BLTP, JINR

Kerr black holes with synchronized hairs and boson stars

Ya Shnir

Thanks to my collaborators:
J Kunz, C.Herdeiro, I.Perapechka,
E Radu and A. Wereszczynski

JHEP 1810 (2018) 119
JHEP 1902 (2019) 111
JHEP 1907 (2019) 109
PLB 797 (2019) 134845

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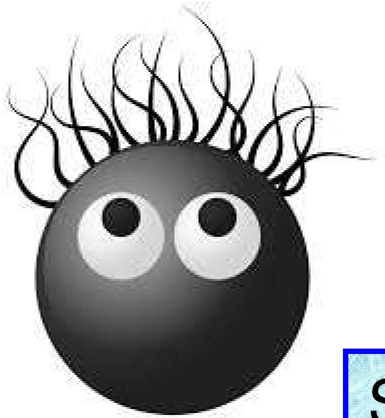
Outline

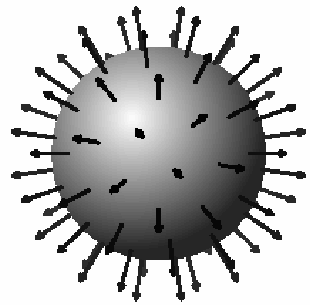
- Hairy Black Holes
- Skyrmions & Gravity
  - Skerrmions
  - Clouds
- Kerr black holes with spinning massive scalar hair
- Kerr black holes with O(3) sigma-model hair
- Dirac stars
- Summary

S Coleman: “Can a *black hole* have *colored hair*?”



J Wheeler: “Black holes have no hair”

S Hawking: “Black holes have hair “



1970s- 1980s :

Israel’s theorem: Static Einstein-Maxwell black holes are spherically symmetric

‘No-hair’ theorem: Stationary black holes are completely characterized by their mass **M**, charge **Q** and angular momentum **J**

More no-go statements:

1970s-1990s:

Bekenstein: “No free massive scalar, vector, or spin 2 hairs”

Chase: “No massless scalar hairs“

Teitelboim: “No weakly and strongly interacting hairs”

Sudarsky: “No Einstein-Higgs hair with arbitrary number of scalar fields and potential

Finster, Smoller & Yau: „No Dirac hair“

Heusler: „No sigma-model hair“

No-hair theorem (beginning of 1990s):

“The only allowed characteristics of black holes are those associated with Gauss law”

From 1990s…

Black holes may have hairs!

The hairs are:

Other observables apart the mass **M**, charge **Q** and angular momentum **J**

- **Primary hairs: New global charges**
- **Secondary hairs: Fields which are not associated with a Gauss law**

Examples:

- Einstein gravity coupled to Yang-Mills fields
- Einstein gravity coupled to self-interacting scalar fields
- Modified models of gravity
- **Einstein-Skyrme theory**
- Higher dimensional theories
- Models in AdS spacetime
- Spinning black holes with matter fields

Localised solitons: Gravity vs Yang-Mills

Pure gravity (attraction)

$$L = -\frac{R}{16\pi G}$$

Lichenrowitz: there are black holes but there are no gravitational solitons, the only globally regular, asymptotically flat, static vacuum solution to the Einstein eqs with finite energy is Minkowski space.

Pure Yang-Mills (attraction/repulsion)

$$L = \frac{1}{2} \mathrm{Tr}\ F_{\mu\nu}^2$$

Derrek theorem: Classical Yang-Mills theory in 3+1 dim do not admit localised soliton solutions

Einstein-Yang-Mills model

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{(R-2\Lambda) - \mathrm{Tr}\ \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\}$$

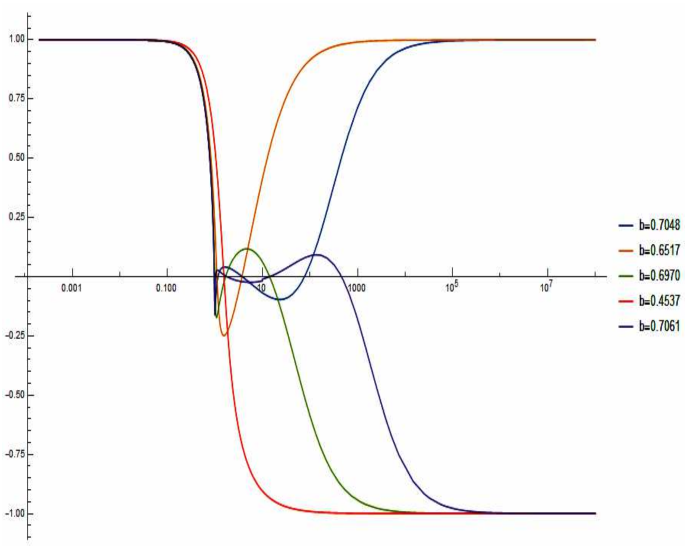
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}; \qquad D_\mu F^\mu_\nu = \nabla_\mu F^\mu_\nu + [A_\mu, F^\mu_\nu] = 0$$

Spherical symmetry:

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Static asymptotically flat solution

$$A^a_k = \varepsilon_{iak}\frac{x^k}{r^2}(w(r)-1)$$



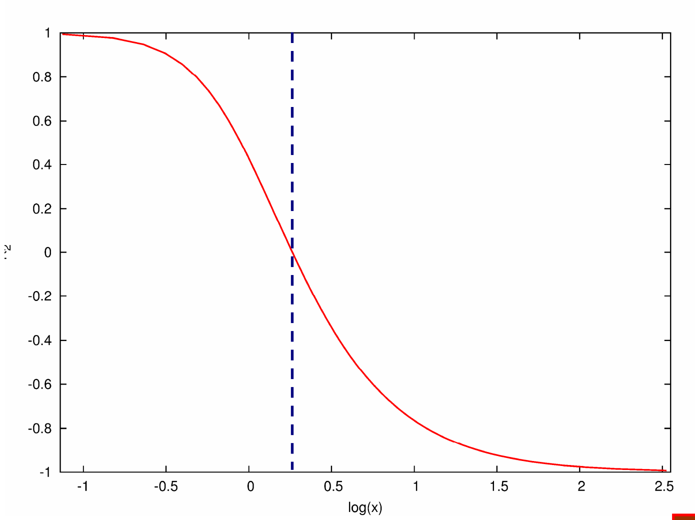
The Bartnik-McKinnon solitons (1988)

- Found numerically by the shooting method;
- The solution is globally regular;
- Analytic proof of existence of solutions of the differential equation;
- Gauge function $\boldsymbol{\omega}(\mathbf{r})$ has at least one zero, the solutions are characterized by the number of nodes of the $\boldsymbol{\omega}(\mathbf{r})$

Properties of the solutions

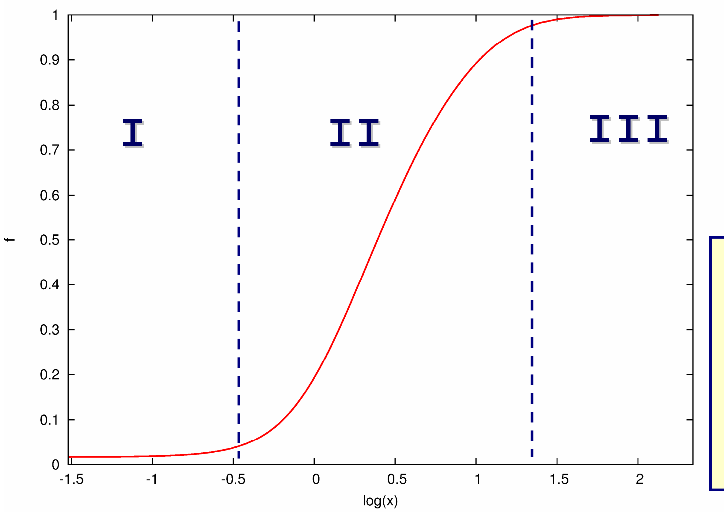
Dimensionless variables: $x = \frac{e}{\sqrt{4\pi G}} r \sim \frac{r}{l_{Pl}}; \quad \tilde{M} = eM\sqrt{\frac{G}{4\pi}} \sim \frac{M}{M_{Pl}}$

$$M_{Pl} \sim 1/\sqrt{G}; \quad l_{Pl} \sim \sqrt{G}$$



- **Region I:** Yang-Mills field is almost trivial, the metric is close to Schwarzchild
- **Region II:** Yang-Mills field corresponds to monopole the metric is almost Reissner–Nordström
- **Region III:** Yang-Mills field is almost trivial, the metric is asymptotically Schwarzchild

