On exotic six-dimensional supergravity theories

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Introduction

Field theories with maximal susy are special

They always contain gravity

They are unique, with the exception of the ten-dimensional case

On the other hand, in six dimensions three different maximal susy algebras exist

Only in the non-chiral case we know the corresponding supergravity theory

What can we say about the other two algebras?

Introduction

$\mathcal{N} = (2,2)$: R-symmetry $USp(4) \times USp(4)$

Supergravity multiplet:

$$\underset{g_{\mu\nu}}{(\mathbf{3},\mathbf{3};\mathbf{1},\mathbf{1})} + \underset{B^{-ab}_{\mu\nu}}{(\mathbf{1},\mathbf{3};\mathbf{5},\mathbf{1})} + \underset{B^{+\alpha\beta}_{\mu\nu}}{(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{5})} + \underset{A^{a\alpha}_{\mu}}{(\mathbf{2},\mathbf{2};\mathbf{4},\mathbf{4})} + \underset{\phi^{ab\alpha\beta}}{(\mathbf{1},\mathbf{1};\mathbf{5},\mathbf{5})}$$

$$\underset{\psi^{a}_{\mu}}{(\mathbf{2},\mathbf{3};\mathbf{4},\mathbf{1})} + \underset{\psi^{\alpha}_{\mu}}{(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{4})} + \underset{\chi^{a\alpha\beta}}{(\mathbf{2},\mathbf{1};\mathbf{4},\mathbf{5})} + \underset{\chi^{ab\alpha}}{(\mathbf{1},\mathbf{2};\mathbf{5},\mathbf{4})}$$

where first two representations are of little group $SU(2) \times SU(2)$

Theory is maximal sugra in six dimensions

Global symmetry $SO(5,5)$

Scalars parametrise coset manifold $SO(5,5)/[SO(5) \times SO(5)]$

Introduction

$\mathcal{N} = (3,1)$: R-symmetry $USp(6) \times USp(2)$

$$\underset{h^+_{\mu\nu,\rho}}{(\mathbf{4},\mathbf{2};\mathbf{1},\mathbf{1})} + \underset{B^{+a\alpha}_{\mu\nu}}{(\mathbf{3},\mathbf{1};\mathbf{6},\mathbf{2})} + \underset{A^{ab}_{\mu}}{(\mathbf{2},\mathbf{2};\mathbf{14},\mathbf{1})} + \underset{\phi^{abc\alpha}}{(\mathbf{1},\mathbf{1};\mathbf{14}',\mathbf{2})}$$

$$\underset{\psi^a_\mu}{(\mathbf{3},\mathbf{2};\mathbf{6},\mathbf{1})} + \underset{\chi^\alpha_{\mu\nu}}{(\mathbf{4},\mathbf{1};\mathbf{1},\mathbf{2})} + \underset{\chi^{ab\alpha}}{(\mathbf{2},\mathbf{1};\mathbf{14},\mathbf{2})} + \underset{\chi^{abc}}{(\mathbf{1},\mathbf{2};\mathbf{14}',\mathbf{1})}$$

Scalars compatible with coset manifold $F_{4(4)}/[USp(6) \times USp(2)]$

$\mathcal{N} = (4,0)$: R-symmetry $USp(8)$

$$\underset{h^+_{\mu\nu,\rho\sigma}}{(\mathbf{5},\mathbf{1};\mathbf{1})} + \underset{B^{+ab}_{\mu\nu}}{(\mathbf{3},\mathbf{1};\mathbf{27})} + \underset{\phi^{abcd}}{(\mathbf{1},\mathbf{1};\mathbf{42})} \qquad + \underset{\chi^a_{\mu\nu}}{(\mathbf{4},\mathbf{1};\mathbf{8})} + \underset{\chi^{abc}}{(\mathbf{2},\mathbf{1};\mathbf{48})}$$

Theory compatible with global symmetry $E_{6(6)}$

None of these two multiplets contain the graviton

Introduction

All these multiplets give the five-dimensional maximal sugra multiplet when dimensionally reduced

