Generalisation of the Poincare Group and of the Yang-Mills theory

George Savvidy

Demokritos National Research Centre
Athens, Greece

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1.G.Savvidy, Generalization of Yang-Mills theory
Phys. Lett. B 625 (2005) 341

2.G.Savvidy, Extension of Poincaré Group and Tensorgluons
Int.J.Mod.Phys. A25 (2010) 5765-5785

3.I.Antoniadis, L. Brink, G. Savvidy
Supersymmetric Extensions of the Poincare group
J.Math.Phys. 52 (2011) 072303,

4.G.Savvidy, Invariant scalar product on extended Poincare algebra
arXiv:1308.2695

5.G.Savvidy, Asymptotic Freedom of Tensorgluons
Phys. Lett. B 732 (2014) 150

6. R. Kirschner and G.Savvidy,
High Spin Parton Distribution Functions (PDF)
Mod.Phys.Lett. A32 (2017) 1750121

Space-Time symmetry.

$* \cdot *$ The Lorentz and Poincaré groups were discovered during the investigating of the symmetries of the Maxwell equations

$$\nabla \cdot \vec{E} = 0, \qquad \nabla \cdot \vec{B} = 0,$$
$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \qquad \nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

invariant with respect to the transformations of space-time coordinates

$$x^{'} = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad t^{'} = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - v^2/c^2}}, \quad y^{'} = y + a, \quad z^{'} = z + b$$

and fields

$$E^{'}_y = \frac{E_y + \frac{v}{c}B_z}{\sqrt{1 - v^2/c^2}}, \quad E^{'}_z = \frac{E_z - \frac{v}{c}B_y}{\sqrt{1 - v^2/c^2}},$$
$$B^{'}_y = \frac{B_y - \frac{v}{c}E_z}{\sqrt{1 - v^2/c^2}}, \quad B^{'}_z = \frac{B_z + \frac{v}{c}B_y}{\sqrt{1 - v^2/c^2}}$$

The Poincaré algebra $L(\mathcal{P})$

$$[P^{\mu},\ P^{\nu}] = 0,$$
$$[M^{\mu\nu},\ P^{\lambda}] = i(\eta^{\lambda\nu}\ P^{\mu} - \eta^{\lambda\mu}\ P^{\nu}),$$
$$[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),$$

$*\cdot*$ It have been found that The Maxwell equations are invariant with respect to the conformal group H. Bateman, "The Transformation of the Electrodynamics Equations" . Proc. of London Math.Soc.. 8 (1910) 223-264.

$$x_\mu \to \lambda x_\mu,$$
$$x_\mu \to \frac{x_\mu}{x^2}$$

$*\cdot*$The other important symmetry exhibited by Electrodynamic Equations is the gauge symmetry

$$A_\mu \to A_\mu + \partial_\mu \phi, \quad F_{\mu\nu} \to F_{\mu\nu}, \quad Action \to Action$$

The Faraday-Maxwell equations were the indispensable source of fundamental symmetries !

In Quantum Field Theory

One of the main requirement imposed on quantum field theories is their invariance with respect to the Poincaré group $\mathcal{P}$.

It is of great interest to study extended Poincaré algebras and groups, the invariance with respect to which may impose limitations on the form of the Fundamental interactions and became the guiding principle in formulating theories beyond the Standard Model.

Supersymmetry transformation

EXTENSION OF THE ALGEBRA OF POINCARE GROUP GENERATORS AND VIOLATION OF P INVARIANCE

Yu.A. Gol'fand and E.P. Likhtman
Physics Institute, USSR Academy of Sciences
Submitted 10 March 1971
ZhETF Pis. Red. 13, No. 8, 452 - 455 (20 April 1971)

One of the main requirements imposed on quantum field theory is invariance of the theory to the Poincare group [1]. However, only a fraction of the interactions satisfying this requirement is realized in nature. It is possible that these interactions, unlike others, have a higher degree of symmetry. It is therefore of interest to study different algebras and groups, the invariance with respect to which imposes limitations on the form of the elementary particle interaction. In the present paper we propose, in constructing the Hamiltonian formulation of the quantum field theory, to use as the basis a special algebra $\mathcal{R}$, which is an extension of the algebra $\mathcal{P}$ of the Poincare group generators. The purpose of the paper is to find such a realization of the algebra $\mathcal{R}$, in which the Hamiltonian operator describes the interaction of quantized fields.

