Three-pronged junctions on $SO(2N)/U(N)$ and $Sp(N)/U(N)$

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Plan

- Introduction

- Hyper-Kähler (HK) nonlinear sigma models (NLSMs) on $T^*G_{N+M,M}$
On shell, the models in the $\mathcal{N}=1$ superspace formalism, in the harmonic superspace formalism and in the projective superspace formalism are the same.

- Models obtained by imposing quadratic constraints on the HK NLSM on $T^*G_{2N,N}$
The HK NLSMs on $T^*SO(2N)/U(N)$ and $T^*Sp(N)/U(N)$ as the quotient with respect to the gauge group are not known yet.

- Three-pronged junctions, which are the simplest observables we have in the NLSMs with complex mass parameters.

Introduction

- The HK NLSM on the cotangent bundle of the Grassmann manifold $T^*G_{N+M,M}$ is constructed in the $\mathcal{N}=1$ superspace formalism, in the harmonic superspace formalism and in the projective superspace formalism and they are equivalent on shell [A.Galperin & E.Ivanov & V.Ogievetsky & P.K.Townsend (86)][A.Galperin & E.Ivanov & V.Ogievetsky & E.Sokatchev (86)][S.Ketov & C.Unkmeir (98)][GIOS(01)][M.Arai & M.Naganuma & M.Nitta & N.Sakai (03)][M.Arai & M.Nitta & N.Sakai (03)][M.Arai & S.M.Kuzenko & U.Lindstrom (06)][T.Kim & SS (ongoing)].

- $SO(2N)/U(N)$ and $Sp(N)/U(N)$ are quadrics of $G_{2N,N}$.

- HK NLSMs on the Hermitian symmetric spaces including $SO(2N)/U(N)$ and $Sp(N)/U(N)$ are constructed in the projective superspace formalism [M.Arai & S.M.Kuzenko & U.Lindstrom (06)][M.Arai & S.M.Kuzenko & U.Lindstrom (07)].

Introduction (cont'd)

- HK NLSM on $T^*G_{N+M,M}$ in the harmonic superspace formalism: homogeneous coordinates as the quotient with respect to the gauge group, Fayet-Ilopoulos (FI) parameters.

- HK NLSM on T^*[Hermitian symmetric spaces] in the projective superspace formalism: inhomogeneous coordinates, no parameters corresponding to the FI parameters.

- Due to the $U(N)$ gauge fixing and the $SU(2)_R$ symmetry, the direct comparison between the HK NLSMs on the quadrics in the harmonic superspace and the HK NLSMs on the quadrics in the projective superspace would be a nontrivial task.

- SUSY vacua, 1/2 BPS walls of the mass-deformed NLSMs are on the Kähler manifold [Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai (04)].

- 1/4 BPS objects require complex masses of hypermultiplets $\rightarrow$ HK NLSM.

Introduction (cont'd)

- Other 1/4 BPS objects are discussed in the models, which are constructed by introducing complex mass parameters to the mass-deformed Kähler NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ [Eto & Fujimori & Gudnason & Jiang & Konishi & Nitta & Ohashi (11)]. The models are justified by the fact that there is a bundle structure when $b = 0 = b^*$, so that the chiral field of the hypermultiplet in the cotangent space does not contribute to the SUSY vacua and BPS equations.

Introduction (cont'd)

- Three-pronged junctions are the simplest 1/4 BPS objects.

- We use the moduli matrix formalism [Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai (04)] to study the SUSY vacua and the BPS objects.

- Quadratic constraints are embedded into the moduli matrices for SUSY vacua and 1/2 BPS walls on the Grassmann manifold [M.Arai & SS (11)][M.Eto & T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (11)][B-H.Lee & C.Park & SS (17)][M.Arai & A.Golubtosva & C.Park & SS (18)].

- Three-pronged junctions of the mass-deformed NLSM on $T^*\mathbf{C}P^{N-1}$ are constructed in the moduli matrix formalism [M.Eto & Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai (05)].

Introduction (cont'd)

- Three-pronged junctions of the mass-deformed NLSM on $T^*G_{N_F,N_C}$ are studied by embedding the base manifold G_{N_F,N_C} into $\mathbf{C}P^{{}^{N_F}C_{N_C}-1}$ by using the Plüker embedding [M.Eto & Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai (05)]. This resolves some complications of the moduli matrix formalism but the method is not applicable to the quadrics of the Grassmann manifold.

- An alternative method has been proposed and applied to the three-pronged junctions of the mass-deformed NLSMs on $T^*G_{N+M,M}$ [SS (18,19)]. This method is applicable to the quadrics of the Grassmann manifold.

Introduction (cont'd)

• We construct models by imposing the F-term constraints, which are vanishing $O(2N)$ or $USp(2N)$-invariants on the HK NLSM on $T^*G_{2N,N}$
$\rightarrow$ possible HK NLSMs on $T^*SO(2N)/U(N)$ or $T^*Sp(N)/U(N)$?

• We construct three-pronged junctions of the models and compute the positions of the junctions.
$\rightarrow$ the simplest physical observables that the models have.

Actions of the HK NLSM on $T^*G_{N+M,M}$

- Action in the $\mathcal{N}=1$ superspace [M.Rocek & P.K.Townsend (80)] [U.Lindstrom & M.Rocek (83)]

$$
\begin{aligned}
S = \int d^4x \Big\{ \int d^4\theta \Big[\mathrm{Tr}(\Phi\bar{\Phi}e^{V}) + \mathrm{Tr}(\bar{\Psi}\Psi e^{-V}) - c\,\mathrm{Tr}\,V \Big] \\
+ \int d^2\theta \Big[\mathrm{Tr}\left(\Xi(\Phi\Psi - bI_M)\right) + (\text{conjugate transpose}) \Big] \Big\}, \\
(c \in \mathbf{R}_{\geq 0},\ b \in \mathbf{C}). \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&\Phi_A{}^{I}(y) = \mathcal{A}_A{}^{I}(y) + \sqrt{2}\theta\zeta_A{}^{I}(y) + \theta\theta F_A{}^{I}(y), \quad (y^{\mu} = x^{\mu} + i\theta\sigma^{\mu}\bar{\theta}), \\
&\Psi_I{}^{A}(y) = \mathcal{B}_I{}^{A}(y) + \sqrt{2}\theta\eta_I{}^{A}(y) + \theta\theta G_I{}^{A}(y), \\
&V_A{}^{B}(x) = 2\theta\sigma^{\mu}\bar{\theta}A_{\mu A}{}^{B}(x) + i\theta\theta\bar{\theta}\bar{\lambda}_A{}^{B}(x) - i\bar{\theta}\bar{\theta}\theta\lambda_A{}^{B}(x) + \theta\theta\bar{\theta}\bar{\theta}D_A{}^{B}(x), \\
&\Xi_A{}^{B}(y) = -\mathcal{S}_A{}^{B}(y) + \theta\omega_A{}^{B}(y) + \theta\theta K_A{}^{B}(y), \\
&(i = 1, \cdots, N+M;\ A = 1, \cdots, M). \qquad (2)
\end{aligned}
$$

