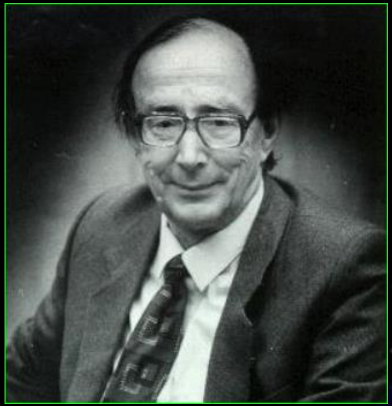


Entropy/Area Quantization of Schwarzschild-like Black Holes in Bumblebee Gravity Model

Presented by

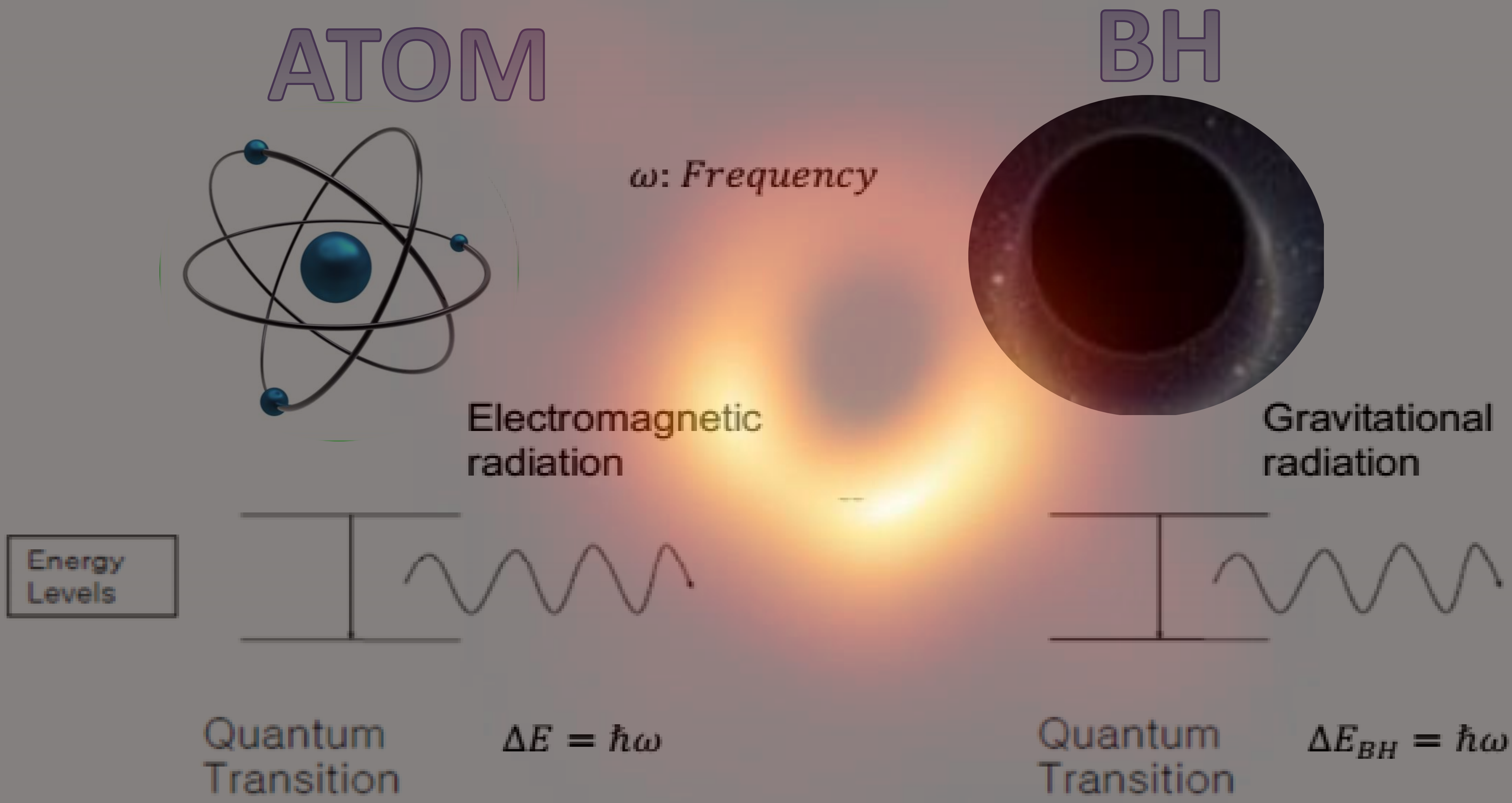
İZZET SAKALLI

Eastern Mediterranean University
Cyprus-Famagusta
26 August 2019

OUTLINE

- ❖ BRIEF REVIEW ABOUT THE QUANTIZATION OF BHs
- ❖ CAGED BHs and THEIR “BOXED QNMs”
- ❖ LSBBH SPACETIME & 1D WAVE EQ. FROM MASSLESS KGE
- ❖ COMPUTATION OF BQNMs BY USING THE NEAR HORIZON SCHRÖDINGER LIKE 1D WAVE EQUATION
- ❖ ENTROPY/AREA SPECTRA OF THE LSBBH
- ❖ CONCLUSION

Analogy between Atom and Quantum BH

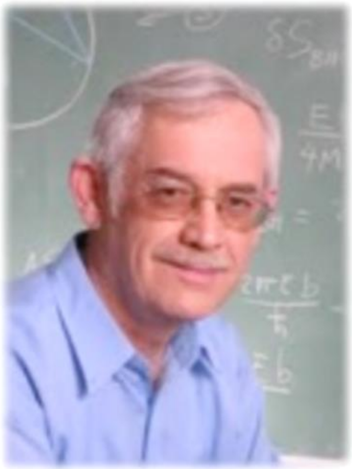


QUANTIZATION

Adiabatic Invariant : A physical quantity that is almost constant when changes are made very slowly (applied to Plasma physics..)–Chandrasekhar

Ehrenfest Theorem : Any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum

Bekenstein (1974) : Black hole horizon area can be regarded as an adiabatic invariant



Based on Christodoulou's point particle model:

"D. Christodoulou, Phys. Rev. Lett. **25**, 1596 (1970)"

Bekenstein:

1. J.D. Bekenstein, Lett. Nuovo Cimento **4**, 737 (1972)
2. J.D. Bekenstein, Phys. Rev. D **7**, 2333 (1973)
