 = {\rho_2}^2 = {\rho_3}^2 = {\rho_4}^2$$

# Summary

- Gauge invariant formualtion of massive higher spin fields give systematic approach to construction of massive higher spin supermultiplets both in Minkowski and AdS space.
- In such formulation massive supermultiplets is described as the system of massless ones which are mixed with each other. Massive deformation fixes the arbitrariness of mixing.
- To construct massive deformations for supertransformations we need add terms without derivatives for fermionic fields only.
- General scheme has been successfully applied by us to construction of 4D $N = 1$ partially massless supermultiplets arXiv:1904.01959 and infinite spin supermultiplets arXiv:1904.05580.