HK NLSM on $T^*G_{N+M,M}$ in the $\mathcal{N}=1$ superspace formalism

- Constraints

$$\Phi\bar{\Phi}e^{V} - e^{-V}\bar{\Psi}\Psi - cI_M = 0, \tag{3}$$
$$\Phi\Psi - bI_M = 0, \quad \text{(c.t.)}. \tag{4}$$

- We can eliminate the vector field V. Then the potential for the HK manifold [U.Lindstrom & M.Rocek (83)] is

$$K = \mathrm{Tr}\sqrt{c^2 I_M + 4\Phi\bar{\Phi}\bar{\Psi}\Psi} - c\,\mathrm{Tr}\ln\left(cI_M + \sqrt{c^2 I_M + 4\Phi\bar{\Phi}\bar{\Psi}\Psi}\right)$$
$$+ c\,\mathrm{Tr}\ln(\Phi\bar{\Phi}). \tag{5}$$

- The constraints (4) can be solved by two cases $b=0$ and $b\neq 0$ with proper gauge fixing.

$\underline{b=0}$

$$\Phi = \begin{pmatrix} I_M & f \end{pmatrix}, \quad \Psi = \begin{pmatrix} -fg \\ g \end{pmatrix}. \tag{6}$$

$\underline{b\neq 0}$

$$\Phi = Q\begin{pmatrix} I_M & s \end{pmatrix}, \; \Psi = \begin{pmatrix} I_M \\ t \end{pmatrix} Q,$$
$$Q = \sqrt{b}\,(I_M + st)^{-\frac{1}{2}}. \tag{7}$$

HK NLSM on $T^*G_{N+M,M}$ in the $\mathcal{N}=1$ superspace formalism (cont'd)

$b=0$: with the parametrisation (6), the potential (5) becomes

$$K = \mathrm{Tr}\sqrt{c^2 I_M + 4(I_M + f\bar{f})\bar{g}(I_N + \bar{f}f)g} \\ - c\,\mathrm{Tr}\ln\left(cI_M + \sqrt{c^2 I_M + 4(I_M + f\bar{f})\bar{g}(I_N + \bar{f}f)g}\right) \\ + c\,\mathrm{Tr}\ln(I_M + f\bar{f}). \tag{8}$$

For $g=0$, the potential (8) becomes the potential of the Grassmann manifold. Therefore f parametrises the base Grassmann manifold whereas g parametrises the cotangent space as the fiber [U.Lindstrom & M.Rocek (83)][M.Arai & M.Nitta & N.Sakai (03)]. The potential (8) with $M=1$ and $c=1$, is equivalent to the potential, which is constructed in the projective superspace formalism [M.Arai & S.M.Kuzenko & U.Lindstrom (06)].

Kähler NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$

- Adding the F-term constraints, which are vanishing $O(2N)$ or $USp(2N)$ invariants to the Kähler NLSM on $G_{2N,N}$ [Higashijima & Nitta (00)]

$$
\begin{aligned}
S = \int d^4x \Big\{ \int d^4\theta \Big[\mathrm{Tr}(\Phi\bar{\Phi}e^V) - c\,\mathrm{Tr}\,V \Big] \\
+ \int d^2\theta \Big[\mathrm{Tr}\left(\Phi_0 \Phi J \Phi^T\right) + (\text{conjugate transpose}) \Big] \Big\}, \\
(c \in \mathbf{R}_{\geq 0}). \qquad (9)
\end{aligned}
$$

$$
J = \begin{pmatrix} 0 & I_N \\ \epsilon I_N & 0 \end{pmatrix}, \quad \epsilon = \begin{cases} 1, & \text{for } SO(2N)/U(N) \\ -1, & \text{for } Sp(N)/U(N). \end{cases} \qquad (10)
$$

- Isomorphism

$$
\begin{aligned}
&\mathbf{C}P^1 \sim SO(4)/U(2) \sim Sp(1)/U(1) \sim Q^1, \\
&\mathbf{C}P^3 \sim SO(6)/U(3). \qquad (11)
\end{aligned}
$$

It is shown that the Kähler potentials are equivalent [Higashijima & Kimura & Nitta (01)].

Adding F-term constraints to the HK NLSM on $T^*G_{2N,N}$

Since there is a bundle structure, we guess that we can find a(n) (anti)holomorphic embedding into $T^*G_{2N,N}$ when $b = 0 = b^*$. The hypermultiplet consists of Φ and $\bar{\Psi}$. The HK NLSMs on the cotangent bundles of $SO(2N)/U(N)$ and $Sp(N)/U(N)$ should incorporate the F-term constraints of the Kähler NLSMs, which are discussed in [Higashijima & Nitta (99)] and have the same number of F-term constraints for Φ and $\bar{\Psi}$. Therefore there exists only one possible action for the NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ as a submanifolds of $T^*G_{2N,N}$ in the $\mathcal{N} = 1$ superspace formalism:

$$S = \int d^4x \Big\{ \int d^4\theta \Big[\mathrm{Tr}(\Phi\bar{\Phi}e^V) + \mathrm{Tr}(\bar{\Psi}\Psi e^{-V}) - c\,\mathrm{Tr}\,V \Big] + \int d^2\theta \Big[\mathrm{Tr}\left(\Xi\Phi\Psi + \Phi_0\Phi J\Phi^T + \Psi_0\Psi^T J\Psi \right) + (\mathrm{c.t.}) \Big] \Big\}, \quad (c \in \mathbf{R}_{\geq 0}). \tag{12}$$

Φ and $\bar{\Psi}$ are $N \times 2N$ chiral matrix fields. Φ_0 and Ψ_0 are $N \times N$ chiral matrix fields, which are introduced as Lagrange multipliers.