3. J.D. Bekenstein, Lett. Nuovo Cimento **11**, 467 (1974)
4. J.D. Bekenstein, Quantum black holes as atoms (1997). arXiv:gr-qc/9710076
5. J.D. Bekenstein, Black holes: classical properties, thermodynamics and heuristic quantization (1998). arXiv:gr-qc/9808028

Minimum possible change in horizon area of a non-extremal BH is

$$\Delta A = 8\pi l_p^2 = 8\pi\hbar.$$

The above result relies on the Ehrenfest principle:

"Any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum."

Ehrenfest: Annalen der Physik 36, 91-111, (1911).

✓ In the case of the radiation in a cavity, the invariant quantity is:

$$\frac{E_\nu}{\nu},$$

where E_ν is the energy of a mode of vibration, and ν its frequency.

BOHR-SOMMERFELD QUANTIZATION RULE WITH EHRENFEST PRINCIPLE

Kunstater: PRL 90, 161301 (2003).

$$I = \int \frac{dE}{\omega(E)} \tag{3}$$

is an adiabatic invariant, which via Bohr-Sommerfeld quantization has an equally spaced spectrum in the semi-classical (large n) limit:

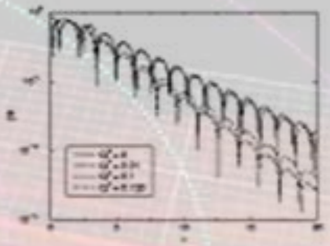
$$I \approx n\hbar \tag{4}$$

Maggiore

"Phys. Rev. Lett. **100**, 141301 (2008)"

showed that the equally spaced area spectrum of Bekenstein is the same as that obtained through the standard QNM frequencies.