All Bartnik-McKinnon configurations are sphalerons



Galtsov, Volkov: **There are EYM black hole solutions with long-range non-abelian fields (hairy black holes)**

BM solutions are static asymptotically flat gravitationally bound EYM sphaleron solutions; the exterior of the limiting solution approaches RN black hole

Skyrme model

(Skyrme, 1961)

- **The Skyrme field:** $U(\vec{r},t) \underset{r\to\infty}{\longrightarrow} \mathbb{I}$

  $U: \; S^3 \to S^3$

f_π = 186 MeV, m_π = 136 MeV

$$L = \frac{f_\pi^2}{4}\mathrm{Tr}\;\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{Tr}\;\left(\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2\right) + \frac{m_\pi^2 f_\pi^2}{8}\mathrm{Tr}\;\left(U - \mathbb{I}\right)$$

Sigma-model term — Skyrme term — Potential term

- **The topological charge:** $Q = \frac{1}{24\pi^2}\varepsilon_{ijk}\int d^3x \mathrm{Tr}\;\left[(U^\dagger\partial^i U)(U^\dagger\partial^j U)(U^\dagger\partial^k U)\right]$

- **The su(2) current:** $R_i = (\partial_i U)U^\dagger$ ⇒ $Q = -\frac{1}{24\pi^2}\varepsilon_{ijk}\int d^3x \mathrm{Tr}(R_i R_j R_k)$

Rescaling: $x_\mu \to 2x_\mu/(ef_\pi); \quad m = 2m_\pi/(f_\pi e)$

$$E = \frac{f_\pi}{4e}\int d^3x \left\{-\frac{1}{2}\mathrm{Tr}\;(R_i R^i) - \frac{1}{16}\mathrm{Tr}\;([R_i, R_j])^2 + m^2\mathrm{Tr}(\mathrm{U} - \mathbb{I})\right\}$$

Gravitating Skyrmions

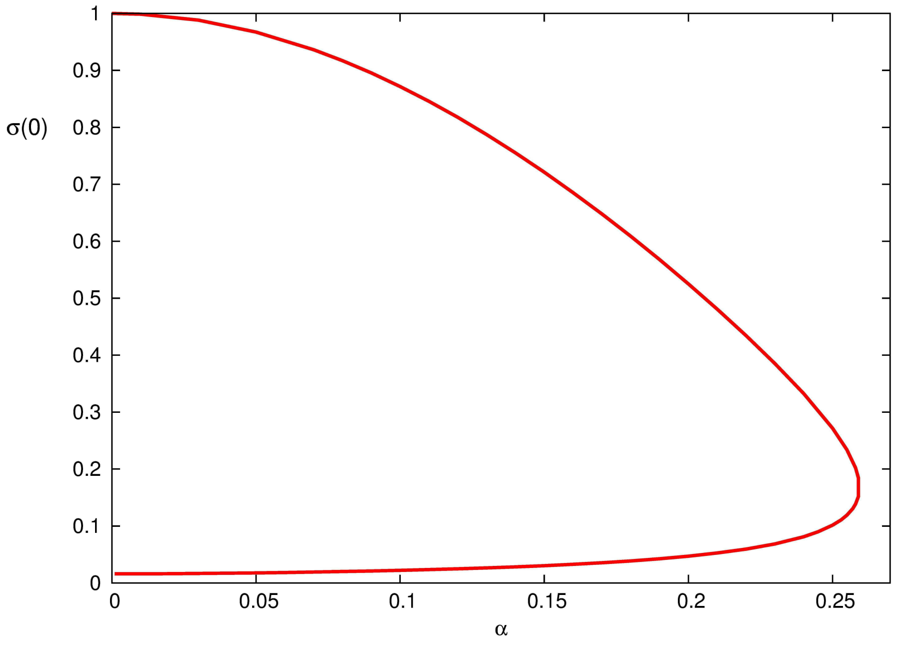
$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g} d^4x$$

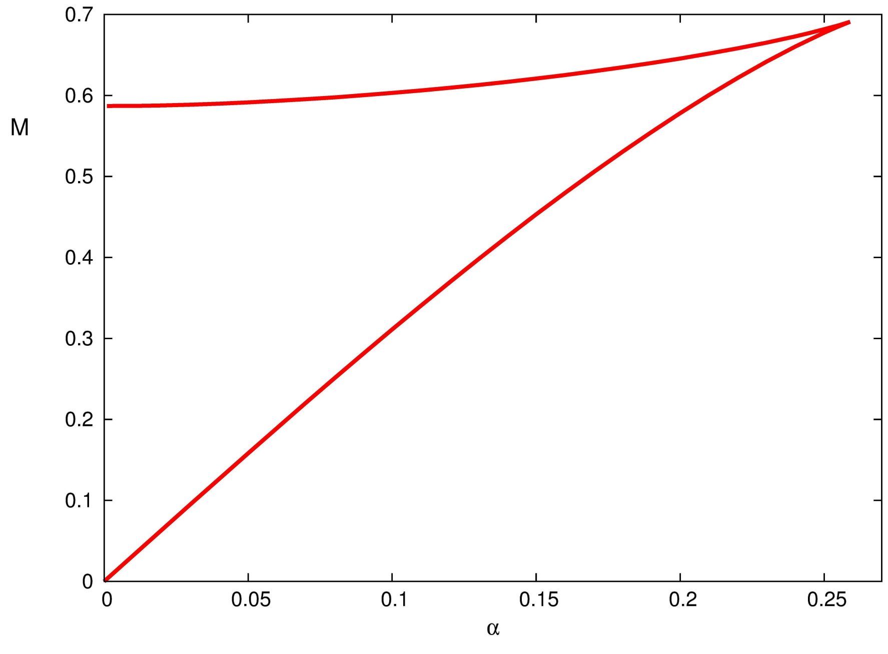
Dimensionless variables:

Spherical symmetry:

$$ds^2 = -\sigma^2(r) N(r) dt^2 + \frac{1}{N(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$L_2 + L_4$$





Gravitating Skyrmions

$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g} d^4x$$

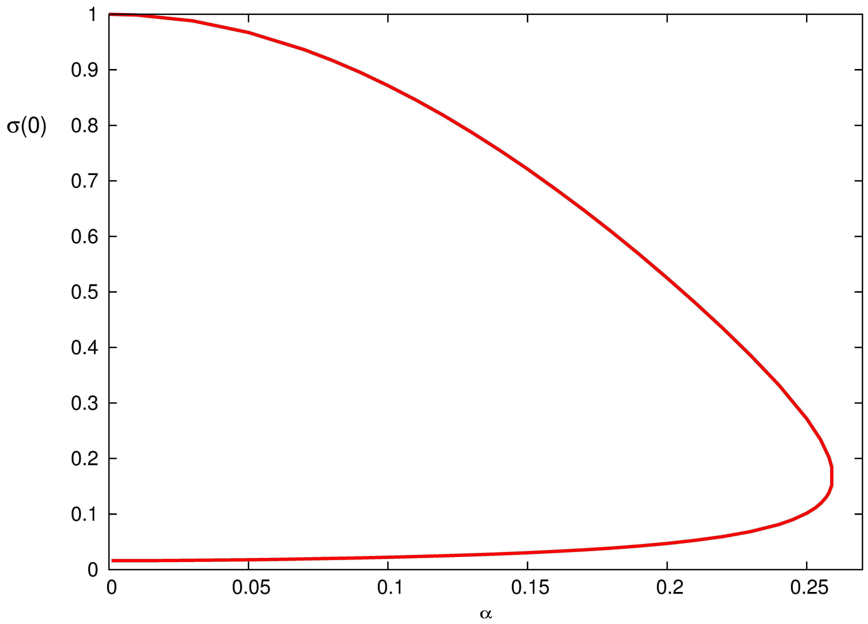
Dimensionless variables:

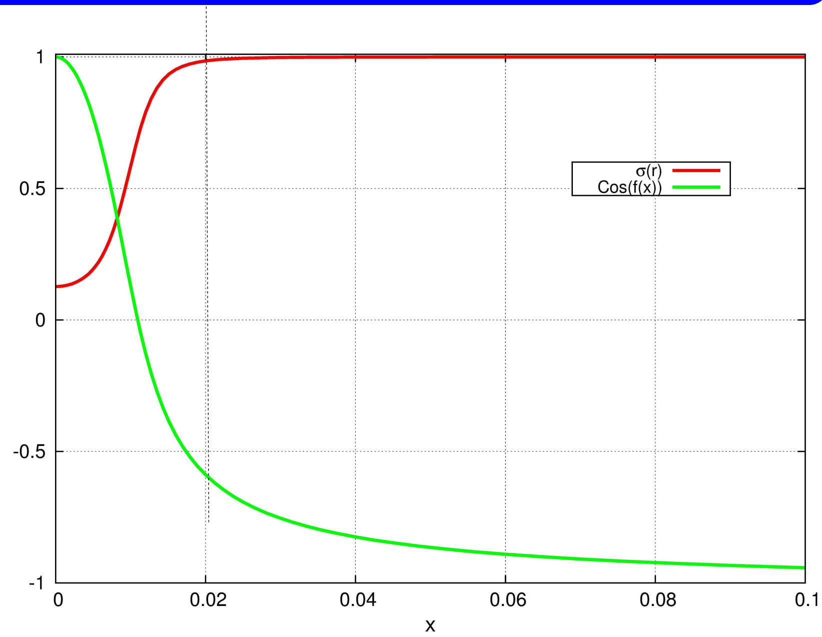
$$x = erf_\pi/2; \quad \alpha^2 = 4\pi G f_\pi$$

Spherical symmetry:

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$L_2 + L_4$ Branch of gravitating Skyrmions is linked to the BM solutions

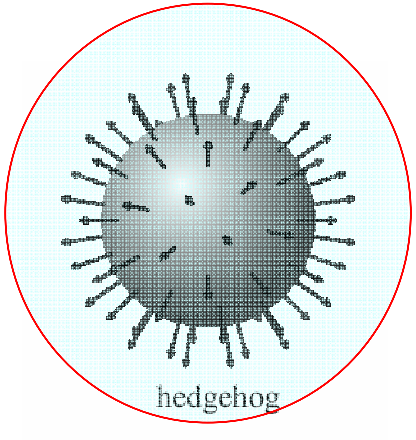




$$\cos[f(x)] \rightleftharpoons \omega(r)$$

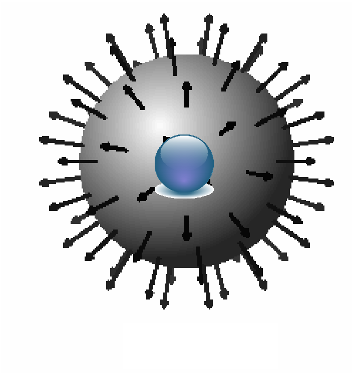
Black holes with skyrmionic hairs

(Luckock and Moss, 1986, Bison 1992, Shiiki and Sawado, 2005)

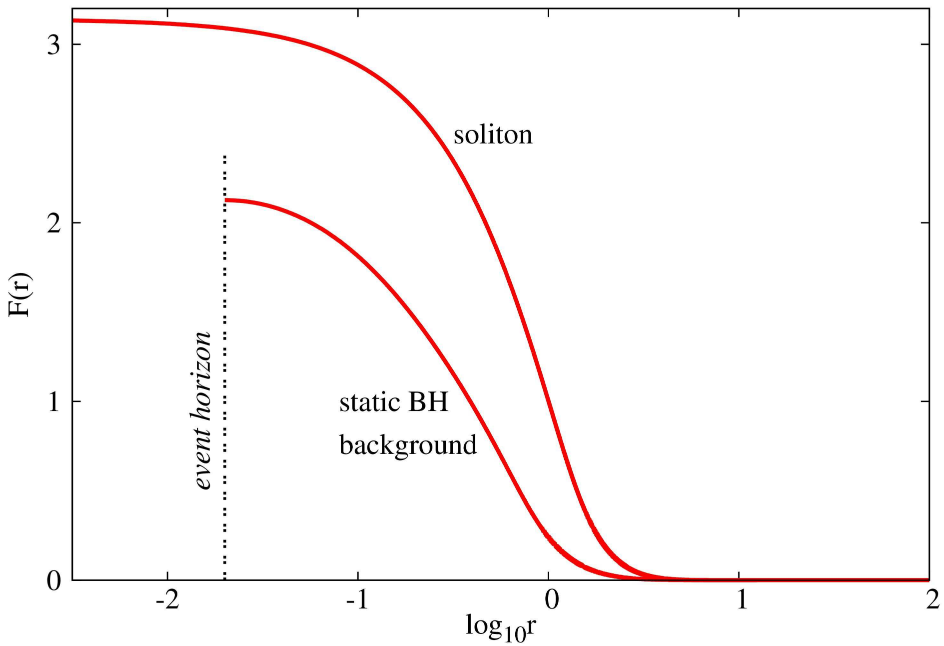


Skyrmion size $R_{Sk} \sim (eF_\pi)^{-1}$ vs Schwarzschild radius $R_{Sch} = 2M_{Sk}G$;

$M_{Sk} \sim F_\pi e^{-1}$ ⟶ $R_{Sk} \sim R_{Sch}$ as $F_\pi \sim M_{Pl} = G^{-1/2}$



Hairy black hole – event horizon *inside* Skyrmion



Gravitating isospinning Skyrmions

$$U(r) = \sigma + \pi^a \cdot \tau^a$$

T.Ioannidou, B.Kleihaus and J.Kunz
Phys.Lett. B643 (2006) 213

$$\pi_1 = \phi_1 \cos(n\varphi + \omega t); \quad \pi_2 = \phi_1 \sin(n\varphi + \omega t); \quad \pi_3 = \phi_2; \quad \sigma = \phi_3$$

Q=1

Pion clouds:

$$\phi_1 = \sin H(r,\theta); \quad \phi_2 = 0; \quad \phi_3 = \cos H(r,\theta)$$

Q=0

- **Lewis-Papapetrou parametrization:**

$$ds^2 = -f dt^2 + \frac{m}{f}\left(dr^2 + r^2 d\theta^2\right) + l r^2 \sin^2\theta \left(d\varphi - \frac{o}{r} dt\right)^2$$