Adding F-term constraints to the HK NLSM on $T^*G_{2N,N}$ (cont'd)

- Φ and Ψ are constrained by the invariant tensor J of $O(2N)$ or $USp(2N)$:

$$\Phi J \Phi^T = 0, \tag{13}$$
$$\Psi^T J \Psi = 0, \tag{14}$$

$$J = \begin{pmatrix} 0 & I_N \\ \epsilon I_N & 0 \end{pmatrix}, \quad \epsilon = \begin{cases} 1, & \text{for } SO(2N)/U(N) \\ -1, & \text{for } Sp(N)/U(N). \end{cases} \tag{15}$$

- $\Phi J \Phi^T$ and $\Psi^T J \Psi$ are symmetric (antisymmetric) for $SO(2N)$ ($USp(2N)$)
$\Rightarrow$ Φ_0 and Ψ_0 are symmetric (antisymmetric) rank-2 tensors
- $U(1)$ gauge: charge of Φ_0 is -2 whereas the $U(1)$ charge of Ψ_0 is $+2$
- $SU(N)$ gauge: The $G^{\mathbf{C}}$-invariants (13) and (14) vanish, so that they are consistent with the gauge symmetry. $\rightarrow$ the "consistency condition with a gauge symmetry" in [Higashijima & Nitta (99)].

Adding F-term constraints to the HK NLSM on $T^*G_{2N,N}$ (cont'd)

- $b = 0,\ M = N$

$$\Phi = \left(I_N \quad f \right), \quad \Psi = \left(\begin{array}{c} -fg \\ g \end{array} \right). \tag{16}$$

$$\Phi J \Phi^T = f^T + \epsilon f = 0, \tag{17}$$

$$\Psi^T J \Psi = -g^T (f^T + \epsilon f) g = 0. \tag{18}$$

Constraint (17), for Φ guarantees that Ψ obeys the other constraint (18). It should be noted that field g, which parametrises the cotangent space as the fiber, is not constrained by the invariant tensor (15). Since g is not constrained by J, the bosonic component fields of Ψ do not contribute to the continuous vacuum. Therefore the bosonic component fields of the homogeneous Ψ before the gauge fixing, also do not contribute to the vacuum.

Adding F-term constraints to the HK NLSM on $T^*G_{2N,N}$ (cont'd)

- Mass-deformed action

$$S = \int d^4x \Big\{ \int d^4\theta \Big[\mathrm{Tr}(\Phi\bar{\Phi}e^V) + \mathrm{Tr}(\bar{\Psi}\Psi e^{-V}) - c\,\mathrm{Tr}\,V \Big] + \int d^2\theta \Big[\mathrm{Tr}\left(\Xi\Phi\Psi + \Phi M \Psi + \Phi_0 \Phi J \Phi^T + \Psi_0 \Psi^T J \Psi \right) + (\mathrm{c.t.}) \Big] \Big\}. \quad (19)$$

- Bosonic component fields

$$\begin{aligned} &\Phi_A{}^I(y) = \mathcal{A}_A{}^I(y) + \theta\theta F_A{}^I(y), \quad (y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}), \\ &\Psi_I{}^A(y) = \mathcal{B}_I{}^A(y) + \theta\theta G_I{}^A(y), \\ &V_A{}^B(x) = 2\theta\sigma^\mu\bar{\theta}A_{\mu A}{}^B(x) + \theta\theta\bar{\theta}\bar{\theta}D_A{}^B(x), \\ &\Xi_A{}^B(y) = -\mathcal{S}_A{}^B(y) + \theta\theta K_A{}^B(y), \\ &\Phi_0(y) = \mathcal{A}_0(y) + \theta\theta F_0(y), \\ &\Psi_0(y) = \mathcal{B}_0(y) + \theta\theta G_0(y), \\ &(A = 1, \cdots, N;\ I = 1, \cdots, 2N). \end{aligned} \quad (20)$$

Adding F-term constraints to the HK NLSM on $T^*G_{2N,N}$ (cont'd)

- Bosonic part of the action

$$S = \int d^4x \,\mathrm{Tr}\Big(D_\mu \mathcal{A}\overline{D^\mu \mathcal{A}} + \overline{D_\mu \mathcal{B}} D^\mu \mathcal{B} - |\mathcal{A}M - \mathcal{S}\mathcal{A} + 2\mathcal{B}_0\mathcal{B}^T J|^2 - |M\mathcal{B} - \mathcal{B}\mathcal{S} + 2J\mathcal{A}^T\mathcal{A}_0|^2\Big), \tag{21}$$

with constraints

$$\begin{aligned} &\mathcal{A}\bar{\mathcal{A}} - \bar{\mathcal{B}}\mathcal{B} - cI_M = 0, \\ &\mathcal{A}\mathcal{B} = 0, \quad \text{(c.t)=0}, \\ &\mathcal{A}J\mathcal{A}^T = 0, \quad \text{(c.t)=0}, \\ &\mathcal{B}^T J\mathcal{B} = 0, \quad \text{(c.t)=0}. \end{aligned} \tag{22}$$

- We set $\mathcal{B} = 0 = \bar{\mathcal{B}}$. Then the Lagrangian that describes vacua, walls and junctions of the mass-deformed NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ is

$$\mathcal{L} = \mathrm{Tr}\Big(D_\mu \mathcal{A}\overline{D^\mu \mathcal{A}} - |\mathcal{A}M - \mathcal{S}\mathcal{A}|^2 - |2J\mathcal{A}^T\mathcal{A}_0|^2\Big), \tag{23}$$

with constraints

$$\begin{aligned} &\mathcal{A}\bar{\mathcal{A}} - cI_M = 0, \\ &\mathcal{A}J\mathcal{A}^T = 0, \quad \text{(c.t.)=0}. \end{aligned} \tag{24}$$

Adding F-term constraints to the HK NLSM on $T^*G_{2N,N}$ (cont'd)

The Lagrangian (23) is mass-deformed Kähler NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ with the complex extension of the mass matrix and the scalar matrix field of the $U(N)$ gauge group. This type of extension has been considered to construct dyonic configurations with non-parallel charge vectors [M.Eto & T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (11)].