Do black holes have a characteristic "sound"? ***Yes.!!!!***



QUASINORMAL MODES

IN GENERAL:

- Characteristic ringing
- Relevant to gravitational waves astronomy
- Originally studies for stability of BH's

IN PHYSICAL POINT OF VIEW:

- Small perturbations of a back hole
- Obtained by solving a wave equation for small fluctuations
- The frequency of the oscillations is not real because of damping

MAGGIORE

M. Maggiore, *The physical interpretation of the spectrum of black hole quasinormal modes*, *Phys. Rev. Lett.* **100** (2008) 141301 [arXiv:0711.3145] [SPIRES].

i) Frequency should be read as

$$\omega(E) = \sqrt{|\omega_R|^2 + |\omega_I|^2},$$

ii) Transition frequency should be taken into account in the adiabatic invariant

$$I = \int \frac{dE}{\Delta\omega(E)}.$$

iii) Under the case of highly excited QNMs for which $\omega_R \ll \omega_I$, the frequency of the QNMs becomes $\omega(E) = |\omega_I|$.

For Schwarzchild BH

$$I = \int \frac{dM}{\Delta\omega}.$$ with $$\Delta\omega = (|\omega_I|)_m - (|\omega_I|)_{m-1} = \frac{1}{4M}.$$

Bohr-Sommerfeld quantization

$$I = n\hbar.$$

⟹

for Schwarzschild black hole

$$\mathcal{A}_n = 8\pi\ell_{Pl}^2 \cdot n,$$

$$\Delta A = 8\pi\hbar = 8\pi l_p^2$$

CAGED BHs with SCALAR CLOUDS

PRL **109**, 081102 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

Schwarzschild Black Holes can Wear Scalar Wigs

Juan Barranco,[1] Argelia Bernal,[2] Juan Carlos Degollado,[3] Alberto Diez-Tejedor,[1] Miguel Megevand,[2] Miguel Alcubierre,[2] Darío Núñez,[2] and Olivier Sarbach[4]

arXiv:1904.12783v1

On the growth of massive scalar hair around a Schwarzschild black hole

Katy Clough,[1,*] Pedro G. Ferreira,[1,†] and Macarena Lagos[2,‡]
[1]Astrophysics, University of Oxford, DWB, Keble Road, Oxford OX1 3RH, UK
[2]Kavli Institute for Cosmological Physics, The University of Chicago, Chicago, IL 60637, USA

(Dated: Received April 30, 2019; published – 00, 0000)

Eur. Phys. J. C (2015) 75:142
DOI 10.1140/epjc/s10052-015-3370-4
THE EUROPEAN PHYSICAL JOURNAL C
CrossMark
Regular Article - Theoretical Physics

Scalar clouds in charged stringy black hole-mirror system

Ran Li[a], Junkun Zhao, Xinghua Wu, Yanming Zhang
Department of Physics, Henan Normal University, Xinxiang 453007, China

PHYSICAL REVIEW D **90**, 104032 (2014)

Study of the nonlinear instability of confined geometries

Hirotada Okawa,[1,2,3] Vitor Cardoso,[1,4] and Paolo Pani[1,5]

Eur. Phys. J. C (2014) 74:3137
DOI 10.1140/epjc/s10052-014-3137-3
THE EUROPEAN PHYSICAL JOURNAL C
Regular Article - Theoretical Physics

Resonance spectra of caged black holes

Shahar Hod[1,2,a]

- Carlos A. R. Herdeiro, Eugen Radu

Kerr black holes with scalar hair

Phys.Rev.Lett. 112 (2014) 221101

PHYSICAL REVIEW D **90**, 104032 (2014)

Study of the nonlinear instability of confined geometries

Hirotada Okawa,[1,2,3] Vitor Cardoso,[1,4] and Paolo Pani[1,5]
[1]*CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*
[2]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[3]*Advanced Research Institute for Science and Engineering, Waseda University, Tokyo 169-8555, Japan*
[4]*Perimeter Institute for Theoretical Physics Waterloo, Ontario N2J 2W9, Canada*
[5]*Dipartimento di Fisica, "Sapienza" Università di Roma, P.A. Moro 5, 00185 Roma, Italy*
(Received 20 July 2014; published 21 November 2014)