The extension of the algebra $\mathcal{P}$ is carried out in the following manner: we add to the generators P_μ and $M_{\mu\nu}$ the bispinor generators W_α and $\bar{W}_\beta$, which we shall call the generators of spinor translations. In order to obtain the algebra $\mathcal{R}$, it is necessary to find the Lorentz-invariant form of the permutation relations between the translation generators. In order not to violate subsequently the connection between the spin and statistics, we shall consider anticommutators of the operators W_α and $\bar{W}_\beta$. A generalization of the Jacobi identities imposes stringent limitations on the form of the possible commutation relations between the algebra operators. We confine ourselves to consideration of only those algebras $\mathcal{R}$, in which there are no subalgebras Q such that $\mathcal{P} \subset Q$ and $\mathcal{P} \neq Q$. This choice is governed by the fact that the remaining algebras $\mathcal{R}$ are obtained by further extending the algebras $\mathcal{R}$, and the field theories corresponding to them will have a still higher degree of symmetry.

An investigation of the albegras $\mathcal{R}$ has shown that upon spatial inversion they do not go over into themselves for any choice of the structure constants of the algebra. As a result, in a field theory that is invariant against such an algebra, the parity should not be conserved[1)], and the form of the nonconservation is completely determined by the algebra itself. We shall stop to discuss one of the algebras $\mathcal{R}$:

$$[M_{\mu\nu}, M_{\sigma\lambda}]_- = i(\delta_{\mu\sigma}M_{\nu\lambda} + \delta_{\nu\lambda}M_{\mu\sigma} - \delta_{\mu\lambda}M_{\nu\sigma} - \delta_{\nu\sigma}M_{\mu\lambda}); \quad [P_\mu, P_\nu]_- = 0;$$

$$[M_{\mu\nu}, P_\lambda]_- = i(\delta_{\mu\lambda}P_\nu - \delta_{\nu\lambda}P_\mu); \quad [M_{\mu\nu}, W]_- = \frac{i}{4}\{\gamma_\mu, \gamma_\nu\}_- W; \quad \bar{W} = W^+\gamma_0. \tag{1a}$$

$$[W_\pm, \bar{W}]_+ = \overset{+}{\gamma}_\mu P_\mu; \quad [W, W]_+ = 0; \quad [P_\mu, W]_- = 0, \tag{1b}$$

We propose to formulate a quantum field theory based on algebra $L_G(\mathcal{P}) = L(\mathcal{P}) \otimes L(G)$, an extension of the Poincaré $L(\mathcal{P})$ and Lee $L(G)$ algebras with infinite set of tensor generators $L_a^{\lambda_1 ... \lambda_s}$.

$$
\begin{aligned}
&L_a \\
&L_a^{\lambda_1} \\
&L_a^{\lambda_1,\lambda_2} \\
&...................... \\
&L_a^{\lambda_1 ... \lambda_s} \\
&...................... \quad \lambda = (0,1,2,3) \qquad s = 0,1,2,....
\end{aligned}
$$

where $L_a \in G$ and the extended current algebra is defined as

$$
\begin{aligned}
[L_a, L_b] &= if_{abc} L_c \\
[L_a, L_b^{\lambda_1}] &= if_{abc} L_c^{\lambda_1} \\
[L_a^{\lambda_1}, L_b^{\lambda_2}] &= if_{abc} L_c^{\lambda_1 \lambda_2} \\
&......................
\end{aligned}
$$

We add to the generators P_μ and $M_{\mu\nu}$ the new generators $L_a^{\lambda_1 \ldots \lambda_s}$
The extension of the Poincaré algebra $L_G(\mathcal{P})$ is:

$$
\begin{aligned}
&[P^{\mu},\ P^{\nu}] = 0,\\
&[M^{\mu\nu},\ P^{\lambda}] = i(\eta^{\lambda\nu}\ P^{\mu} - \eta^{\lambda\mu}\ P^{\nu}),\\
&[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),
\end{aligned}
$$

$$
\begin{aligned}
&[P^{\mu},\ L_a^{\lambda_1 \ldots \lambda_s}] = 0,\\
&[M^{\mu\nu},\ L_a^{\lambda_1 \ldots \lambda_s}] = i(\eta^{\lambda_1 \nu} L_a^{\mu\lambda_2 \ldots \lambda_s} + \ldots + -\eta^{\lambda_s \mu} L_a^{\lambda_1 \ldots \lambda_{s-1}\nu}),
\end{aligned}
$$

$$
[L_a^{\lambda_1 \ldots \lambda_n}, L_b^{\lambda_{n+1} \ldots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \ldots \lambda_s} \quad (s = 0, 1, 2, \ldots).
$$