- **Generalized Einstein-Skyrme model:**

I.Perapechka and Ya.Shnir
Phys.Rev. D96 (2017) 125006

$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g}\, d^4x$$

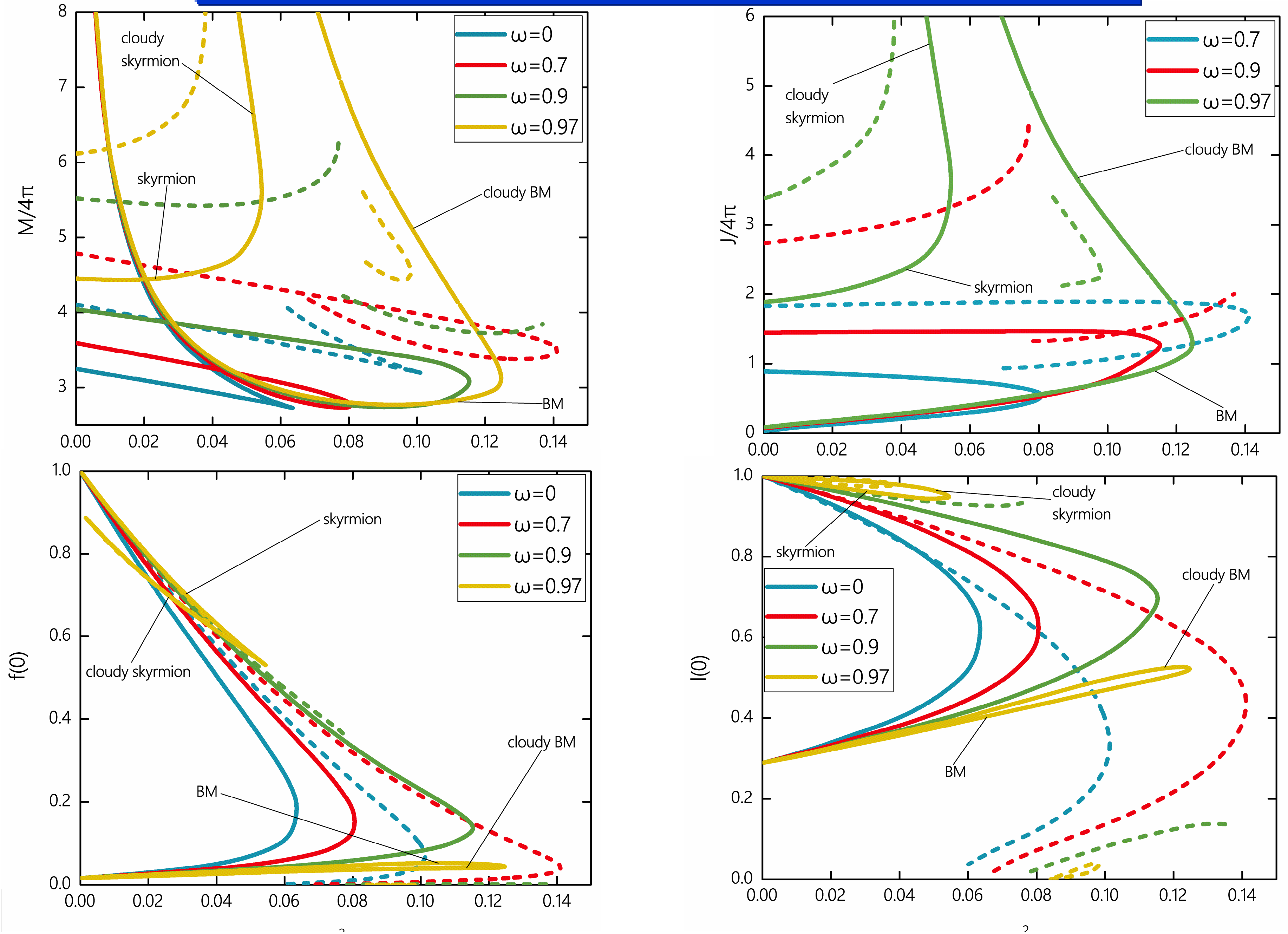
$$\mathcal{L}_{Sk} = L_2 + L_4 + cL_6 + L_0$$

Potential $\mu^2(1-\sigma^2)$

- **Asymptotic expansion:**

$$f \approx 1 - \frac{2MG}{r} + O\left(\frac{1}{r^2}\right), \qquad o \approx -\frac{2JG}{r^2} + O\left(\frac{1}{r^3}\right)$$

Gravitating isospinning Skyrmions



Spinning black holes with Skyrme hair

Line element (fixed Kerr BH geometry):

$$ds^2 = -F_0(r,\theta)dt^2 + F_1(r,\theta)\left(dr^2 + r^2 d\theta^2\right) + F_2(r,\theta)r^2\sin^2\theta\,[d\varphi - W(r,\theta)dt]^2$$

$$F_1(r,\theta) = \frac{2M_{\rm Kerr}^2}{r^2} + \left(1 - \frac{r_H^2}{r^2}\right)^2 + \frac{2M_{\rm Kerr}}{r}\left(1 + \frac{r_H^2}{r^2}\right) - \frac{M_{\rm Kerr}^2 - 4r_H^2}{r^2}\sin^2\theta$$

$$F_2(r,\theta) = \frac{S(r,\theta)}{F_1(r,\theta)} \qquad F_0(r,\theta) = \left(1 - \frac{r_H^2}{r^2}\right)^2 \frac{F_1(r,\theta)}{S(r,\theta)}$$

$$W(r,\theta) = \frac{2M_{Kerr}\sqrt{M_{\rm Kerr}^2 - 4r_H^2}}{r^3}\,\frac{1 + \frac{M_{\rm Kerr}}{r} + \frac{r_H^2}{r^2}}{S(r,\theta)}$$

$$S(r,\theta) = \left[\frac{2M_{\rm Kerr}^2}{r^2} + \left(1 - \frac{r_H^2}{r^2}\right)^2 + \frac{2M_{\rm Kerr}}{r}\left(1 + \frac{r_H^2}{r^2}\right)\right]^2 - \left(1 - \frac{r_H^2}{r^2}\right)^2 \frac{M_{\rm Kerr}^2 - 4r_H^2}{r^2}\sin^2\theta\,,$$

Two input parameters: r_H , M_{Kerr}

Komar integrals: Horizon *vs* asymptotic quantities

- **Komar integrals:**

$$M = \frac{3}{32\pi}\int_{\partial\Sigma} \star d\tilde{\xi}, \quad J = \frac{1}{16\pi}\int_{\partial\Sigma} \star d\tilde{\zeta}$$

1-form, dual to the timelike Killing vector ξ

1-form, dual to the spacelike Killing vector ζ

$\partial\Sigma \to \infty$ → Arnowitt Deser & Misner (ADM) mass and angular momentum (***M, J***)

$\partial\Sigma \to r_h$ → Horizon mass and angular momentum (***M_H, J_H***)

$$M = M_H + 2\int_{\Sigma} dS_\mu \left(T^\mu_\nu \xi^\nu - \frac{1}{2}T^\nu_\nu \xi^\mu\right)$$

$$J = J_H + \int_{\Sigma} dS_\mu \left(T^\mu_\nu \zeta^\nu - \frac{1}{2}T^\nu_\nu \zeta^\mu\right)$$

Fractions of ADM mass and angular momentum stored inside the horizon:

$$p = M_H/M, \quad q = J_H/J$$

p=1 → ***Kerr BH***

Skerrmions

$$U = \sigma \, \mathbb{I} + \tau^a \cdot \pi^a$$

$$\pi^1 + i\pi^2 = \phi^1(r,\theta) e^{i(m\varphi - wt)}, \;\; \pi^3 = \phi^2(r,\theta), \;\; \sigma = \phi^3(r,\theta)$$

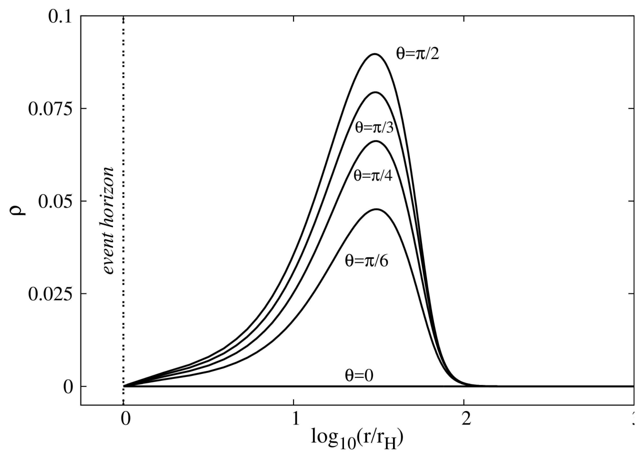
The event horizon angular velocity:

$$\Omega_H = \frac{\sqrt{M_{\rm Kerr}^2 - 4r_H^2}}{2M_{\rm Kerr}(M_{\rm Kerr} + 2r_H)}$$