The discovery of a "weakly turbulent" instability of anti–de Sitter spacetime supports the idea that confined fluctuations eventually collapse to black holes and suggests that similar phenomena might be possible in asymptotically flat spacetime, for example in the context of spherically symmetric oscillations of stars or nonradial pulsations of ultracompact objects. Here we present a detailed study of the evolution of the Einstein-Klein-Gordon system in a cavity, with different types of deformations of the

PRL **107,** 031102 (2011) PHYSICAL REVIEW LETTERS week ending 15 JULY 2011

Weakly Turbulent Instability of Anti–de Sitter Spacetime

Piotr Bizoń[1,2] and Andrzej Rostworowski[1]
[1]*Institute of Physics, Jagiellonian University, Kraków, Poland*
[2]*Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Golm, Germany*
(Received 27 April 2011; published 15 July 2011)

- The study of Okawa et al. provides compelling evidence that spherically symmetric confined scalar fields generically collapse to form caged black holes.

- The observation of these "characteristic complex resonances" may allow one to determine the physical parameters of the newly born black hole.

Eur. Phys. J. C (2014) 74:3137
DOI 10.1140/epjc/s10052-014-3137-3

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Resonance spectra of caged black holes

Reissner–Nordström (RN)

Shahar Hod[1,2,a]

[1] The Ruppin Academic Center, Emeq Hefer 40250, Israel
[2] The Hadassah Institute, Jerusalem 91010, Israel

Eur. Phys. J. C (2015) 75:142
DOI 10.1140/epjc/s10052-015-3370-4

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Scalar clouds in charged stringy black hole-mirror system

Ran Li[a], Junkun Zhao, Xinghua Wu, Yanming Zhang

Department of Physics, Henan Normal University, Xinxiang 453007, China

Received: 8 October 2014 / Accepted: 16 October 2014 / Published online: 6 November 2014

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Ann. Phys. (Berlin) 528, No. 7–8, 612–618 (2016) / DOI 10.1002/andp.201500305

Original Paper

Spectroscopy of rotating linear dilaton black holes from boxed quasinormal modes

I. Sakalli* and G. Tokgoz

Received 4 October 2015, revised 8 December 2015, accepted 17 December 2015
Published online 14 January 2016

annalen der physik

3 Boundary conditions

FOR CHARACTERISTIC RESONANCE SPECTRA

BOXED QUASINORMAL MODES

We shall be interested in solutions of the radial wave equation (7) with the physical requirement (boundary condition) of purely ingoing waves crossing the black-hole horizon [30]:

$$R \sim e^{-i\omega y} \quad \text{as} \quad r \to r_+ \quad (y \to -\infty). \tag{10}$$

In addition, following [25] we shall consider two types of boundary conditions at the surface $r = r_{\mathrm{m}}$ of the confining cavity:

(1) The Dirichlet-type boundary condition implies

$$R(r = r_{\mathrm{m}}) = 0. \tag{11}$$

(2) The Neumann-type boundary condition implies

$$\frac{\mathrm{d}R}{\mathrm{d}r}(r = r_{\mathrm{m}}) = 0. \tag{12}$$

4 The resonance conditions

The boundary conditions (11) and (12) single out two discrete families of complex resonant frequencies $\{\omega(M,Q,r_{\text{m}},l;n)\}$[9] which characterize the late-time dynamics of the composed black-hole-field-cavity system (these characteristic resonances are also known as "boxed quasinormal frequencies" [10, 11]). The main goal of the present paper is to determine these characteristic resonances *analytically*.

Defining the dimensionless variable

$$x = \frac{r - r_+}{r_+}$$

In particular, we have studied the characteristic resonance spectra of confined scalar fields in caged Reissner–Nordström black-hole spacetimes. It was shown that these resonances can be derived *analytically* for caged black holes whose confining mirrors are placed in the vicinity of the black-hole horizon

in the $x \ll 1$ regime

PHYSICAL REVIEW D **97**, 104001 (2018)

Exact Schwarzschild-like solution in a bumblebee gravity model

R. Casana* and A. Cavalcante†
Departamento de Física, Universidade Federal do Maranhão, 65080-805 São Luís, Maranhão, Brazil

F. P. Poulis‡
Coordenação do Curso Interdisciplinar em Ciência e Tecnologia, Universidade Federal do Maranhão, 65080-805 São Luís, Maranhão, Brazil

E. B. Santos§
Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife, Pernambuco, Brazil

(Received 6 November 2017; published 2 May 2018)

bumblebee field strength is defined as

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

where B_μ has mass dimension 1.