The generators $L_a^{\lambda_1 \ldots \lambda_s}$ carry *internal charges* a *and high helicities* .

$$P_\mu = -i\partial^{(x)}_\mu \qquad L_a^{\mu_1,\ldots,\mu_n} = x^{\mu_1}\ldots\, x^{\mu_n} L_a \qquad [L_a, L_b] = i f_{abc} L_c$$

$$[P^\mu, L_a^{\mu_1,\ldots,\mu_n}] = -i(\delta^{\mu\mu_1} L_a^{\mu_2,\ldots,\mu_n} + \ldots + \delta^{\mu\mu_n} Q_a^{\mu_1,\ldots,\mu_{n-1}})$$

$$[L_a^{\mu_1,\ldots,\mu_k}, L_b^{\mu_{k+1},\ldots,\mu_n}] = i f_{abc} L_c^{\mu_1,\ldots,\mu_n}$$

E.A. Ivanov, V.I. Ogievetsky
Lett.Math.Phys. 1 (1976) 309-313

Supersymmetric Extension of $L_G(\mathcal{P})$

$$
\begin{aligned}
&[P^{\mu},\ P^{\nu}] = 0,\\
&[M^{\mu\nu},\ P^{\lambda}] = i(\eta^{\lambda\nu}\ P^{\mu} - \eta^{\lambda\mu}\ P^{\nu}),\\
&[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),
\end{aligned}
$$

$$
\begin{aligned}
&[P^{\mu},\ L_a^{\lambda_1 \ldots \lambda_s}] = 0,\\
&[P^{\mu},\ Q^i_{\alpha}] = 0,\\
&[M^{\mu\nu},\ L_a^{\lambda_1 \ldots \lambda_s}] = i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2\ldots\lambda_s} - \eta^{\lambda_1\mu} L_a^{\nu\lambda_2\ldots\lambda_s} + \ldots + \eta^{\lambda_s\nu} L_a^{\lambda_1\ldots\lambda_{s}\cdots}\\
&[M^{\mu\nu},\ Q^i_{\alpha}] = \frac{i}{2}(\gamma^{\mu\nu} Q^i)_{\alpha}, \qquad \gamma^{\mu\nu} = \frac{1}{2}[\gamma^{\mu}, \gamma^{\nu}]
\end{aligned}
$$

$$
\begin{aligned}
&[L_a^{\lambda_1 \ldots \lambda_n}, L_b^{\lambda_{n+1} \ldots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \ldots \lambda_s}, \quad s = 0, 1, 2, \ldots\\
&\{Q^i_{\alpha},\ Q^j_{\beta}\} = -2\ \delta^{ij} (\gamma^{\mu} C)_{\alpha\beta} P_{\mu}, \qquad i = 1, \ldots, N\\
&[L_a^{\lambda_1 \ldots \lambda_s}, Q^i_{\alpha}] = 0\ .
\end{aligned}
$$

Automorphism of the extended current algebra

$$
\begin{aligned}
& L_a \to L_a \\
& L_a^{\lambda_1} \to L_a^{\lambda_1} + P^{\lambda_1} L_a \\
& L_a^{\lambda_1\lambda_2} \to L_a^{\lambda_1\lambda_2} + P^{\lambda_1} L_a^{\lambda_2} + P^{\lambda_2} L_a^{\lambda_1} + P^{\lambda_1} P^{\lambda_2} L_a \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& P^{\lambda} \quad \to \quad P^{\lambda}, \\
& M^{\mu\nu} \quad \to \quad M^{\mu\nu},
\end{aligned}
$$

The example how the gauge transformation works

$$
\begin{aligned}
& [L_a^{\lambda_1} + P^{\lambda_1} L_a,\ L_b^{\lambda_2} + P^{\lambda_2} L_b] = \\
& = i f_{abc} (L_c^{\lambda_1\lambda_2} + P^{\lambda_1} L_c^{\lambda_2} + P^{\lambda_2} L_c^{\lambda_1} + P^{\lambda_1} P^{\lambda_2} L_c)
\end{aligned}
$$