By introducing real valued matrices M_α and Σ_α, $(\alpha = 1, 2)$

$$\begin{aligned} M &= M_1 + iM_2, \\ \mathcal{S} &= \Sigma_1 + i\Sigma_2, \end{aligned} \tag{25}$$

the Lagrangian (23) can be rewritten as

$$\mathcal{L} = \mathrm{Tr}\left(D_\mu \mathcal{A} \overline{D^\mu \mathcal{A}} - \sum_{\alpha=1,2} |\mathcal{A} M_\alpha - \Sigma_\alpha \mathcal{A}|^2 - |2J\mathcal{A}^T\mathcal{A}_0|^2 \right). \tag{26}$$

BPS solutions in the moduli matrix formalism

- Cartan generators of $SO(2N)$ and $USp(2N)$

$$H_I = e_{I,I} - e_{N+I,N+I}, \quad (I = 1, \cdots, N), \tag{27}$$

- Complex mass matrix

$$\begin{aligned} \underline{l} &:= (m_1 + in_1, m_2 + in_2, \cdots, m_N + in_N), \\ \underline{H} &:= (H_1, H_2, \cdots, H_N), \\ M &= \underline{l} \cdot \underline{H}. \end{aligned} \tag{28}$$

- Matrix field $\mathcal{S}$

$$\mathcal{S} = \mathrm{diag}(\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \cdots, \sigma_N + i\tau_N), \quad (\sigma_i, \tau_i \in \mathbf{R}). \tag{29}$$

- Vacuum conditions

$$\mathcal{A}M - \mathcal{S}\mathcal{A} = 0, \quad \text{(c.t.)}, \quad \mathcal{A}^T \mathcal{A}_0 = 0, \quad \text{(c.t.)}. \tag{30}$$

- Vacuum solutions

$$\begin{aligned} &(\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \cdots, \sigma_N + i\tau_N) \\ &= (\pm(m_1 + in_1), \pm(m_2 + in_2), \cdots, \pm(m_N + in_N)). \end{aligned} \tag{31}$$

BPS solutions in the moduli matrix formalism (cont'd)

We study three-pronged junctions of the mass-deformed NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$. We are interested in a static configurations, which is independent of the x^3-coordinate. We also assume that there is the Poincaré invariance on the worldvolume. Therefore we set $\partial_0 = \partial_3 = 0$ and $A_0 = A_3 = 0$.

- Energy density

$$\mathcal{E} = \mathrm{Tr}\left(\sum_{\alpha=1,2}\left|D_\alpha \mathcal{A} \mp (\mathcal{A}M_\alpha - \Sigma_\alpha \mathcal{A})\right|^2 + |2J\mathcal{A}^T\mathcal{A}_0|^2\right) \pm \mathcal{T} \geq \pm\mathcal{T}. \quad (32)$$

- Tension density

$$\mathcal{T} = \mathrm{Tr}\left(\sum_{\alpha=1,2}\partial_\alpha\left(\mathcal{A}M_\alpha\bar{\mathcal{A}}\right)\right). \quad (33)$$

We use the index $\alpha = 1, 2$ for codimensions and adjoint scalars.

BPS solutions in the moduli matrix formalism (cont'd)

- (Anti)BPS equation

$$D_\alpha \mathcal{A} \mp (\mathcal{A} M_\alpha - \Sigma_\alpha \mathcal{A}) = 0, \quad (\alpha = 1, 2). \tag{34}$$

- BPS solution

$$\begin{aligned} \mathcal{A} &= S^{-1} H_0 e^{M_1 x^1 + M_2 x^2}, \\ S^{-1} \partial_\alpha S &:= \Sigma_\alpha - i A_\alpha, \quad (\alpha = 1, 2). \end{aligned} \tag{35}$$

The coefficient matrix H_0 is the moduli matrix.

- Constraints

$$\mathcal{A} \bar{\mathcal{A}} - c I_M = 0 \;\to\; S \bar{S} = \frac{1}{c} H_0 e^{2M_1 x^1 + 2M_2 x^2} \bar{H}_0, \tag{36}$$

$$\mathcal{A} J \mathcal{A}^T = 0 \;\to H_0 J \bar{H}_0 = 0. \tag{37}$$

- The BPS solution (35), Σ_α and A_α are invariant under the transformations:

$$H_0' = V H_0, \quad S' = V S, \quad V \in GL(N, \mathbf{C}). \tag{38}$$

This equivalent class of (S, H_0) is the worldvolume symmetry in the moduli matrix formalism. Eqs. (38) and (37) show that the moduli matrices H_0's parametrise $SO(2N)/U(N)$ and $Sp(N)/U(N)$ respectively.

BPS solutions in the moduli matrix formalism (cont'd)

In the moduli matrix formalism, walls are constructed from elementary walls. The elementary walls can be identified with the simple roots of the global symmetry group [N.Sakai & D.Tong (05)].

- $SO(2N)$

$$\begin{aligned} E_i &= e_{i,i+1} - e_{i+N+1,i+N}, \quad (i = 1, \cdots, N-1), \\ E_N &= e_{N-1,2N} - e_{N,2N-1}, \\ \underline{\alpha}_i &= \hat{e}_i - \hat{e}_{i+1}, \\ \underline{\alpha}_N &= \hat{e}_{N-1} + \hat{e}_N. \end{aligned} \tag{39}$$

- $USp(2N)$

$$\begin{aligned} E_i &= e_{i,i+1} - e_{i+N+1,i+N}, \quad (i = 1, \cdots, N-1), \\ E_N &= e_{N,2N}, \\ \underline{\alpha}_i &= \hat{e}_i - \hat{e}_{i+1}, \\ \underline{\alpha}_N &= 2\hat{e}_N. \end{aligned} \tag{40}$$

Junctions of the mass-deformed NLSM on $SO(2N)/U(N)$

We start from vacua and walls by setting $M_2 = 0$ and $\Sigma_2 = 0$.

- Vacuum labels [B-H.Lee & C.Park & SS (17)]

$$
\begin{aligned}
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, m_2, \cdots, m_{N-1}, m_N), \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, m_2, \cdots, -m_{N-1}, -m_N), \\
&\vdots \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (\pm m_1, -m_2, \cdots, -m_{N-1}, -m_N),
\end{aligned}
\tag{41}
$$

where the sign $\pm$ is $+$ for odd N and $-$ for even N.