Potential V provides a nonvanishing VEV for B_μ, which could have the following general functional form:

$$V \equiv V(B^\mu B_\mu \pm b^2),$$

b^2 is a positive real constant

VEV of the bumblebee field is determined when $V(B^\mu B_\mu \pm b^2) = 0$, implying that the condition

$$B^\mu B_\mu \pm b^2 = 0$$

This is solved when the field B^μ acquires a nonzero VEV given by

$$\langle B^\mu \rangle = b^\mu,$$

where the vector b^μ is a function of the spacetime coordinates such that $b^\mu b_\mu = \mp b^2 = \text{const}$; then, the nonzero vector background b^μ spontaneously breaks the Lorentz symmetry.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu},$$

$$T_{\mu\nu} = T^{\mathrm{M}}_{\mu\nu} + T^{B}_{\mu\nu},$$

with $T^{\mathrm{M}}_{\mu\nu} = 0.$ vacuum solution

$$\begin{aligned} T^B_{\mu\nu} = {} & -B_{\mu\alpha} B^\alpha{}_\nu - \frac{1}{4} B_{\alpha\beta} B^{\alpha\beta} g_{\mu\nu} - V g_{\mu\nu} + 2V' B_\mu B_\nu \\ & + \frac{\xi}{\kappa} \Big[\frac{1}{2} B^\alpha B^\beta R_{\alpha\beta} g_{\mu\nu} - B_\mu B^\alpha R_{\alpha\nu} - B_\nu B^\alpha R_{\alpha\mu} \\ & + \frac{1}{2} \nabla_\alpha \nabla_\mu (B^\alpha B_\nu) + \frac{1}{2} \nabla_\alpha \nabla_\nu (B^\alpha B_\mu) \\ & - \frac{1}{2} \nabla^2 (B_\mu B_\nu) - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \nabla_\beta (B^\alpha B^\beta) \Big]. \end{aligned}$$

ξ is the real coupling constant (with mass dimension -1) that controls the non-minimal gravity-bumblebee interaction.

Setting Lorentz-violating parameter: $\mathsf{L} = \xi b^2$

Finally, we write down the Lorentz-violating spherically symmetric solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + (1+\mathsf{L})\left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{42}$$

we compute the Kretschmann scalar

$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{4(12M^2 + 4\mathsf{L}Mr + \mathsf{L}^2 r^2)}{r^6(\mathsf{L}+1)^2}, \tag{43}$$

which clearly differs from the Schwarzschild Kretschmann invariant for non-null L. This ensures that the metric (42) is a true solution containing Lorentz-violating corrections, i.e., there exists no coordinate transformation connecting the metric (42) to the Schwarzschild one; otherwise, the scalar invariant (43) would be the same for both metrics.

$$T_H = \frac{\hbar\kappa}{2\pi} = \frac{\hbar}{4\pi\sqrt{-g_{tt}g_{rr}}}\frac{dg_{tt}}{dr}\bigg|_{r=r_h} = \frac{\hbar}{2\pi\sqrt{1+L}}\frac{M}{r^2}\bigg|_{r=r_h} = \frac{\hbar}{8\pi M\sqrt{1+L}}.$$

$$\kappa = \frac{1}{2r_h\sqrt{1+L}}.$$

$$S^{BH} = \frac{\mathcal{A}}{4\hbar} = \frac{\pi r_h^2}{\hbar}.$$

$r_h = 2M$: event horizon

$$T_H dS^{BH} = d\mathcal{M}$$

$$\mathcal{M} = \frac{M}{\sqrt{1+L}}.$$

IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. **35** (2018) 235003 (10pp) https://doi.org/10.1088/1361-6382/aaea96