The algebra $L_G(\mathcal{P})$ is gauge invariant:

$$L_a^{\lambda_1 ... \lambda_s} \rightarrow L_a^{\lambda_1 ... \lambda_s} + \\ + \sum_1 P^{\lambda_1} L_a^{\lambda_2 ... \lambda_s} + \sum_2 P^{\lambda_1} P^{\lambda_2} L_a^{\lambda_3 ... \lambda_s} + ... + P^{\lambda_1} ... P^{\lambda_s} L_a,$$

$$P^{\lambda} \quad \rightarrow \quad P^{\lambda},$$

$$M^{\mu\nu} \quad \rightarrow \quad M^{\mu\nu},$$

These generators are transforming similar to the Abelian tensor gauge field.

This is "off-shell" symmetry, the operator P^2 has any value.

All representations of the $L_a^{\lambda_1 ... \lambda_s}, \; s = 1, 2, ...$ *are defined modulo longitudinal terms.*

This extended authomorphism of the algebra is a property intrinsic to the algebroids

The reducible representation of $L_G(\mathcal{P})$ has the following form:

$$P^{\mu} = k^{\mu},$$
$$M^{\mu\nu} = i(k^{\mu}\ \frac{\partial}{\partial k_{\nu}} - k^{\nu}\ \frac{\partial}{\partial k_{\mu}}) + i(e^{\mu}\ \frac{\partial}{\partial e_{\nu}} - e^{\nu}\ \frac{\partial}{\partial e_{\mu}}),$$
$$L_a^{\lambda_1...\lambda_s} = e^{\lambda_1}...e^{\lambda_s} \otimes L_a,$$

The gauge transformation of $L_a^{\lambda_1\cdots\lambda_s}$ induces the transformation

$$e^{\mu} \to e^{\mu} + \alpha k^{\mu},$$

which is a gauge transformation of the photon polarisation vector. The generators $L_a^{\lambda_1\cdots\lambda_s}$ are indeed gauge generators.

What is the helicity content of the $L_a^{\lambda_1 \dots \lambda_s}$ operators ?

The Pauli-Lubanski operators W^μ can be used to investigate the helicity content

$$W^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} M_{\nu\lambda} P_\rho$$

The Longitudinal representation

$$L_a^{||\lambda_1 \ldots \lambda_s} = P^{\lambda_1} \ldots\ldots P^{\lambda_s} L_a$$

it is trivial representation as far as the helicity content of this representation is empty

$$[W^{\mu}, L_a^{||\lambda_1 \ldots \lambda_s}] = 0$$

The Transversal representation

the following commutation relation take place with L_a^λ

$$[W_\mu, L_\nu^a] = -i\varepsilon_{\mu\nu\lambda\rho} L_a^\lambda P_\rho$$

for massless states $P_\mu = (\omega, 0, 0, \omega)$ $\quad W^\mu = \omega(-h, \pi^{''}, -\pi^{'}, h),$

$$h = J_3, \;\; \pi^{'} = K_1 - J_2, \;\; \pi^{''} = K_2 + J_1,$$

defining $L_\pm^a = L_1^a \pm L_2^a,$ we shall get

$$[h, L_\pm^a] = \pm L_\pm^a,$$

therefore the helicity content is $h = (+1, -1)$

The commutation relation with second level generator $L_a^{\lambda_1 \lambda_2}$ is

$$[W_\mu, L^a_{\nu\lambda}] = -i(\varepsilon_{\mu\nu\rho\sigma} L^a_{\rho\lambda} + \varepsilon_{\mu\lambda\rho\sigma} L^a_{\rho\nu}) P_\sigma$$

defining

$$\begin{aligned} L^a_{++} &= L^a_{11} + 2i L^a_{12} - L^a_{22}, \\ L^a_{--} &= L^a_{11} - 2i L^a_{12} - L^a_{22}, \\ L^a_{+-} &= L^a_{11} + L^a_{22} \end{aligned}$$

we shall get

$$[h, L^a_{\pm\pm}] = \pm 2 \; L^a_{\pm\pm}, \qquad [h, L^a_{+-}] = 0,$$

therefore the helicity content is $h = (+2, 0, -2)$

The rank-s generators

$$(L_a^{+....+}, ..., L_a^{-....-})$$

carry the helicities:

$$h = (+s, s-2, ..., -s+2, -s)$$

in total $s+1$ states

G.S. J.Phys.A 47 (2014) 055204

The explicit Transversal representation of the generators $L_a^{\perp\lambda_1\ldots\lambda_s}$

$$L_a^{\perp\lambda_1\ldots\lambda_s} = \prod_{n=1}^{s}(\xi k^{\lambda_n} + e^{i\varphi}e_+^{\lambda_n} + e^{-i\varphi}e_-^{\lambda_n}) \oplus L_a,$$

where the helicity vectors are $e_\pm^\lambda = (e_1^\lambda \mp i e_2^\lambda)/2$,