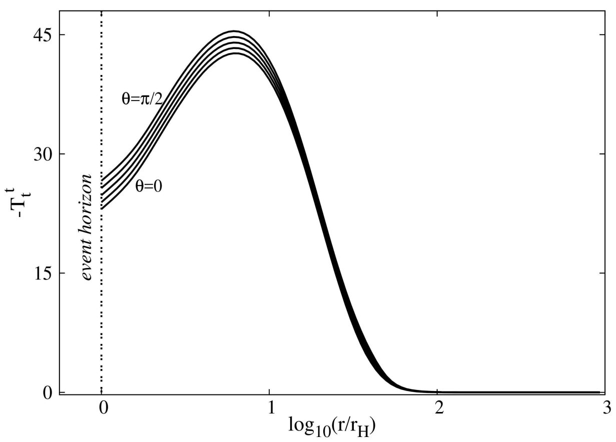
Synchronisation condition: $w = m\Omega_H$

$$L_m = L_2 + L_4$$

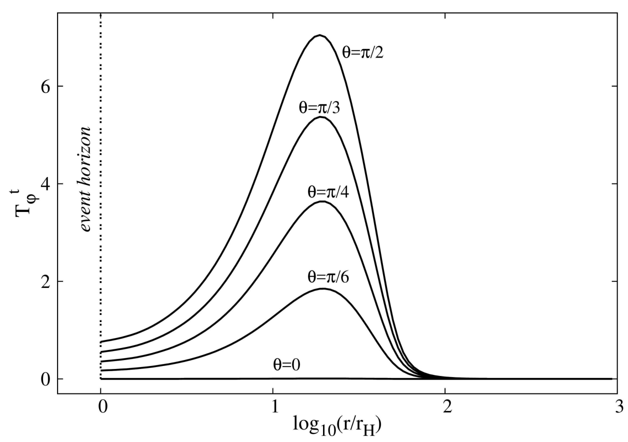
$$\Omega_H = 0.95, \quad M_{\bf Kerr} = 0.04$$



Q density



E density



J density

Skerrmions (Topological sector)

(Herdeiro, Perapechka, Radu & Ya S 2018)

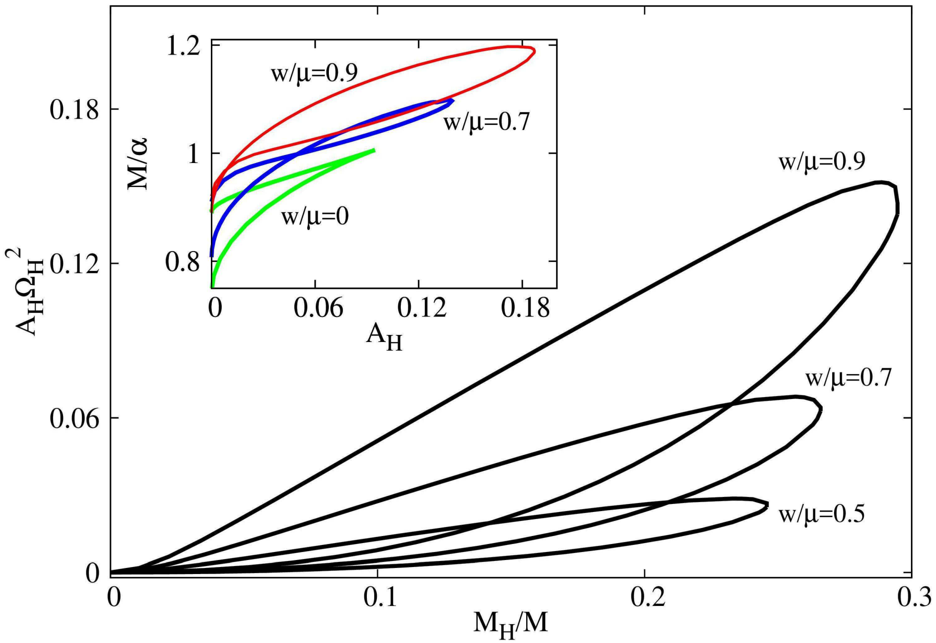
Line element (with backreaction):

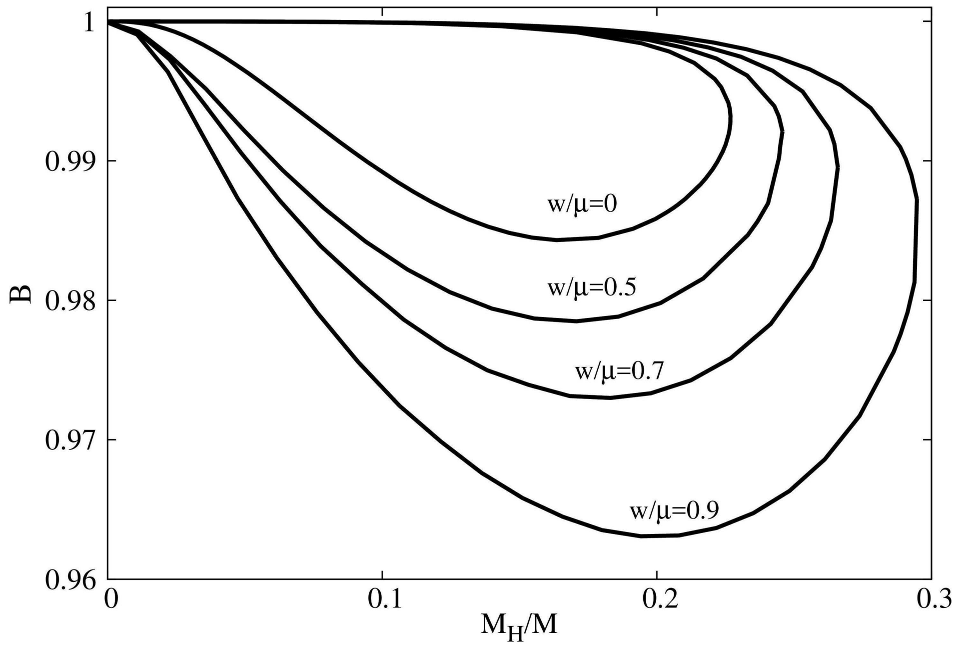
$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g} d^4x$$

$$ds^2 = -F_0(r,\theta)dt^2 + F_1(r,\theta)\left(dr^2 + r^2 d\theta^2\right) + F_2(r,\theta)r^2 \sin^2\theta \left[d\varphi - W(r,\theta)dt\right]^2$$