Tom Złośnik[1], Federico Urban[1], Luca Marzola[2] and Tomi Koivisto[2,3]

Spacetime and dark matter from spontaneous breaking of Lorentz symmetry

PHYSICAL REVIEW D **99**, 024042 (2019)

Exact traversable wormhole solution in bumblebee gravity

Ali Övgün,[1,2,*] Kimet Jusufi,[3,4,†] and İzzet Sakallı[2,‡]

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Nuclear Physics B 946 (2019) 114703

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

GUP modified Hawking radiation in bumblebee gravity

Sara Kanzi, İzzet Sakallı *

massless scalar field Ψ satisfying the KGE:

$$\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\mu\nu}\partial_\mu\Psi\right)=0.$$

$$\Psi=\frac{1}{r}F(r)e^{-i\omega t}Y_l^m(\theta,\varphi),\quad Re(\omega)>0,$$

ω and $Y_l^m(\theta,\varphi)$ represent the frequency of the propagating scalar wave and the spheroidal harmonics with the eigenvalue $\lambda=-l(l+1)$, respectively.

$$\left[-\frac{d^2}{dr^{*2}}+V(r)\right]F(r)=\omega^2F(r),$$

Zerilli equation

$$r^*=\sqrt{1+L}\left[r+r_h\ \ln\left(r-r_h\right)\right].$$

$$r^*=\sqrt{1+L}\int\frac{dr}{f(r)},$$

$$\lim_{r\to r_h} r^*=-\infty,\quad\text{and}\quad \lim_{r\to\infty} r^*=\infty.$$

The effective or the so-called Zerilli potential $V(r)$ is given by

$$V(r)=\frac{f(r)}{r^2}\left[l(l+1)+\frac{2M}{r(1+L)}\right].$$

BQNM Frequencies

$$f(r) = 1 - \frac{r_h}{r} \rightarrow f(x) = \frac{x}{x+1},$$

$$x = \frac{r}{r_h} - 1.$$

$$f(x) \cong x + O(x^2),$$ as $x \rightarrow 0$

$$r^* = \sqrt{1+L}\,r_h \int \frac{dx}{f(x)} \cong r_h\sqrt{1+L}\,\ln(x) = \frac{1}{2\kappa}\ln x,$$

$$x = e^{2y},$$

$$y = \kappa r^*.$$

$$V_{NH}(x) = \frac{\rho x}{r_h^2(1+L)} + O(x^2),$$

$$\rho = l(l+1)(L+1)+1.$$

NH form of the Zerilli equation

$$\left[-\frac{d^2}{dy^2} + 4\rho e^{2y}\right] F(y) = \widetilde{\omega}^2 F(y).$$

$$\omega = \widetilde{\omega}\kappa,$$

Two linearly independent solutions:

$$F(y) = A_1 J_{-i\tilde{\omega}}(2i\sqrt{\rho}e^y) + A_2 Y_{-i\tilde{\omega}}(2i\sqrt{\rho}e^y),$$

and correspondingly

$$F(x) = A_1 J_{-i\tilde{\omega}}(2i\sqrt{\rho x}) + A_2 Y_{-i\tilde{\omega}}(2i\sqrt{\rho x}),$$

Bessel functions of the first and second kinds

The following limiting forms (when v is fixed and $z \to 0$) of the Bessel functions are needed for our analysis.

$$J_v(z) \sim \frac{\left(\frac{1}{2}z\right)^v}{\Gamma(1+v)}, \qquad (v \neq -1, -2, -3, \ldots),$$

$$Y_v(z) \sim -\frac{1}{\pi}\Gamma(v)\left(\frac{1}{2}z\right)^{-v}, \quad (\Re v > 0).$$

M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions*, (Dover, New York, 1965).