$\mathcal{N} = (4,0)$ multiplet is a representation of the superconformal group $OSp^*(8|8)$

The corresponding theory would be superconformal

Conjecture: the $\mathcal{N} = (4,0)$ theory is a particular limit of the five-dimensional maximal theory in which gravity is strongly coupled

Hull (2000)

Also the linearised field equations give the ones of the five-dimensional theory when dimensionally reduced

But what are the field equations that the self-dual mixed-symmetry potentials $h^+_{\mu\nu,\rho}$ and $h^+_{\mu\nu,\rho\sigma}$ satisfy?

Introduction

Potential $h^{+}_{2,\tilde{1}}$: gauge-invariant second order 'Riemann' tensor

$$R_{3,\tilde{2}} = d\tilde{d}h^{+}_{2,\tilde{1}}$$

Self-duality:

$$R_{3,\tilde{2}} = *R_{3,\tilde{2}}$$

Similarly, from the potential $h^{+}_{2,\tilde{1}}$ one gets the gauge-invariant second-order 'Riemann' tensor

$$R_{3,\tilde{3}} = d\tilde{d}h^{+}_{2,\tilde{2}}$$

Self-duality:

$$R_{3,\tilde{3}} = *R_{3,\tilde{3}} = *R_{\tilde{3},3} = *\tilde{*}R_{3,\tilde{3}}$$

Hull (2001)

Lagrangian

If we want to write down a (free) lagrangian, we have two issues

First issue is writing down a lagrangian for a mixed-symmetry potential

Curtright (1986)

de Medeiros, Hull (2002)

$\mathcal{L} = h^{\mu_1 \ldots \mu_p, \nu_1 \ldots \nu_1} G_{\mu_1 \ldots \mu_p, \nu_1 \ldots \nu_q}$

where G is the ‘Einstein’ tensor

Second issue is that if the field is self-dual the ‘Einstein’ tensor is identically zero

Lagrangian

As we know, this problem already arises for self-dual p-forms in $D = 2p + 2$ dimensions, that is 2-forms is 6 dimensions

A non-manifestly covariant action principle for self-dual p-forms was originally given by Henneaux, Teitelboim (1988)

This has been recently generalised to construct an action for the $\mathcal{N} = (4, 0)$ and $\mathcal{N} = (3, 1)$ theories

Henneaux, Lekeu, Leonard (2016)

Henneaux, Lekeu, Leonard (2017)

Henneaux, Matuilich, Prohaka (2018)

In the case of the $h^{+}_{\mu\nu,\rho\sigma}$ potential, one defines the electic and magnetic fields

$$\mathcal{E}_{ij,kl} = G_{ij,kl} \qquad \mathcal{B}_{ij,kl} = \frac{1}{6} R_{0ij,abc}\epsilon_{abckl}$$

Self-duality implies $\mathcal{E} = \mathcal{B}$

Lagrangian

This equation is shown to be equivalent to a differential equation of order four acting on a 'prepotential'

This means that one can integrate the equation to get a lagrangian

The self-dual potential is given in terms of a differential operator of order one acting on the prepotential, up to gauge transformations

Also the susy transformations are given in terms of the prepotential

Lagrangian

For self-dual p-forms, we know that there is another way of writing down an action

Pasti, Sorokin, Tonin (1995)

$$\mathcal{L} = -\frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{\partial^{\mu}\phi \partial_{\sigma}\phi}{2(\partial\phi)^2} H^{-}_{\mu\nu\rho} H^{-\sigma\nu\rho}$$

It possesses the following additional gauge invariances:

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\phi \, \Lambda_{\nu]}$$

$$\delta B_{\mu\nu} = \frac{\Lambda}{(\partial\phi)^2} H^{-}_{\mu\nu\rho} \partial^{\rho}\phi \qquad \delta\phi = \Lambda$$

Using the first invariance, one can fix $F^{-}_{\mu\nu\rho} = 0$, so that on-shell only the self-dual part remains

The equation for ϕ does not propagate new degrees of freedom

Can we use the PST formulation to write down an action for the exotic supergravity theories in six dimensions?