BH hairiness: $p = M_H/M$

- **p=1 → Kerr BH**
- **p=0 → GraviSkyrme**



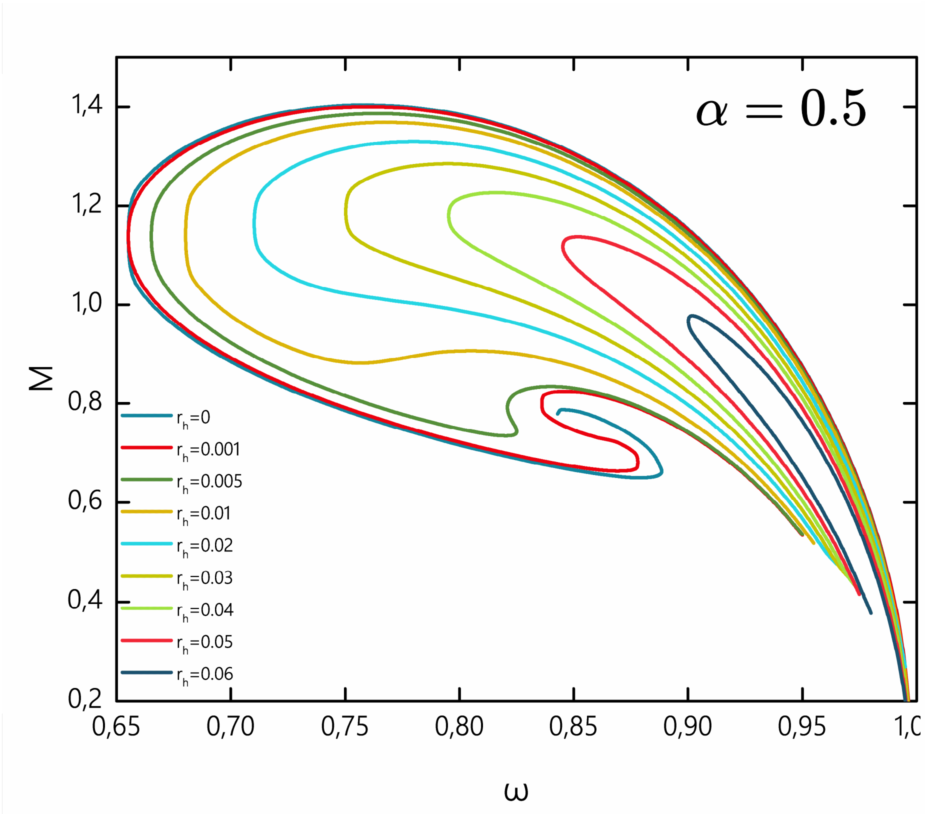


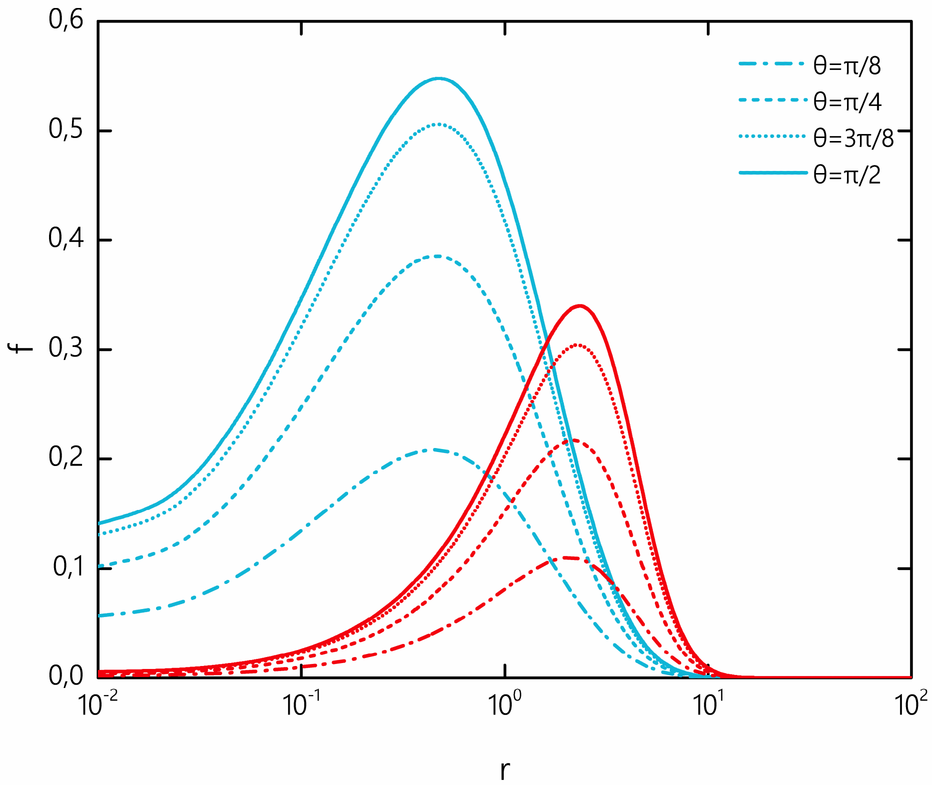
Skerrmions (Pion clouds)

$$U = \sigma\,\mathbb{I} + \tau^a \cdot \pi^a$$

$$\pi^1 + i\pi^2 = \sin f(r,\theta) e^{i(m\varphi - wt)},\ \ \pi^3 = 0,\ \ \sigma = \cos f(r,\theta)$$

- **U(1) Noether charge:** $J = mQ$





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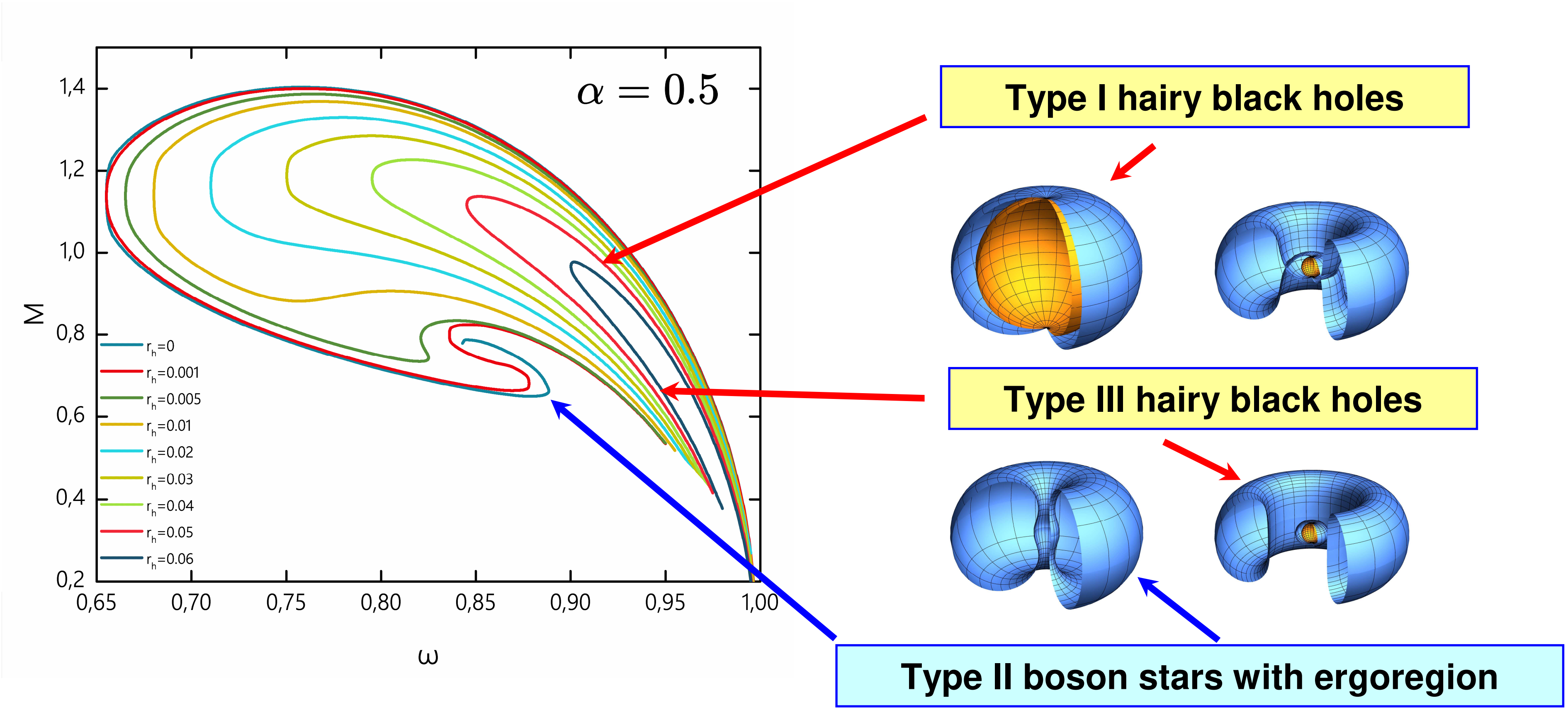
- **Ergosurfaces:** $g_{tt} = F_0^2 - r^2 \sin^2\theta F_2^2 W^2 = 0$