NH ($e^y \ll 1$) behavior of the solution

$$F \sim A_1 \frac{(i\sqrt{\rho})^{-i\tilde{\omega}}}{\Gamma(1-i\tilde{\omega})} e^{-i\tilde{\omega}y} - A_2 \frac{1}{\pi}\Gamma(-i\tilde{\omega})\left(i\sqrt{\rho}\right)^{i\tilde{\omega}} e^{i\tilde{\omega}y},$$

$$= A_1 \frac{\left(i\sqrt{\rho}\right)^{-i\tilde{\omega}}}{\Gamma(1-i\tilde{\omega})} e^{-i\omega r^*} - A_2 \frac{1}{\pi}\Gamma(-i\tilde{\omega})\left(i\sqrt{\rho}\right)^{i\tilde{\omega}} e^{i\omega r^*},$$

Ingoing

Outgoing

Boundary Conditions of Confining Cavity

acceptable solution of the radial equation

$$F(x) = A_1 J_{-i\tilde{\omega}}(2i\sqrt{\rho x}).$$

DBC

$$F(x)\big|_{x=x_m} = 0.$$

Thus, we have

$$J_{-i\tilde{\omega}}(2i\sqrt{\rho x_m}) = 0.$$

$$Y_v(z) = J_v(z)\cot(v\pi) - J_{-v}(z)\csc(v\pi),$$

0

M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions*, (Dover, New York, 1965).

$$\tan(i\tilde{\omega}\pi) = \frac{J_{i\tilde{\omega}}(2i\sqrt{\rho x_m})}{Y_{i\tilde{\omega}}(2i\sqrt{\rho x_m})},$$

Boundary of the confining cavity is located at the vicinity of the event horizon. Setting

$$u_m \equiv \rho x_m \ll 1, \qquad \rightarrow \qquad r_m \approx r_+,$$

DBC's Resonance condition:

$$\tan(i\tilde{\omega}\pi) = \frac{J_{i\tilde{\omega}}(2i\sqrt{\rho x_m})}{Y_{i\tilde{\omega}}(2i\sqrt{\rho x_m})},$$

$u_m \equiv \rho x_m \ll 1$

$$\tan(i\tilde{\omega}\pi) \sim -\frac{\pi\left(i\sqrt{u_m}\right)^{i\tilde{\omega}}}{\Gamma\left(i\tilde{\omega}\right)\Gamma^2\left(i\tilde{\omega}+1\right)\left(i\sqrt{u_m}\right)^{-i\tilde{\omega}}},$$
$$= i\frac{\pi e^{-\pi\tilde{\omega}}}{\tilde{\omega}\Gamma^2\left(i\tilde{\omega}\right)}u_m^{i\tilde{\omega}}.$$

NBC

Using the derivative features of the Bessel functions:

$$\left.\frac{dF(x)}{dx}\right|_{x=x_m} = 0.$$

$$J_{-i\tilde{\omega}-1}(2i\sqrt{u_m}) - J_{-i\tilde{\omega}+1}(2i\sqrt{u_m}) = 0.$$

Extra Feature (Abramowitz & Stegun):

$$Y_{v+1}(z) - Y_{v-1}(z) = \cot(v\pi)\left[J_{v+1}(z) - J_{v-1}(z)\right]$$
$$- \csc(v\pi)\left[J_{-v-1}(z) - J_{-v+1}(z)\right].$$

0

$$\tan(i\widetilde{\omega}\pi) = \frac{J_{i\widetilde{\omega}-1}(2i\sqrt{u_m})}{Y_{i\widetilde{\omega}+1}(2i\sqrt{u_m})}\left[\frac{-1+\frac{J_{i\widetilde{\omega}+1}(2i\sqrt{u_m})}{J_{i\widetilde{\omega}-1}(2i\sqrt{u_m})}}{1-\frac{Y_{i\widetilde{\omega}-1}(2i\sqrt{u_m})}{Y_{i\widetilde{\omega}+1}(2i\sqrt{u_m})}}\right].$$

$$\frac{J_{i\widetilde{\omega}+1}(2i\sqrt{u_m})}{J_{i\widetilde{\omega}-1}(2i\sqrt{u_m})} \equiv \frac{Y_{i\widetilde{\omega}-1}(2i\sqrt{u_m})}{Y_{i\widetilde{\omega}+1}(2i\sqrt{u_m})} \sim O\left(u_m\right),$$

(Abramowitz & Stegun)