$L_a \in SU(N)$

k_μ is the momentum vector $k^2 = 0$, $k \cdot e_\pm = 0$

the ξ and φ are Wigner variables on the cylinder

$$\varphi \in S^1, \xi \in R^1.$$

The $L_a^{\perp \mu_1 \ldots \mu_s}$ are high helicity generators

$$L_a^{\perp \lambda_1 \ldots \lambda_s} = \\ = (e^{is\varphi} e_+^{\lambda_1} .... e_+^{\lambda_s} + e^{-is\varphi} e_-^{\lambda_1} .... e_-^{\lambda_s} + ... + \xi^s k^{\lambda_1} ... k^{\lambda_s}) \oplus L_a,$$

indeed the helicity operator has the form

$$h = -i \frac{\partial}{\partial \varphi}$$

and the operator $L_a^{\perp \mu_1 \ldots \mu_s}$ carries the helicities

$$h = \pm s, \pm (s-2), ...,$$

we have also the transversality properties

$$k_{\mu_1} L_a^{\perp \mu_1, \ldots, \mu_s} = 0, \quad k^2 = 0, \quad k^\mu e_\mu = 0$$

Using transversal representation one can calculate Killing forms:

$$L_G: \qquad tr(L_a L_b) = \delta_{ab},$$

$$L_G(\mathcal{P}): \qquad tr(L_a^{\lambda_1 \ldots \lambda_n} L_b^{\lambda_{n+1} \ldots . \lambda_{2s}}) = \delta_{ab} s! (\bar{\eta}^{\lambda_1 \lambda_2} \bar{\eta}^{\lambda_3 \lambda_4} \ldots \bar{\eta}^{\lambda_{2s-1} \lambda_{2s}} + \mathrm{per})$$

where

$$\bar{\eta}^{\lambda_1 \lambda_2} = \eta^{\lambda_1 \lambda_2} - \frac{k^{\lambda_1} \bar{k}^{\lambda_2} + \bar{k}^{\lambda_1} k^{\lambda_2}}{k \bar{k}}, \quad k_{\lambda_1} \bar{\eta}^{\lambda_1 \lambda_2} = 0$$

$$tr(L_a^{\lambda_1 \ldots \lambda_n} L_b^{\lambda_{n+1} \ldots . \lambda_{2s}}) = \delta_{ab} \int e^{\lambda_1} \ldots e^{\lambda_{2s}} e^{-\frac{1}{4} F^2_{\lambda\rho}(e)} \mathcal{D} e_{\lambda}.$$

The Yang-Mills theory is defined using Killing form on the Lee algebra $L(G)$

$$tr(L_a L_b) = \delta_{ab}$$

and the Lagrangian is

$$\mathcal{L} = -\frac{1}{4} tr(G_{\mu\nu} G_{\mu\nu})$$

where

$$G_{\mu\nu} = L_a G^a_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig[A_\mu(x)\ A_\nu(x)]$$

The Yang-Mills non-Abelian tensor gauge fields

$$\mathcal{A}_\mu(x,L) = \sum_{s=0}^{\infty} \frac{1}{s!}\ A^a_{\mu\lambda_1\ldots\lambda_s}(x)\ L_a^{\lambda_1\ldots\lambda_s} = A^a_\mu L_a + A^a_{\mu\lambda} L^\lambda_a + ...$$

The field strength tensor

$$\mathcal{G}_{\mu\nu}(x,e) = \partial_\mu \mathcal{A}_\nu(x,e) - \partial_\nu \mathcal{A}_\mu(x,e) - ig[\mathcal{A}_\mu(x,e)\ \mathcal{A}_\nu(x,e)]$$

and the Lagrangian density is

$$\mathcal{L}(x) = tr\, \mathcal{G}_{\mu\nu}(x,e)\mathcal{G}_{\mu\nu}(x,e) = \int \mathcal{G}^a_{\mu\nu}(x,e)\mathcal{G}^a_{\mu\nu}(x,e) e^{-\frac{1}{4}F^2_{\lambda\rho}(e)} \mathcal{D}e_\lambda .$$