First order formulation

First order formulation for linearised gravity: consider at the linearised level $e_\mu{}^a = \delta_\mu^a + h_\mu{}^a$

The potential $h_{\mu,\nu}$ is no longer symmetric

Gauge transformations:

$$\delta h_{\mu,\nu} = \partial_\mu \xi_\nu \qquad \delta h_{\mu,\nu} = \Lambda_{\mu\nu}$$

We define

$$f_{\mu\nu,\rho} = \partial_\mu h_{\nu,\rho} - \partial_\nu h_{\mu,\rho}$$

which is invariant under the first gauge transformations

Action:

$$\mathcal{L} = \frac{1}{8} f_{\mu\nu,\rho} f^{\mu\nu,\rho} + \frac{1}{4} f_{\mu\nu,\rho} f^{\mu\rho,\nu} - \frac{1}{2} f_{\mu\nu}{}^{\nu} f^{\mu\rho}{}_{\rho}$$

Unique combination which is invariant under local Lorentz

First order formulation

This can be generalised to the potentials $h_{\mu\nu,\rho}$ and $h_{\mu\nu,\rho\sigma}$ (not self-dual yet)

Zinoviev (2003)

In the first case we have

$$f_{\mu\nu\rho,\sigma} = 3\partial_{[\mu} h_{\nu\rho],\sigma}$$

and the lagrangian

$$\mathcal{L} = -\frac{1}{6} f_{\mu\nu\rho,\sigma} f^{\mu\nu\rho,\sigma} - \frac{1}{4} f_{\mu\nu\rho,\sigma} f^{\mu\nu\sigma,\rho} + \frac{3}{4} f_{\mu\nu\rho}{}^{\rho} f^{\mu\nu\sigma}{}_{\sigma}$$

is invariant under the 'local Lorentz' transformation

$$\delta h_{\mu\nu,\rho} = \Lambda_{\mu\nu\rho}$$

First order formulation

In the second case

$f_{\mu\nu\rho,\sigma\delta} = 3\partial_{[\mu} h_{\nu\rho],\sigma\delta}$

Lagrangian:

$\mathcal{L} = \frac{1}{6} f_{\mu\nu\rho,\sigma\delta} f^{\mu\nu\rho,\sigma\delta} + \frac{1}{2} f_{\mu\nu\rho,\sigma\delta} f^{\mu\nu\sigma,\rho\delta} + \frac{1}{2} f_{\mu\nu\rho,\sigma\delta} f^{\mu\sigma\delta,\nu\rho}$

$-\frac{3}{2} f_{\mu\nu\rho,\sigma}{}^{\rho} f^{\mu\nu\delta,\sigma}{}_{\delta} - 3 f_{\mu\nu\rho,\sigma}{}^{\rho} f^{\mu\sigma\delta,\nu}{}_{\delta} + \frac{3}{2} f_{\mu\nu\rho}{}^{\nu\rho} f^{\mu\sigma\delta}{}_{\sigma\delta}$

It is invariant under the ‘local Lorentz’ symmetry

$\delta h_{\mu\nu,\rho\sigma} = \Lambda_{\mu\nu[\rho,\sigma]}$

where $\Lambda_{\mu\nu\rho,\sigma}$ is reducible

First order formulation

Duality relation: start from parent action with dual field being the lagrange multiplier for the Bianchi identities, determine equation for $f_{\mu\nu\rho,\sigma}$ and then impose that the dual field coincides with the field itself

One finds

$$-\frac{2}{3} f_{\mu\nu\rho,\sigma} - f_{[\mu\nu|\sigma,|\rho]} + 3\eta_{\sigma[\mu} f_{\nu\rho]\delta}{}^{\delta} = \frac{1}{6}\epsilon_{\mu\nu\rho\delta\tau\xi} f^{\delta\tau\xi}{}_{\sigma}$$

We find that this relation is consistent, in the sense that contracting with epsilon and suitably manipulating the indices one find the same equation (non-trivial check!)