Kerr black holes with parity odd hairs

$$\mathcal{L}_m = |\partial_\mu \phi|^2 + \mu^2|\phi|^2$$

$$(\Box - \mu^2)\phi = 0$$

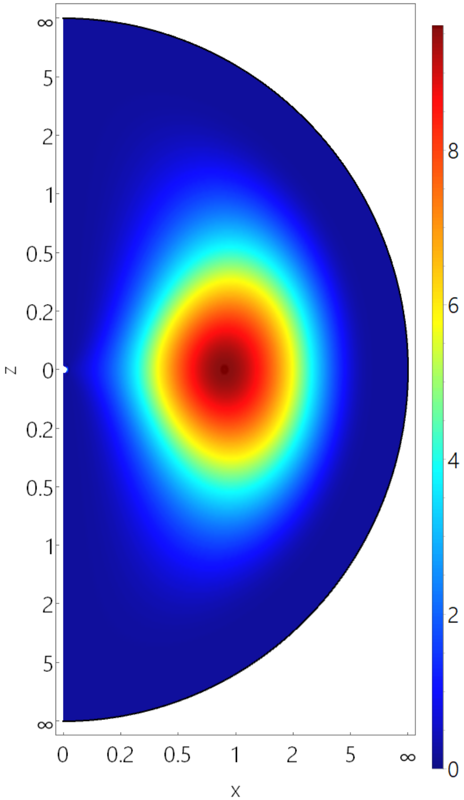
(Herdeiro & Radu 2014)

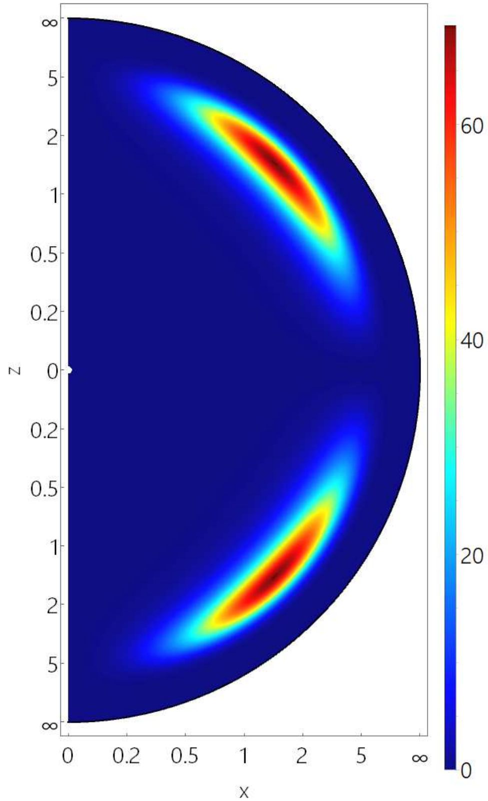
(Kunz, Perapechka & Ya S 2019)

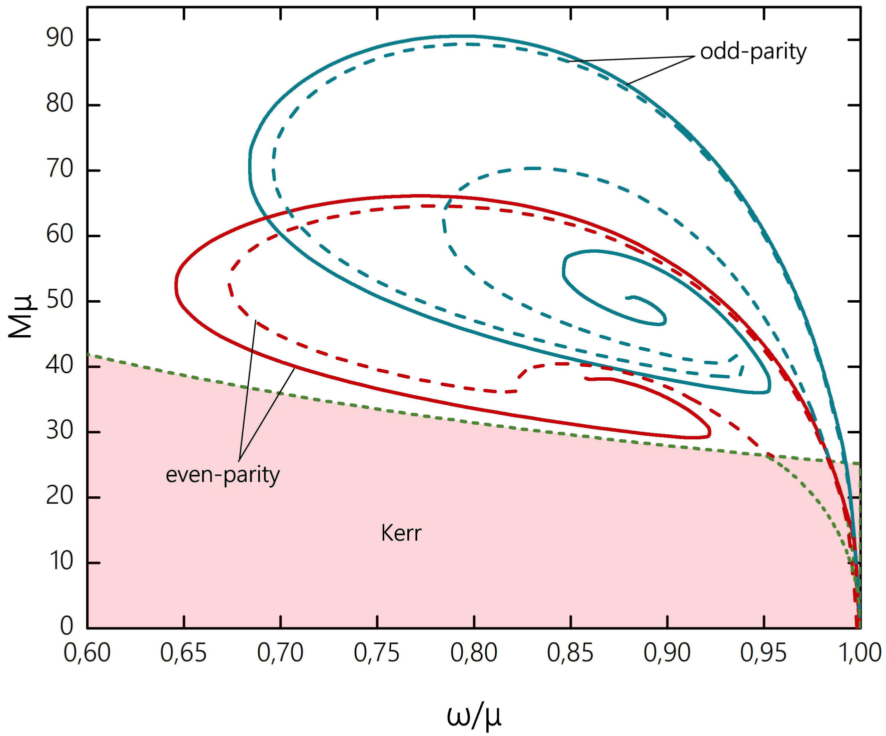
- **Parity-even solutions:** $f(r,\theta) = f(r,\pi-\theta)$
- **Parity-odd solutions:** $f(r,\theta) = -f(r,\pi-\theta)$

- **U(1) current:** $j_\mu = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$

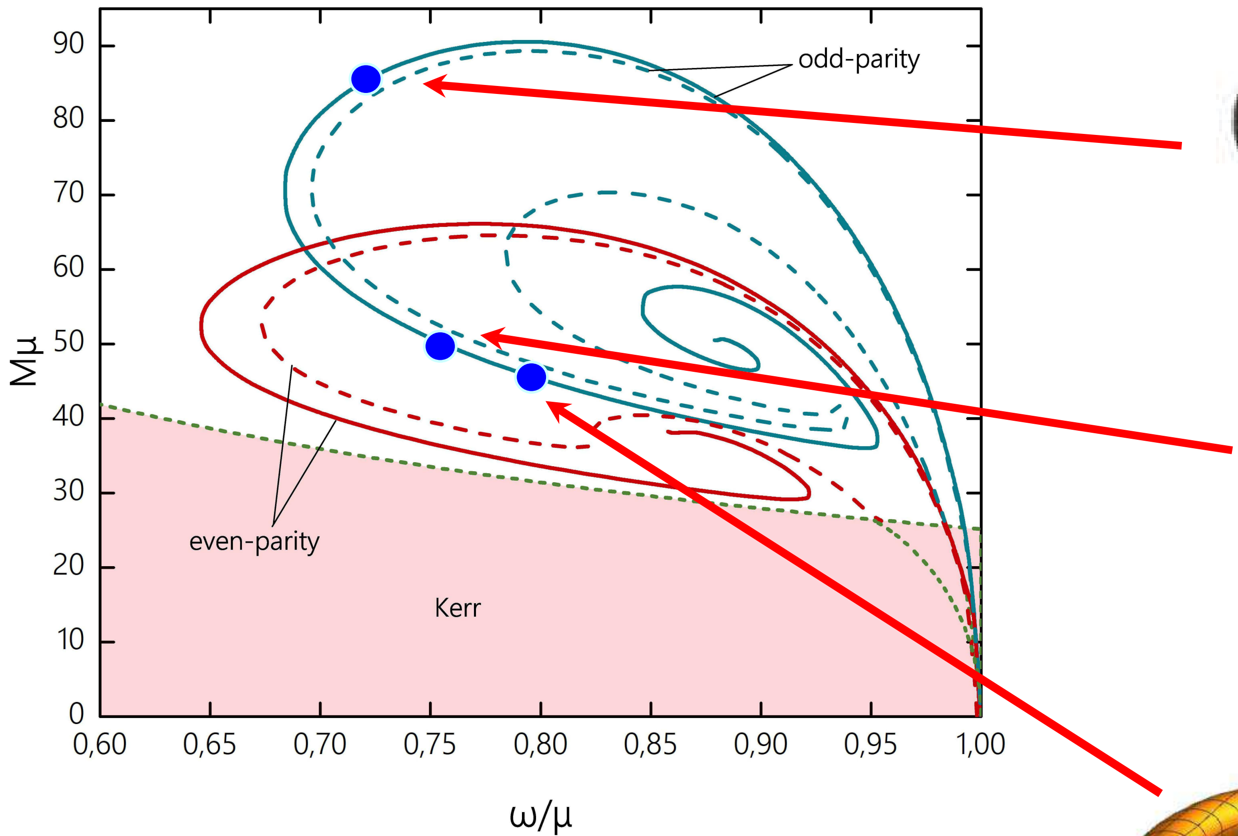
$$\phi = f(r,\theta)e^{i(\omega t + n\varphi)}$$