In components the Lagrangian is

$$\begin{aligned}\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + = & - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \\ & - \frac{1}{4} G^a_{\mu\nu,\bar{\lambda}} G^a_{\mu\nu,\bar{\lambda}} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\bar{\lambda}\bar{\lambda}} + \ldots\end{aligned}$$

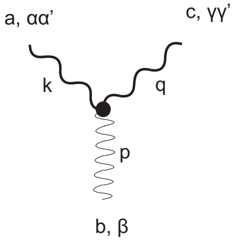
where the field strength tensors are:

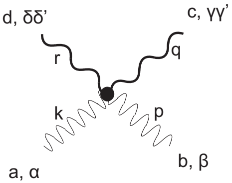
$$\begin{aligned}G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}\ A^b_\mu\ A^c_\nu, \\ G^a_{\mu\nu,\lambda} &= \partial_\mu A^a_{\nu\lambda} - \partial_\nu A^a_{\mu\lambda} + g f^{abc}(\ A^b_\mu\ A^c_{\nu\lambda} + A^b_{\mu\lambda}\ A^c_\nu\ ),\end{aligned}$$

....

Helicity spectrum of the tensorgluons

$$
\begin{array}{llllll}
\pm 1 & & & & & \\
\pm 2, & 0 & & & & \\
\pm 3, & \pm 1, & \pm 1 & & & \\
\pm 4, & \pm 2, & \pm 2, & 0 & & \\
\pm 5, & \pm 3, & \pm 3, & \pm 1, & \pm 1 & \\
\pm 6, & \pm 4, & \pm 4, & \pm 2, & \pm 2, & 0 \\
\multicolumn{6}{l}{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,}
\end{array}
$$





Interaction Vertices of gluons and tensorgluons

Tree level scattering amplitudes of (n-2)-gluons and 2-tensorgluons calculated using BCFW formalism.G.Georgiou and G.S. IJMP 2011.

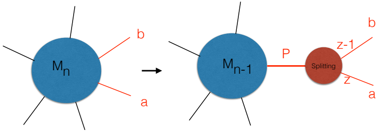
$$\hat{M}_n(1^+, ..i^-, ...k^{+s}, ..j^{-s}, ..n^+) =$$

$$= ig^{n-2}(2\pi)^4 \delta^{(4)}(P^{ab}) \frac{< ij >^4}{\prod_{l=1}^{n} < ll+1 >} \Big(\frac{< ij >}{< ik >} \Big)^{2s-2},$$

They reduce to the Parke-Taylor formula when $s = 1$.

The collinear behavior:

$$M_n^{tree}(..., a^{\lambda_a}, b^{\lambda_b}, ...) \xrightarrow{a \parallel b} \sum_{\lambda=\pm1} Split_{-\lambda}^{tree}(a^{\lambda_a}, b^{\lambda_b}) \times M_{n-1}^{tree}(..., P^{\lambda}, ..),$$



Antoniadis and Savvidy, Mod.Phys.Lett.(2012)

The splitting probabilities are:

$$\begin{aligned}
P_{TG}(z) &= C_2(G)\left[\frac{z^{2s+1}}{(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z^{2s-1}}\right], \\
P_{GT}(z) &= C_2(G)\left[\frac{1}{z(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z}\right], \\
P_{TT}(z) &= C_2(G)\left[\frac{1}{(1-z)z^{2s-1}} + \frac{z^{2s+1}}{1-z}\right].
\end{aligned}$$

s=0,1,2,3,......

The quark and gluon splitting probabilities of Altarelli-Parisi:

$$\begin{aligned}
P_{qq}(z) &= C_2(R)\frac{1+z^2}{1-z},\\
P_{Gq}(z) &= C_2(R)\frac{1+(1-z)^2}{z},\\
P_{qG}(z) &= T(R)[z^2+(1-z)^2],\\
P_{GG}(z) &= C_2(G)\left[\frac{1}{z(1-z)}+\frac{z^4}{z(1-z)}+\frac{(1-z)^4}{z(1-z)}\right],
\end{aligned}$$

where $C_2(G) = N, C_2(R) = \frac{N^2-1}{2N}, T(R) = \frac{1}{2}$ for the SU(N) groups.

$$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta = \pm 1.$$

The formula describes all known splitting probabilities found earlier in QFT and the generalised Yang-Mills theory.