Moreover, contracting with ∂^{μ} one recovers the second order field equations

Problem: how can this be compatible with local Lorentz transformations?

First order formulation

The antisymmetric part of $h_{\mu\nu,\rho}$ in six dimensions can be split in self-dual and antiself-dual part, and one finds that in the duality relation the antiself-dual part disappears

We define

$$F^{-}_{\mu\nu\rho,\sigma} = -\frac{2}{3} f_{\mu\nu\rho,\sigma} - f_{[\mu\nu|\sigma,|\rho]} + 3\eta_{\sigma[\mu} f_{\nu\rho]\delta}{}^{\delta} - \frac{1}{6}\epsilon_{\mu\nu\rho\delta\tau\xi} f^{\delta\tau\xi}{}_{\sigma}$$

We want $F^{-}_{\mu\nu\rho,\sigma} = 0$ up to self-dual local Lorentz transformations $\Lambda^{+}_{\mu\nu\rho}$

The PST action must be obtained adding terms of the form

$$\frac{\partial\phi\partial\phi}{(\partial\phi)^2} F^{-} F^{-}$$

in such a way that the full action is invariant under a generalisation of the PST transformations and under $\Lambda^{+}_{\mu\nu\rho}$

Work in progress!

First order formulation

Similarly, in the $\mathcal{N} = (4, 0)$ case we find

$$F^{-}_{\mu\nu\rho,\sigma\delta} = \frac{1}{3} f_{\mu\nu\rho,\sigma\delta} + f_{\mu\nu\delta,\rho\sigma} + f_{\mu\sigma\delta,\nu\rho} - 3\eta_{\mu\delta} f_{\nu\rho\gamma,\sigma}{}^{\gamma}$$

$$-6\eta_{\mu\delta} f_{\nu\sigma\gamma,\rho}{}^{\gamma} + 9\eta_{\mu\sigma}\eta_{\nu\delta} f_{\rho\gamma\xi}{}^{\gamma\xi} - \frac{1}{6}\epsilon_{\mu\nu\rho\gamma\xi\tau} f^{\gamma\xi\tau}{}_{\sigma\delta}$$

Again, we find that out of the component $h_{\mu\nu\rho,\sigma}$ (reducible) only the self-dual part survives

We want $F^{-}_{\mu\nu\rho,\sigma\delta} = 0$ up to self-dual local Lorentz transformations $\Lambda^{+}_{\mu\nu\rho,\sigma}$

Similar considerations for the PST action apply to this case

First order formulation

Although we do not have an action yet, in the $\mathcal{N} = (4, 0)$ case we have studied the algebra that results from the commutator of two linearised susy transformations of the fields

$$\delta B^{ab}_{\mu\nu} = \bar{\epsilon}^{[a}\chi^{b]}_{\mu\nu} - \frac{1}{8}\Omega^{ab}\bar{\epsilon}^{c}\chi^{d}_{\mu\nu}\Omega_{cd} + \bar{\epsilon}_c\gamma_{\mu\nu}\chi^{abc}$$

$$\delta\phi^{abcd} = \bar{\epsilon}^{[a}\chi^{bcd]} - \frac{3}{4}\Omega^{[ab}\bar{\epsilon}^{e}\chi^{cd]f}\Omega_{ef}$$

$$\delta h_{ab,cd} = \bar{\epsilon}_a\gamma_{cd}\chi^{a}_{ab}$$

We find that the algebra closes provided that all the first order duality relations are imposed

Conclusions

- PST formulation must be possible

  Can we embed these theories in E_{11}?

- The linear $\mathcal{N} = (4, 0)$ theory can be obtained as the square of the $\mathcal{N} = (2, 0)$ theory (tensor multiplets)

  Borsten (2017)

- We also expect that the $\mathcal{N} = (3, 1)$ theory should arise from the product $(2, 0) \times (1, 1)$

- Still, we do not know whether these theories are actually consistent at the interacting level