- Elementary wall $\langle A \leftarrow B \rangle$

$$
2c[M, E_i] = 2c(\underline{m} \cdot \underline{\alpha}_i)E_i = T_{\langle A \leftarrow B \rangle} E_i, \quad (i = 1, \cdots, N). \tag{42}
$$

E_i: positive root generator of the simple root of $SO(2N)$ $\rightarrow$ elementary wall operator

c: FI parameter

$T_{\langle A \leftarrow B \rangle}$: tension

$H_{0\langle A \leftarrow B \rangle} = H_{0\langle A \rangle} e^{E_i(r)}$, $E_i(r) \equiv e^r E_i$, $(r \in \mathbf{C})$: moduli matrix of the elementary wall

$\underline{g}_{\langle A \leftarrow B \rangle} \equiv 2c\underline{\alpha}_i, \quad (i = 1, \cdots, N)$: elementary wall

Junctions of the mass-deformed NLSM on $SO(2N)/U(N)$ (cont'd)

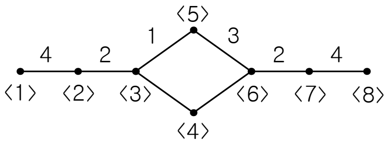
- Compressed wall of level s

$$\underline{g}_{\langle\cdots\rangle} = 2c\underline{\alpha}_{i_1} + 2c\underline{\alpha}_{i_2} + \cdots + 2c\underline{\alpha}_{i_s}, \quad (i_m = 1, \cdots, N;\ m \leq s). \tag{43}$$

- Pair of penetrable walls

$$\underline{g}_{\langle\cdots\rangle} \cdot \underline{g}_{\langle\cdots\rangle} = 0. \tag{44}$$

- $SO(8)/U(4)$ [B-H.Lee & C.Park & SS (17)]

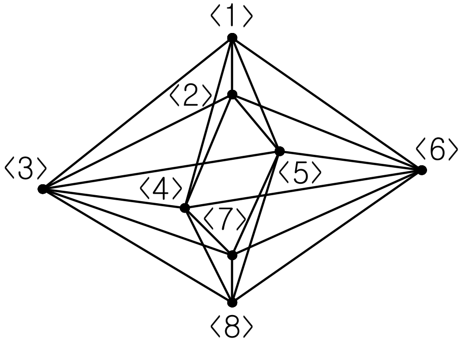


$$\underline{\alpha}_1 \cdot (\underline{\alpha}_2 + \underline{\alpha}_4) \neq 0 \Rightarrow \langle 1 \leftarrow 5\rangle,\ \langle 4 \leftarrow 8\rangle. \tag{45}$$

$$\underline{\alpha}_3 \cdot (\underline{\alpha}_2 + \underline{\alpha}_4) \neq 0 \Rightarrow \langle 1 \leftarrow 4\rangle,\ \langle 5 \leftarrow 8\rangle. \tag{46}$$

Junctions of the mass-deformed NLSM on $SO(8)/U(4)$

- $M_2 \neq 0$, $\Sigma_2 \neq 0$, $m_1 + in_1 \neq m_2 + in_2 \neq \cdots \neq m_N + in_N$



Junctions of the mass-deformed NLSM on $SO(8)/U(4)$ (cont'd)

The three-pronged junction that divides vacua $\{\langle 1\rangle, \langle 2\rangle, \langle 3\rangle\}$:

- Vacuum labels

$$\begin{aligned}
\langle 1\rangle \ &: \ (\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \sigma_3 + i\tau_3, \sigma_4 + i\tau_4) \\
&\quad = (m_1 + in_1, m_2 + in_2, m_3 + in_3, m_4 + in_4), \\
\langle 2\rangle \ &: \ (\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \sigma_3 + i\tau_3, \sigma_4 + i\tau_4) \\
&\quad = (m_1 + in_1, m_2 + in_2, -m_3 - in_3, -m_4 - in_4), \\
\langle 3\rangle \ &: \ (\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \sigma_3 + i\tau_3, \sigma_4 + i\tau_4) \\
&\quad = (m_1 + in_1, -m_2 - in_2, m_3 + in_3, -m_4 - in_4).
\end{aligned} \tag{47}$$

- Single walls

$$\begin{aligned}
H_{0\langle 1\leftarrow 2\rangle} &= H_{0\langle 1\rangle} e^{e^r E_4}, \\
H_{0\langle 2\leftarrow 3\rangle} &= H_{0\langle 2\rangle} e^{e^r E_2}, \\
H_{0\langle 1\leftarrow 3\rangle} &= H_{0\langle 1\rangle} e^{e^r [E_4, E_2]}.
\end{aligned} \tag{48}$$

Junctions of the mass-deformed NLSM on $SO(8)/U(4)$ (cont'd)

- Moduli matrices of walls $h_{AI} = \exp(a_{AI} + ib_{AI})$, $a_{AI}, b_{AI} \in \mathbf{R}$

$$
H_{0\langle 1\leftrightarrow 2\rangle} = \left(\begin{array}{cccccccc}
1 & & & & 0 & & & \\
& 1 & & & & 0 & & \\
& & h_{33} & & & & 0 & h_{38} \\
& & & h_{33} & & & -h_{38} & 0
\end{array}\right),
$$

$$
H_{0\langle 2\leftrightarrow 3\rangle} = \left(\begin{array}{cccccccc}
1 & & & & 0 & & & \\
& h_{22} & h_{23} & & & 0 & & \\
& & 0 & & & -h_{23} & h_{22} & \\
& & & 0 & & & & 1
\end{array}\right),
$$

$$
H_{0\langle 3\leftrightarrow 1\rangle} = \left(\begin{array}{cccccccc}
1 & & & & 0 & & & \\
& h_{22} & & & & 0 & & -h_{28} \\
& & 1 & & & & 0 & 0 \\
& & & h_{22} & & h_{28} & & 0
\end{array}\right). \qquad (49)
$$

As the moduli matrices have the worldvolume symmetry (38), only one of h_{AI} parameters or the ratio of the two parameters in each moduli matrix is independent.