R. Kirschner, G.S.,Mod.Phys.Lett.A32(2017)1750121

Generalisation of DIGLAP evolution equations

$$\dot{q}^i(x,t) = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}[q^j(y,t)\ P_{q^i q^j}(\frac{x}{y}) + G(y,t)\ P_{q^i G}(\frac{x}{y})],$$
$$\dot{G}(x,t) = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}[q^j(y,t)P_{Gq^j}(\frac{x}{y}) + G(y,t)P_{GG}(\frac{x}{y}) + T(y,t)P_{GT}(\frac{x}{y}$$
$$\dot{T}(x,t) = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}[G(y,t)\ P_{TG}(\frac{x}{y}) + T(y,t)\ P_{TT}(\frac{x}{y})].$$

The $\alpha(t)$ is the running coupling constant ($\alpha = g^2/4\pi$)

$$\alpha(t) = \frac{\alpha}{1 + b_1\ \alpha\ t},$$

where

$$b_1 = b_{quarks} + b_{gluons} + b_{tensorgluons}$$

Both calculations gave the identical result :

$$\alpha(t) = \frac{\alpha}{1 + b_1 \alpha \ t} \ ,$$

where

$$b_1 = \frac{\sum_{s=1} (12 s^2 - 1) C_2(G) - 4 n_f T(R)}{12\pi} \qquad s = 0, 1, 2.....$$

at s=1 it reproduces the Gross-Wilczek-Politzer result

Tensorgluons "accelerate" the asymptotic freedom !

Generalisation of the YM effective action $\Gamma(A)$ gives 1-loop effective action similar to G.S., Phys.Lett.B 1977

Summing the spectrum of the tensorgluons in external field

$$k_0^2 = (2n + 1 \pm 2s) gH + k_{\|}^2$$

one can get

$$V(H) = \frac{H^2}{2} + \frac{(gH)^2}{4\pi}\ b_1\ [\ln \frac{gH}{\mu^2} - \frac{1}{2}],$$

where now

$$b_1 = \frac{12 s^2 - 1}{12\pi} C_2(G),$$

How the contribution of tensor-gluons changes the high energy behavior of the coupling constants of the SM ?
The coupling constants evolve with scale as

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(\mu)} + 2b_i \ln\frac{M}{\mu}, \quad i = 1,2,3,$$

consider only the contribution of $s = 2$ tensor-bosons:
For the $SU(3)_c \times SU(2)_L \times U(1)$ group with its coupling constants α_3, α_2 and α_1 and six quarks $n_f = 6$ and $SU(5)$ unification group we will get

$$2b_3 = \frac{1}{2\pi}54, \quad 2b_2 = \frac{1}{2\pi}\frac{10}{3}, \quad 2b_1 = -\frac{1}{2\pi}4,$$

the solution of the system of equations (5) gives

$$\ln\frac{M}{\mu} = \frac{\pi}{58}\left(\frac{1}{\alpha_{el}(\mu)} - \frac{8}{3}\frac{1}{\alpha_s(\mu)}\right),$$

If one takes $\alpha_{el}(M_Z) = 1/128$ and $\alpha_s(M_Z) = 1/10$ one can get that coupling constants have equal strength at energies of order

$$M \sim 4 \times 10^4 GeV = 40 \; TeV,$$

it is much smaller than the previous GU scale $M \sim 10^{14} GeV$ the value of the weak angle remains intact :

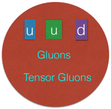
$$\sin^2 \theta_W = \frac{1}{6} + \frac{5}{9} \frac{\alpha_{el}(M_Z)}{\alpha_s(M_Z)},$$

the coupling constant at the unification scale is of order $\bar{\alpha}(M) = 0,01.$

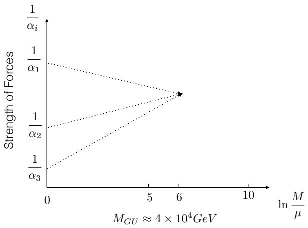
Summary

Asymptotic Freedom of Tensorgluons of spin s=1,2,.....

$$\beta_{GYM} = -\frac{\sum_{s=1}(12s^2-1)C_2(G) - 4n_f T(R)}{48\pi^2} g^3$$



PROTON



Thank you!