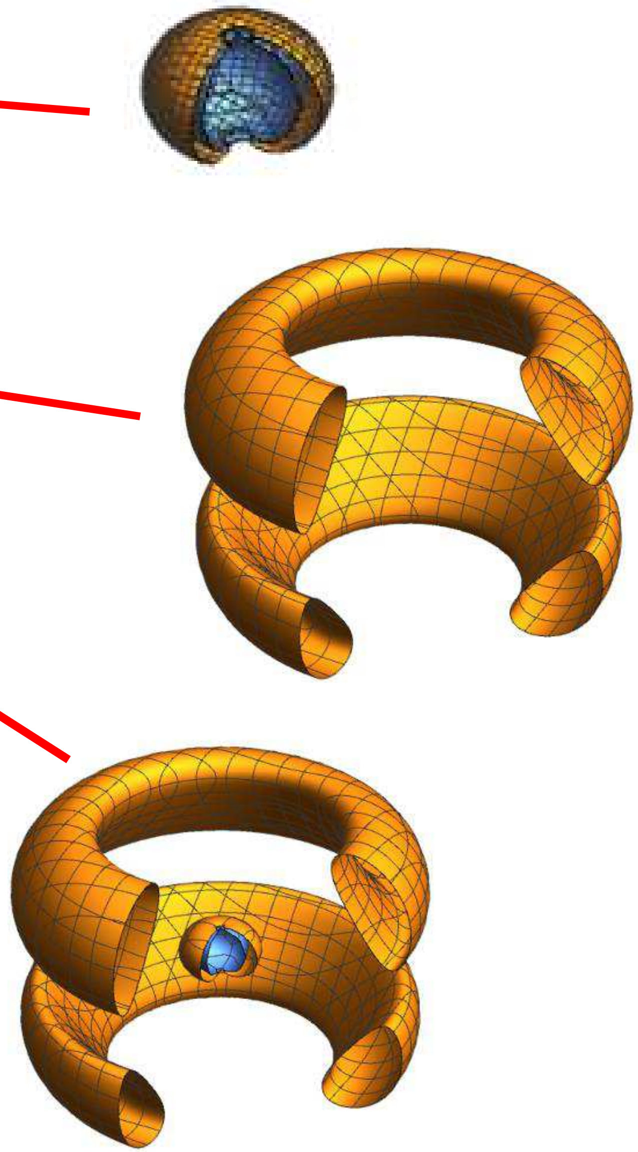






Ergosurfaces





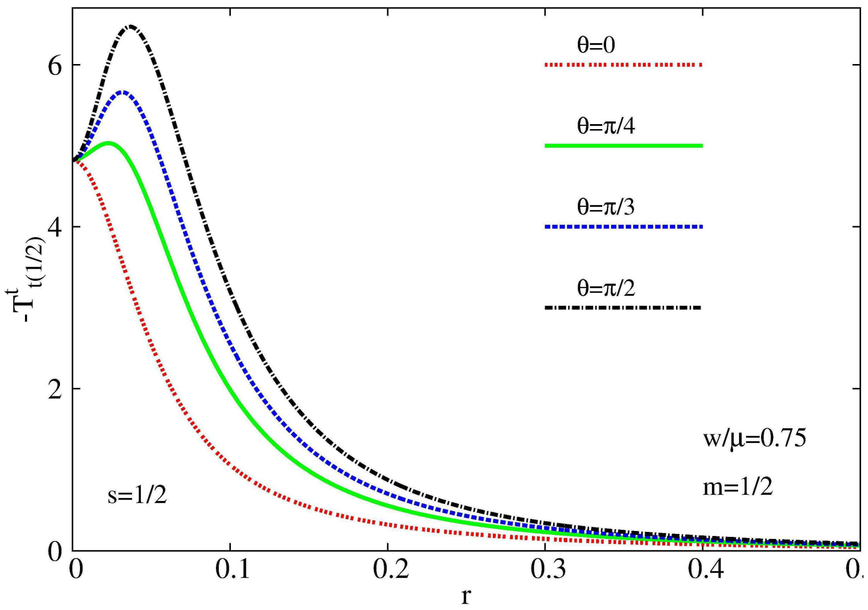
$$g_{tt} = F_0^2 - r^2 \sin^2 \theta F_2^2 W^2 = 0$$

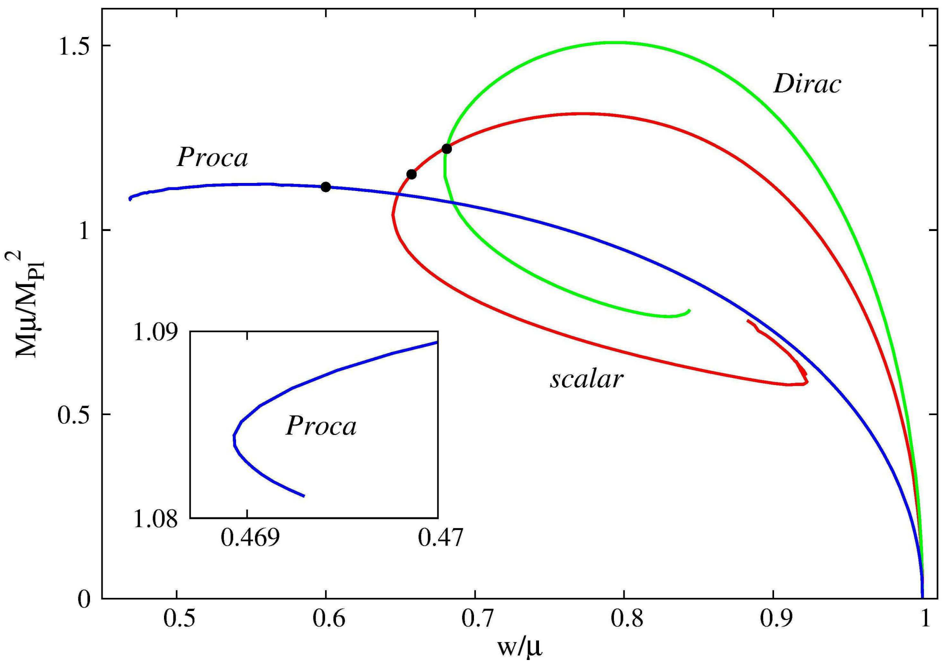
Dirac stars

(Herdeiro, Perapechka, Radu & Ya S 2019)

$$\mathcal{L}_m = -i\frac{1}{2}\left(\gamma^\mu D_\mu \bar{\Psi}\Psi - \bar{\Psi}\gamma^\mu D_\mu \Psi\right) + \mu\bar{\Psi}\Psi$$

- **Fermionic current:** $j_\mu = \bar{\Psi}\gamma_\mu \Psi$ $\qquad \bar{\Psi} = e^{i(m\varphi - \omega t)}(\psi_1, \psi_2, -i\psi_1^*, -i\psi_2^*)$





Summary

- There are Skyrme-type models which support self-gravitating regular topological solitons which are linked to hairy black holes
- The hairy black holes are necessarily spinning, the internal rotation (isorotation) must be synchronous with the rotational angular velocity of the event horizon.
- There is a family of rotating BHs with synchronised Skyrme hair, which are continously connected to the Kerr solution
- We constructed new family of non-topological Skerrmions which do not carry topological charge and bifurcate from a subset of Kerr solutions
- We constructed new families of parity-odd spinning boson stars and hairy BHs
- Similar pattern for axially symmetric boson stars and hairy Q-balls
- Fermion and boson stars show similar behavior

"How the Universe Works"
(Discovery channel, 2018)

Thank you!