Junctions of the mass-deformed NLSM on $SO(8)/U(4)$ (cont'd)

- Wall $\langle 1 \leftrightarrow 2 \rangle$

$$\phi_{\langle 1\leftrightarrow 2\rangle} = \begin{pmatrix} \sqrt{c} & & & & 0 & & & \\ & \sqrt{c} & & & & 0 & & \\ & & \frac{p_{33}}{\sqrt{s_3}} & & & & 0 & \frac{p_{38}}{\sqrt{s_3}} \\ & & & \frac{p_{44}}{\sqrt{s_4}} & & & \frac{p_{47}}{\sqrt{s_4}} & 0 \end{pmatrix},$$

$$\begin{aligned} p_{33} &= \exp(m_3 x^1 + n_3 x^2 + a_{33} + i b_{33}), \\ p_{38} &= \exp(-m_4 x^1 - n_4 x^2 + a_{38} + i b_{38}), \\ p_{44} &= \exp(m_4 x^1 + n_4 x^2 + a_{33} + i b_{33}), \\ p_{47} &= \exp(-m_3 x^1 - n_3 x^2 + a_{38} + i b_{38} + \frac{i\pi}{2}), \\ s_3 &= \frac{1}{c}\left[\exp(2m_3 x^1 + 2n_3 x^2 + 2a_{33}) + \exp(-2m_4 x^1 - 2n_4 x^2 + 2a_{38})\right], \\ s_4 &= \frac{1}{c}\left[\exp(2m_4 x^1 + 2n_4 x^2 + 2a_{33}) + \exp(-2m_3 x^1 - 2n_3 x^2 + 2a_{38})\right]. \end{aligned} \quad (50)$$

Junctions of the mass-deformed NLSM on $SO(8)/U(4)$ (cont'd)

- Wall $\langle 2 \leftrightarrow 3 \rangle$

$$
\phi_{\langle 2\leftrightarrow 3\rangle} = \begin{pmatrix} \sqrt{c} & & & 0 & & & \\ & \frac{q_{22}}{\sqrt{t_2}} & \frac{q_{23}}{\sqrt{t_2}} & & 0 & & \\ & & 0 & & \frac{q_{36}}{\sqrt{t_3}} & \frac{q_{37}}{\sqrt{t_3}} & \\ & & & 0 & & & \sqrt{c} \end{pmatrix},
$$

$$
\begin{aligned}
q_{22} &= \exp(m_2 x^1 + n_2 x^2 + a_{22} + i b_{22}), \\
q_{23} &= \exp(m_3 x^1 + n_3 x^2 + a_{23} + i b_{23}), \\
q_{36} &= \exp(-m_2 x^1 - n_2 x^2 + a_{23} + i b_{23} + \frac{i\pi}{2}), \\
q_{37} &= \exp(-m_3 x^1 - n_3 x^2 + a_{22} + i b_{22}), \\
t_2 &= \frac{1}{c}\left[\exp(2m_2 x^1 + 2n_2 x^2 + 2a_{22}) + \exp(2m_3 x^1 + 2n_3 x^2 + 2a_{23})\right], \\
t_3 &= \frac{1}{c}\left[\exp(-2m_2 x^1 - 2n_2 x^2 + 2a_{23}) + \exp(-2m_3 x^1 - 2n_3 x^2 + 2a_{22})\right]. \quad (51)
\end{aligned}
$$

Junctions of the mass-deformed NLSM on $SO(8)/U(4)$ (cont'd)

- Wall $\langle 1 \leftrightarrow 3 \rangle$

$$
\phi_{\langle 1\leftrightarrow 3\rangle} = \begin{pmatrix} \sqrt{c} & & & & 0 & & & \\ & \frac{r_{22}}{\sqrt{u_2}} & & & & 0 & & \frac{r_{28}}{\sqrt{u_2}} \\ & & \sqrt{c} & & & & 0 & \\ & & & \frac{r_{44}}{\sqrt{u_4}} & & \frac{r_{46}}{\sqrt{u_4}} & & 0 \end{pmatrix},
$$

$$
\begin{aligned}
r_{22} &= \exp(m_2 x^1 + n_2 x^2 + a_{22} + i b_{22}), \\
r_{28} &= \exp(-m_4 x^1 - n_4 x^2 + a_{28} + i b_{28} + \frac{i\pi}{2}), \\
r_{44} &= \exp(m_4 x^1 + n_4 x^2 + a_{22} + i b_{22}), \\
r_{46} &= \exp(-m_2 x^1 - n_2 x^2 + a_{28} + i b_{28}), \\
u_2 &= \frac{1}{c}\left[\exp(2m_2 x^1 + 2n_2 x^2 + 2a_{22}) + \exp(-2m_4 x^1 - 2n_4 x^2 + 2a_{28})\right], \\
u_4 &= \frac{1}{c}\left[\exp(2m_4 x^1 + 2n_4 x^2 + 2a_{22}) + \exp(-2m_2 x^1 - 2n_2 x^2 + 2a_{28})\right]. \qquad (52)
\end{aligned}
$$

Junctions of the mass-deformed NLSM on $SO(8)/U(4)$ (cont'd)

- Position of wall $\langle 1 \leftrightarrow 2 \rangle$: $\mathrm{Re}(p_{33}) = \mathrm{Re}(p_{38})$ and $\mathrm{Re}(p_{44}) = \mathrm{Re}(p_{47})$
  $\rightarrow (m_3 + m_4)x^1 + (n_3 + n_4)x^2 + (a_{33} - a_{38}) = 0.$
- Position of wall $\langle 2 \leftrightarrow 3 \rangle$: $\mathrm{Re}(q_{22}) = \mathrm{Re}(q_{23})$ and $\mathrm{Re}(q_{36}) = \mathrm{Re}(q_{37})$
  $\rightarrow (m_2 - m_3)x^1 + (n_2 - n_3)x^2 + (a_{22} - a_{23}) = 0.$
- Position of wall $\langle 1 \leftrightarrow 3 \rangle$: $\mathrm{Re}(r_{22}) = \mathrm{Re}(r_{28})$ and $\mathrm{Re}(r_{44}) = \mathrm{Re}(r_{46})$
  $\rightarrow (m_2 + m_4)x^1 + (n_2 + n_4)x^2 + (a_{22} - a_{28}) = 0.$
- Consistency condition $\rightarrow a_{23} - a_{33} = a_{28} - a_{38}$.

Therefore there are two independent a_{AI} parameters to describe the junction position as expected.