NBC's Resonance condition:

$$\begin{aligned}\tan(i\widetilde{\omega}\pi) &\sim -\frac{J_{i\widetilde{\omega}-1}(2i\sqrt{u_m})}{Y_{i\widetilde{\omega}+1}(2i\sqrt{u_m})},\\ &= -i\frac{\pi e^{-\pi\widetilde{\omega}}}{\widetilde{\omega}\Gamma^2\left(i\widetilde{\omega}\right)}u_m^{i\widetilde{\omega}}.\end{aligned}$$

Recall the DBC's Resonance condition:

$$\tan(i\widetilde{\omega}\pi) \sim i\frac{\pi e^{-\pi\widetilde{\omega}}}{\widetilde{\omega}\Gamma^2\left(i\widetilde{\omega}\right)}u_m^{i\widetilde{\omega}}.$$

0^{th} order resonance equation:

$$\tan(i\widetilde{\omega}_n^{(0)}\pi) = 0,$$

$$\widetilde{\omega}_n^{(0)} = -in, \qquad (n = 0, 1, 2.......).$$

1^{st} order resonance equation:

$$\tan(i\widetilde{\omega}_n^{(1)}\pi) = \pm i \frac{\pi e^{i\pi n}}{(-in)\,\Gamma^2(n)} u_m^n,$$

$$= \mp n \frac{\pi(-u_m)^n}{(n!)^2},$$

Obtained by 0^{th} order resonance condition: Iteration

minus (plus) stands for the DBC (NBC)

Using:

$$\omega = \widetilde{\omega}\kappa,$$

$$\tan(v + n\pi) = \tan(v) \approx v, \qquad \text{for } v \ll \text{regime}$$

$$i\widetilde{\omega}_n\pi = n\pi \mp n \frac{\pi(-u_m)^n}{(n!)^2}.$$

$$\widetilde{\omega}_n = -in\left[1 \mp \frac{(-u_m)^n}{(n!)^2}\right].$$

$$\omega_n = -i\kappa n\left[1 \mp \frac{(-u_m)^n}{(n!)^2}\right], \qquad (n = 0, 1, 2.......).$$

$\omega_n \approx -i\kappa n.$ For the highly excited states ($n \to \infty$)

transition frequency becomes

$$\Delta\omega_I = \kappa = \frac{2\pi T_H}{\hbar}.$$


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Editors' Suggestion

Physical Interpretation of the Spectrum of Black Hole Quasinormal Modes

Michele Maggiore

Phys. Rev. Lett. **100**, 141301 – Published 8 April 2008

$$I_{adb} = \int \frac{T dS}{\Delta\omega},$$

$$I_{adb} = \frac{S^{BH}}{2\pi}\hbar.$$

Bohr-Sommerfeld quantization rule ($I_{adb} = \hbar n$)

J.D. Bekenstein
Lett. Nuovo Cimento, 11 (1974), p. 467

$$S_n^{BH} = 2\pi n.$$

$$A_n = 8\pi\hbar n.$$

the minimum area spacing becomes

$$\Delta A_{\min} = 8\pi\hbar.$$

Spectral-spacing coefficient becomes

$$\epsilon = 8\pi$$

YES!

fully agreement with
the Bekenstein's original result!

Conclusions and Outlook

- ❖ IT WAS SHOWN THAT BQNMS CAN BE ANALYTICALLY DERIVED FOR CAGED LSBBH WHICH CONFINED BY SCALAR CLOUDS.

- ❖ TO HAVE THE BQNMS DBC AND NBC CONDITIONS WERE APPLIED.

- ❖ MAGGIORE'S METHOD WERE EMPLOYED FOR THE BQNMS AND ENTROPY/AREA SPECTRA OF LSBBH WERE DERIVED.

- ❖ THE RESULT OBTAINED ARE IN AGREEMENT WITH THE BEKENSTEIN'S CONJECTURE.

- ❖ WE PLAN TO EXTEND OUR ANALYSIS FOR THE OTHER TYPE OF PERTURBATIONS : FERMIONS (DIRAC EQ.), PHOTONS (MAXWELL EQ.) MASSIVE BOSONS (PROCA EQ.), GRAVITINOS (RS EQ.) ETC.