- Junction position

$$
\begin{aligned}
(x, y) &= \left(\frac{S_1}{S_3}, \frac{S_2}{S_3} \right), \\
S_1 &= (-a_{33} + a_{38})n_2 + (a_{22} - a_{28})n_3 + (a_{22} - a_{23})n_4, \\
S_2 &= (a_{33} - a_{38})m_2 + (-a_{22} + a_{28})m_3 + (-a_{22} + a_{23})m_4, \\
S_3 &= (m_3 + m_4)(n_2 - n_3) - (m_2 - m_3)(n_3 + n_4).
\end{aligned} \tag{53}
$$

Junctions of the mass-deformed NLSM on $Sp(N)/U(N)$

We start from vacua and walls by setting $M_2 = 0$ and $\Sigma_2 = 0$.

- Vacuum labels [M.Arai & A.Golubtsova & C.Park & SS (18)]

$$
\begin{aligned}
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, m_2, \cdots, m_{N-1}, m_N), \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, m_2, \cdots, m_{N-1}, -m_N), \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, m_2, \cdots, -m_{N-1}, m_N), \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, m_2, \cdots, -m_{N-1}, -m_N), \\
&\vdots \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (m_1, -m_2, \cdots, -m_{N-1}, -m_N), \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (-m_1, m_2, \cdots, m_{N-1}, m_N), \\
&\vdots \\
(\sigma_1, \sigma_2, \cdots, \sigma_{N-1}, \sigma_N) &= (-m_1, -m_2, \cdots, -m_{N-1}, -m_N). \qquad (54)
\end{aligned}
$$

Junctions of the mass-deformed NLSM on $Sp(N)/U(N)$ (cont'd)

- Elementary walls

$$
\begin{aligned}
2c[M, E_i] &= 2c(\underline{m} \cdot \underline{\alpha}_i)E_i = T_{\langle A \leftarrow B \rangle} E_i, \quad (i = 1, \cdots, N-1), \\
c[M, E_N] &= c(\underline{m} \cdot \underline{\alpha}_N)E_N = T_{\langle A \leftarrow B \rangle} E_N.
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
\underline{g}_{\langle A \leftarrow B \rangle} &\equiv 2c\underline{\alpha}_i, \quad (i = 1, \cdots, N-1), \\
\underline{g}_{\langle A \leftarrow B \rangle} &\equiv c\underline{\alpha}_N.
\end{aligned} \tag{56}
$$

- Compressed wall of level s

$$
\underline{g}_{\langle \cdots \rangle} = 2c\underline{\alpha}_{i_1} + 2c\underline{\alpha}_{i_2} + \cdots 2c\underline{\alpha}_{i_s}, \quad (i_m \leq N-1;\ m \leq s). \tag{57}
$$

- Compressed wall sector of unequal length simple roots

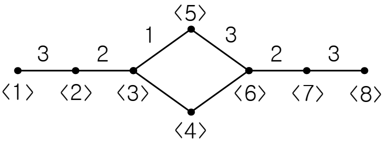
$$
\underline{g}_{\langle \cdots \rangle} = 2c\underline{\alpha}_{N-1} + c\underline{\alpha}_N. \tag{58}
$$

- Pair of penetrable walls

$$
\underline{g}_{\langle \cdots \rangle} \cdot \underline{g}_{\langle \cdots \rangle} = 0. \tag{59}
$$

Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$

- $Sp(3)/U(3)$ [M.Arai & A.Golubtsova & C.Park & SS (18)]

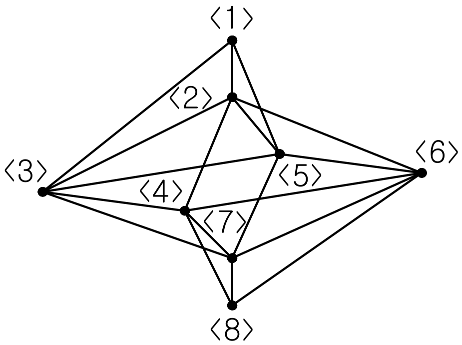


$$(2\underline{\alpha}_2 + \underline{\alpha}_3) \cdot \underline{\alpha}_3 = 0 \Rightarrow \langle 1 \nleftarrow 4 \rangle, \ \langle 5 \nleftarrow 8 \rangle. \tag{60}$$

$$(2\underline{\alpha}_1 + 2\underline{\alpha}_2 + \underline{\alpha}_3) \cdot \underline{\alpha}_3 = 0 \Rightarrow \langle 1 \nleftarrow 6 \rangle, \ \langle 3 \nleftarrow 8 \rangle. \tag{61}$$

Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$ (cont'd)

- $M_2 \neq 0$ and $\Sigma_2 \neq 0$, and set $m_1 + in_1 \neq m_2 + in_2 \neq \cdots \neq m_n + in_n$



Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$ (cont'd)

The three-pronged junction that divides vacua $\{\langle 1\rangle, \langle 2\rangle, \langle 3\rangle\}$:

- Vacuum label

$$\begin{aligned}
\langle 1\rangle \ &: \ (\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \sigma_3 + i\tau_3) \\
&\quad = (m_1 + in_1, m_2 + in_2, m_3 + in_3), \\
\langle 2\rangle \ &: \ (\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \sigma_3 + i\tau_3) \\
&\quad = (m_1 + in_1, m_2 + in_2, -m_3 - in_3), \\
\langle 3\rangle \ &: \ (\sigma_1 + i\tau_1, \sigma_2 + i\tau_2, \sigma_3 + i\tau_3) \\
&\quad = (m_1 + in_1, -m_2 - in_2, m_3 + in_3).
\end{aligned} \tag{62}$$

- Single walls

$$\begin{aligned}
H_{0\langle 1\leftarrow 2\rangle} &= H_{0\langle 1\rangle}\, e^{e^r E_3}, \\
H_{0\langle 2\leftarrow 3\rangle} &= H_{0\langle 2\rangle}\, e^{e^r E_2}, \\
H_{0\langle 1\leftarrow 3\rangle} &= H_{0\langle 1\rangle}\, e^{e^r [[E_3, E_2], E_2]}.
\end{aligned} \tag{63}$$

Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$ (cont'd)

- Moduli matrices of walls ($h_{AI} = \exp(a_{AI} + ib_{AI}),\ a_{AI}, b_{AI} \in \mathbf{R}$)

$$H_{0\langle 1\leftrightarrow 2\rangle} = \begin{pmatrix} 1 & & & 0 & & \\ & 1 & & & 0 & \\ & & h_{33} & & & h_{36} \end{pmatrix},$$

$$H_{0\langle 2\leftrightarrow 3\rangle} = \begin{pmatrix} 1 & & & 0 & & \\ & h_{22} & h_{23} & & 0 & \\ & & 0 & & -h_{23} & h_{22} \end{pmatrix},$$

$$H_{0\langle 3\leftrightarrow 1\rangle} = \begin{pmatrix} 1 & & & 0 & & \\ & h_{22} & & & h_{25} & \\ & & 1 & & & 0 \end{pmatrix}. \tag{64}$$

As the moduli matrices have the worldvolume symmetry (38), only one of h_{AI} parameters or the ratio of the two parameters in each moduli matrix is independent.

Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$ (cont'd)

- Wall $\langle 1 \leftrightarrow 2 \rangle$

$$\phi_{\langle 1\leftrightarrow 2\rangle} = \begin{pmatrix} \sqrt{c} & & & 0 & & \\ & \sqrt{c} & & & 0 & \\ & & \frac{p_{33}}{\sqrt{s_3}} & & & \frac{p_{36}}{\sqrt{s_3}} \end{pmatrix}, \tag{65}$$

$$\begin{aligned} p_{33} &= \exp(m_3 x^1 + n_3 x^2 + a_{33} + i b_{33}), \\ p_{36} &= \exp(-m_3 x^1 - n_3 x^2 + a_{36} + i b_{36}), \\ s_3 &= \frac{1}{c}\left[\exp(2m_3 x^1 + 2n_3 x^2 + 2a_{33}) + \exp(-2m_3 x^1 - 2n_3 x^2 + 2a_{36})\right]. \end{aligned} \tag{66}$$

Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$ (cont'd)

- Wall $\langle 2 \leftrightarrow 3 \rangle$

$$\phi_{\langle 2\leftrightarrow 3\rangle} = \begin{pmatrix} \sqrt{c} & & & 0 & & \\ & \frac{q_{22}}{\sqrt{t_2}} & \frac{q_{23}}{\sqrt{t_2}} & & 0 & \\ & & 0 & & \frac{q_{35}}{\sqrt{t_3}} & \frac{q_{36}}{\sqrt{t_3}} \end{pmatrix},$$

$$q_{22} = \exp(m_2 x^1 + n_2 x^2 + a_{22} + ib_{22}),$$
$$q_{23} = \exp(m_3 x^1 + n_3 x^2 + a_{23} + ib_{23}),$$
$$q_{35} = \exp(-m_2 x^1 - n_2 x^2 + a_{23} + ib_{23} + \frac{i\pi}{2}),$$
$$q_{36} = \exp(-m_3 x^1 - n_3 x^2 + a_{22} + ib_{22}),$$
$$t_2 = \frac{1}{c}\left[\exp(2m_2 x^1 + 2n_2 x^2 + 2a_{22}) + \exp(2m_3 x^1 + 2n_3 x^2 + 2a_{23})\right],$$
$$t_3 = \frac{1}{c}\left[\exp(-2m_2 x^1 - 2n_2 x^2 + 2a_{23}) + \exp(-2m_3 x^1 - 2n_3 x^2 + 2a_{22})\right]. \quad (67)$$

- Wall $\langle 1 \leftrightarrow 3 \rangle$

$$\phi_{\langle 1\leftrightarrow 3\rangle} = \begin{pmatrix} \sqrt{c} & & & 0 & & \\ & \frac{r_{22}}{\sqrt{u_2}} & & & \frac{r_{25}}{\sqrt{u_2}} & \\ & & \sqrt{c} & & & 0 \end{pmatrix},$$

$$r_{22} = \exp(m_2 x^1 + n_2 x^2 + a_{22} + i b_{22}),$$

$$r_{25} = \exp(-m_2 x^1 - n_2 x^2 + a_{25} + i b_{25}),$$

$$u_2 = \frac{1}{c}\left[\exp(2m_2 x^1 + 2n_2 x^2 + 2a_{22}) + \exp(-2m_2 x^1 - 2n_2 x^2 + 2a_{25})\right]. \quad (68)$$

Junctions of the mass-deformed NLSM on $Sp(3)/U(3)$ (cont'd)

- Position of wall $\langle 1 \leftrightarrow 2 \rangle$: $\mathrm{Re}(p_{33}) = \mathrm{Re}(p_{36})$
  $\rightarrow 2m_3x^1 + 2n_3x^2 + (a_{33} - a_{36}) = 0.$
- Position of wall $\langle 2 \leftrightarrow 3 \rangle$: $\mathrm{Re}(q_{22}) = \mathrm{Re}(q_{23})$ and $\mathrm{Re}(q_{35}) = \mathrm{Re}(q_{36})$
  $\rightarrow (m_2 - m_3)x^1 + (n_2 - n_3)x^2 + (a_{22} - a_{23}) = 0.$
- Position of wall $\langle 1 \leftrightarrow 3 \rangle$: $\mathrm{Re}(r_{22}) = \mathrm{Re}(r_{25})$
  $\rightarrow 2m_2x^1 + 2n_2x^2 + (a_{22} - a_{25}) = 0.$
- Consistency condition $\rightarrow (a_{22} - a_{25}) - (a_{33} - a_{36}) - 2(a_{22} - a_{23}) = 0.$

Therefore there are two independent a_{AI} parameters to describe the junction position as expected.

- Junction position

$$(x, y) = \left(\frac{T_1}{T_3}, \frac{T_2}{T_3}\right),$$
$$T_1 = 2(-a_{33} + a_{36})n_2 + 2(a_{22} - a_{25})n_3,$$
$$T_2 = 2(a_{33} - a_{36})m_2 + 2(-a_{22} + a_{25})m_3,$$
$$T_3 = -4m_2n_3 + 4m_3n_2. \tag{69}$$

Summary/Outlook

- We have proposed NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$, which might be the HK NLSMs on $T^*SO(2N)/U(N)$ and $T^*Sp(N)/U(N)$.

- We have constructed three-pronged junctions of the mass-deformed NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$. The method is generically applicable.

- We need to check the supersymmetry of the NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$.

- We hope to get a better understanding of the $\mathcal{N} = 2$ supersymmetric models that are realised as the quotient with respect to the gauge group.

Summary/Outlook

- We have proposed NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$, which might be the HK NLSMs on $T^*SO(2N)/U(N)$ and $T^*Sp(N)/U(N)$.

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- We need to check the supersymmetry of the NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$.

- We hope to get a better understanding of the $\mathcal{N} = 2$ supersymmetric models that are realised as the quotient with respect to the gauge group.

Thank you!